11.3: The Response Function and Energy Absorption

Let's investigate the relationship between the linear response function and the absorption of energy from the external agent—in this case an electromagnetic field. We will relate this to the absorption coefficient $\alpha = \dot{E}/I$ which we have described previously. For this case,

$$H = H_0 - f(t)A = H_0 - \mu \cdot E(t) \tag{11.3.1}$$

This expression gives the energy of the system, so the rate of energy absorption averaged over the nonequilibrium ensemble is described by:

$$\dot{E} = \frac{\partial \overline{H}}{\partial t} = -\frac{\partial f}{\partial t}\overline{A(t)} \tag{11.3.2}$$

We will want to cycle-average this over the oscillating field, so the time-averaged rate of energy absorption is

$$\dot{E} = \frac{1}{T}\int_0^T dt \left[-\frac{\partial f}{\partial t}\overline{A(t)}\right] \tag{11.3.3}$$

$$= \frac{1}{T}\int_0^T dt \frac{\partial f(t)}{\partial t}\left[\langle A\rangle + \int_0^\infty d\tau R(\tau) f(t-\tau)\right] \tag{11.3.4}$$

Here the response function is

$$R(\tau) = -i\langle[\mu(\tau), \mu(0)]\rangle/\hbar. \tag{11.3.5}$$

For a monochromatic electromagnetic field, we can write (and expand)

$$f(t) = E_0 \cos\omega t \tag{11.3.6}$$

$$= \frac{1}{2}\left[E_0 e^{-i\omega t} + E_0^* e^{i\omega t}\right] \tag{11.3.7}$$

which leads to the following for the second term in Equation 11.3.4:

$$\frac{1}{2}\int_0^\infty d\tau R(\tau)\left[E_0 e^{-i\omega(t-\tau)} + E_0^* e^{i\omega(t-\tau)}\right] = \frac{1}{2}\left[E_0 e^{-i\omega t}\chi(\omega) + E_0^* e^{i\omega t}\chi(-\omega)\right] \tag{11.3.8}$$

By differentiating Equation 11.3.7, and using it with Equation 11.3.8in Equation 11.3.4, we have

$$\dot{E} = -\frac{1}{T}\langle A\rangle[f(T) - f(0)] - \frac{1}{4T}\int_0^T dt\left[-i\omega E_0 e^{-i\omega t} + i\omega E_0^* e^{i\omega t}\right]\left[E_0 e^{-i\omega t}\chi(\omega) + E_0^* e^{i\omega t}\chi(-\omega)\right] \tag{11.3.9}$$

We will now cycle-average this expression, setting $T = 2\pi/\omega$. The first term vanishes and the cross terms in second integral vanish, because

$$\frac{1}{T}\int_0^T dt e^{-i\omega t} e^{+i\omega t} = 1 \tag{11.3.10}$$

and

$$\int_0^T dt e^{-i\omega t} e^{-i\omega t} = 0. \tag{11.3.11}$$

The rate of energy absorption from the field is

$$\dot{E} = \frac{i}{4}\omega|E_0|^2[\chi(-\omega) - \chi(\omega)]$$

$$= \frac{\omega}{2}|E_0|^2\chi''(\omega)$$

So, the absorption of energy by the system is related to the imaginary part of the susceptibility. Now, from the intensity of the incident field,

$$I = \frac{c|E_0|^2}{8\pi} \tag{11.3.12}$$

the absorption coefficient is

$$\alpha(\omega) = \frac{\dot{E}}{I} = \frac{4\pi\omega}{c}\chi''(\omega) \tag{11.3.13}$$

Readings

1. McQuarrie, D. A., Statistical Mechanics. Harper & Row: New York, 1976.

Contributors and Attributions

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