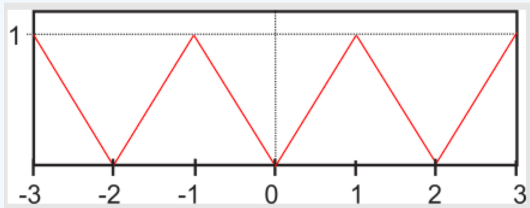
7.4: Problems

Note: You will use some of these results in Chapter 12. Keep a copy of your work handy so you can use it again when needed.

Problem 7.4.1

Consider the following periodic function:



- Is the function odd, even or neither?
- Calculate all the coefficients of the Fourier series of the function by hand (i.e. not in *Mathematica*). Express the function as a Fourier series.
- In the lab: Use the Manipulate function in *Mathematica* to plot the Fourier series. Observe how the finite sum gets closer to the actual triangular wave as you increase the upper bound of the sum.

Problem 7.4.2

Consider the periodic function formed by the periodic extension of:

$$f(x)=\begin{cases}-1/2 & -1\leq x\leq 0\\ 1/2 & 0<x\leq 1\end{cases}$$

- Is the function odd, even or neither?
- Calculate all the coefficients of the Fourier series of the function by hand (i.e. not in *Mathematica*). Express the function as a Fourier series.
- In the lab: Use the Manipulate function in *Mathematica* to plot the Fourier series.

Observe how the finite sum gets closer to the actual triangular wave as you increase the upper bound of the sum.

Problem 7.4.3

The following functions are encountered in quantum mechanics:

$\Phi_m(\phi)=\frac{1}{\sqrt{2\pi}}e^{im\phi},\ m=0,\pm1,\pm2,\pm3...\ and\ 0\leq\phi\leq 2\pi$

Prove that these functions are all normalized, and that any two functions of the set are mutually orthogonal.

Hint: Consider the cases $m=0$ and $m\neq0$ separately, and remember that $e^{im\phi}=1$ when $m=0$. Don't forget to take into account the complex conjugate in the normalization condition!

Hint 2: Check Chapter 2. You may have already solved this problem before!