

7.20: Adiabatic Expansions of An Ideal Gas

Consider an ideal gas that undergoes a reversible adiabatic expansion from an initial state, specified by known values V_1 and T_1 , to a new state in which the value of the volume, V_2 , is known but the value of the temperature, T_2 , is not known. For an adiabatic reversible process, $q = 0$, and $w = \Delta E$. Since $(\partial E / \partial T)_V = C_V$, we have $dE = C_V dT$, so that

$$w = \Delta E = \int_{T_1}^{T_2} C_V dT$$

For any gas, we can assume that C_V is approximately constant over a small temperature range. Taking C_V to be constant in the interval $(T_1 < T_2)$, we have $w = \Delta E = C_V (T_2 - T_1)$. We obtain the enthalpy change from

$$\Delta H = \Delta E + \Delta(PV) = \Delta E + \Delta(RT) = C_V (T_2 - T_1) + R(T_2 - T_1) = C_P (T_2 - T_1)$$

where we use our ideal-gas result from [Section 7.16](#), $C_P = C_V + R$.

While these relationships yield the values of the various thermodynamic quantities in terms of the temperature difference, $T_2 - T_1$, we have yet to find the final temperature, T_2 . To find T_2 , we return to the first law: $dE = dq + dw$. Substituting for dE , dq , and dw , and making use of the ideal gas equation, we have

$$C_V dT = -PdV = -\frac{RT}{V} dV$$

from which, by separation of variables, we have

$$\int_{T_1}^{T_2} C_V \frac{dT}{T} = -R \int_{V_1}^{V_2} \frac{dV}{V} \quad (\text{one mole ideal gas, reversible adiabatic expansion})$$

If we know C_V as a function of temperature, we can integrate to find a relationship among T_1 , T_2 , V_1 , and V_2 . Given any three of these quantities, we can use this relationship to find the fourth. If C_V is independent of temperature, as it is for a monatomic ideal gas, we have

$$\ln \frac{T_2}{T_1} = -\frac{R}{C_V} \ln \frac{V_2}{V_1} = \frac{R}{C_V} \ln \frac{V_1}{V_2} = \ln \left(\frac{V_1}{V_2} \right)^{R/C_V}$$

so that

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{R/C_V}$$

(monatomic ideal gas, reversible adiabatic expansion)

For the spontaneous adiabatic expansion of an ideal gas against a constant applied pressure, we have $dq = 0$, so that $dE = dw$, and $C_V dT = -P_{\text{applied}} dV$. Given the initial conditions, we can find the final temperature from

$$\int_{T_1}^{T_2} C_V dT = \int_{V_1}^{V_2} -P_{\text{applied}} dV = -P_{\text{applied}} \left(\frac{RT_2}{P_{\text{applied}}} - \frac{RT_1}{P_1} \right) = R \left(\frac{P_{\text{applied}} T_1}{P_1} - T_2 \right)$$

(spontaneous adiabatic process)

The changes in the remaining state functions can then be calculated from the relationships above. In this spontaneous adiabatic process, all of the other thermodynamic quantities are different from those of a reversible adiabatic process that reaches the same final volume.

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