21.3: Compounding Periods

Learning Objectives

- If interest compounds other than annually, how does one calculate PV and FV?

Interest does not always compound annually, as assumed in the problems already presented in this chapter. Sometimes it compounds quarterly, monthly, daily, even continuously. The more frequent the compounding period, the more valuable the bond or other instrument, all else constant. The mathematics remains the same (though a little more difficult when compounding is continuous), but you must be careful about what you plug into the equation for *i* and n. For example, $1,000 invested at 12 percent for a year compounded annually would be worth $1,000 × $(1.12)^1$ = $1,120.00. But that same sum invested for the same term at the same rate of interest but compounded *monthly* would grow to $1,000 × $(1.01)^{12}$ = $1,126.83 because the interest paid each month is capitalized, earning interest at 12 percent. Note that we represent *i* as the interest paid *per period* (.12 interest/12 months in a year = .01) and *n* as the number of periods (12 in a year; 12 × 1 = 12), rather than the number of years. That same sum, and so forth with interest compounded quarterly (4 times a year) would grow to $1,000 × $(1.03)^4$ = $1,125.51. The differences among annual, monthly, and quarterly compounding here are fairly trivial, amounting to less than $7 all told, but are important for bigger sums, higher interest rates, more frequent compounding periods, and longer terms. One million dollars at 4 percent for a year compounded annually comes to $1,000,000 × (1.04) = $1,040,000, while on the same terms compounded quarterly, it produces $1,000,000 × $(1.01)^4$ = $1,040,604.01. (I'll take the latter sum over the former any day and "invest" the surplus in a very nice dinner and concert tickets.) Likewise, $100 at 300 percent interest for 5 years compounded annually becomes 100 × $(4)^5$ = $102,400. Compounded quarterly, that $100 grows to $100 × $(1.75)^{20}$ = $7,257,064.34! A mere $1 at 6 percent compounded annually for 100 years will be worth $1 × $(1.06)^{100}$ = $339.30. The same buck at the same interest compounded monthly swells in a century to $1 × $(1.005)^{1200}$ = $397.44. *This all makes good sense because interest is being received sooner than the end of the year and hence is more valuable because, as we know, money now is better than money later*.

Do a few exercises now to make sure you get it.

EXERCISES

For all questions in this set, interest rates are stated in annual terms, but the interest compounds quarterly (four times a year). Also, assume there are no transaction fees, defaults, etc.

1. On your seventieth birthday, you learn that your grandma, bless her soul, deposited $50.00 for you on the day of your birth in a savings account bearing 5 percent interest. How much is in the account?
2. You won $1 million in the lottery but unfortunately the money is payable in a year and you want to start spending it right away. If interest is at 8 percent, how much can you receive today in exchange for that $1 million in year?
3. As a freshman, you hoped to save $2,500 to "pimp your ride" as a college graduation present to yourself. You put $2,012.98 from your high school graduation haul in the bank at 5 percent interest. Will you meet your goal if you graduate in four years?
4. You've won a scholarship for your senior year worth $1,500, but it is payable only after graduation, a year hence. If interest is at 15 percent, how much is your scholarship worth today?
5. You won the lottery and netted a million bucks, but you need $5 million to buy a little island that you have had your eye on. If interest is at 12 percent, will you be able to buy your island in 30 years, assuming its price is unchanged at that time?

KEY TAKEAWAYS

- Present and future value can be calculated for any compounding period (except continuously) using the same formulas presented in this chapter.
- Care must be taken, however, to ensure that the *i* and *n* terms are adjusted appropriately.