4.4: Problems

Problem [Math Processing Error]

In each case,

- Identify the dependent and independent variables.
- Determine if the differential equation is separable.
- Determine if the differential equation is linear.
- Find the general solution.
- Find the particular solution using the given initial condition.
- Verify that your solution satisfies the differential equation by substitution.

1. [Math Processing Error]
2. [Math Processing Error]
3. [Math Processing Error]
4. [Math Processing Error]
5. [Math Processing Error]

Problem [Math Processing Error]

Consider the reaction [Math Processing Error]. The rate of disappearance of A is proportional to the concentration of A, so:

[Math Processing Error]

1) Obtain [A][Math Processing Error] and [B][Math Processing Error].

2) Using the definition of half-life [Math Processing Error], obtain an expression for [Math Processing Error] for this mechanism. Your result will be a function of [Math Processing Error].

3) Sketch [Math Processing Error] and [Math Processing Error] for the case [Math Processing Error], [Math Processing Error] and [Math Processing Error]. Remember that you are expected to do this without the help of a calculator.

Problem [Math Processing Error]

Consider the reaction [Math Processing Error]. This mechanism is called a bi-molecular reaction, because the reaction involves the collision of two molecules of reactant. In this case, the rate of disappearance of A is proportional to the square of the concentration of A, so:

[Math Processing Error]

Notice that the rate is proportional to the square of the concentration, so this is a second-order reaction.
Assume that the initial concentration of [A] is [A][Math Processing Error], and the initial concentration of [B] is zero.

1. Obtain an expression for [A][Math Processing Error].
2. Write down a mass balance (a relationship relating [A](t), [B](t), [A][Math Processing Error] and [B][Math Processing Error]) and obtain [B][Math Processing Error].
3. Using the definition of half-life [Math Processing Error], obtain an expression for [Math Processing Error] for this mechanism. Your result will be a function of [Math Processing Error] and [A][Math Processing Error].

Problem [Math Processing Error]

Obtain [Math Processing Error] and[Math Processing Error] for the following mechanism:

[Math Processing Error]

Assume [Math Processing Error], and [Math Processing Error]

Note that this problem is identical to the one solved in Section 4.2 but with [Math Processing Error]. Be sure you identify the step where the two problems become different.

Problem [Math Processing Error]

Consider the reaction

[Math Processing Error]

modeled mathematically by the following ODE

[Math Processing Error]

The constants, [Math Processing Error] and [Math Processing Error] represent the kinetic constants in the forward and backward direction respectively, and [A] and [B] represent the molar concentration of A and B. Assume you start with initial concentrations [Math Processing Error] and [Math Processing Error].

Mass conservation requires that [Math Processing Error]

1. Obtain [A](t) and [B](t) in terms of [Math Processing Error] and [Math Processing Error].
2. Obtain expressions for the concentrations of A and B in equilibrium: [Math Processing Error]) and [Math Processing Error]).
3. Prove that the equilibrium constant of the reaction, [Math Processing Error], can be expressed as [Math Processing Error]
4. Assume that [Math Processing Error], [Math Processing Error], [Math Processing Error] and [Math Processing Error], calculate [Math Processing Error] and [Math Processing Error], and sketch [Math Processing Error] and [Math Processing Error] to the best of your abilities.