

19.4: Stirling's Approximation

The polynomial coefficient, C , is a function of the factorials of large numbers. Since $N!$ quickly becomes very large as N increases, it is often impractical to evaluate $N!$ from the definition,

$$N! = (N)(N-1)(N-2)\dots(3)(2)(1)$$

Fortunately, an approximation, known as **Stirling's formula** or **Stirling's approximation** is available. Stirling's approximation is a product of factors. Depending on the application and the required accuracy, one or two of these factors can often be taken as unity. Stirling's approximation is

$$N! \approx N^N (2\pi N)^{1/2} \exp(-N) \exp\left(\frac{1}{12N}\right) \approx N^N (2\pi N)^{1/2} \exp(-N) \approx N^N \exp(-N)$$

In many statistical thermodynamic arguments, the important quantity is the natural logarithm of $N!$ or its derivative, $d \ln N! / dN$. In such cases, the last version of Stirling's approximation is usually adequate, even though it affords a rather poor approximation for $N!$ itself.

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