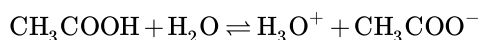# 7.5: Calculating the pH of Weak Acid Solutions

Thus far, we have been discussing problems and answers in equilibria--perhaps the most popular type of problem being how to find the pH of a weak acid solution given a certain concentration of a molecule. However, those problems in particular usually only involve what is called a monoprotic acid. "Mono" in the word "monoprotic" indicates that there is only one ionizeable hydrogen atom in an acid when immersed in water, whereas the concept of Polyprotic Acids and Bases allows for two or more ionizeable hydrogen atoms.

Introduction

Consider the following chemical equation as the molecule acetic acid equilibrates in the solution:

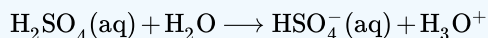$$CH_3COOH + H_2O \rightleftharpoons H_3O^+ + CH_3COO^-$$

Although acetic acid carries a four hydrogen atoms, only a single becomes ionized. Not to get into too much detail between monoprotic and polyprotic acids, but if you desire to find the pH given a concentration of a weak acid (in this case, acetic acid), you would create and complete an ICE Table adjusting for how much acetic acid disassociates. However, if you wish to find the pH of a solution after a **polyprotic acid** disassociates, there are extra steps that would need to be done. Let's first take a look at a unique example:

> ✔ **Example 7.5.1: Finding the pH of sulfuric Acid**
>
> What is the pH of 0.75 M sulfuric acid?
>
> **Solution**
>
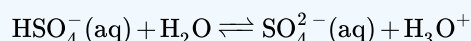> In sulfuric acid (H_2SO_4), there are two ionizable hydrogen atoms. What makes this molecule interesting is that its ionization constant for the first hydrogen (K_{a1}) ionized is significantly larger than is the second ionization constant (K_{a2}). The K_{a1} constant for sulfuric acid is conveniently dubbed "very large" while the K_{a2} constant is 1.1 x 10^{-2}. As such, the sulfuric acid will completely disassociate into HSO_4^- and H_3O^+ ions (as a strong acid).
>
> $$H_2SO_4(aq) + H_2O \longrightarrow HSO_4^-(aq) + H_3O^+$$
>
> Since the sulfuric acid completely disassociates in the solution, we can skip the ICE table process for sulfuric acid, and assert that the concentration HSO_4^- and H_3O^+ are the same as that of H_2SO_4, that is 0.75 M. (This neglects the background concentration of H_3O^+ in water of $1 \times 10^{-7} M$).
>
> Equation:
>
> $$HSO_4^-(aq) + H_2O \rightleftharpoons SO_4^{2-}(aq) + H_3O^+$$
>
> ICE Table:
>
> | | HSO_4^- | SO_4^{2-} | H_3O^+ |
> | --- | --- | --- | --- |
> | **I**nitial | 0.75 M | 0 M | 0.75 M |
> | **C**hange | -x M | +x M | +x M |
> | **E**quilibrium | (0.75 - x) M | +x M | (0.75 + x) M |
>
> $$K_{a2} = \frac{[SO_4^{2-}][H_3O^+]}{[HSO_4^-]} = 1.1 \times 10^{-2} = 0.011 = \frac{x(0.75+x)}{0.75-x}$$
>
> Assume x in the denominator is negligible. Therefore,
>
> $$x = 0.011 M = [SO_4^{2-}]$$
>
> Since we know the value of x, we can use the equation from the ICE table to find the value of [HSO4⁻].
>
> $$[HSO_4^-] = 0.75\ M - x = 0.75 - 0.011 = 0.74\ M$$

We can also find [H₃O⁺] using the equation from the ICE table.

$$[H_3O+] = 0.75 \ M + x = 0.75 + 0.011 = 0.76 \ M$$
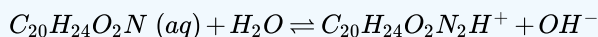
We can then find the pH from the calculated [H₃O⁺] value.

$$pH = -log[H_3O^+] = -log0.76 = 0.119$$

Let's say our task is to find the pH given a polyprotic base which gains protons in water. Thankfully, the process is essentially the same as finding the pH of a polyprotic acid except in this case we deal with the concentration of OH⁻ instead of H₃O⁺.

✔ Example 7.5.2: Finding the pH of a polyprotic base

Let's take a look at how to find the pH of $C_{20}H_{24}O_2N_2$, a diprotic base with a concentration of 0.00162 M, and a K_{b1} of 10^{-6} and a K_{b2} of $10^{-9.8}$.

Equation:

$$C_{20}H_{24}O_2N \ (aq) + H_2O \rightleftharpoons C_{20}H_{24}O_2N_2H^+ + OH^-$$

	$C_{20}H_{24}O_2N_2$	$C_{20}H_{24}O_2N_2H^+$	OH^-
Initial	0.00162 M	0 M	0 M
Change	-x M	+x M	+x M
Equilibrium	0.00162 M	x M	x M

$$K_{b1} = \frac{[C_{20}H_{24}O_2N_2H^+][OH^-]}{[C_{20}H_{24}O_2N_2]} = 10^{-6} = 0.011 = \frac{(x)(x)}{0.00162 - x}$$

Again, assume x in the denominator is negligible. Therefore,

$$0.011 \approx \frac{x^2}{0.00162}$$
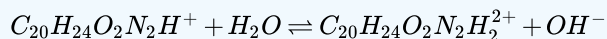
Then,

$$x \approx 4 \times 10^{-5}$$

We can then find the pH.

$$pOH = -log(4 \times 10^{-5}) = 4.4$$
$$pH = 14 - 4.4 = 9.6$$

As we determine the pH of the solution, we realize that the OH⁻ gained using the second ionization constant is so insignificant that it does not impact the final pH value. For good measure, the following is the process to determine the pH in case the second use of the ICE table would indeed make a difference.

Equation:

$$C_{20}H_{24}O_2N_2H^+ + H_2O \rightleftharpoons C_{20}H_{24}O_2N_2H_2^{2+} + OH^-$$

ICE Table:

	$C_{20}H_{24}O_2N_2H^+$	$C_{20}H_{24}O_2N_2H_2^{2+}$	OH^-
Initial	4 x 10⁻⁵ M	0 M	4 x 10⁻⁵ M
Change	-x M	+x M	+x M
Equilibrium	(4 x 10⁻⁵ - x) M	x M	(4 x 10⁻⁵ + x) M

$$K_{b2} = \frac{[C_{20}H_{24}O_2N_2H_2^{2+}][OH^-]}{[C_{20}H_{24}O_2N_2H^+]} = 10^{-9.8}$$

$$10^{-9.8} = \frac{(4 \times 10^{-5} + x)(x)}{(4 \times 10^{-5} - x)}$$

$$10^{-9.8} = \frac{0.00004\, x + x^2}{0.00004 - x}$$

$$10^{-9.8}(0.00004 - x) = 0.00004x + x^2$$

$$x^2 + (4 \times 10^{-5})x - 6.3 \times 10^{-15} = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \times 10^{-5} \pm \sqrt{(4 \times 10^{-5})^2 - 4(1)(6.3 \times 10^{-15})}}{2(1)}$$

$$= 0$$

References

1. Petrucci, et al. General Chemistry: Principles & Modern Applications; Ninth Edition. Pearson/Prentice Hall; Upper Saddle River, New Jersey 07.