9.8: The Entropy of the Universe

In Section 9.7, we conclude that the entropy change is positive for any spontaneous change in an isolated system. Since we can consider the universe to be an isolated system, it follows that $\Delta S_{universe} > 0$ for any spontaneous process.

To reach this conclusion by a more detailed argument, let us consider an arbitrary system that is in contact with its surroundings. We can subdivide these surroundings into subsystems. As diagrammed in Figure 8, we define a surroundings subsystem (Surroundings 1) that interacts with the system and a more remote surroundings subsystem (Surroundings 2) that does not. That is, we assume that we can define Surroundings 2 so that it is unaffected by the process. Then we define an augmented system consisting of the original system plus Surroundings 1. The augmented system is isolated from the remote portion of the surroundings, so that the entropy change for the augmented system is positive by the argument in the previous section. Denoting entropy changes for the system, Surroundings 1, Surroundings 2, and the augmented system by ΔS, $\Delta \hat{S}_1$, $\Delta \hat{S}_2$, and $\Delta S_{augmented}$, respectively, we have $\Delta S_{augmented} = \Delta S + \Delta \hat{S}_1 > 0$, and

$\Delta \hat{S} = \Delta \hat{S}_1 + \Delta \hat{S}_2 > 0$. Since the remote portion of the surroundings is unaffected by the change, we have $\Delta \hat{S}_2 = 0$. For any spontaneous change, whether the system is isolated or not, we have

$$\Delta S_{universe} = \Delta S + \Delta \hat{S}_1 + \Delta \hat{S}_2 = \Delta S_{augmented} + \Delta \hat{S}_2 = \Delta S_{augmented} > 0$$

(any spontaneous change)

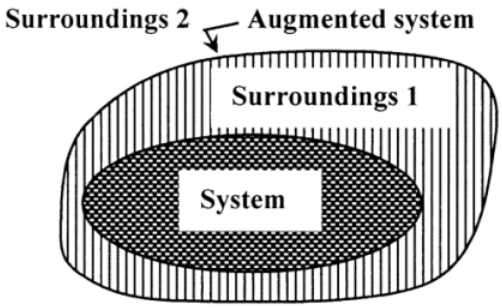

Figure 8. Expanding a system to create a new, augmented, isolated system.

This statement is an essential part of the entropy-based statement of the second law. We have now developed it from the machine-based statement of the second law by convincing, but not entirely rigorous arguments. In Section 9.6 we find that $\Delta S_{universe} = \Delta S + \Delta \hat{S} = 0$ for any reversible process. Thus, for any possible process, we have

$$\Delta S_{universe} = \Delta S + \Delta \hat{S} \geq 0$$

The equality applies when the process is reversible; the inequality applies when it is spontaneous.

Because entropy is a state function, ΔS and $\Delta \hat{S}$ change sign when the direction of a process is reversed. We say that a process for which $\Delta S + \Delta \hat{S} < 0$ is an impossible process. Our definitions mean that these classifications—reversible, spontaneous, and impossible—are exhaustive and mutually exclusive. We conclude that $\Delta S_{universe} = \Delta S + \Delta \hat{S} = 0$ is necessary and sufficient for a process to be reversible; $\Delta S_{universe} = \Delta S + \Delta \hat{S} > 0$ is necessary and sufficient for a process to be spontaneous. (See problem 19.)