# 8.5: Titrations and pH Curves

$K_a$ is an **acid dissociation constant**, also known as the **acid ionization constant**. It describes the likelihood of the compounds and the ions to break apart from each other. As we already know, strong acids completely dissociate, whereas weak acids only partially dissociate. A big $K_a$ value will indicate that you are dealing with a very strong acid and that it will completely dissociate into ions. A small $K_a$ will indicate that you are working with a weak acid and that it will only partially dissociate into ions.

## General Guide to Solving Problems involving $K_a$

Generally, the problem usually gives an initial acid concentration and a $K_a$ value. From there you are expected to know:

1. How to write the $K_a$ formula
2. Set up in an ICE table based on the given information
3. Solve for the concentration value, x.
4. Use x to find the equilibrium concentration.
5. Use the concentration to find pH.

## How to write the $K_a$ formula

The general formula of an acid dissociating into ions is

$$HA_{(aq)} + H_2O_{(l)} \rightleftharpoons H_3O^+_{(aq)} + A^-_{(aq)} \tag{8.5.1}$$

with

- $HA$ is the acid,
- $A^-$ is the conjugate base and
- $H_3O^+$ is the the hydronium ion

By definition, the $K_a$ formula is written as the products of the reaction divided by the reactants of the reaction

$$K_a = \frac{[Products]}{[Reactants]} \tag{8.5.2}$$

Based off of this general template, we plug in our concentrations from the chemical equation. The concentrations on the right side of the arrow are the products and the concentrations on the left side are the reactants. Using this information, we now can plug the concentrations in to form the $K_a$ equation. We then write:

$$K_a = \frac{[H_3O^+][A^-]}{[HA]} \tag{8.5.3}$$

> *The concentration of the hydrogen ion ($[H^+]$) is often used synonymously with the hydrated hydronium ion ($[H_3O^+]$).*

To find a concentration of hydronium ions in solution from a pH, we use the formula:

$$[H_3O^+] = 10^{-pH} \tag{8.5.4}$$

This can be flipped to calculate pH from hydronium concentration:

$$pH = -\log[H_3O^+] \tag{8.5.5}$$

- An acidic solution is one that has an excess of $H_3O^+$ ions compared to $OH^-$ ions.
- An basic (or alkaline) solution is one that has an excess of $OH^-$ ions compared to $H_3O^+$ ions.
- A neutral solution is one that has equal concentrations of $OH^-$ ions and $H_3O^+$ ions.

At 25 °C, we can correlate whether a solution is acidic, basic, or neutral based off of the measured pH of the solutions:

- pH = 7 is neutral
- pH > 7 is basic
- pH < 7 is acidic

However, these relationships are not valid at temperatures outside 25 °C.

> ✔ cid
>
> Calculate the pH of a weak acid solution of 0.2 M HOBr, given:
>
> $$HOBr + H_2O \rightleftharpoons H_3O^+ + OBr^- \tag{8.5.6}$$
>
> $$K_a = 2 \times 10^{-9} \tag{8.5.7}$$
>
> **Solution**
>
> Step 1: The ICE Table
>
> Since we were given the initial concentration of HOBr in the equation, we can plug in that value into the Initial Concentration box of the ICE chart. Considering that no initial concentration values were given for $H_3O^+$ and OBr-, we can assume that none was present initially, and we indicate this by placing a zero in the corresponding boxes. M stands for molarity.
>
> |                           | HOBr   | $H_3O^+$ | OBr⁻ |
> |---------------------------|--------|----------|------|
> | Initial Concentration     | 0.2 M  | 0        | 0    |
> | Change in Concentration   |        |          |      |
> | Equilibrium Concentration |        |          |      |
>
> Because we started off without an initial concentration of $H_3O^+$ and OBr⁻, it has to come from somewhere. In the Change in Concentration box, we add a +x because while we do not know what the numerical value of the concentration is at the moment, we do know that it has to be added and not taken away. In contrast, since we did start off with a numerical value of the initial concentration, we know that it has to be taken away to reach equilibrium. Because of this, we add a -x in the HOBr box.
>
> |                           | HOBr   | $H_3O^+$ | OBr⁻  |
> |---------------------------|--------|----------|-------|
> | Initial Concentration     | 0.2 M  | 0        | 0     |
> | Change in Concentration   | -x M   | +x M     | +x M  |
> | Equilibrium Concentration |        |          |       |
>
> Now its time to add it all together! Go from top to bottom and add the Initial concentration boxes to the Change in concentration boxes to get the Equilibrium concentration.
>
> |                           | HOBr        | $H_3O^+$ | OBr⁻  |
> |---------------------------|-------------|----------|-------|
> | Initial Concentration     | 0.2 M       | 0        | 0     |
> | Change in Concentration   | -x M        | +x M     | +x M  |
> | Equilibrium Concentration | (0.2 - x) M | x M      | x M   |
>
> Step 2: Create the $K_a$ equation using this equation: $K_a = \dfrac{[Products]}{[Reactants]}$
>
> $$K_a = \frac{[H_3O^+][OBr-]}{[HOBr-]}$$
>
> Step 3: Plug in the information we found in the ICE table
>
> $$K_a = \frac{(x)(x)}{(0.2 - x)}$$
>
> Step 4: Set the new equation equal to the given Ka

