8.8: Solubility Equilibria and the Solubility Product

Considering the relation between solubility and K_{sq} is important when describing the solubility of slightly ionic compounds. However, this article discusses ionic compounds that are difficult to dissolve; they are considered "slightly soluble" or "almost insoluble." Solubility product constants (K_{sq}) are given to those solutes, and these constants can be used to find the molar solubility of the compounds that make the solute. This relationship also facilitates finding the K_{sq} of a slightly soluble solute from its solubility.

Introduction

Solubility is the ability of a substance to dissolve. The two participants in the dissolution process are the solute and the solvent. The solute is the substance that is being dissolved, and the solvent is the substance that is doing the dissolving. For example, sugar is a solute and water is a solvent. Solubility is defined as the maximum amount of solute that can be dissolved in a solvent at equilibrium. Equilibrium is the state at which the concentrations of products and reactant are constant after the reaction has taken place.

The solubility product constant (K_{sq}) describes the equilibrium between a solid and its constituent ions in a solution. The value of the constant identifies the degree to which the compound can dissociate in water. For example, the higher the K_{sq}, the more soluble the compound is. K_{sq} is defined in terms of activity rather than concentration because it is a measure of a concentration that depends on certain conditions such as temperature, pressure, and composition. It is influenced by surroundings. K_{sq} is used to describe the saturated solution of ionic compounds. (A saturated solution is in a state of equilibrium between the dissolved, dissociated, undissolved solid, and the ionic compound.)

✔ Example 1: Barium Carbonate

Consider the compound barium carbonate $BaCO_3$ (an ionic compound that is not very soluble):

$$\mathrm{BaCO_3}_{\,(s)} \rightleftharpoons \mathrm{Ba^{2+}(aq)^{+}CO_3^{2-}(aq)}$$

Solution

First, write down the equilibrium constant expression:

$$K_c = \frac{[\mathrm{Ba^{2+}}][\mathrm{CO_3^{2-}}]}{[\mathrm{BaCO_3}]}$$

The activity of solid \(\ce{BaCO$_{3}$}\) is 1, and considering that the concentrations of these ions are small, the activities of the ions are approximated to their molar concentrations. K_{sq} is therefore equal to the product of the ion concentrations:

$$\begin{aligned} K_{sp} &= [\mathrm{Ba^{2+}}][\mathrm{CO_3^{2-}}] \\ &= 5.1 \times 10^{-9} \end{aligned}$$

How are K_{sp} and Solubility Related?

The relation between solubility and the solubility product constants is that one can be used to derive the other. In other words, there is a relationship between the solute's molarity and the solubility of the ions because K_{sq} is the product of the solubility of each ion in moles per liter.

For example, to find the K_{sq} of a slightly soluble compound from its solubility, the solubility of each ion must be converted from mass per volume to moles per liter to find the molarity of each ion. These numbers can then be substituted into the K_{sq} formula, which is the product of the solubility of each ion. An example of this process is given below:

✔ Example 2: Lead Iodide

Suppose the aqueous solubility for compound PbI_2 is 0.54 grams/100 ml at 25 °C and calculate the K_{sq} of PbI_2 at 25°C.

$$PbI_{2(s)} \rightleftharpoons Pb^{2+}_{(aq)} + 2I^{-}_{(aq)}$$

Solution

a. Convert 0.54 grams of PbI_2 to moles:

$$0.54\ grams \times \frac{1\ mol\ PbI_2}{461.0\ grams} = 0.001171\ mol\ PbI_2$$

b. Convert ml to L:

$$\frac{100\ mL}{1000\ L} = 0.100\ L$$

c. Find the molarity:

$$\frac{0.001171\ mol}{0.100\ L} = 0.01171\ M\ PbI_2$$

d. Now find the molarity of each ion by using the stoichiometric ratio (remember there are two I$^-$ ions for each Pb^{2+} ion):

$$[Pb^{2+}] = \frac{0.01171\ M}{1\ L} \times \frac{1\ mol\ Pb}{1\ mol\ PbI_2} \tag{8.8.1}$$
$$= 0.011714\ M\ Pb^{2+} \tag{8.8.2}$$
$$[I^-] = \frac{0.01171\ M}{1\ L} \times \frac{2\ mol\ I^-}{1\ mol\ PbI_2} \tag{8.8.3}$$
$$= 0.23427\ M\ I^- \tag{8.8.4}$$

e. Finally, plug in the molarity to find K_{sq}:

$$K_{sp} = [Pb^{2+}][I^-]^2 \tag{8.8.5}$$
$$= (0.011714\ M)(0.023427\ M)^2 \tag{8.8.6}$$
$$= 6.4 \times 10^{-6} \tag{8.8.7}$$

This relation facilitates solving for the molar solubility of the ionic compounds when the K_{sq} is given to us. The process involves working backwards from K_{sq} to the molarity of the ionic compound.

Example 3

Suppose the K_{sq} at 25 ^{0}C is 8.5 x 10^{-17} for the compound AgI. What is the molar solubility?

$$AgI_{(s)} \rightleftharpoons Ag^+_{(aq)} + I^-_{(aq)}$$

Solution

a. Let "g" represent the number of moles:

$$K_{sp} = [Ag^{2+}][I^-] \tag{8.8.8}$$
$$= g^2 \tag{8.8.9}$$
$$= 8.5 \times 10^{-17} \tag{8.8.10}$$

b. Solve for "g":

$$g^2 = 8.5 \times 10^{-17} \tag{8.8.11}$$
$$g = (8.5 \times 10^{-17})\frac{1}{2} \tag{8.8.12}$$
$$= 9.0 \times 10^{-9} \tag{8.8.13}$$

The molar solubility of AgI is 9.0 x 10^{-9}