15.9: Problems

Problem 15.9.1

Given

$$\mathbf{A} = \begin{pmatrix} 2 & 3 & -1 \\ -5 & 0 & 6 \\ 0 & 2 & 3 \end{pmatrix} ; \mathbf{B} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} ; \mathbf{C} = \begin{pmatrix} 0 & 1 \\ 2 & 0 \\ -1 & 3 \end{pmatrix}$$

Multiply all possible pairs of matrices.

Problem 15.9.2

The matrix representation of a spin $1/2$ system was introduced by Pauli in 1926. The Pauli spin matrices are the matrix representation of the angular momentum operator for a single spin $1/2$ system and are defined as:

$$\sigma_{\mathbf{x}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} ; \sigma_{\mathbf{y}} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} ; \sigma_{\mathbf{z}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

1. Show that $\sigma_{\mathbf{x}}\sigma_{\mathbf{y}} = i\sigma_{\mathbf{z}}$,$\sigma_{\mathbf{y}}\sigma_{\mathbf{z}} = i\sigma_{\mathbf{x}}$ and $\sigma_{\mathbf{z}}\sigma_{\mathbf{x}} = i\sigma_{\mathbf{y}}$
2. Calculate the commutator $[\sigma_{\mathbf{x}}, \sigma_{\mathbf{y}}]$.
3. Show that ${\sigma_{\mathbf{x}}}^2 = {\sigma_{\mathbf{y}}}^2 = {\sigma_{\mathbf{z}}}^2 = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. Hint: as with numbers, the square of a matrix is the matrix multiplied by itself.

Problem 15.9.3

The inversion operator, $\hat{i}$ transforms the point (x, y, z) into $(-x, -y, -z)$. Write down the matrix that corresponds to this operator.

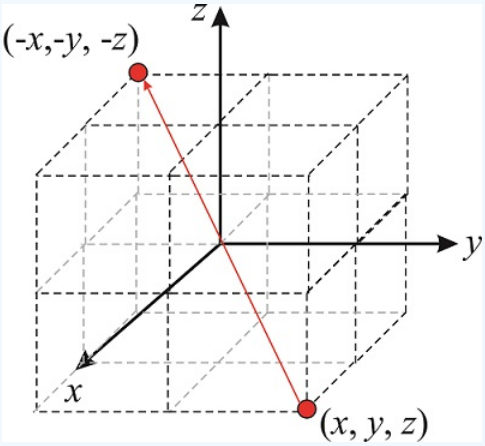

Figure 15.9.1: The inversion operator (CC BY-NC-SA; Marcia Levitus)

Problem 15.9.4

Calculate the inverse of $\mathbf{A}$ by definition.

$$\mathbf{A} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$$

Problem 15.9.5

Calculate the inverse of $\mathbf{A}$ by definition.

$$\mathbf{A} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Problem 15.9.6

Find the eigenvalues and nomalized eigenvectors of

$$\mathbf{M_1} = \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$$

$$\mathbf{M_2} = \begin{pmatrix} 1 & 1+i \\ 1-i & 1 \end{pmatrix}$$

Problem 15.9.7

Given,

$$\mathbf{M_3} = \begin{pmatrix} 1 & 1-i \\ 1+i & 1 \end{pmatrix}$$

1. Show that the matrix is Hermitian.
2. Calculate the eigenvectors and prove they are orthogonal.