

4.2: The Standard Normal Distribution

The **standard normal distribution** is a normal distribution of **standardized values called z-scores**. A **z-score is measured in units of the standard deviation**.

The mean for the standard normal distribution is zero, and the standard deviation is one. What this does is dramatically simplify the mathematical calculation of probabilities. The transformation $z = \frac{x - \mu}{\sigma}$ produces the distribution $Z \sim N(0, 1)$. The value x in the given equation comes from a known normal distribution with known mean μ and known standard deviation σ . The z-score tells how many standard deviations a particular x is away from the mean.

Z-Scores

If X is a normally distributed random variable and $X \sim N(\mu, \sigma)$, then the z-score for a particular x is:

$$z = \frac{x - \mu}{\sigma}$$

The z-score tells you how many standard deviations the value x is above (to the right of) or below (to the left of) the mean, μ . Values of x that are larger than the mean have positive z-scores, and values of x that are smaller than the mean have negative z-scores. If x equals the mean, then x has a z-score of zero.

Suppose $X \sim N(5, 6)$. This says that X is a normally distributed random variable with mean $\mu = 5$ and standard deviation $\sigma = 6$. Suppose $x = 17$. Then:

$$z = \frac{x - \mu}{\sigma} = \frac{17 - 5}{6} = 2$$

This means that $x = 17$ is two standard deviations (2σ) above or to the right of the mean $\mu = 5$.

Now suppose $x = 1$. Then: $z = \frac{x - \mu}{\sigma} = \frac{1 - 5}{6} = -0.67$ (rounded to two decimal places). This means that $x = 1$ is 0.67 standard deviations (-0.67σ) below or to the left of the mean $\mu = 5$.

The Empirical Rule

If X is a random variable and has a normal distribution with mean μ and standard deviation σ , then **the Empirical Rule** states the following:

- About 68% of the x values lie between -1σ and $+1\sigma$ of the mean μ (within one standard deviation of the mean).
- About 95% of the x values lie between -2σ and $+2\sigma$ of the mean μ (within two standard deviations of the mean).
- About 99.7% of the x values lie between -3σ and $+3\sigma$ of the mean μ (within three standard deviations of the mean). Notice that almost all the x values lie within three standard deviations of the mean.
- The z-scores for $+1\sigma$ and -1σ are $+1$ and -1 , respectively.
- The z-scores for $+2\sigma$ and -2σ are $+2$ and -2 , respectively.
- The z-scores for $+3\sigma$ and -3σ are $+3$ and -3 respectively.


 This frequency curve illustrates the empirical rule. The normal curve is shown over a horizontal axis. The axis is labeled with points $-3s$, $-2s$, $-1s$, m , $1s$, $2s$, $3s$. Vertical lines connect the axis to the curve at each labeled point. The peak of the curve aligns with the point m .

Figure 4.2.1

Example 4.2.1

Suppose x has a normal distribution with mean 50 and standard deviation 6.

- About 68% of the x values lie within one standard deviation of the mean. Therefore, about 68% of the x values lie between $-1\sigma = (-1)(6) = -6$ and $1\sigma = (1)(6) = 6$ of the mean 50. The values $50 - 6 = 44$ and $50 + 6 = 56$ are within one standard deviation from the mean 50. The z-scores are -1 and $+1$ for 44 and 56, respectively.
- About 95% of the x values lie within two standard deviations of the mean. Therefore, about 95% of the x values lie between $-2\sigma = (-2)(6) = -12$ and $2\sigma = (2)(6) = 12$. The values $50 - 12 = 38$ and $50 + 12 = 62$ are within two standard deviations from the mean 50. The z-scores are -2 and $+2$ for 38 and 62, respectively.

- About 99.7% of the x values lie within three standard deviations of the mean. Therefore, about 99.7% of the x values lie between $-3\sigma = (-3)(6) = -18$ and $3\sigma = (3)(6) = 18$ of the mean 50. The values $50 - 18 = 32$ and $50 + 18 = 68$ are within three standard deviations from the mean 50. The z-scores are -3 and $+3$ for 32 and 68, respectively.

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