2.1: Algebra with Complex Numbers

The imaginary unit *[Math Processing Error]* is defined as the square root of -1: *[Math Processing Error]*. If *[Math Processing Error]* and *[Math Processing Error]* are real numbers, then the number *[Math Processing Error]* is said to be complex. The real number *[Math Processing Error]* is the real part of the complex number *[Math Processing Error]*, and the real number *[Math Processing Error]* is its imaginary part. If *[Math Processing Error]*, then the number is pure imaginary. All the rules of ordinary arithmetic apply with complex numbers, you just need to remember that *[Math Processing Error]* For example, if *[Math Processing Error]* and *[Math Processing Error]*:

- *[Math Processing Error]*
- *[Math Processing Error]*
- *[Math Processing Error]*
- *[Math Processing Error]* (remember that *[Math Processing Error]*!)
- *[Math Processing Error]*

In order to divide complex numbers we will introduce the concept of complex conjugate.

Complex Conjugate

The complex conjugate of a complex number is defined as the number that has the same real part and an imaginary part which is the negative of the original number. It is usually denoted with a star: If *[Math Processing Error]*, then *[Math Processing Error]*

For example, the complex conjugate of *[Math Processing Error]* is *[Math Processing Error]*. Notice that the product *[Math Processing Error]* is always real:

[Math Processing Error]

We'll use this result in a minute. For now, let's see how the complex conjugate allows us to divide complex numbers with an example:

Example *[Math Processing Error]*: Complex Division

Given *[Math Processing Error]* and *[Math Processing Error]* obtain *[Math Processing Error]*

Solution

[Math Processing Error]

Multiply the numerator and denominator by the complex conjugate of the denominator:

[Math Processing Error]

This "trick" ensures that the denominator is a real number, since *[Math Processing Error]* is always real. In this case,

[Math Processing Error]

The numerator is

[Math Processing Error]

Therefore,

[Math Processing Error]

Example *[Math Processing Error]*

Calculate *[Math Processing Error]* and express your result in cartesian form (*[Math Processing Error]*)

Solution

[Math Processing Error]

The concept of complex conjugate is also useful to calculate the real and imaginary part of a complex number. Given *[Math Processing Error]* and *[Math Processing Error]*, it is easy to see that *[Math Processing Error]* and *[Math Processing Error]*. Therefore:

[Math Processing Error]

and

[Math Processing Error]

You may wonder what is so hard about finding the real and imaginary parts of a complex number by visual inspection. It is certainly not a problem if the number is expressed as *[Math Processing Error]*, but it can be more difficult when dealing with more complicated expressions.

algebra with complex numbers

- Dividing complex numbers: http://tinyurl.com/lkhztm5

External Links:

- Multiplying and dividing complex numbers: http://www.youtube.com/watch?v= KPSj4-76eEc
- Dividing complex numbers: http://www.youtube.com/watch?v=9I4QsSV1XDg

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