3.4: Annuities

The examples previously discussed are for situations where we have a specific amount today and want to know what it is worth at some point in the future (FV) or when we plan to receive a certain amount at some point in the future and want to know what it is worth today (PV). These are referred to as lump sum situations because there is only one cash flow that we are discounting or compounding.

An annuity is different in that with an annuity we have the same exact amount being received (or paid) at the end of each period over a number of periods. For example, if we win the lottery that is usually paid in equal installments over a twenty-five year period. A $1,000,000 jackpot would be paid at $40,000 per year for twenty-five years. This is said to be an annuity. (Technically the lottery is an "annuity due" because the first payment is paid today, or the beginning of the period, as opposed to the end of the period).

Timelines: Let us pause here for a moment to introduce an important tool used in time value of money – timelines. Timelines provide an aid that helps us better visualize what the cash flow stream looks like. Consider an annuity that pays $2000 per year for 4 years with an 8% discount rate. We can illustrate this on a timeline as follows:

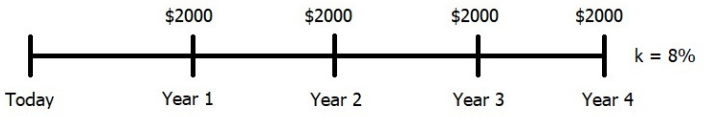

Figure 3.4.1: Annuity timeline

Note that the hashmarks represent the end of the time increment and the space between the hashmarks represent the time increment itself. In other words, the year 1 hashmark represents the end of year 1 where the annuity makes its first $2000 payment. Some students find timelines very helpful and use them for most time value of money problems while others use them less frequently. However, when we get to the section on complex time value of money problems later in this chapter, most students will find timelines quite beneficial.

Solving for Present Value of an Annuity

We have three ways to solve for the PV of an annuity: formula, financial table, and financial calculator.

Method 1: Using a Formula to Find the PV of an Annuity

The first is directly with a formula. Under this method, we use the following formula:

$$PVA = PMT\left(\frac{1-\frac{1}{(1+k)^n}}{k}\right)$$

where

PVA is the present value of the anticipated cash flow stream (annuity)
PMT is the annuity payment (how much we receive or save each period)
k is the rate of return we can earn elsewhere (also referred to as the compound rate, required return, or discount rate)
n is the number of periods which we will have to wait before receiving the money.

Method 2: Using a Table to Find the PV of an Annuity

The second method is to use financial tables and will not be covered in this text.

Method 3: Using a Financial Calculator to Find the PV of an Annuity

The third method is to use the financial calculator (or spreadsheet) which is what we will focus on. Let's walk through an example with the financial calculator. An investment that pays $100 at the end of each year for 4 years is an annuity (note that a clue for annuities is to look for the word "each' or "every" to indicate that the same cash flow is being repeated multiple times). If we wanted to know what that investment is worth to us today and we had a 10% discount rate, we would be finding the present value of that annuity.

Calculator Steps to Compute PV of an Annuity

HP10BII	TI-BAII+	TI-83/84
Step 1: 4 N Step 2: 10 I/YR Step 3: 100 PMT Step 4: 0 FV Step 5: PV⇒	Step 1: 4 N Step 2: 10 I/YR Step 3: 100 PMT Step 4: 0 FV Step 5: Press the CPT key Step 6: Press the PV key	Go to APPS⇒Finance⇒ TVM_Solver Step 1: 4 N Step 2: 10 I/YR Step 3: 100 PMT Step 4: 0 FV Step 5: Move to PV line and press the ALPHA SOLVE key

Note: The order of steps 1-4 is not important. The PV answer will appear as a negative number, ignore the negative sign for now.

You should get a solution of $316.99

Solving for Future Value of an Annuity

As with the other TVM calculations we have encountered, there are 3 basic methods to solve for the FV of an annuity: formula, financial table, and financial calculator.

Method 1: Using a Formula to Find the FV of an Annuity

The first is directly with a formula. Under this method, we use the following formula:

$$FVA = PMT\left(\frac{(1+k)^n - 1}{k}\right) \tag{3.4.1}$$

where

FVA is the future value that our cash flow stream will grow to at the end of n periods
PMT is the annuity payment (how much we receive or save each period)
k is the rate of return we can earn elsewhere (also referred to as the compound rate, required return, or discount rate)
n is the number of periods which we will have to wait before receiving the money.

Method 2: Using a Table to Find the FV of an Annuity

The second method is to use financial tables. These tables are included in Appendix A and will not be covered in this text.

Method 3: Using a Financial Calculator to Find the FV of an Annuity

The third method is the financial calculator (or spreadsheet) approach. Let's walk through an example using the financial calculator to solve for the future value of an annuity. We want to save $1000 per year (at the end of each year) for 10 years at 12%. How much will this be worth at the end of the 10th year?

Calculator Steps to Compute FV of an Annuity

HP10BII	TI-BAII+	TI-83/84

HP10BII	TI-BAII+	TI-83/84
Step 1: 10 N Step 2: 12 I/YR Step 3: 1000 PMT Step 4: 0 PV Step 5: FV	Step 1: 10 N Step 2: 12 I/YR Step 3: 1000 PMT Step 4: 0 PV Step 5: Press the CPT key Step 6: Press the FV key	Go to APPS⇒Finance⇒ TVM_Solver Step 1: 10 N Step 2: 12 I/YR Step 3: 1000 PMT Step 4: 0 PV Step 5: Move to FV line and press the ALPHA SOLVE key

Note: The order of steps 1-4 is not important. The FV answer will appear as a negative number, ignore the negative sign for now.

You should get a solution of $17,548.74

Note: Ordinary annuities (both present value and future value) assume that cash flows will arrive at the end of each period. Occasionally, you might encounter an annuity due (which means that cash flows arrive at the BEGINNING of each period). It is easy to adjust for this when using a financial calculator by changing the calculator from END of period cash flows to BEGINNING of period cash flows. This process is described in Setting up Your Financial Calculator in Appendix B (for the TI-83/84, it is just part of the onscreen display in the TVM_Solver).