$$2 \times 10^{-9} = \frac{(x)(x)}{(0.2 - x)} \tag{8.5.8}$$

Step 5: Solve for x

$$x^2 + (2 \times 10^{-9})x - (4 \times 10^{-10}) = 0 \tag{8.5.9}$$

To solve for x, we use the quadratic formula

- $a = 1$
- $b = 2 \times 10^{-9}$
- $c = -4 \times 10^{-10}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \times 10^{-9} \pm \sqrt{(2 \times 10^{-9})^2 - 4(1)(-4 \times 10^{-10})}}{2(1)} \tag{8.5.10}$$

$$x = 2.0 \times 10^{-5} \tag{8.5.11}$$

Step 6: Plug x back into the ICE table to find the concentration

$$x = [H_3O^+] = 2 \times 10^{-5} \ M \tag{8.5.12}$$

Step 7: Use the formula using the concentration to find pH

$$pH = -\log[H_3O^+] = -\log(2 \times 10^{-5}) = -(-4.69) = 4.69 \tag{8.5.13}$$

$$pH = 4.69 \tag{8.5.14}$$

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✔ Example 8.5.2: Concentrated Solution of Acetic Acid (Vineger)

For acetic acid, $HC_2H_3O_2$, the $K_a$ value is $1.8 \times 10^{-5}$. Calculate the concentration of $H_3O^+$ in a 0.3 M solution of $HC_2H_3O_2$.

**Solution**

Step 1: The ICE Table

Since we were given the initial concentration of $HC_2H_3O_2$ in the original equation, we can plug in that value into the Initial Concentration box of the ICE chart. Considering that no initial concentration values were given for $H_3O^+$ and $C_2H_3O_2^-$, we assume that none was present initially, and we indicate this by placing a zero in the corresponding boxes.

| | $HC_2H_3O_2$ | $H_3O^+$ | $C_2H_3O_2$ |
|---|---|---|---|
| Initial Concentration | 0.3 M | 0 | 0 |
| Change in Concentration | | | |
| Equilibrium Concentration | | | |

Because we started off without any initial concentration of $H_3O^+$ and $C_2H_3O_2$-, is has to come from somewhere. For the Change in Concentration box, we add a +x because while we do not know what the numerical value of the concentration is at the moment, we do know that it has to be added and not taken away. In contrast, since we did start off with a numerical value of the initial concentration, we know that it has to be taken away to reach equilibrium. Because of this, we add a -x in the $HC_2H_3O_2$ box.

| | $HC_2H_3O_2$ | $H_3O^+$ | $C_2H_3O_2$ |
|---|---|---|---|
| Initial Concentration | 0.3 M | 0 | 0 |
| Change in Concentration | -x M | +x M | +x M |
| Equilibrium Concentration | | | |

Now its time to add it all together! Go from top to bottom and add the Initial concentration boxes to the Change in concentration boxes to get the Equilibrium concentration.

| | $HC_2H_3O_2$ | $H_3O^+$ | $C_2H_3O_2$ |
|---|---|---|---|
| Initial Concentration | 0.3 M | 0 | 0 |
| Change in Concentration | -x M | +x M | +x M |
| Equilibrium Concentration | (0.3 - x) M | x M | x M |

