

10.4: Linear Equations

Linear regression for two variables is based on a linear equation with one independent variable. The equation has the form:

$$Y = a + bX$$

where a and b are constant numbers.

The variable X is **the independent variable**, and Y is **the dependent variable**. Another way to think about this equation is a statement of cause and effect. The X variable is the cause and the Y variable is the hypothesized effect. Typically, you choose a value to substitute for the independent variable and then solve for the dependent variable.

Example 10.4.1

The following examples are linear equations.

$$Y = 3 + 2X$$

$$Y = -0.01 + 1.2X$$

The graph of a linear equation of the form $Y = a + bX$ is a **straight line**. Any line that is not vertical can be described by this equation

Example 10.4.2

Graph the equation $Y = -1 + 2X$.

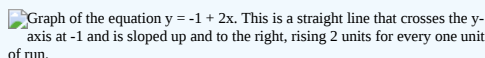
Graph of the equation $y = -1 + 2x$. This is a straight line that crosses the y -axis at -1 and is sloped up and to the right, rising 2 units for every one unit of run.

Figure 10.4.1

Exercise 10.4.1

Is the following an example of a linear equation? Why or why not?

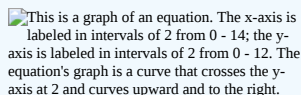
This is a graph of an equation. The x -axis is labeled in intervals of 2 from 0 - 14; the y -axis is labeled in intervals of 2 from 0 - 12. The equation's graph is a curve that crosses the y -axis at 2 and curves upward and to the right.

Figure 10.4.2

Example 10.4.3

Aaron's Word Processing Service (AWPS) does word processing. The rate for services is \$32 per hour plus a \$31.50 one-time charge. The total cost to a customer depends on the number of hours it takes to complete the job.

Find the equation that expresses the **total cost** in terms of the **number of hours** required to complete the job.

Answer

Let X = the number of hours it takes to get the job done.

Let Y = the total cost to the customer.

The \$31.50 is a fixed cost. If it takes X hours to complete the job, then $(32)(X)$ is the cost of the word processing only. The total cost is: $Y = 31.50 + 32X$

Slope and Y-Intercept of a Linear Equation

For the linear equation $Y = a + bX$, b = slope and a = Y -intercept. From algebra recall that the slope is a number that describes the steepness of a line, and the Y -intercept is the Y coordinate of the point $(0, a)$ where the line crosses the y -axis. From calculus

the slope is the first derivative of the function. For a linear function, the slope is $dY/dX = b$ where we can read the mathematical expression as "the change in Y (dY) that results from a unit change in X (dX) equals b ".



Figure 10.4.3 Three possible graphs of $Y = a + bX$. (a) If $b > 0$, the line slopes upward to the right. (b) If $b = 0$, the line is horizontal. (c) If $b < 0$, the line slopes downward to the right.

Example 10.4.4

Sven tutors to make extra money for college. For each tutoring session, he charges a one-time fee of \$25 plus \$15 per hour of tutoring. A linear equation that expresses the total amount of money Sven earns for each session he tutors is $Y = 25 + 15X$.

What are the independent and dependent variables? What is the Y -intercept and what is the slope? Interpret them using complete sentences.

Answer

The independent variable (X) is the number of hours Sven tutors each session. The dependent variable (Y) is the amount, in dollars, Sven earns for each session.

The Y -intercept is 25 ($a = 25$). At the start of the tutoring session, Sven charges a one-time fee of \$25 (this is when $X = 0$). The slope is 15 ($b = 15$). For each session, Sven earns \$15 for each hour he tutors.

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