7.4: Expected Return and Standard Deviation of a Portfolio

Expected Return

The expected return for a portfolio is simply the weighted average of each stock held in the portfolio. The formula here is

$$\overline{k_p} = \sum_{i=1}^{n} W_i \bar{k}_i$$

or

$$\overline{k_p} = W_1\overline{k_1} + W_2\overline{k_2} + \ldots + W_n\overline{k_n}$$

where

- $\overline{k_p}$ represents the expected return for the portfolio
- W_1 represents the weight (proportion of portfolio) of stock 1
- $\overline{k_1}$ represents the expected return for stock 1
- W_n represents the weight (proportion of portfolio) of stock n
- $\overline{k_n}$ represents the expected return for stock n

Again, let's consider an example. What is the expected return of a portfolio made up of 60\% Stock A and 40\% Stock B when the expected return for Stock A is 10\% and the expected return for Stock B is 20\%?

$$\overline{k_p} = (.60)(10\%) + (.40)(20\%)$$

$$\overline{k_p} = 6\% + 8\%$$

$$\overline{k_r} = 14\% v$$

Video [Expected Return of a Two-Stock Portfolio]

Standard Deviation

The standard deviation of a portfolio becomes more complicated. It depends not only on the standard deviation and weightings of each stock, but also on the correlation between pairs of stocks.

Correlation

The correlation between a pair of stocks measures how closely the returns for each stock are related. A negative correlation means that the price of one stock tends to fall while the other rises (prices/returns are inversely related). A positive correlation means that the price of one stock tends to rise while the other rises (prices/returns are positively related). Correlations can range from -1.0 to

1.0. Correlations for real-world variables are almost never at the extremes (perfect positive correlation, no correlation, or perfect negative correlation).

See the Observed Betas, Correlations, and Standard Deviations Table in Appendix B for some standard deviations and correlations from actual companies over the past five years.

Observations from the Table

- The last two rows/columns are for an aggregate US bond fund and the S&P 500 ETF. The purpose of these is to provide a proxy for the US bond market and the US stock market.
- The vast majority (65 of 66) of correlations between pairs of stocks are positive. This is because all stocks are impacted by general economic factors.
- The average correlation across pairs is a low, positive value (0.31 in our sample). While general economic factors cause a tendency for stock to move together, firm-specific factors push correlations towards zero.
- Stocks in similar industries tend to have higher correlations than the average stock (0.50 vs. 0.31).
- Each stock has a positive correlation with the overall stock market.
- The stock and bond market have a negative (although essentially zero) correlation during this time-period.
- The bond market has a much lower standard deviation than the stock market, but also generated much lower returns during this time-period.
- Most individual stocks have a standard deviation that is higher than the overall stock market. This is because the stock market represents a diversified portfolio which has eliminated most of the firm-specific risk. The exception is Pepsi in our sample which is essentially the same standard deviation.
- The historical annualized returns are not the same as the expected returns. It is unrealistic to expect 30%+ annual returns for Deere, Boston Beer or Amazon going forward (which is not the same as saying that these stocks can't generate those returns). Also, it is unreasonable to expect people to buy stock in Molson Coors and Exxon with the anticipation of losing nearly 12% or 8% each year.
- While standard deviations will vary depending on overall market conditions, large cap stocks over this 5-year window had standard deviations of approximately 15-42%. Note that the standard deviations will be bigger or smaller depending on both the stock and the time period and that Boston Beer's 42.5% is noticeably larger than the next closest company.

Standard Deviation for a two-stock Portfolio

In order to calculate the standard deviation of a two-stock portfolio, we will use the following formula:

$$\sigma_p = \sqrt{W_1^2 \sigma_1^2 + W_2^2 \sigma_2^2 + 2 W_1 W_2 \sigma_1 \sigma_2 \operatorname{corr}_{1,2}}$$

where

σ_p represents the standard deviation of the portfolio
W_1 represents the weight (proportion of portfolio) of stock 1
σ_1 represent the standard deviation of stock 1
W_2 represents the weight (proportion of portfolio) of stock 2
σ_2 represent the standard deviation of stock 2
corr $_{1,2}$ represents the correlation between the returns of stocks 1 and 2

Again, let's work through an example. Consider a two-stock portfolio in which 60\% of your money is invested in stock A and 40% of your money is invested in stock B. Stock A has a standard deviation of 50% and stock B has a standard deviation of 70%. The correlation between the returns for stock A and stock B are 0.30 . You want to find the standard deviation of this portfolio.

$$\sigma_p = \sqrt{(0.6)^2(50)^2 + (0.4)^2(70)^2 + 2(0.6)(0.4)(50)(70)(0.3)}$$

$$\sigma_p = \sqrt{(0.36)(2500) + (0.16)(4900) + (504)}$$

$$\sigma_p = \sqrt{900 + 784 + 504}$$

$$\sigma_p = \sqrt{2188}$$

$$\sigma_p = 46.78\%$$

Note that in this example, the standard deviation of the portfolio is less than the standard deviation of either stock separately. This illustrates the concept of diversification. As long as the correlation is less than 1.0 (which it will be for any two stocks), the risk of the portfolio is less than the weighted average risk of the two securities which make up the portfolio (and sometimes — like here — even less than the lowest risk stock in the portfolio). While we will not cover the process of calculating the expected return and standard deviation for larger portfolios in this class, in practice, most portfolios hold far more securities.

Video **Standard Deviation of a Two-Stock Portfolio**