10.5: Problems

Problem 10.5.1

The wave function describing the state of an electron in the 1s orbital of the hydrogen atom is:

$$\psi_{1s} = Ae^{-r/a_0},$$

where a_0 is Bohr's radius (units of distance), and A is a normalization constant.

1. Calculate A
2. calculate $\langle r \rangle$, the average value of the distance of the electron from the nucleus.
3. The radius of the hydrogen atom is taken as the most probable value of r for the 1s orbital. Calculate the radius of the hydrogen atom.
4. What is the probability that the electron is found at a distance from the nucleus equal to $a_0/2$?
5. What is the probability that the electron is found at a distance from the nucleus less than $a_0/2$?
6. We know that the probability that the electron is found at a distance from the nucleus $0 < r < \infty$ is 1. Using this fact and the result of the previous question, calculate the probability that the electron is found at a distance from the nucleus greater than $a_0/2$.

Hint: $\int x^2 e^{ax} dx = e^{ax} \frac{(2 - 2ax + a^2x^2)}{a^3}$

Note: Be sure you show all the steps!

Problem 10.5.2

The wave function describing the state of an electron in the 2s orbital of the hydrogen atom is:

$$\psi_{2s} = Ae^{-r/2a_0}\left(2 - \frac{r}{a_0}\right)$$

where a_0 is Bohr's radius (units of distance), and A is a normalization constant.

- Calculate A
- Calculate $\langle r \rangle$, the average value of the distance of the electron from the nucleus.

Problem 10.5.3

Calculate the normalization constant of each of the following orbitals:

$$\psi_{2p+1} = A_1 re^{-r/2a_0} \sin\theta e^{i\phi}$$

$$\psi_{2p-1} = A_2 re^{-r/2a_0} \sin\theta e^{-i\phi}$$

[1]The integral in r was solved using the formula sheet

[2]If you find this strange think about a situation where 20 18-year olds gather in a room with 4 60-year olds. The average age in the room is 25, but the most probable age is 18

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