3.5: Problems

Problem 3.5.1

Expand the following functions around the value of x indicated in each case.

In each case, write down at least four terms of the series, and write down the result as an infinite sum.

- $\sin(ax)$, $x = 0$, a is a constant
- $\cos(ax)$, $x = 0$, a is a constant
- e^{ax}, $x = 0$, a is a real constant
- e^{-ax}, $x = 0$, a is a real constant
- $\ln(ax)$, $x = 1$, a is a real constant

Problem 3.5.2

Use the results of the previous problem to prove Euler's relationship:

$$e^{ix} = \cos x + i \sin x$$

Problem 3.5.3

The osmotic pressure (π) of a solution is given by

$$-RT \ln x_A = \pi V_m$$

where V_m is the molar volume of the pure solvent, and x_a is the mole fraction of the solvent.

Show that in the case of a dilute solution

$$RT x_B \approx \pi V_m$$

where x_B is the mole fraction of the solute. Remember that the mole fractions of the solute and the solvent need to add up to 1.

Note: you may use any of the results you obtained in Problem $3.5.1$.

Problem 3.5.4

The following expression is known as the Butler-Volmer equation, and it is used in electrochemistry to describe the kinetics of an electrochemical reaction controlled solely by the rate of the electrochemical charge transfer process.

$$j = j_0(e^{(1-\alpha)f\eta} - e^{-\alpha f\eta}),\ 0 < \alpha < 1 \text{ and } f > 0, \eta > 0$$

Show that $j \approx j_0 f\eta$ when $f\eta << 1$.

Note: you may use any of the results you obtained in Problem $3.5.1$.

Problem 3.5.5

The energy density of black-body radiation (ρ) at temperature T is given by Plank's formula:

$$\rho(\lambda) = \frac{8\pi hc}{\lambda^5}[e^{hc/\lambda kT} - 1]^{-1}$$

where λ is the wavelength, h is Plank's constant, and c is the speed of light. Show that the formula reduces to the classical Rayleigh-Jeans law $\rho = 8\pi kT/\lambda^4$ for long wavelengths ($\lambda \to \infty$).

Hint: Define a variable $\nu = \lambda^{-1}$ and solve the problem for $\nu \to 0$.

Note: you may use any of the results you obtained in Problem $3.5.1$.

Problem 3.5.6

Use series to prove that $\sum_{k=0}^{\infty} \frac{\lambda^k e^{-\lambda}}{k!} = 1$, λ is a positive real constant.

Problem 3.5.7

Write down the equation of a straight line that provides a good approximation of the function e^x at values close to $x = 2$.

Problem 3.5.8

Use a Taylor expansion around a to prove that $\ln x = \ln a + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n a^n} (x-a)^n$

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