26.7: Appendix D. Some Important Definite Integrals

We frequently need the values of the definite integrals below. These values are available in standard tables. Note that integrands involving even powers of the argument are even functions; integrands involving odd powers are odd functions. (A function, $f(x)$, is even if $f(x) = f(-x)$; it is odd if $f(x) = -f(-x)$.) The integrals are given over the interval $0 < x < \infty$. For integrands that are even functions, the integrals over the interval $-\infty < x < \infty$ are twice the integrals over the interval $0 < x < \infty$. For integrands that are odd functions, the integrals over the interval $-\infty < x < \infty$ are zero.

$$\int_0^\infty \exp\left(-ax^2\right) dx = \tfrac{1}{2}\sqrt{\tfrac{\pi}{a}}$$
$$\int_0^\infty x \exp\left(-ax^2\right) dx = \tfrac{1}{2a}$$
$$\int_0^\infty x^2 \exp\left(-ax^2\right) dx = \tfrac{1}{4}\sqrt{\tfrac{\pi}{a^3}}$$
$$\int_0^\infty x^3 \exp\left(-ax^2\right) dx = \tfrac{1}{2a^2}$$
$$\int_0^\infty x^4 \exp\left(-ax^2\right) dx = \tfrac{3}{8}\sqrt{\tfrac{\pi}{a^5}}$$
$$\int_0^\infty x^6 \exp\left(-ax^2\right) dx = \tfrac{15}{16}\sqrt{\tfrac{\pi}{a^7}}$$