4.7: Experimental Test of the Maxwell-Boltzmann Probability Density

There are numerous applications of the Maxwell-Boltzmann equation. These include predictions of collision frequencies, mean-free paths, effusion and diffusion rates, the thermal conductivity of gases, and gas viscosities. These applications are important, but none of them is a direct test of the validity of the Maxwell-Boltzmann equation.

The validity of the equation has been demonstrated directly in experiments in which a gas of metal atoms is produced in an oven at a very high temperature. As sketched in Figure 4, the gas is allowed to escape into a vacuum chamber through a very small hole in the side of the oven. The escaping atoms impinge on one or more metal plates. Narrow slits cut in these plates stop any metal atoms whose flight paths do not pass though the slits. This produces a beambeam:of metal atoms of metal atoms whose velocity distribution is the same as that of the metal-atom gas inside the oven. The rate at which metal atoms arrive at a detector is measured. Various methods are used to translate the atom-arrival rate into a measurement of their speed.

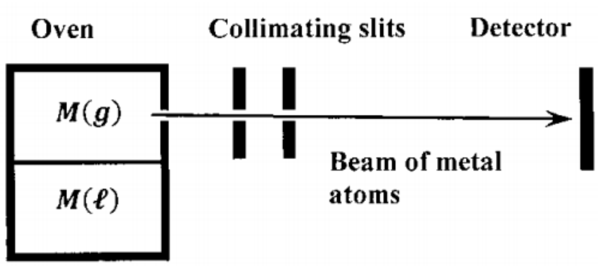

Figure 4. Producing a beam of metal atoms.

One device uses a solid cylindrical drum, which rotates on its cylindrical axis. As sketched in Figure 5, a spiral groove is cut into the cylindrical face of this drum. This groove is cut with a constant pitch. When the drum rotates at a constant rate, an atom traveling at a constant

velocity parallel to the cylindrical axis can traverse the length of the drum while remaining within the groove. That is, for a given rotation rate, there is one critical velocity at which an atom can travel in a straight line while remaining in the middle of the groove all the way from one end of the drum to the other. If the atom moves significantly faster or slower than this critical velocity, it collides with—and sticks to—one side or the other of the groove.

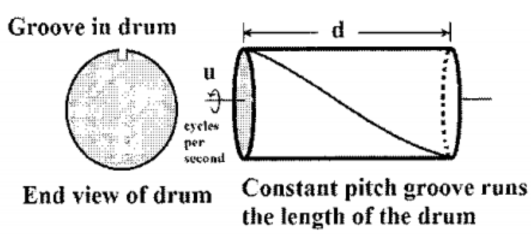

Figure 5. Device to select metal atoms having a specified velocity.

Since the groove has a finite width, atoms whose velocities lie in a narrow range about the critical velocity can traverse the groove without hitting one of the sides.

Let us assume that the groove is cut so that the spiral travels half way around the cylinder. That is, if we project the spiral onto one of the circular faces of the drum, the projection traverses an angle of 180^o on the face. In order to remain in the middle of this groove all the way from one end of the drum to the other, the atom must travel the length of the cylindrical drum in exactly the same time that it takes the drum to make a half-rotation. Let the critical velocity be $v_{critical}$. Then the time required for the atom to traverse the length, d, of the drum is $d/v_{critial}$. If the drum rotates at u cycles/sec, the time required for the drum to make one-half rotation is $1/2u$. Thus, the atom will remain in the middle of the groove all the way through the drum if

$$v_{critial}=2ud$$

By varying the rotation rate, we can vary the critical velocity.

Because the groove has a finite width, atoms whose velocities are in a range \(v_{min}<v_{max}\) can successfully traverse the groove. Whether or not a particular atom can do so depends on its velocity, where it enters the groove, and the width of the groove. Let the width of the groove be w and the radius of the drum be r, where the drum is constructed with $r \gg w$. A slower atom that

enters the groove at the earliest possible time—when the leading edge of the groove first encounters the beam of atoms—can traverse the length of the groove in a longer time, t_{max}. A point on the circumference of the drum travels with speed $2\pi ru$. The slowest atom traverses the length of the drum while a point on the circumference of the drum travels a distance $\pi r+w$. (To intercept the slowest atom, the trailing edge of the groove must travel a distance equal to half the circumference of the drum, πr, plus the width of the groove, w.) The time required for this rotation is the maximum time a particle can take to traverse the length, so

$$t_{max} = \left(\pi r+w\right)/\left(2\pi ru\right)$$

and

$$v_{min} = d/t_{max} = 2\pi rud/\left(\pi r+w\right)$$

A fast atom that enters the groove at the last possible moment—when the trailing edge of the grove just leaves the beam of atoms—can still traverse the groove if it does so in the time, t_{min} that it takes the trailing edge of the groove to travel a distance $\pi r-w$. So,

$$t_{min} = \left(\pi r-w\right)/\left(2\pi ru\right)$$

and

$$v_{max} = d/t_{min} = 2\pi rud/\left(\pi r-w\right)$$

At a given rotation rate, the drum will pass atoms whose speeds are in the range

$$\Delta v = v_{max}-v_{min} = 2ud\left(\frac{\pi r}{\pi r-w}-\frac{\pi r}{\pi r+w}\right) = 2ud\left(\frac{2\pi rw}{\left(\pi r\right)^2-w^2}\right) \approx v_{critical}\left(\frac{2w}{\pi r}\right)$$

So that

$$\frac{\Delta v}{v_{critial}} \approx \frac{2w}{\pi r}$$

The fraction of the incident atoms that successfully traverse the groove is equal to the fraction that have velocities in the interval Δv centered on the critical velocity, $v_{critical} = 2ud$.