4.7: Chapter 4 Homework

4.2 The Standard Normal Distribution

1. A bottle of water contains 12.05 fluid ounces with a standard deviation of 0.01 ounces. Define the random variable X in words. $X =$ ____________.

2. A normal distribution has a mean of 61 and a standard deviation of 15. What is the median?

3. $X \sim N(1, 2)$

$\sigma =$ _______

4. A company manufactures rubber balls. The mean diameter of a ball is 12 cm with a standard deviation of 0.2 cm. Define the random variable X in words. $X =$ ______________.

5. $X \sim N(-4, 1)$

What is the median?

6. $X \sim N(3, 5)$

$\sigma =$ _______

7. $X \sim N(-2, 1)$

$\mu =$ _______

8. What does a *z*-score measure?

9. What does standardizing a normal distribution do to the mean?

10. Is $X \sim N(0, 1)$ a standardized normal distribution? Why or why not?

11. What is the *z*-score of $x = 12$, if it is two standard deviations to the right of the mean?

12. What is the *z*-score of $x = 9$, if it is 1.5 standard deviations to the left of the mean?

13. What is the *z*-score of $x = -2$, if it is 2.78 standard deviations to the right of the mean?

14. What is the *z*-score of $x = 7$, if it is 0.133 standard deviations to the left of the mean?

15. Suppose $X \sim N(2, 6)$. What value of x has a *z*-score of 3?

16. Suppose $X \sim N(8, 1)$. What value of x has a *z*-score of –2.25?

17. Suppose $X \sim N(9, 5)$. What value of x has a *z*-score of –0.5?

18. Suppose $X \sim N(2, 3)$. What value of x has a *z*-score of –0.67?

19. Suppose $X \sim N(4, 2)$. What value of x is 1.5 standard deviations to the left of the mean?

20. Suppose $X \sim N(4, 2)$. What value of x is two standard deviations to the right of the mean?

21. Suppose $X \sim N(8, 9)$. What value of x is 0.67 standard deviations to the left of the mean?

22. Suppose $X \sim N(-1, 2)$. What is the *z*-score of $x = 2$?

23. Suppose $X \sim N(12, 6)$. What is the *z*-score of $x = 2$?

24. Suppose $X \sim N(9, 3)$. What is the *z*-score of $x = 9$?

25. Suppose a normal distribution has a mean of 6 and a standard deviation of 1.5. What is the *z*-score of $x = 5.5$?

26. In a normal distribution, $x = 5$ and $z = -1.25$. This tells you that $x = 5$ is ____ standard deviations to the ____ (right or left) of the mean.

27. In a normal distribution, $x = 3$ and $z = 0.67$. This tells you that $x = 3$ is ____ standard deviations to the ____ (right or left) of the mean.

28. In a normal distribution, $x = -2$ and $z = 6$. This tells you that $x = -2$ is ____ standard deviations to the ____ (right or left) of the mean.

29. In a normal distribution, $x = -5$ and $z = -3.14$. This tells you that $x = -5$ is ____ standard deviations to the ____ (right or left) of the mean.

30. In a normal distribution, $x = 6$ and $z = -1.7$. This tells you that $x = 6$ is ____ standard deviations to the ____ (right or left) of the mean.

31. About what percent of x values from a normal distribution lie within one standard deviation (left and right) of the mean of that distribution?

32. About what percent of the x values from a normal distribution lie within two standard deviations (left and right) of the mean of that distribution?

33. About what percent of x values lie between the second and third standard deviations (both sides)?

34. Suppose $X \sim N(15, 3)$. Between what x values does the middle 68.27% of the data lie?

35. Suppose $X \sim N(-3, 1)$. Between what x values does the middle 95.45% of the data lie?

36. Suppose $X \sim N(-3, 1)$. Between what x values does the middle 34.14% of the data lie?

37. About what percent of x values lie between the mean and three standard deviations?

38. About what percent of x values lie between the mean and one standard deviation?

39. About what percent of x values lie between the first and second standard deviations from the mean (both sides)?

40. About what percent of x values lie between the first and third standard deviations (both sides)?

Use the following information to answer the next two exercises: The life of a particular gaming console is normally distributed with mean of 4.1 years and a standard deviation of 1.3 years. This gaming console is guaranteed for three years. We are interested in the length of time a given gaming console lasts.

41. Define the random variable X in words. $X =$ _______________.

42. $X \sim$ _____(_____,_____)

4.3 Using the Normal Distribution

43. How would you represent the area to the left of 1 in a probability statement?

Figure 4.7.1

44. What is the area to the right of 1?

Figure 4.7.2

45. Is $P(x < 1)$ equal to $P(x \leq 1)$? Why?

46. How would you represent the area to the left of 3 in a probability statement?

Figure 4.7.3

47. What is the area to the right of 3?

Figure 4.7.4

48. If the area to the left of x in a normal distribution is 0.123, what is the area to the right of x?

49. If the area to the right of x in a normal distribution is 0.543, what is the area to the left of x?

Use the following information to answer the next two exercises: $X \sim N(54, 8)$

50. Find the probability that $x > 56$.

51. Find the probability that $x < 30$.

52. $X \sim N(6, 2)$

Find the probability that x is between 3 and 9.

