6.11: Reversible Motion of A Mass in A Constant Gravitational Field

Let us explore our ideas about reversibility further by considering the familiar case of a bowling ball ball that can move vertically in the effectively constant gravitational field near the surface of the earth.

We begin by observing that we develop our description by abstracting from reality. We consider idealized models because we want to develop theories that capture the most important features of real systems. We ignore less important features. In the present example, we know that the behavior of the bowling ball will be slightly influenced by its frictional interaction with the surrounding atmosphere. (We attribute these interactions to a property of air that we call viscosity.) We assume that this effect can be ignored. This causes no difficulty so long as our experiments are too insensitive to observe the effects of this atmospheric drag. If necessary, of course, we could do our experiments inside a vacuum chamber, so that the system we study experimentally better meets the assumptions we make in our analysis. Alternatively, we could expand our theory to include the effects of atmospheric drag.

To raise an initially stationary bowling ball to a greater height requires that we apply a vertical upward force that exceeds the downward gravitational force on the ball. Let height increase in the upward direction, and let $h\left(t\right)$ and $v\left(t\right)$ be the height and (vertical) velocity of the ball at time t. Let the mass of the ball be m, and let the ball be at rest at time zero. Representing the initial velocity and height as v_0 and h_0, we have $v_0=v\left(0\right)=0$ and $h_0=h\left(0\right)=0$. Letting the gravitational acceleration be g, the gravitational force on the ball is $f_{gravitation}=-mg$. To raise the ball, we must apply a vertical force, $f_{applied}>0$, that makes the net force on the ball greater than zero. That is, we require

$$f_{net}=f_{applied}-mg>0$$

so that

$$m\frac{d^2h}{dt^2}=f_{net}$$

If $f_{applied}$ is constant, f_{net} is constant; we find for the height and velocity of the ball at any later time t,

$$v\left(t\right)=\left(\frac{f_{net}}{m}\right)t$$

and

$$h\left(t\right)=\left(\frac{f_{net}}{m}\right)\frac{t^2}{2}$$

Let us consider the state of the system when the ball reaches a particular height, h_S. Let the corresponding time, velocity, kinetic energy, and potential energy at h_S, be t_S, v_S, τ_S, and υ_S, respectively. Since

$$v_S=\left(\frac{f_{net}}{m}\right)t_S$$

and

$$h_S=\left(\frac{f_{net}}{m}\right)\frac{t^2_S}{2}$$

we have

$$\tau\left(h_S\right)=\frac{mv^2_S}{2}=\frac{m}{2}\left(\frac{f_{net}}{m}\right)^2t^2_S=f_{net}h_S$$

The energy we must supply to move the ball from height zero to h_S is equal to the work done by the surroundings on the ball. The increase in the energy of the ball is $-\hat{w}$. At h_S this input energy is present as the kinetic and potential energy of the ball. We have

$$-\hat{w}=\int^{h_S}_{h=o}f_{applied}dh=\int^{h_S}_{h=0}\left(f_{net}+mg\right)dh=f_{net}h_S+mgh_S=\tau\left(h_S\right)+\upsilon\left(h_S\right)$$

where the kinetic and potential energies are $\tau\left(h_S\right)=f_{net}h_S$ and $\upsilon\left(h_S\right)=mgh_S$, respectively.

The ball rises only if the net upward force is positive: $f_{net}=f_{applied}-mg>0$. Then the ball arrives at h_S with a non-zero velocity and kinetic energy. If we make f_{net} smaller and smaller, it takes the ball longer and longer to reach h_S; when it arrives, its velocity and kinetic energy are smaller and smaller. However, no matter how long it takes the ball to reach h_S, when it arrives, its potential energy is $\upsilon\left(h_S\right)=mgh_S$,

Now, let us consider the energy change in a process in which the ball begins at rest at height zero and ends at rest at h_S. At the end, we have $\tau\left(h_S\right)=0$. To effect this change in a real system, we must apply a net upward force to the ball to get it moving; later we must apply a net downward force to slow the ball in such a way that its velocity becomes zero at exactly the time that it reaches h_S. There are infinitely many ways we could apply forces to meet these conditions. The net change in the ball's energy is the same for all of them.

We find it useful to use a hypothetical process to calculate this energy change. In this hypothetical process, the upward force is always just sufficient to oppose the gravitational force on the ball. That is, $f_{net}=0$ so that $f_{applied}=mg$, and from the development above $v_S=0$ and $\tau\left(h_S\right)=0$. Of course, $t_{\infty}=\infty$. This is a hypothetical process, because the ball would not actually move under these conditions. We see that the hypothetical process is the limiting case in a series of real processes in which we make $f_{net}>0$ smaller and smaller. In all of these processes, the potential energy change is

$$\upsilon\left(h_S\right)=\int_{h=0}^{h_S} mgdh = mgh_S$$

If the ball is stationary and $f_{applied}=mg$, the ball remains at rest, whatever its height. If we make $f_{applied}>mg$, the ball rises. If we make $f_{applied}<mg>$, the ball falls. If $f_{net}\approx 0$ and the ball is moving only slowly in either direction, a very small change in $f_{net}=f_{applied}-mg$ can be enough to reverse the direction of motion. These are the characteristics of a reversible process: an arbitrarily small change in the applied force changes the direction of motion.

The advantage of working with the hypothetical reversible process is that the integral of the applied force over the distance through which it acts is the change in the potential energy of the system. While we cannot actually carry out a reversible process, we can compute the work that must be done if we know the limiting force that is required in order to effect the change. This is true because the velocity and kinetic energy of the ball are zero throughout the process. When the process is reversible, the change in the potential energy of the ball is equal to the work done on the ball; we have

$$-\hat{w}\left(h_S\right)=w\left(h_S\right)=\upsilon\left(h_S\right)$$

Gravitational potential energy is an important factor in some problems of interest in chemistry. Other forms of potential energy are important much more often. Typically, our principal interest is in the potential energy change associated with a change in the chemical composition of a system. We are seldom interested in the kinetic energy associated with the motion of a macroscopic system as a whole. We can include effects that arise from gravitational forces or from the motion of the whole system in our thermodynamic models, but we seldom find a need to do so. For systems in which the motion of the whole system is important, the laws of mechanics are usually sufficient; we find out what we want to know about such systems by solving their equations of motion.

When we discuss the first law of thermodynamics, we write $E=q+w$ (or $dE=dq+dw$) for the energy change that accompanies some physical change in a system. Since chemical applications rarely require that we consider the location of the system or the speed with which it may be moving, "w" usually encompasses only work that changes the energy of the system itself. Then, E designates the energy of the macroscopic system itself. As noted earlier, we often recognize this by calling the energy of the system its ***internal energy***. Some writers use the symbol U to represent the internal energy, intending thereby to make it explicit that the energy under discussion is independent of the system's location and motion.