2.6 Half-lives and the Rate of Radioactive Decay

Skills to Develop

- To know how to use half-lives to describe the rates of first-order reactions

Radioactive Decay Rates

Radioactivity, or radioactive decay, is the emission of a particle or a photon that results from the spontaneous decomposition of the unstable nucleus of an atom. The rate of radioactive decay is an intrinsic property of each radioactive isotope that is independent of the chemical and physical form of the radioactive isotope. The rate is also independent of temperature. Because there are so many unstable nuclei that decay, we need a method to describe and compare the rates at which these nuclei decay. One approach to describing reaction rates is based on the time required for the number of unstable nuclei to decrease to one-half the initial value. This period of time is called the **half-life** of the process, written as $t_{1/2}$. Thus the half-life of a nuclear decay process is the time required for the number of unstable nuclei to decrease from $[A]_0$ to $1/2[A]_0$.

Number of Half-Lives	Percentage of Reactant Remaining	
1	$\frac{100\%}{2} = 50\%$	$\frac{1}{2}(100\%) = 50\%$
2	$\frac{50\%}{2} = 25\%$	$\frac{1}{2}\left(\frac{1}{2}\right)(100\%) = 25\%$
3	$\frac{25\%}{2} = 12.5\%$	$\frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(100\%) = 12.5\%$
n	$\frac{100\%}{2^n}$	$\left(\frac{1}{2}\right)^n(100\%) = \left(\frac{1}{2}\right)^n\%$

As you can see from this table, the amount of reactant left after n half-lives of a first-order reaction is $(1/2)^n$ times the initial concentration.

> *For a first-order reaction, the concentration of the reactant decreases by a constant with each half-life and is independent of [A].*

For a given number of atoms, isotopes with shorter half-lives decay more rapidly, undergoing a greater number of radioactive decays per unit time than do isotopes with longer half-lives. The half-lives of several isotopes are listed in Table 14.6, along with some of their applications.

Table 2*: Half-Lives and Applications of Some Radioactive Isotopes*

Radioactive Isotope	Half-Life	Typical Uses
hydrogen-3 (tritium)	12.32 yr	biochemical tracer
carbon-11	20.33 min	positron emission tomography (biomedical imaging)
carbon-14	5.70×10^3 yr	dating of artifacts
sodium-24	14.951 h	cardiovascular system tracer
phosphorus-32	14.26 days	biochemical tracer
potassium-40	1.248×10^9 yr	dating of rocks
iron-59	44.495 days	red blood cell lifetime tracer
cobalt-60	5.2712 yr	radiation therapy for cancer
technetium-99*m**	6.006 h	biomedical imaging
iodine-131	8.0207 days	thyroid studies tracer
*The *m* denotes metastable, where an excited state nucleus decays to the ground state of the same isotope.		

Radioactive Isotope	Half-Life	Typical Uses
radium-226	1.600×10^3 yr	radiation therapy for cancer
uranium-238	4.468×10^9 yr	dating of rocks and Earth's crust
americium-241	432.2 yr	smoke detectors
*The *m* denotes metastable, where an excited state nucleus decays to the ground state of the same isotope.		

Note

Radioactive decay is a first-order process.

Example 1

If you have a 120 gram sample of a radioactive element, how many grams of that element will be left after 3 half-lives have passed?

Solution

Given: mass of radioactive sample of an element, number of half-lives

Asked to Solve For: mass of radioactive element after so many half-lives

Solve:

All radioactive samples lose half of their mass after each half-life. Thus, one solution is to calculate the mass after each half-life. (This method only works if you are asked to solve for a whole number of half-lives). Let the passing of time equal to one half-life be represented by and arrow, →. Then the solution is:

120 g → 60 g → 30 g → 15 g

If you want to solve for any number of half-lives, including fractional half-lives, then you use the equation: amount remaining = $\left(\frac{1}{2}\right)^n (amount\ at\ start)$, where n= number of half-lives. Then the solution is:

$$\text{amount remaining} = \left(\frac{1}{2}\right)^3 (120g) = 15g$$

Exercise 1

If you have a 300. gram sample of a radioactive element, how many grams of that element will be left after 4.30 half-lives have passed?

Answer

$$\text{amount left} = \left(\frac{1}{2}\right)^{4.30} (300.g) = 15.2g$$

Exercise 2

A certain radioactive nuclide has a half-life of 5.25 days. If you start with 100. grams of this nuclide, how many grams of the nuclide will be left after 20.0 days?

Answer

$$20.0\ days \times \frac{1\ half-life}{5.25\ days} = 3.81\ half-lives$$

$$\text{amount left} = \left(\frac{1}{2}\right)^{3.81} (100.g) = 7.13g$$

Radioisotope Dating Techniques

In our earlier discussion, we used the half-life of a first-order reaction to calculate how long the reaction had been occurring. Because nuclear decay reactions follow first-order kinetics and have a rate constant that is independent of temperature and the chemical or physical environment, we can perform similar calculations using the half-lives of isotopes to estimate the ages of geological and archaeological artifacts. The techniques that have been developed for this application are known as radioisotope dating techniques.

The most common method for measuring the age of ancient objects is carbon-14 dating. The carbon-14 isotope, created continuously in the upper regions of Earth's atmosphere, reacts with atmospheric oxygen or ozone to form $^{14}CO_2$. As a result, the CO_2 that plants use as a carbon source for synthesizing organic compounds always includes a certain proportion of $^{14}CO_2$ molecules as well as nonradioactive $^{12}CO_2$ and $^{13}CO_2$. Any animal that eats a plant ingests a mixture of organic compounds that contains approximately the same proportions of carbon isotopes as those in the atmosphere. When the animal or plant dies, the carbon-14 nuclei in its tissues decay to nitrogen-14 nuclei by a radioactive process known as beta decay, which releases low-energy electrons (β particles) that can be detected and measured:

$$^{14}\mathrm{C} \rightarrow {}^{14}\mathrm{N} + \beta^{-} \tag{1}$$

The half-life for this reaction is 5700 ± 30 yr.

The $^{14}C/^{12}C$ ratio in living organisms is 1.3×10^{-12}, with a decay rate of 15 dpm/g of carbon. Comparing the disintegrations per minute per gram of carbon from an archaeological sample with those from a recently living sample enables scientists to estimate the age of the artifact, as illustrated in Example 11.Using this method implicitly assumes that the $^{14}CO_2/^{12}CO_2$ ratio in the atmosphere is constant, which is not strictly correct. Other methods, such as tree-ring dating, have been used to calibrate the dates obtained by radiocarbon dating, and all radiocarbon dates reported are now corrected for minor changes in the $^{14}CO_2/^{12}CO_2$ ratio over time.

Summary

- The half-life of a first-order reaction is independent of the concentration of the reactants.
- The half-lives of radioactive isotopes can be used to date objects.

The rate of decay, or activity, of a sample of a radioactive substance is the rate of decrease in the number of radioactive nuclei per unit time. The half-life of a reaction is the time required for the reactant concentration to decrease to one-half its initial value. Radioactive decay reactions are first-order reactions.

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