9.4: A Mathematical Toolbox

In Chapter 8 we learned some important properties of partial derivatives, and in this chapter we learned about exact and inexact differentials. We saw many examples where these properties can be used to create relationships between thermodynamic variables. Usually we will try to calculate what we want from information we have (which is usually the information we can access experimentally). We just saw how we can calculate a change in entropy from quantities that are easy to measure in the lab: volume, temperature and pressure. In Chapter 8 we saw how we can get an expression of a partial derivative from partial derivatives that are much easier to calculate.

I will summarize some of these mathematical relationships, and call it our "toolbox". The more comfortable you get using these relationships, the easier it will be for you to derive the thermodynamic relationships you will come across in your advanced physical chemistry courses.

1. The Euler reciprocity rule (Equation $8.1.1$): $\left(\frac{\partial^2 f}{\partial x \partial y}\right)=\left(\frac{\partial^2 f}{\partial y \partial x}\right)$
2. The inverse rule (Equation $8.1.2$): $\left(\frac{\partial y}{\partial x}\right)=\frac{1}{\left(\frac{\partial x}{\partial y}\right)}$
3. The cycle rule (Equation $8.1.3$) $\left(\frac{\partial y}{\partial x}\right)_z\left(\frac{\partial x}{\partial z}\right)_y\left(\frac{\partial z}{\partial y}\right)_x=-1$
4. The chain rule (Equations $8.3.4$ and $8.3.5$):
   $$\left(\frac{\partial u}{\partial r}\right)_\theta=\left(\frac{\partial u}{\partial x}\right)_y\left(\frac{\partial x}{\partial r}\right)_\theta+\left(\frac{\partial u}{\partial y}\right)_x\left(\frac{\partial y}{\partial r}\right)_\theta$$
   $$\left(\frac{\partial u}{\partial \theta}\right)_r=\left(\frac{\partial u}{\partial x}\right)_y\left(\frac{\partial x}{\partial \theta}\right)_r+\left(\frac{\partial u}{\partial y}\right)_x\left(\frac{\partial y}{\partial \theta}\right)_r$$
5. The definition of the total differential (Equation $9.1.7$): $du=\left(\frac{\partial u}{\partial x_1}\right)_{x_2 \ldots x_n} dx_1+\left(\frac{\partial u}{\partial x_2}\right)_{x_1, x_3 \ldots x_n} dx_2+\ldots+\left(\frac{\partial u}{\partial x_n}\right)_{x_1 \ldots x_{n-1}} dx_n$
6. The concept of exact differential (Section 9.2)

This page titled 9.4: A Mathematical Toolbox is shared under a CC BY-NC-SA 4.0 license and was authored, remixed, and/or curated by Marcia Levitus via source content that was edited to the style and standards of the LibreTexts platform.