7.2: Expected Return and Standard Deviation of a Single Security

Expected Return

The expected return of a security is based on the probability distribution of returns. Before we get into the details of the expected return, let's briefly introduce the concept of a probability distribution. A probability distribution is a representation of possible outcomes (states of nature) that may occur and the likelihood (probability) of each outcome. If you think of a coin flip, the probability distribution has two possible outcomes (heads or tails) and each outcome has a 50% chance of occurring (technically, this is not true as even in a "fair" coin flip, the side that starts up has about a 51% chance of occurring). When dealing with financial securities, the number of possible outcomes is nearly infinite and it is not possible to know the exact outcomes or probabilities of those outcomes. Therefore, we are really only approximating the true probability distribution.

Video Probability Distribution

Specifically, the expected return is the probability of a specific state of nature occurring times the return under that state of nature summed across all possible states of nature. In formula terms, it is

$$\bar{k} = \sum_{i=1}^{n} P_i k_i$$

or

$$\bar{k} = P_1 k_1 + P_2 k_2 + \ldots + P_n k_n$$

where

$\bar{k}$ represents the expected return of the stock
P_i represents the probability of the ith possible outcome (state of nature)
k_i represents the return under the ith outcome (state of nature)
P_n represents the probability of the nth possible outcome (state of nature) k_n represents the return under the nth outcome (state of nature)

Don't worry if the formula and definition seem intimidating, the process is relatively simple. Consider the following example. After researching Stock A we have determined that there are 3 possible outcomes for the next year (3 states of nature). The first possibility is the economy enters a recession causing the stock to have a return of -15%. The probability of this occurring is 20%. The second possibility is that the economy goes smoothly, but does not experience rapid growth causing the stock to rise and offer a 10% return. The probability of this occurring is 50%. The third possibility is that the economy booms, causing the stock to provide a 35% rate of return. The probability of the economy booming is 30% (note that the probabilities must sum to 1.0 and the states of nature should be mutually exclusive).

State of Nature	Probability	Return
Recession	0.20	-15%

State of Nature	Probability	Return
Normal	0.50	10%
Boom	0.30	35%

What is the expected rate of return?

$$\bar{k} = (.20)(-15\%) + (.50)(10\%) + (.30)(35\%)$$

$$\bar{k} = -3\% + 5\% + 10.5\%$$

$$\bar{k} = 12.5\%$$

Video Expected Return of a Single Security

Standard Deviation

The standard deviation measures the variability of possible returns and is represented by the lower-case Greek symbol sigma. The smaller the standard deviation, the more likely we are going to earn something "close" to our expected return. The greater the standard deviation, the greater the chance that we may earn something far more (good) or far less (bad) than our expected return. The formula for this is (remember that $\bar{k}$is our symbol for expected return):

$$\sigma = \sqrt{\sum_{i=1}^{n} P_i \left(k_i - \bar{k}\right)^2}$$

or

$$\sigma = \sqrt{P_1 \left(k_1 - \bar{k}\right)^2 + P_2 \left(k_2 - \bar{k}\right)^2 + \ldots + P_n \left(k_n - \bar{k}\right)^2}$$

where

sigma (σ) represents the standard deviation
P_i represents the probability of the ith outcome (state of nature)
k_i represents the return under the ith outcome (state of nature)
$\bar{k}$ represent the expected return for the stock
P_n represents the probability of the nth outcome (state of nature)
k_n represents the return under the nth outcome (state of nature)

Calculation Notes:

It is easy to get confused with decimals and percents. The best way to do these calculations is to always leave the weights as decimals and the returns as a regular number. For instance, if you have a probability of 0.10 and a return of 15%, you would put the probability into your calculator as 0.10 and the return as 15.

Be careful with your order of operations.

1. Do $\left(\mathrm{k}_1 - \bar{k}\right)$ first
2. Then square that
3. Then multiply by P_1
4. Repeat for all n states of nature
5. Add them up
6. Finally, take the square root

Consider our previous example. What is the standard deviation for stock A?

$$\sigma = \sqrt{0.2(-15-12.5)^2 + 0.5(10-12.5)^2 + 0.3(35-12.5)^2}$$

$$\sigma = \sqrt{0.2(756.25) + 0.5(6.25) + 0.3(506.25)}$$

$$\sigma = \sqrt{151.25 + 3.13 + 151.88}$$

$$\sigma = \sqrt{306.25}$$

$$\sigma = 17.50\%$$

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