

10.8: Finding Equilibrium Using Algebra

Learning Objectives

- Find the macro equilibrium using algebra

In the income-expenditure model, the equilibrium occurs at the level of GDP where aggregate expenditures equal national income (or GDP). We can identify this equilibrium using algebra as well as graphically. Given algebraic equations for the aggregate expenditure line and the income=expenditure line, the point where they cross can be readily calculated.

USING AN ALGEBRAIC APPROACH TO THE EXPENDITURE-OUTPUT MODEL

Imagine an economy defined by the following:

$$C = 140 + 0.9(Y_d)$$

This is the consumption function where 140 is autonomous consumption, 0.9 is the marginal propensity to consume, and Y_d is disposable (i.e. after tax income).

$Y_d = Y - T$, where Y is national income (or GDP) and $T = \text{Tax Revenues} = 0.3Y$; note that 0.3 is the average income tax rate.

$I = \text{Investment} = 400$

$G = \text{Government spending} = 800$

$X = \text{Exports} = 600$

$M = \text{Imports} = 0.15Y$

Step 1. Determine the aggregate expenditure function. Using the numbers from above, it is:

AE	=	$C + I + G + X - M$
AE	=	$140 + 0.9(Y - T) + 400 + 800 + 600 - 0.15Y$

Step 2. The equation for the 45-degree line is the set of points where GDP or national income on the horizontal axis is equal to aggregate expenditure on the vertical axis. Thus, the equation for the 45-degree line is: $AE = Y$.

Step 3. The next step is to solve these two equations for Y (or AE , since they will be equal to each other). Substitute Y for AE :

$$Y = AE = 140 + 0.9(Y - T) + 400 + 800 + 600 - 0.15Y$$

Step 4. Insert the term $0.3Y$ for the tax rate T . This produces an equation with only one variable, Y .

Step 5. Work through the algebra and solve for Y .

Y	=	$140 + 0.9(Y - 0.3Y) + 400 + 800 + 600 - 0.15Y$
Y	=	$140 + 0.9Y - 0.27Y + 1800 - 0.15Y$
Y	=	$1940 + 0.48Y$
$Y - 0.48Y$	=	1940
$0.52Y$	=	1940
$\frac{0.52Y}{0.52}$	=	$\frac{1940}{0.52}$
Y	=	3730

This algebraic framework is flexible and useful in predicting how economic events and policy actions will affect real GDP.

Say, for example, that because of changes in the relative prices of domestic and foreign goods, the marginal propensity to import falls to 0.1. Calculate the equilibrium output when the marginal propensity to import is changed to 0.10.

Y	$=$	$140 + 0.9(Y - 0.3Y) + 400 + 800 + 600 - 0.1Y$
Y	$=$	$1940 - 0.53Y$
$0.47Y$	$=$	1940
Y	$=$	4127

Alternatively, suppose because of a surge of business confidence, investment rises to 500. Calculate the equilibrium output.

Y	$=$	$140 + 0.9(Y - 0.3Y) + 500 + 800 + 600 - 0.15Y$
Y	$=$	$2040 + 0.48Y$
$Y - 0.48Y$	$=$	2040
$0.52Y$	$=$	2040
Y	$=$	3923

Exercise: Consumption in the Income-Expenditure Model

Let's work through another example. Suppose that the amount of autonomous consumption is \$20. Assume that taxes are 0.2 of real GDP. Let the marginal propensity to save of after-tax income be 0.1. The level of investment is \$70, the level of government spending is \$80, and the level of exports is \$50. Imports are 0.2 of after-tax income. Given these values, you need to complete the table and then answer these questions: What is the consumption function? What is the equilibrium? Why is a national income of \$300 not at equilibrium? How do expenditures and output compare at this point?

National Income	Taxes	After-tax income	Consumption	$I + G + X$	Imports	Aggregate Expenditures
\$300			\$236			
\$400						
\$500						
\$600						
\$700						

Step 1. Calculate the amount of taxes for each level of national income (reminder: $GDP = \text{national income}$) for each level of national income using the following as an example:

$$\text{National Income (Y)} = \$300 \quad \text{Taxes} = 0.2 \text{ or } 20\% \times \text{Tax amount (T)} = \$60$$

Step 2. Calculate after-tax income by subtracting the tax amount from national income for each level of national income using the following as an example:

$$\text{National income minus taxes} = \$300 - \$60 = \text{After-tax income} = \$240$$

Step 3. Calculate consumption. The marginal propensity to save is given as 0.1. This means that the marginal propensity to consume is 0.9, since $MPS + MPC = 1$. Therefore, multiply 0.9 by the after-tax income amount using the following as an example:

$$\text{After-tax Income} = \$240 \quad \text{MPC} \times \text{After-tax Income} = 0.9 \times \$240 = \text{Consumption} = \$216$$

Step 4. Consider why the table shows consumption of \$236 in the first row. As mentioned earlier, the Keynesian model assumes that there is some level of consumption even without income. That amount is $\$236 - \$216 = \$20$.

