

### 3.13: Problems

#### ? Problem 3.13.1

Determine the answer to each of the following questions.

- a. Find the Future Value of \$2500 invested today at 11% for 10 years.
- b. Find the Future Value of \$2500 invested today at 11% for 30 years.
- c. Find the Present Value of \$6000 received 10 years from today if the discount rate is 5%.
- d. Find the Present Value of \$6000 received 10 years from today if the discount rate is 10%.
- e. Find the Future Value of \$3000 per year (at the end of each year) invested at 6% for 30 years.
- f. Find the Future Value of \$3000 per year (at the end of each year) invested at 12% for 30 years.
- g. Find the Present Value of \$4000 per year (at the end of each year) if the discount rate is 15% for 20 years.
- h. Find the Present Value of \$4000 per year (at the end of each year) if the discount rate is 15% for 40 years.

#### Answer

##### Part 1a

Step 1: 10 N  
Step 2: 11 I/Y  
Step 3: 2500 PV  
Step 4: 0 PMT  
Step 5: FV⇒ \$7,098.55

##### Part 1b

Step 1: 30 N  
Step 2: 11 I/Y  
Step 3: 2500 PV  
Step 4: 0 PMT  
Step 5: FV⇒ \$57,230.74

##### Part 1c

Step 1: 10 N  
Step 2: 5 I/Y  
Step 3: 6000 FV  
Step 4: 0 PMT  
Step 5: PV⇒ \$3,683.48

##### Part 1d

Step 1: 10 N  
Step 2: 10 I/Y  
Step 3: 6000 FV  
Step 4: 0 PMT  
Step 5: PV⇒ \$2,313.26

##### Part 1e

Step 1: 30 N  
Step 2: 6 I/Y  
Step 3: 0 PV  
Step 4: 3000 PMT  
Step 5: FV⇒ \$237,174.56

### Part 1f

Step 1: 30 N  
Step 2: 12 I/Y  
Step 3: 0 PV  
Step 4: 3000 PMT  
Step 5: FV  $\Rightarrow$  \$723,998.05

### Part 1g

Step 1: 20 N  
Step 2: 15 I/Y  
Step 3: 4000 PMT  
Step 4: 0 FV  
Step 5: PV  $\Rightarrow$  \$25,037.33

### Part 1h

Step 1: 40 N  
Step 2: 15 I/Y  
Step 3: 4000 PMT  
Step 4: 0 FV  
Step 5: PV  $\Rightarrow$  \$26,567.11

### ? Problem 3.13.2

Find the interest rates implied by each of the following:

- You borrow \$1500 today and promise to repay the loan by making a single payment of \$2114.00 in 5 years.
- You invest \$500 today and receive a promise of receiving back \$193.50 for each of the next 4 years.

**Answer**

#### Part 2a

Step 1: 5 N  
Step 2: 2114 FV  
Step 3: -1500 PV  
Step 4: 0 PMT  
Step 5: I/Y  $\Rightarrow$  7.10%

#### Part 2b

Step 1: 4 N  
Step 2: 0 FV  
Step 3: -500 PV  
Step 4: 193.50 PMT  
Step 5: I/Y  $\Rightarrow$  20.10%

### ? Problem 3.13.3

If \$2000 is invested today at a 12% nominal interest rate, how much will it be worth in 15 years if interest is compounded

- Annually
- Quarterly
- Monthly
- Daily (365-days per year)

## Answer

### Part 3a

Step 1: 15 N  
Step 2: 12 I/Y  
Step 3: 2000 PV  
Step 4: 0 PMT  
Step 5: FV  $\Rightarrow$  \$10,947.13

### Part 3b

Step 1: Set P/YR to 4  
Step 2: 60 N  
Step 3: 12 I/Y  
Step 4: 2000 PV  
Step 5: 0 PMT  
Step 6: FV  $\Rightarrow$  \$11,783.21

### Part 3c

Step 1: Set P/YR to 12  
Step 2: 180 N  
Step 3: 12 I/Y  
Step 4: 2000 PV  
Step 5: 0 PMT  
Step 6: FV  $\Rightarrow$  \$11,991.60

### Part 3d

Step 1: Set P/YR to 365  
Step 2: 5475 N  
Step 3: 12 I/Y  
Step 4: 2000 PV  
Step 5: 0 PMT  
Step 6: FV  $\Rightarrow$  \$12,095.72  
Remember to Set P/YR back to 1.

## ? Problem 3.13.4

How long will it take your money to triple given the following interest rates?

