

5.7: Chapter 5 Homework

5.3 Using the Central Limit Theorem

Use the following information to answer the next eight exercises: The length of time a particular smartphone's battery lasts follows an normal distribution with a mean of 10 months and a standard deviation of 10 months. A sample of 64 of these smartphones is taken.

1. a. What is the mean of the sampling distribution?
- b. What is the standard deviation of the sampling distribution?
2. What is the distribution for the length of time one battery lasts?
3. What is the distribution for the mean length of time 64 batteries last?
4. What is the distribution for the total length of time 64 batteries last?
5. Find the probability that the sample mean is between seven and 11.
6. Find the 80th percentile for the total length of time 64 batteries last.
7. Find the *IQR* for the mean amount of time 64 batteries last.
8. Find the middle 80% for the total amount of time 64 batteries last.
9. A population has a mean of 25 and a standard deviation of 2. If it is sampled repeatedly with samples of size 49, what is the mean and standard deviation of the sample means?
10. A population has a mean of 48 and a standard deviation of 5. If it is sampled repeatedly with samples of size 36, what is the mean and standard deviation of the sample means?
11. A population has a mean of 90 and a standard deviation of 6. If it is sampled repeatedly with samples of size 64, what is the mean and standard deviation of the sample means?
12. A population has a mean of 120 and a standard deviation of 2.4. If it is sampled repeatedly with samples of size 40, what is the mean and standard deviation of the sample means?
13. A population has a mean of 17 and a standard deviation of 1.2. If it is sampled repeatedly with samples of size 50, what is the mean and standard deviation of the sample means?
14. A population has a mean of 17 and a standard deviation of 0.2. If it is sampled repeatedly with samples of size 16, what is the expected value of the mean and the standard deviation of the sample means?
15. A population has a mean of 38 and a standard deviation of 3. If it is sampled repeatedly with samples of size 48, what is the expected value of the mean and the standard deviation of the sample means?
16. A population has a mean of 14 and a standard deviation of 5. If it is sampled repeatedly with samples of size 60, what is the expected value of the mean and the standard deviation of the sample means?
17. A fishing boat has 1,000 fish on board, with an average weight of 120 pounds and a standard deviation of 6.0 pounds. If sample sizes of 50 fish are checked, what is the probability the fish in a sample will have mean weight within 2.0 pounds of the true mean of the population?
18. An experimental garden has 500 sunflower plants. The plants are being treated so they grow to unusual heights. The average height is 9.3 feet with a standard deviation of 0.5 foot. If sample sizes of 60 plants are taken, what is the probability the plants in a given sample will have an average height within 0.1 foot of the true mean of the population?
19. A company has 800 employees. The average number of workdays between absence for illness is 123 with a standard deviation of 14 days. Samples of 50 employees are examined. What is the probability a sample has a mean of workdays with no absence for illness of at least 124 days?
20. Cars pass an automatic speed check device that monitors 2,000 cars on a given day. This population of cars has an average speed of 67 miles per hour with a standard deviation of 2 miles per hour. If samples of 30 cars are taken, what is the probability a given sample will have an average speed within 0.50 mile per hour of the population mean?

21. A town keeps weather records. From these records it has been determined that it rains on an average of 37% of the days each year. If 30 days are selected at random from one year, what is the probability that at least 5 and at most 11 days had rain?
22. A maker of yardsticks has an ink problem that causes the markings to smear on 4% of the yardsticks. The daily production run is 2,000 yardsticks. What is the probability if a sample of 100 yardsticks is checked, there will be ink smeared on at most 4 yardsticks?
23. A school has 300 students. Usually, there are an average of 21 students who are absent. If a sample of 30 students is taken on a certain day, what is the probability that at most 2 students in the sample will be absent?
24. A college gives a placement test to 5,000 incoming students each year. On the average 1,213 place in one or more developmental courses. If a sample of 50 is taken from the 5,000, what is the probability at most 12 of those sampled will have to take at least one developmental course?