Step 5. There is now enough information to write the consumption function. The consumption function is found by figuring out the level of consumption that will happen when income is zero. Remember that:

$$C = \text{Consumption when national income is zero} + \text{MPC (after-tax income)}$$

Let C represent the consumption function, Y represent national income, and T represent taxes.

$$\begin{array}{l} \text{C} = \$20 + 0.9(Y - T) \\ \text{C} = \$20 + 0.9(\$300 - \$60) \\ \text{C} = \$236 \end{array}$$

Step 6. Use the consumption function to find consumption at each level of national income.

Step 7. Add investment (I), government spending (G), and exports (X). Remember that these do not change as national income changes:

Step 8. Find imports, which are 0.2 of after-tax income at each level of national income. For example:

$$\begin{array}{l} \text{After-tax income} = \$240 \\ \text{Imports of 0.2 or 20\% of Y} = \$240 \times 0.2 \\ \text{Imports} = \$48 \end{array}$$

Step 9. Find aggregate expenditure by adding $C + I + G + X - I$ for each level of national income. Your completed table should look like this:

National Income (Y)	Tax = 0.2 × Y (T)	After-tax income (Y – T)	Consumption C = \$20 + 0.9(Y – T)	I + G + X	Minus Imports (M)	Aggregate Expenditures AE = C + I + G + X – M
\$300	\$60	\$240	\$236	\$200	\$48	\$388
\$400	\$80	\$320	\$308	\$200	\$64	\$444
\$500	\$100	\$400	\$380	\$200	\$80	\$500
\$600	\$120	\$480	\$452	\$200	\$96	\$556
\$700	\$140	\$560	\$524	\$200	\$112	\$612

Step 10. Answer the question: What is equilibrium? Equilibrium occurs where $AE = Y$. This table shows that equilibrium occurs where national income equals aggregate expenditure at \$500.

Step 11. Find equilibrium mathematically, knowing that national income is equal to aggregate expenditure. Step 10. Answer the question: What is equilibrium? Equilibrium occurs where $AE = Y$. The table shows that equilibrium occurs where national income equals aggregate expenditure at \$500.

$$\begin{array}{l} Y = AE \\ Y = C + I + G + X - M \\ Y = \$20 + 0.9(Y - T) + \$70 + \$80 + \$50 - 0.2(Y - T) \\ Y = \$220 + 0.9(Y - T) - 0.2(Y - T) \end{array}$$

Since T is 0.2 of national income, substitute T with 0.2 Y so that:

$$\begin{array}{l} Y = \$220 + 0.9(Y - 0.2Y) - 0.2(Y - 0.2Y) \\ Y = \$220 + 0.9(0.8Y) - 0.2(0.8Y) \\ Y = \$220 + 0.72Y - 0.16Y \\ Y = \$220 + 0.56Y \end{array}$$

Solve for Y.

$$\begin{array}{l} Y = \$220 + 0.56Y \\ Y - 0.56Y = \$220 \\ 0.44Y = \$220 \\ Y = \frac{\$220}{0.44} \\ Y = \$500 \end{array}$$

Step 12. Answer this question: Why is a national income of \$300 not an equilibrium? At national income of \$300, aggregate expenditures are \$388.

Step 13. Answer this question: How do expenditures and output compare at this point? Aggregate expenditures cannot exceed output (GDP) in the long run, since there would not be enough goods to be bought.

Try It

<https://assessments.lumenlearning.co...sessments/7573>

CC licensed content, Shared previously

- Appendix B. **Authored by:** OpenStax College. **Located at:** <http://Appendix%20B>. **License:** CC BY: Attribution. **License Terms:** Download for free at <http://cnx.org/contents/4061c832-098...93a2cb31@11.11>

10.8: Finding Equilibrium Using Algebra is shared under a [not declared](#) license and was authored, remixed, and/or curated by LibreTexts.