53. $X \sim N(-3, 4)$

Find the probability that x is between 1 and 4.

54. $X \sim N(4, 5)$

Find the maximum of x in the bottom quartile.

55. *Use the following information to answer the next three exercises:* The life of a particular gaming console is normally distributed with mean of 4.1 years and a standard deviation of 1.3 years. This gaming console is guaranteed for three years. We are interested in the length of time a given gaming console lasts. Find the probability that a gaming console will break down during the guarantee period.

1. Sketch the situation. Label and scale the axes. Shade the region corresponding to the probability.

Empty normal distribution curve.

Figure 4.7.5

2. $P(0 < x <$ ____________) = ___________ (Use zero for the minimum value of x.)

56. Find the probability that a gaming console will last between 2.8 and six years.

1. Sketch the situation. Label and scale the axes. Shade the region corresponding to the probability.

Empty normal distribution curve.

Figure 4.7.6

2. $P($__________ $< x <$ __________) = __________

Use the following information to answer the next two exercises: The patient recovery time from a particular surgical procedure is normally distributed with a mean of 5.3 days and a standard deviation of 2.1 days.

65. What is the median recovery time?

a. 2.7
b. 5.3
c. 7.4
d. 2.1

66. What is the *z*-score for a patient who takes ten days to recover?

a. 1.5
b. 0.2
c. 2.2
d. 7.3

67. The length of time to find an open parking space at 9 A.M. follows a normal distribution with a mean of five minutes and a standard deviation of two minutes. If the mean is significantly greater than the standard deviation, which of the following statements is true?

I. The data follows a normal distribution.
II. The data follows a right-skewed distribution.
III. The data follows a left-skewed distribution.

a. I only
b. II only
c. III only
d. I, II, and III

68. The heights of the 430 National Basketball Association players were listed on team rosters at the start of the 2005–2006 season. The heights of basketball players have an approximate normal distribution with mean, $\mu = 79$ inches and a standard deviation, $\sigma = 3.89$ inches. For each of the following heights, calculate the z-score and interpret it using complete sentences.

1. 77 inches
2. 85 inches
3. If an NBA player reported his height had a *z*-score of 3.5, would you believe him? Explain your answer.

69. The systolic blood pressure (given in millimeters) of males has an approximately normal distribution with mean $\mu = 125$ and standard deviation $\sigma = 14$. Systolic blood pressure for males follows a normal distribution.

1. Calculate the z-scores for the male systolic blood pressures 100 and 150 millimeters.
2. If a male friend of yours said he thought his systolic blood pressure was 2.5 standard deviations below the mean, but that he believed his blood pressure was between 100 and 150 millimeters, what would you say to him?

70. Kyle's doctor told him that the *z*-score for his systolic blood pressure is 1.75. Which of the following is the best interpretation of this standardized score? The systolic blood pressure (given in millimeters) of males has an approximately normal distribution with mean $\mu = 125$ and standard deviation $\sigma = 14$.

1. Which answer(s) is/are correct?
   - Kyle's systolic blood pressure is 175.
   - Kyle's systolic blood pressure is 1.75 times the average blood pressure of men his age.
   - Kyle's systolic blood pressure is 1.75 above the average systolic blood pressure of men his age.
   - Kyles's systolic blood pressure is 1.75 standard deviations above the average systolic blood pressure for men.
2. Calculate Kyle's blood pressure.

71. Height and weight are two measurements used to track a child's development. The World Health Organization measures child development by comparing the weights of children who are the same height and the same gender. In 2009, weights for all 80 cm girls in the reference population had a mean $\mu = 10.2$ kg and standard deviation $\sigma = 0.8$ kg. Weights are normally distributed. $X \sim$N(10.2, 0.8). Calculate the *z*-scores that correspond to the following weights and interpret them.

1. 11 kg
2. 7.9 kg
3. 12.2 kg

72. In 2005, 1,475,623 students heading to college took the SAT. The distribution of scores in the math section of the SAT follows a normal distribution with mean $\mu = 520$ and standard deviation $\sigma = 115$.

1. Calculate the *z*-score for an SAT score of 720. Interpret it using a complete sentence.
2. What math SAT score is 1.5 standard deviations above the mean? What can you say about this SAT score?
3. For 2012, the SAT math test had a mean of 514 and standard deviation 117. The ACT math test is an alternate to the SAT and is approximately normally distributed with mean 21 and standard deviation 5.3. If one person took the SAT math test and scored 700 and a second person took the ACT math test and scored 30, who did better with respect to the test they took?