- a. 5%
- b. 10%
- c. 15%

## Answer

### Part 4a

Step 1: 3 FV  
Step 2: 5 I/Y  
Step 3: -1 PV  
Step 4: 0 PMT  
Step 5: N  $\Rightarrow$  22.52 years

### Part 4b

Step 1: 3 FV  
Step 2: 10 I/Y  
Step 3: -1 PV  
Step 4: 0 PMT  
Step 5:  $N \Rightarrow 11.53$  years

### Part 4c

Step 1: 3 FV  
Step 2: 15 I/Y  
Step 3: -1 PV  
Step 4: 0 PMT  
Step 5:  $N \Rightarrow 7.86$  years

### ? Problem 3.13.5

After graduating from college you make it big — all because of your success in business finance. You decide to endow a scholarship for needy finance students that will provide \$5000 per year indefinitely, beginning 1 year from now. How much must be deposited today to fund the scholarship under the following conditions.

- The interest rate is 10%
- The interest rate is 10% and the first payment is made 6 years from today instead of 1 year from today.

#### Answer

#### Part 5a

$PV = PMT / k$   
 $PV = \$5000 / .10$   
 $PV = \$50,000$

#### Part 5b

Now, the first payment is in year 6, so when we solve for the perpetuity we get the amount we need to have at the end of year 5  $\Rightarrow$  \$50,000. In order to find out how much we need to invest now to have \$50,000 at the end of year 5, we solve for PV

Step 1: 5 N  
Step 2: 10 I/Y  
Step 3: 50000 FV  
Step 4: 0 PMT  
Step 5:  $PV \Rightarrow \$31,046.07$

### ? Problem 3.13.6

Find the present value of the following cash flow stream if the discount rate is 12%:

Years 1-10 \$4000 per year  
Years 11-15 \$6000 per year  
Years 16-20 \$8000 per year

#### Answer

Solution: \$34,833.37. Calculator steps are below.

HP10BII+

TI-BAII+

TI-83/84

HP10BII+	TI-BAIL+	TI-83/84
Step 1: Clear All Step 2: 0 CFj Step 3: 4000 CFj Step 4: 10 Nj Step 5: 6000 CFj Step 6: 5 Nj Step 7: 8000 CFj Step 8: 5 Nj Step 9: 12 I/YR Step 10: NPV	Step 1: CF CLR Work Step 2: 0 Enter ↓ Step 3: 4000 Enter ↓ Step 4: 10 Enter ↓ Step 5: 6000 Enter ↓ Step 6: 5 Enter ↓ Step 7: 8000 Enter ↓ Step 8: 5 Enter Step 9: NPV 12 Enter ↓ Step 10: CPT	Go to APPS⇒Finance⇒ Step 1: Select npv( Step 2: Enter the given information in the following format: npv(InterestRate, CF0, {CF Stream}, {CF Frequencies}) npv(12,0,{4000,6000,8000},{10,5,5}) Step 3: Press the SOLVE key

### ? Problem 3.13.7

Add exercises text here.

#### Answer

Solution: \$77,129.07. Calculator steps are below.

HP10BII+	TI-BAIL+	TI-83/84

HP10BII+	TI-BAII+	TI-83/84
Step 1: Clear All Step 2: 0 CFj Step 3: 3000 CFj Step 4: 5 Nj Step 5: 7500 CFj Step 6: 9000 CFj Step 7: 9 Nj Step 8: 12,000 CFj Step 9: 15 Nj Step 10: 8.75 I/YR Step 11: NPV	Step 1: CF CLR Work Step 2: 0 Enter ↓ Step 3: 3000 Enter ↓ Step 4: 5 Enter ↓ Step 5: 7500 Enter ↓↓ Step 6: 9000 Enter ↓ Step 7: 9 Enter ↓ Step 8: 12,000 Enter ↓ Step 9: 15 Enter Step 10: NPV 8.75 Enter ↓ Step 11: CPT	Go to APPS⇒Finance⇒ Step 1: Select npv( Step 2: Enter the given information in the following format: npv(InterestRate, CF0, {CF Stream}, {CF Frequencies}) npv(8.75,0,{3000,7500,9000,12000}, {5,1,9,15}) Step 3: Press the SOLVE key

Second, solve for FV using 5-key approach:

Step 1: 30 N  
 Step 2: 8.75 I/Y  
 Step 3: 77,129.07 PV  
 Step 4: 0 PMT  
 Step 5: FV⇒ \$955,203.85

### ? Problem 3.13.8

Find the value of the following cash flow stream at the end of year 30 if the rate of return is 8.75%:

Years 1-5 \$3000 per year  
 Year 6 \$7500  
 Years 7-15 \$9000 per year  
 Years 16-30 \$12,000 per year

