

## 4.3: Calculating the Relationship of Time and Value

### Learning Objectives

1. Identify the factors you need to know to relate a present value to a future value.
2. Write the algebraic expression for the relationship between present and future value.
3. Discuss the use of the algebraic expression in evaluating the relationship between present and future values.
4. Explain the importance of understanding the relationships among the factors that affect future value.

Financial calculation is not often a necessary skill since it is easier to use calculators, spreadsheets, and software. However, understanding the calculations is important in understanding the relationships between time, risk, opportunity cost, and value.

To do the math, you need to know

- what the future cash flows (CF) will be,
- when the future cash flows will be,
- the rate at which time affects value (e.g., the costs per time period, or the magnitude [the size or amount] of the effect of time on value).

It is usually not difficult to forecast the timing and amounts of future cash flows. Although there may be some uncertainty about them, gauging the rate at which time affects money can require some judgment. That rate, commonly called the **discount rate** because time discounts value, is the opportunity cost of not having liquidity. Opportunity cost derives from forgone choices or sacrificed alternatives, and sometimes it is not clear what those might have been (see Chapter 2). It is an important judgment call to make, though, because the rate will directly affect the valuation process.

At times, the alternatives are clear: you could be putting the liquidity in an account earning 3 percent, so that's your opportunity cost of not having it. Or you are paying 6.5 percent on a loan, which you wouldn't be paying if you had enough liquidity to avoid having to borrow; so that's your opportunity cost. Sometimes, however, your opportunity cost is not so clear.

Say that today is your twentieth birthday. Your grandparents have promised to give you \$1,000 for your twenty-first birthday, one year from today. If you had the money today, what would it be worth? That is, how much would \$1,000 worth of liquidity one year from now be worth today?

That depends on the cost of its not being liquid today, or on the opportunity costs and risks created by not having liquidity today. If you had \$1,000 today, you could buy things and enjoy them, or you could deposit it in an interest-bearing account. So on your twenty-first birthday, you would have more than \$1,000. You would have the \$1,000 plus whatever interest it had earned. If your bank pays 4 percent per year (interest rates are always stated as annual rates) on your account, then you would earn \$40 of interest in the next year, or  $\$1,000 \times .04$  (Table 4.3.1 ). So on your twenty-first birthday you would have \$1,040.

$$\begin{aligned} & \$1,000 + (\$1,000 \times 0.04) \\ &= \$1,000 \times (1 + 0.04) \\ &= \$1,040 \end{aligned}$$

Table 4.3.1 : Future Value of \$1,000 in One Year

Today	Interest Rate	Time (years)	One Year from Now
\$ 1,000	0.04	1	$1040 = 1000 \times (1 + 0.04)$

If you left that amount in the bank until your twenty-second birthday, you would have

$$\begin{aligned} & \$1,040 + (\$1,040 \times 0.04) \\ &= [\$1,040 \times (1 + 0.04)] \times (1 + 0.04) \\ &= \$1,040 \times (1 + 0.04)^2 \end{aligned}$$

$$= \$1,081.60$$

To generalize the computation, if your **present value**, or PV, is your value today,  $r$  is the rate at which time affects value or discount rate (in this case, your interest rate), and if  $t$  is the number of time periods between you and your liquidity, then the **future value**, or FV, of your wealth would be (Table 4.3.2 )

Table 4.3.2 : Future Value of \$1,000 in Two Years

Today	Interest Rate	Time (years)	One Year from Now
\$ 1,000	0.04	1	$\$1,040 = \$1,000 \times (1 + 0.04)^1$
\$ 1,000	0.04	2	$\$1,081.60 = \$1,000 \times (1 + 0.04)^2$
PV	$r$	$t$	$FV = PV \times (1 + r)^t$

$$FV = PV \times (1 + r)^t$$

$$\text{in this case } \$1,000 \times (1 + 0.04)^1 = \$1,040$$

$$\text{and } \$1,000 \times (1 + 0.04)^2 = \$1,081.60$$

Assuming there is little chance that your grandparents will not be able to give this gift, there is negligible risk. Your only cost of not having liquidity now is the opportunity cost of having to delay consumption or not earning the interest you could have earned.

The cost of delayed consumption is largely derived from a subjective valuation of whatever is consumed, or its **utility** or satisfaction. The more value you place on having something, the more it “costs” you not to have it, and the more the time that you are without it affects its value.

Assuming that if you had the money today you would save it (as it’s much harder to quantify your joy from consumption), by having to wait to get it until your twenty-first birthday—and *not* having it today—you miss out on \$40 it could have earned.