Use the following information to answer the next two exercises: The patient recovery time from a particular surgical procedure is normally distributed with a mean of 5.3 days and a standard deviation of 2.1 days.

73. What is the probability of spending more than two days in recovery?

a. 0.0582
b. 0.8447
c. 0.0553
d. 0.9418

Use the following information to answer the next three exercises: The length of time it takes to find a parking space at 9 A.M. follows a normal distribution with a mean of five minutes and a standard deviation of two minutes.

74. Based upon the given information and numerically justified, would you be surprised if it took less than 1 minute to find a parking space?

a. Yes
b. No
c. Unable to determine

75. Find the probability that it takes at least 8 minutes to find a parking space.

a. 0.0001
b. 0.9270
c. 0.1862

d. 0.0668

76. Seventy percent of the time, it takes more than how many minutes to find a parking space?

a. 1.24
b. 2.41
c. 3.95
d. 6.05

77. According to a study done by Gettysburg students, the height for Asian adult males is normally distributed with an average of 66 inches and a standard deviation of 2.5 inches. Suppose one Asian adult male is randomly chosen. Let $X =$ height of the individual.

1. $X \sim$ _____(_____,_____)
2. Find the probability that the person is between 65 and 69 inches. Include a sketch of the graph, and write a probability statement.
3. Would you expect to meet many Asian adult males over 72 inches? Explain why or why not, and justify your answer numerically.
4. The middle 40% of heights fall between what two values? Sketch the graph, and write the probability statement.

78. IQ is normally distributed with a mean of 100 and a standard deviation of 15. Suppose one individual is randomly chosen. Let $X =$ IQ of an individual.

1. $X \sim$ _____(_____,_____)
2. Find the probability that the person has an IQ greater than 120. Include a sketch of the graph, and write a probability statement.
3. MENSA is an organization whose members have the top 2% of all IQs. Find the minimum IQ needed to qualify for the MENSA organization. Sketch the graph, and write the probability statement.

79. The number of advertisements that a person in America views each day is normally distributed with a mean of about 36 and a standard deviation of 10. Suppose that one individual is randomly chosen. Let $X =$ number of advertisements seen per day.

1. $X \sim$ _____(_____,_____)
2. Find the probability that the number of advertisements seen per day is more than 40. Graph the situation. Shade in the area to be determined.
3. Find the maximum number for the lower quarter of number of advertisements seen per day. Sketch the graph and write the probability statement.

80. Suppose that the distance of fly balls hit to the outfield (in baseball) is normally distributed with a mean of 250 feet and a standard deviation of 50 feet.

1. If $X =$ distance in feet for a fly ball, then $X \sim$ _____(_____,_____)
2. If one fly ball is randomly chosen from this distribution, what is the probability that this ball traveled fewer than 220 feet? Sketch the graph. Scale the horizontal axis X. Shade the region corresponding to the probability. Find the probability.

81. Four-year-olds average three hours a day of screen time (e.g., on electronic devices). Suppose that the standard deviation is 1.5 hours and the amount of screen time is normally distributed. We randomly select one four-year-old. We are interested in the amount of screen time the child experiences per day.

1. In words, define the random variable X.
2. $X \sim$ _____(_____,_____)
3. Find the probability that the child has less than one hour of screen time per day. Sketch the graph, and write the probability statement.
4. What percent of the children have over ten hours of screen time per day?
5. Seventy percent of the children have at least how much screen time per day?

82. In the 1992 presidential election, Alaska's 40 election districts averaged 1,956.8 votes per district for President Clinton. The standard deviation was 572.3. (There are only 40 election districts in Alaska.) The distribution of the votes per district for President Clinton was bell-shaped. Let $X =$ number of votes for President Clinton for an election district.

1. State the approximate distribution of X.
2. Is 1,956.8 a population mean or a sample mean? How do you know?

3. Find the probability that a randomly selected district had fewer than 1,600 votes for President Clinton. Sketch the graph and write the probability statement.
4. Find the probability that a randomly selected district had between 1,800 and 2,000 votes for President Clinton.
5. Find the third quartile for votes for President Clinton.

83. Suppose that the duration of a particular type of criminal trial is known to be normally distributed with a mean of 21 days and a standard deviation of seven days.

1. In words, define the random variable X.
2. $X \sim$ _____(_____,_____)
3. If one of the trials is randomly chosen, find the probability that it lasted at least 24 days. Sketch the graph and write the probability statement.
4. Sixty percent of all trials of this type are completed within how many days?

84. Terri Vogel, an amateur motorcycle racer, averages 129.71 seconds per 2.5 mile lap (in a seven-lap race) with a standard deviation of 2.28 seconds. The distribution of her race times is normally distributed. We are interested in one of her randomly selected laps.