**Answer**

#### Part 8a

$k_{\text{eff}}=9.00\%$

#### Part 8b

Solution: 9.31%

Formula:  $k_{\text{eff}}=[1+(.09/4)]^4-1=9.31\%$  or

Calculator:

HP10BII+	TI-BAII+	TI-83/84

HP10BII+	TI-BAII+	TI-83/84
Step 1: 4 SHIFT P/YR Step 2: 9 SHIFT NOM% Step 3: SHIFT EFF%	Step 1: 2nd I Conv Step 2: 9 Enter ↓↓ Step 3: 4 Enter ↑ Step 4: Press the CPT key	Go to APPS⇒Finance⇒ Step 1: Select EFF( Step 2: Enter the given information in the following format: EFF(NOMINAL      PERIODS RATE,COMPOUNDING PER YEAR) EFF(9,4) Step 3: Press SOLVE

## Part 8c

Solution: 9.38%

Formula:  $k_{\text{eff}} = [1 + (.09/12)]^{12} - 1 = 9.38\%$  or

Calculator:

HP10BII+	TI-BAII+	TI-83/84
Step 1: 12 SHIFT P/YR Step 2: 9 SHIFT NOM% Step 3: SHIFT EFF%	Step 1: 2nd I Conv Step 2: 9 Enter ↓↓ Step 3: 12 Enter ↑ Step 4: Press the CPT key	Go to APPS⇒Finance⇒ Step 1: Select EFF( Step 2: Enter the given information in the following format: EFF(NOMINAL      PERIODS RATE,COMPOUNDING PER YEAR) EFF(9,12) Step 3: Press SOLVE

## Part 8d

Solution: 9.42%

Formula:  $k_{\text{eff}} = [1 + (.09/365)]^{365} - 1 = 9.42\%$  or

Calculator:

HP10BII+	TI-BAII+	TI-83/84
Step 1: 365 SHIFT P/YR Step 2: 9 SHIFT NOM% Step 3: SHIFT EFF%	Step 1: 2nd I ConvI Step 2: 9 Enter ↓↓ Step 3: 365 Enter ↑ Step 4: Press the CPT key	Go to APPS⇒Finance⇒ Step 1: Select EFF( Step 2: Enter the given information in the following format: EFF(NOMINAL      PERIODS RATE,COMPOUNDING PER YEAR) EFF(9,365) Step 3: Press SOLVE

Note: If you are using the HP, be sure to set P/YR back to 1 after finishing 8d.

### ? Problem 3.13.9

Your firm has a retirement plan that matches all contributions on a one-to-two basis. That is, if you contribute \$3000 per year, the company will add \$1500 to make it \$4500. The firm guarantees a 9% return on your investment. Alternatively, you can “do-it-yourself” and you think you can earn 12% on your money by doing it this way. The first contribution will be made 1 year from today. At that time, and every year thereafter, you will put \$3000 into the retirement account. If you want to retire in 25 years, which way are you better off?

#### Answer

Company Plan	Do-it-Yourself Plan
Step 1: 25 N	Step 1: 25 N
Step 2: 9 I/Y	Step 2: 12 I/Y
Step 3: 0 PV	Step 3: 0 PV
Step 4: 4500 PMT	Step 4: 3000 PMT
Step 5: FV⇒ \$381,154.03	Step 5: FV⇒ \$400,001.61

### ? Problem 3.13.10

Jen is planning for retirement. She plans to work for 32 more years. She currently has \$15,000 saved and, for the next 15 years, she can save \$6,000 at the end of each year. Fifteen years from now, she wants to buy a weekend vacation home that she estimates will require her to withdraw \$100,000. How much will she have to save in years 16 through 32 so that she has exactly \$750,000 saved when she retires? Assume she can earn 9% throughout the 32-year period.

#### Answer

Step 1 ⇒ How much will Jen have saved immediately before purchasing vacation home?

15 N  
 9 I/Y  
 -15000 PV  
 -6000 PMT  
 FV⇒ \$230,802.73

Note that I made both the 15,000 and the 6000 negative. That is because Jen’s 15,000 that she has already saved is equivalent to a cash outflow (set aside today so it can compound) and the \$6000 she is saving at the end of each year are also effectively outflows (set aside so they can compound until 15 years from now). After 15 years, we will have \$230,802.73 available for us to withdraw (equivalent to a cash inflow). While I made the \$15,000 and \$6000 negative, you could also leave them both positive...just make sure the both are the same sign.

Step 2 ⇒ How much will Jen have immediately after withdrawing \$100,000 for the purchase of a vacation home?