So, what would that nominal \$1,000 (that future value that you get one year from now) actually be worth today? The rate at which time affects your value is 4 percent because that’s what having a choice (spend it or invest it) could earn for you if only you had received the \$1,000. That’s your opportunity cost. That’s what it costs you to not have liquidity. Since

$$\text{If } FV = PV \times (1 + r)^t ,$$

$$\text{then } PV = \frac{FV}{(1 + r)^t}$$

$$\text{so } PV = \frac{\$1,000}{1.04^1}$$

$$PV = \$961.5385$$

Your gift is worth \$961.5385 today (its present value). If your grandparents offered to give you your twenty-first birthday gift on your twentieth birthday, they could give you \$961.5385 today, which would be the equivalent value to you of getting \$1,000 one year from now.

It is important to understand the relationships between time, risk, opportunity cost, and value. This equation describes that relationship:

$$PV \times (1 + r)^t = FV$$

The “r” is more formally called the “discount rate” because it is the rate at which your liquidity is discounted by time, and it includes not only opportunity costs but also risk. (On some financial calculators, “r” is displayed as “I” or “i.”)

The “t” is how far away you are from your liquidity over time.

Studying this equation yields valuable insights into the relationship it describes. Looking at the equation, you can observe the following relationships.

The more time (t) separating you from your liquidity, the more time affects value. The less time separating you from your liquidity, the less time affects value (as t decreases, PV increases).

A the number of periods	The present value of your future value liquidity
increases	decreases
decreases	increases

The greater the rate at which time affects value (r), or the greater the opportunity cost and risk, the more time affects value. The less your opportunity cost or risk, the less your value is affected.

As the interest rate	The present value of your future liquidity
increases	decreases
decreases	increases

Table 4.3.3 presents examples of these relationships.

Table 4.3.3 : Present Values, Interest Rates, Time, and Future Values

	Today	Interest Rate	Time (years)	Future Value
<b>Example A</b>	\$ 1,000	0.04	1	\$1,040 = PV x 104%
Greater effect r increases	\$ 1,000	0.10	1	\$ 1,100 = PV x 110%
More time t increases	\$ 1,000	0.04	3	\$ 1,124.86 = PV x 104 <sup>3</sup> %
Less effect r decreases	\$ 1,000	0.01	1	\$ 1,010 = PV x 101%
less time t decreases	\$ 1,000	0.04	5	\$ 1,019.80 = PV x 104 <sup>0.5</sup> %
<b>Example B</b>	961.54 = FV / 104%	0.04	1	\$ 1,000
Greater effect r increases	909.09 = FV / 110%	0.10	1	\$ 1,000
More time t increases	889.00 = FV / 104 <sup>3</sup> %	0.04	3	\$ 1,000
Less effect r decreases	990.1 = FV / 101%	0.01	1	\$ 1,000
Less time t decreases	980.58 = FV / 104 <sup>0.5</sup> %	0.04	0.5	\$ 1,000

The strategy implications of this understanding are simple, yet critical. All things being equal, it is more valuable to have liquidity (get paid, or have positive cash flow) *sooner* rather than later and give up liquidity (pay out, or have negative cash flow) *later* rather than sooner.

If possible, accelerate incoming cash flows and decelerate outgoing cash flows: get paid sooner, but pay out later. Or, as Popeye’s pal Wimpy used to say, “I’ll give you 50 cents tomorrow for a hamburger today.”

## Summary

- To relate a present (liquid) value to a future value, you need to know
  - what the present value is or the future value will be,
  - when the future value will be,
  - the rate at which time affects value: the costs per time period, or the magnitude of the effect of time on value.
- The relationship of present value (PV), future value (FV), risk and opportunity cost (the discount rate,  $r$ ), and time ( $t$ ), may be expressed as:
  - $PV \times (1 + r)^t = FV$  .
- The above equation yields valuable insights into these relationships:
  - The more time ( $t$ ) creates distance from liquidity, the more time affects value.
  - The greater the rate at which time affects value ( $r$ ), or the greater the opportunity cost and risk, the more time affects value.
  - The closer the liquidity, the less time affects value.
  - The less the opportunity cost or risk, the less value is affected.
- To maximize value, get paid sooner and pay later.

## Exercises

1. Watch [The Time Value of Money](#) (8:16 minutes) from Khan Academy for an alternative explanation of this sometimes confusing topic. The video shows you how to calculate the present and future values of money. How is each part of the formula used in that lesson equivalent to the formula presented in this text?
2. In your financial planning journal, identify a future cash flow. Calculate its present value and then calculate its future value based on the discount rate and time to liquidity. Repeat the process for other future cash flows you identify. What pattern of relationships do you observe between time and value?
3. Use the [Present Value Calculator](#) and [Future Value Calculator](#) to compare your results with your manual calculations in Exercise 1.
4. To have liquidity, when should you increase positive cash flows and decrease negative cash flows, and why?

This page titled [4.3: Calculating the Relationship of Time and Value](#) is shared under a [CC BY-NC-SA 3.0](#) license and was authored, remixed, and/or curated by [Anonymous](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.

- [4.3: Calculating the Relationship of Time and Value](#) by Anonymous is licensed [CC BY-NC-SA 3.0](#). Original source: <https://2012books.lardbucket.org/books/individual-finance>.