6.1: Introduction to Power Series Solutions of Differential Equations

In Chapter 5 we discussed a method to solve linear homogeneous second order differential equations with constant coefficients. Many important differential equations in physical chemistry are second order homogeneous linear differential equations, but do not have constant coefficients. The following examples are all important differential equations in the physical sciences:

- Hermite equation: *[Math Processing Error]*
- Laguerre equation: *[Math Processing Error]*
- Legendre equation: *[Math Processing Error]*

These equations do not have constant coefficients because some of the terms multiplying *[Math Processing Error]* and *[Math Processing Error]* are functions of *[Math Processing Error]*. In order to solve these differential equations, we will assume that the solution, *[Math Processing Error]*, can be expressed as a Maclaurin series:

[Math Processing Error]

This method will give us a series as the solution, but at this point we know that an infinite series is one way of representing a function, so we will not be too surprised. For example, instead of obtaining *[Math Processing Error]* as the solution, we will get the series *[Math Processing Error]*, which of course represents the same thing. Does it mean that we need to know all the series to be able to recognize which function is represented by the series we got as the answer? Not really. We will see that this method is useful when the solution can be expressed only as a series, but not as a known function. Even if this is the case, for simplicity we will see how the method works with a problem whose solution is a known function. We will then move to a problem whose solution can be expressed as a series only.