5.2 The Central Limit Theorem for Sample Means

25. Previously, Gettysburg College statistics students estimated that the amount of change statistics students carry is normally distributed with a mean of \$0.88 and a standard deviation of \$0.31. Suppose that we randomly pick 25 statistics students.

1. In words, $X =$ _____
2. $X \sim$ _____(_____, _____)
3. In words, $\bar{x} =$ _____
4. $\bar{x} \sim$ _____(_____, _____)
5. Find the probability that an individual had between \$0.80 and \$1.00. Graph the situation, and shade in the area to be determined.
6. Find the probability that the average of the 25 students was between \$0.80 and \$1.00. Graph the situation, and shade in the area to be determined.
7. Explain why there is a difference in part e and part f.

26. Suppose that the distance of fly balls hit to the outfield (in baseball) is normally distributed with a mean of 250 feet and a standard deviation of 50 feet. We randomly sample 49 fly balls.

1. If \bar{x} = average distance in feet for 49 fly balls, then $\bar{x} \sim$ _____(_____, _____)
2. What is the probability that the 49 balls traveled an average of less than 240 feet? Sketch the graph. Scale the horizontal axis for \bar{x} . Shade the region corresponding to the probability. Find the probability.
3. Find the 80th percentile of the distribution of the average of 49 fly balls.

27. According to the Internal Revenue Service, the average length of time for an individual to complete (keep records for, learn, prepare, copy, assemble, and send) IRS Form 1040 is 10.53 hours (without any attached schedules). The distribution is unknown. Let us assume that the standard deviation is two hours. Suppose we randomly sample 36 taxpayers.

1. In words, $X =$ _____
2. In words, $\bar{x} =$ _____
3. $\bar{x} \sim$ _____(_____, _____)
4. Would you be surprised if the 36 taxpayers finished their Form 1040s in an average of more than 12 hours? Explain why or why not in complete sentences.
5. Would you be surprised if one taxpayer finished his or her Form 1040 in more than 12 hours? In a complete sentence, explain why.

28. Suppose that a category of world-class runners are known to run a marathon (26 miles) in an average of 145 minutes with a standard deviation of 14 minutes. Consider 49 of the races. Let \bar{x} the average of the 49 races.

1. $\bar{x} \sim$ _____(_____, _____)
2. Find the probability that the runner will average between 142 and 146 minutes in these 49 marathons.
3. Find the 80th percentile for the average of these 49 marathons.
4. Find the median of the average running times.

29. The length of songs in a collector's Spotify collection is normally distributed with a mean of 2.75 minutes and a standard deviation of 0.43 minutes. Suppose we randomly pick five albums from the collection. There are a total of 43 songs on the five albums.

1. In words, $X =$ _____
2. $X \sim$ _____
3. In words, $\bar{x} =$ _____
4. $\bar{x} \sim$ _____ (_____, _____)
5. Find the first quartile for the average song length.
6. The *IQR* (interquartile range) for the average song length is from _____ – _____.

30. In 1940 the average size of a U.S. farm was 174 acres. Let's say that the standard deviation was 55 acres. Suppose we randomly survey 38 farmers from 1940.

1. In words, $X =$ _____
2. In words, $\bar{x} =$ _____
3. $\bar{x} \sim$ _____ (_____, _____)
4. The *IQR* for \bar{x} is from _____ acres to _____ acres.

31. Determine which of the following are true and which are false. Then, in complete sentences, justify your answers.

1. When the sample size is large, the mean of \bar{x} is approximately equal to the population mean of X .
2. When the sample size is large, the sampling distribution of \bar{x} is approximately normally distributed.
3. When the sample size is large, the standard deviation of \bar{x} is approximately the same as the standard deviation of X .

32. The percent of calories from protein that a person in America consumes each day is normally distributed with a mean of about 36 and a standard deviation of about ten. Suppose that 16 individuals are randomly chosen. Let \bar{x} = average percent of calories from protein.

- a. $\bar{x} \sim$ _____ (_____, _____)
- b. For the group of 16, find the probability that the average percent of calories from protein consumed is more than five. Graph the situation and shade in the area to be determined.
- c. Find the first quartile for the average percent of calories from protein.

