5.5: Problems

Problem *[Math Processing Error]*

Solve the following initial value problems:

1. *[Math Processing Error]*
2. *[Math Processing Error]*
3. *[Math Processing Error]*
4. *[Math Processing Error]*

Problem *[Math Processing Error]*

The simple harmonic oscillator consists of a body moving in a straight line under the influence of a force whose magnitude is proportional to the displacement *[Math Processing Error]* of the body from the point of equilibrium, and whose direction is towards this point. *[Math Processing Error]* The force acts in the direction opposite to that of the displacement. The constant *[Math Processing Error]* is a measure of how hard or soft the spring is.

Newton's law of motion states that the force applied on an object equals its mass multiplied by its acceleration. The variable *[Math Processing Error]* represents the displacement of the spring from its undistorted length, and the acceleration is the second derivative of the displacement. Therefore: *[Math Processing Error]*

Combining equations *[Math Processing Error]* and *[Math Processing Error]* we obtain: *[Math Processing Error]*

which is a second order differential equation. Notice that *[Math Processing Error]* (the mass of the body) and *[Math Processing Error]* (the spring constant) are not functions of *[Math Processing Error]*.

Assume that the displacement *[Math Processing Error]* and the velocity *[Math Processing Error]* at time *[Math Processing Error]* are: *[Math Processing Error]* and *[Math Processing Error]*. Physically, this means that the displacement at time zero is *[Math Processing Error]*, and the body is at rest.

[Math Processing Error] Obtain an expression for *[Math Processing Error]*.

[Math Processing Error] What is the period of the function you found above?

In the example above we assumed that the forces due to friction were negligible. If the oscillator moves in a viscous medium, we need to include a frictional term in Newton's equation. The force due to friction is proportional to the velocity of the mass (*[Math Processing Error]*), and the direction is opposite to the displacement. Therefore:

[Math Processing Error]

where *[Math Processing Error]* is a constant that depends on the viscosity of the medium.

[Math Processing Error] Obtain an expression for *[Math Processing Error]*. You will have to consider the cases *[Math Processing Error]*, *[Math Processing Error]* and *[Math Processing Error]* separately. The answers are printed below so you can check your results. Be sure you show all your work step by step.

- *[Math Processing Error]*:

  [Math Processing Error]

- *[Math Processing Error]*:

  [Math Processing Error]

- *[Math Processing Error]*:

  [Math Processing Error]

Problem *[Math Processing Error]*

Find the eigenfunctions (*[Math Processing Error]*) and eigenvalues *[Math Processing Error]* of the following boundary value problems:

- *[Math Processing Error]*, *[Math Processing Error]*
- *[Math Processing Error]*, *[Math Processing Error]*