$\$230,802.73 - \$100,000 = \$130,802.73$

Step 3 ⇒ How much will Jen have to save at the end of each year for the remaining years (17) to accumulate \$750,000?

17 N  
 9 I/Y  
 -130,802.73 PV  
 750,000 FV  
 PMT ⇒ \$4,974.72

Note that it is 17 years (not 16) as we have the initial 15 + the subsequent 17 to give us 32 years (the full time horizon). Also, note that the PV needs to be negative (we are setting aside the 130,802.73 at the start of the last 17 years) and the FV positive (so we can get \$750,000 at the end of our time horizon). While I didn’t put a negative sign in front of it, you should note that your PMT is also negative as you are giving up the \$4974.72 per year (along with the \$130,802.73) in order to get the \$750,000 at the end.



### ? Problem 3.13.11

You are a recent college graduate and want to start saving for retirement. You plan to save \$2000 per year for the next 15 years. After that you will stop contributing and just allow your savings to accumulate for another 20 years. Your twin brother would rather wait awhile before he starts saving. He is not going to put away anything for the next ten years, then he will start making contributions at the end of each year for the final 25 years. You both anticipate earning a 9.5% rate of return on your investments. How much must your brother put away at the end of each year to have the same amount of money for retirement as you?

#### Answer

Step 1  $\Rightarrow$  How much will you have at the end of year 15?

15 N  
9.5 I/Y  
0 PV  
2000 PMT  
FV  $\Rightarrow$  61,080.46

Step 2  $\Rightarrow$  How much will this \$61,080.46 grow to over the remaining 20 years?

20 N  
9.5 I/Y  
61,080.46 PV  
0 PMT  
FV 375,132.49

Step 3  $\Rightarrow$  Since your brother will save nothing for the first 10 years, he will start at the end of year 10 with nothing and have 25 years to accumulate \$375,132.49. How much must he save each year to accomplish this?

25 N  
9.5 I/Y  
0 PV  
375,132.49 FV  
PMT  $\Rightarrow$  \$4,111.22

### ? Problem 3.13.12

You are considering purchasing a new home. The house you are looking at costs \$120,000 and you plan to make a 10% down payment. You checked with a bank and they have two mortgage loan options for you. The first is a 15-year mortgage at 6.25%. The second is a 30-year mortgage at 6.50%.

- What are your monthly payments for each loan?
- What is the total you will pay over the life of the loan for each loan?
- After one year you get a job transfer and have to sell the house. What is the payoff value of your remaining loan balance (hint: find PV of remaining payments)?
- Over the first year, how much did you pay in principal and how much did you pay in interest?

#### Answer

#### Part 12a

15-Year Mortgage	30-Year Mortgage

#### 15-Year Mortgage

Set P/YR to 12  
 Step 1: 180 N  
 Step 2: 6.25 I/Y  
 Step 3: 108,000 PV  
 Step 4: 0 FV  
 Step 5: PMT  $\Rightarrow$  \$926.02

#### 30-Year Mortgage

Set P/YR to 12  
 Step 1: 360 N  
 Step 2: 6.5 I/Y  
 Step 3: 108,000 PV  
 Step 4: 0 FV  
 Step 5: PMT  $\Rightarrow$  \$682.63

### Part 12b

15-Year Mortgage  $\Rightarrow$   $\$926.02 \times 180 = \$166,683.60$

30-Year Mortgage  $\Rightarrow$   $\$682.63 \times 360 = \$245,746.80$

### Part 12c

#### 15-Year Mortgage

Set P/YR to 12  
 Step 1: 168 N  
 Step 2: 6.25 I/Y  
 Step 3: 926.02 PMT  
 Step 4: 0 FV  
 Step 5: PV  $\Rightarrow$  \$103,511.02

#### 30-Year Mortgage

Set P/YR to 12  
 Step 1: 348 N  
 Step 2: 6.5 I/Y  
 Step 3: 682.63 PMT  
 Step 4: 0 FV  
 Step 5: PV  $\Rightarrow$  \$106,792.31

### Part 12d

#### 15-Year Mortgage

Total First Year Payments (15-Year) =  $\$926.02 \times 12 = \$11,112.24$

Principal Paid (15-Year) =  $\$108,000 - \$103,511.02 = \$4488.98$

Interest Paid (15-Year) =  $\$11,112.24 - \$4488.98 = \$6623.26$

#### 30-Year Mortgage

Total First Year Payments (30-Year) =  $\$682.63 \times 12 = \$8191.56$

Principal Paid (30-Year) =  $\$108,000 - \$106,792.31 = \$1207.69$

Interest Paid (30-Year) =  $\$8191.56 - \$1207.69 = \$6983.87$

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