33. The distribution of income in some Third World countries is considered wedge-shaped (many very poor people, very few middle income people, and even fewer wealthy people). Suppose we pick a country with a wedge-shaped distribution. Let the average salary be \$2,000 per year with a standard deviation of \$8,000. We randomly survey 1,000 residents of that country.

1. In words, $X =$ _____
2. In words, $\bar{x} =$ _____
3. $\bar{x} \sim$ _____ (_____, _____)
4. How is it possible for the standard deviation to be greater than the average?
5. Why is it more likely that the average of the 1,000 residents will be from \$2,000 to \$2,100 than from \$2,100 to \$2,200?

34. Which of the following is NOT TRUE about the distribution for averages?

1. The mean, median, and mode are equal.
2. The area under the curve is one.
3. The curve never touches the x-axis.
4. The curve is skewed to the right.

35. The cost of unleaded gasoline in the Bay Area once followed an unknown distribution with a mean of \$4.59 and a standard deviation of \$0.10. Sixteen gas stations from the Bay Area are randomly chosen. We are interested in the average cost of gasoline for the 16 gas stations. The distribution to use for the average cost of gasoline for the 16 gas stations is:

- a. $\bar{x} \sim N(4.59, 0.10)$
- b. $\bar{x} \sim N\left(4.59, \frac{0.10}{\sqrt{16}}\right)$
- c. $\bar{x} \sim N\left(4.59, \frac{16}{0.10}\right)$
- d. $\bar{x} \sim N\left(4.59, \frac{\sqrt{16}}{0.10}\right)$

5.3 Using the Central Limit Theorem

36. A large population of 5,000 students take a practice test to prepare for a standardized test. The population mean is 140 questions correct, and the standard deviation is 80. What size samples should a researcher take to get a distribution of means of the samples with a standard deviation of 10?
37. A large population has skewed data with a mean of 70 and a standard deviation of 6. Samples of size 100 are taken, and the distribution of the means of these samples is analyzed.
1. Will the distribution of the means be closer to a normal distribution than the distribution of the population?
 2. Will the mean of the means of the samples remain close to 70?
 3. Will the distribution of the means have a smaller standard deviation?
 4. What is that standard deviation?
38. A researcher is looking at data from a large population with a standard deviation that is much too large. In order to concentrate the information, the researcher decides to repeatedly sample the data and use the distribution of the means of the samples? The first effort used sample sized of 100. But the standard deviation was about double the value the researcher wanted. What is the smallest size samples the researcher can use to remedy the problem?
39. A researcher looks at a large set of data, and concludes the population has a standard deviation of 40. Using sample sizes of 64, the researcher is able to focus the mean of the means of the sample to a narrower distribution where the standard deviation is 5. Then, the researcher realizes there was an error in the original calculations, and the initial standard deviation is really 20. Since the standard deviation of the means of the samples was obtained using the original standard deviation, this value is also impacted by the discovery of the error. What is the correct value of the standard deviation of the means of the samples?
40. A population has a standard deviation of 50. It is sampled with samples of size 100. What is the variance of the means of the samples?
41. A company has 1,000 employees. The average number of workdays between absence for illness is 80 with a standard deviation of 11 days. Samples of 80 employees are examined. What is the probability a sample has a mean of workdays with no absence for illness of at least 78 days and at most 84 days?
42. Trucks pass an automatic scale that monitors 2,000 trucks. This population of trucks has an average weight of 20 tons with a standard deviation of 2 tons. If a sample of 50 trucks is taken, what is the probability the sample will have an average weight within one-half ton of the population mean?
43. A town keeps weather records. From these records it has been determined that it rains on an average of 12% of the days each year. If 30 days are selected at random from one year, what is the probability that at most 3 days had rain?
44. A maker of greeting cards has an ink problem that causes the ink to smear on 7% of the cards. The daily production run is 500 cards. What is the probability that if a sample of 35 cards is checked, there will be ink smeared on at most 5 cards?
45. A school has 500 students. Usually, there are an average of 20 students who are absent. If a sample of 30 students is taken on a certain day, what is the probability that at least 2 students in the sample will be absent?

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