Step 2: Create the $K_a$ equation using this equation: $K_a = \dfrac{[Products]}{[Reactants]}$

$$K_a = \dfrac{[H_3O^+][C_2H_3O_2]}{[HC_2H_3O_2]}$$

Step 3: Plug in the information we found in the ICE table

$$K_a = \dfrac{(x)(x)}{(0.3-x)} \tag{8.5.15}$$

Step 4: Set the new equation equal to the given $K_a$

$$1.8x10^{-5} = \dfrac{(x)(x)}{(0.3-x)} \tag{8.5.16}$$

Step 5: Solve for x

$$(x^2) + (1.8 \times 10^{-5}x) - (5.4 \times 10^{-6}) \tag{8.5.17}$$

To solve for x, we use the quadratic formula

- $a = 1$
- $b = 1.8 \times 10^{-5}$
- $c = -5.4 \times 10^{-6}$

$$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-1.8 \times 10^{-5} \pm \sqrt{(1.8 \times 10^{-5})^2 - 4(1)(-5.4 \times 10^{-6})}}{2(1)} \tag{8.5.18}$$

$$x = 0.0023 \tag{8.5.19}$$

Step 6: Plug x back into the ICE table to find the concentration

$$x = [H_3O^+] = 0.0023 \ M \tag{8.5.20}$$

✔ Example 8.5.3: Concentrated Solution of Benzoic Acid

Find the equilibrium concentration of $HC_7H_5O_2$ from a 0.43 M solution of Benzoic Acid, $HC_7H_5O_2$.

**Solution**

Given: $K_a$ for $HC_7H_5O_2$= 6.4 x 10$^{-5}$

Step 1: The ICE Table

| | $HC_7H_5O_2$ | $H_3O^+$ | $C_7H_5O_2^-$ |
|---|---|---|---|
| Initial Concentration | 0.43 M | 0 | 0 |
| Change in Concentration | -x | +x | +x |

| | HC$_7$H$_5$O$_2$ | H$_3$O$^+$ | C$_7$H$_5$O$_2^-$ |
|---|---|---|---|
| Equilibrium Concentration | (0.43-x)M | x M | x M |

Step 2: Create the $K_a$ equation using this equation : $K_a = \dfrac{[Products]}{[Reactants]}$

$$K_a = \dfrac{[H_3O^+][C_7H_5O_2-]}{[HC_7H_5O_2]}$$

Step 3: Plug in the information we found in the ICE table

$$K_a = \dfrac{(x)(x)}{(0.43-x)}$$

Step 4: Set the new equation equal to the given Ka

$$6.4x10^{-5} = \dfrac{(x)(x)}{(0.43-x)}$$

Step 5: Solve for x.

x=0.0052

Step 6: Plug x back into the ICE table to find the concentration

[HC$_7$H$_5$O$_2$]= (0.43-x)M

[HC$_7$H$_5$O$_2$]= (0.43-0.0052)M

Answer: [HC$_7$H$_5$O$_2$]= 0.425 M

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✔ Example 8.5.4: Concentrated Solution of Hypochlorous acid

For a 0.2 M solution of Hypochlorous acid, calculate all equilibrium concentrations.

**Solution**

Given: $K_a = 3.5 \times 10^{-8}$

Step 1: The ICE Table

| | HOCl | H$_3$O$^+$ | OCl$^-$ |
|---|---|---|---|
| Initial Concentration | 0.2 M | 0 | 0 |
| Change in Concentration | -x M | +x M | +x M |
| Equilibrium Concentration | (0.2 - x) M | x M | x M |

Step 2: Create the $K_a$ equation using this equation: $K_a = \dfrac{[Products]}{[Reactants]}$

$$K_a = \dfrac{[H_3O^+][OCl-]}{[HOCl-]}$$

Step 3: Plug in the information we found in the ICE table

$$K_a = \dfrac{(x)(x)}{(0.2-x)}$$

Step 4: Set the new equation equal to the given Ka

$$3.5x10^{-8} = \dfrac{(x)(x)}{(0.2-x)}$$

Step 5: Solve for x

$$x = 8.4 \times 10^{-5}$$

Step 6: Plug x back into the ICE table to find the concentration

$$[HOCl] = [(.2)-(8.4 \times 10^{-5})] = .199$$

$$[H_3O+] = 8.4 \times 10^{-5}$$

$$[OCl-] = 8.4 \times 10^{-5}$$

> ✔ **Example 8.5.5: pH**
>
> Calculate the pH from the equilibrium concentrations of $[H_3O^+]$ in Example 8.5.4.
>
> **Solution**
>
> Given:
>
> $$[HOCl] = 0.199$$
>
> $$[H_3O^+] = 8.4 \times 10^{-5}$$
>
> $$[OCl-] = 8.4 \times 10^{-5}$$
>
> Step 1: Use the formula using the concentration of $[H_3O^+]$ to find pH
>
> $$pH = -\log[H3O+] = -\log(8.4x10^{-5}) = 4.08 \tag{8.5.21}$$

## References

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3. van Lubeck, Henk. "Why Not Replace pH and pOH by Just One Real Acidity Grade, AG?." *J. Chem. Educ.* **1999** *76* 892.

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