1. In words, define the random variable X.
2. $X \sim$ _____(_____,_____)
3. Find the percent of her laps that are completed in less than 130 seconds.
4. The fastest 3% of her laps are under _____.
5. The middle 80% of her laps are from _______ seconds to _______ seconds.

85. Thuy Dau, Ngoc Bui, Sam Su, and Lan Voung conducted a survey as to how long customers at Lucky claimed to wait in the checkout line until their turn. Let $X =$ time in line. Table $4.7.1$ displays the ordered real data (in minutes):

1	5	5	6	7
4	5	6	7	11

Table $4.7.1$

1. Calculate the sample mean and the sample standard deviation.
2. Construct a histogram.
3. Draw a smooth curve through the midpoints of the tops of the bars.
4. In words, describe the shape of your histogram and smooth curve.
5. Let the sample mean approximate μ and the sample standard deviation approximate σ. The distribution of X can then be approximated by $X \sim$ _____(_____,_____)
6. Use the distribution in part e to calculate the probability that a person will wait fewer than 6 minutes.
7. Determine the cumulative relative frequency for waiting less than 6 minutes.
8. Why aren't the answers to part 6 and part 7 exactly the same?
9. Why are the answers to part 6 and part 7 as close as they are?
10. If only 5 customers has been surveyed rather than 10, do you think the answers to part f and part g would have been closer together or farther apart? Explain your conclusion.

86. Suppose that Ricardo and Anita attend different colleges. Ricardo's GPA is the same as the average GPA at his school. Anita's GPA is 0.70 standard deviations above her school average. In complete sentences, explain why each of the following statements may be false.

1. Ricardo's actual GPA is lower than Anita's actual GPA.
2. Ricardo is not passing because his *z*-score is zero.
3. Anita is in the 70^{th} percentile of students at her college.

87. An expert witness for a paternity lawsuit testifies that the length of a pregnancy is normally distributed with a mean of 280 days and a standard deviation of 13 days. An alleged father was out of the country from 240 to 306 days before the birth of the child, so the pregnancy would have been less than 240 days or more than 306 days long if he was the father. The birth was uncomplicated, and the child needed no medical intervention. What is the probability that he was NOT the father? What is the probability that he could be the father? Calculate the *z*-scores first, and then use those to calculate the probability.

88. A NUMMI assembly line, which has been operating since 1984, has built an average of 6,000 cars and trucks a week. Generally, 10% of the cars were defective coming off the assembly line. Suppose we draw a random sample of $n = 100$ cars. Let X represent the number of defective cars in the sample. What can we say about values of x in regard to the Empirical Rule? Assume a normal distribution for the defective cars in the sample.

89. We flip a coin 100 times ($n = 100$) and note that it only comes up heads 20% ($p = 0.20$) of the time. The mean and standard deviation for the number of times the coin lands on heads is $\mu = 20$ and $\sigma = 4$. Solve the following:

1. There is about a 68% chance that the number of heads will be somewhere between ___ and ___.
2. There is about a ____chance that the number of heads will be somewhere between 12 and 28.
3. There is about a ____ chance that the number of heads will be somewhere between eight and 32.

90. A $1 scratch off lotto ticket will be a winner one out of five times. Out of a shipment of $n = 190$ lotto tickets, find the probability for the lotto tickets that there are

1. somewhere between 34 and 54 prizes.
2. somewhere between 54 and 64 prizes.
3. more than 64 prizes.

91. Facebook provides a variety of statistics on its Web site that detail the growth and popularity of the site.

On average, 28 percent of 18 to 34 year olds check their Facebook profiles before getting out of bed in the morning. Suppose this percentage follows a normal distribution with a standard deviation of five percent.

Use the normal distribution to determine the probability that half or more of a sample of *n* = 32 18-to-34 year olds checked Facebook before getting out of bed this the morning.

92. A hospital has 49 births in a year. It is considered equally likely that a birth be a boy as it is the birth be a girl.

1. What is the mean?
2. What is the standard deviation?
3. Use the normal distribution to find the probability that at least 23 of the 49 births were boys.

93. Historically, a final exam in a course is passed with a probability of 0.9. The exam is given to a group of 70 students.

1. What is the mean?
2. What is the standard deviation?
3. Use the normal distribution to find the probability that at least 60 of the students pass the exam?

94. A tree in an orchard has 200 oranges. Of the oranges, 40 are not ripe. Use the normal distribution to determine the probability a box containing 35 oranges has at most two oranges that are not ripe.

95. In a large city one in ten fire hydrants are in need of repair. If a crew examines 100 fire hydrants in a week, what is the probability they will find nine or fewer fire hydrants that need repair?

96. On an assembly line it is determined 85% of the assembled products have no defects. If one day 50 items are assembled, what is the probability at least 4 and no more than 8 are defective?
