7.21: Problems

1. Which of the following differential expressions are exact?

a. $df = ydx + xdy$

b. $df = 2xy^2dx + 2x^2ydy$

c. $df = 2xydx + 2x^2ydy$

d. $df = \left[(1-xy)\,e^{-xy}\right]dx - \left[x^2e^{-xy}\right]dy$

e. $df = (\cos x\,\cos y\,)\,dx - (\sin x \sin y\,)\,dy$

f. $df = (\cos x\,\cos y\,)\,dx - (\sin y\,)\,dy$

2. Show that $df = e^{-y}dx - xe^{-y}dy$ is exact. Find $f\left(x,y\right)$ by integrating the dx term. Find $f\left(x,y\right)$ by integrating the dy term.

3. A marble of mass m is free to move on a surface whose height above the x,y-plane is $h = ax^2 + by^2$.

a. What is the gravitational potential energy of the marble expressed as a function of x and y, $E\left(x,y\right)$?

b. The force experienced by the marble due to gravity is the vector function

$$\overrightarrow{f}\left(x,y\right) = -\nabla E\left(x,y\right) = -\left(\frac{\partial E}{\partial x}\right)_y\overrightarrow{i} - \left(\frac{\partial E}{\partial y}\right)_x\overrightarrow{j}$$

What is $\overrightarrow{f}\left(x,y\right)$ on this surface?

c. What is the differential of E? Is dE exact or inexact?

d. The vector description of a general path, $\{(x,y)\}$, is the position vector, $\overrightarrow{r} = x\overrightarrow{i} + y\overrightarrow{j}$, and so $d\overrightarrow{r} = dx\overrightarrow{i} + dy\overrightarrow{j}$. If we push the marble up the surface from point $(0,0,E\left(0,0\right))$ to point $(2,2,E\left(2,2\right))$ along the path $y = x$, express $d\overrightarrow{r}$ as a vector function of dx.

e. If we push the marble along the path in part d with a force just large enough to overcome the force of gravity, what is the increment of work, dw, associated with an increment of motion, $d\overrightarrow{r}$?

f. How much work must we do if we are to move the marble from $(0,0,E\left(0,0\right))$ to point $(2,2,E\left(2,2\right))$ along the path in part d, using the force in part e? What is the relationship between this amount of work and the change in the energy of the marble during this process?

g. Suppose that we push the marble up the surface from point $(0,0,E\left(0,0\right))$ to point $(2,2,E\left(2,2\right))$ along the path $y = x^2/2$. What is the vector description of this path?

h. How much work must we do if we are to move the marble from point $(0,0,E\left(0,0\right))$ to point $(2,2,E\left(2,2\right))$ along the path in part g using the force in part b? Compare this result to your result in part f. Explain.

4. Consider the plane, $f\left(x,y\right) = 1 - 2x - 3y$. What is df for this surface? Evaluate $\Delta f = f\left(1,1\right) - f\left(-1,-1\right)$ by integrating df along each of the following paths:

a. $y = x$

b. $y = x^3$

c. $y = 1 + x - x^2$

d. $y = \sin\left(\pi x/2\right)$

5. A 2.00 mole sample of a monatomic ideal gas is expanded reversibly and isothermally at 350 K from 5.82 L to 58.20 L. How much work is done on the gas? What are q, ΔE, and ΔH for the gas in this process?

6. A 2.00 mole sample of a monatomic ideal gas is expanded irreversibly from 5.82 L to 58.20 L at a constant applied pressure equal to the final pressure of the gas. The initial and final temperatures are 350 K. How much work is done on the gas? What are q, ΔE, and ΔH for the gas in this process? Compare w, q, ΔE, and ΔH for this process to the corresponding quantities for the process in problem 5. Compare the initial and final states of the gas to the corresponding states in problem 5.

7. A 2.00 mole sample of a monatomic ideal gas is expanded reversibly and adiabatically from 5.82 L to 58.20 L. The initial temperature is 350 K. What is the final temperature? What are the initial and final pressures? How much work is done on the gas? What are q, ΔE, and ΔH for the gas in this process?

8. The equation of state for a "hard-sphere gas" is $P\left(V-nb\right)=nRT$, where n is the number of moles and b is the molar volume of the hard spheres. How much work is done on this gas when n moles of it expand reversibly and isothermally from V_1 to V_2?

9. Strictly speaking, can the spontaneous expansion of a real gas be isothermal? Can it be free? Can it be adiabatic? Can the reversible expansion of a gas be isothermal? Can it be free? Can it be adiabatic?

10. Consider a machine that operates in a cycle and converts heat into a greater amount of work. What would happen to the energy of the universe if this machine could be operated in reverse?

11. Show that the product of pressure and volume has the units of energy.

12. Give a counter-example to prove that each of the following propositions is false:

a. If X is a state function, X is conserved.

b. If X is an extensive quantity that satisfies $X+\hat{X}=0$, X is a state function.

Notes

[1] Since the temperature of the water increases and the process is to be reversible, we must keep the temperature of the thermal reservoir just dT greater than that of the water throughout the process. We can accomplish this by using a quantity of ideal gas as the heat reservoir. By reversibly compressing the ideal gas, we can reversibly deliver the required heat while maintaining the required temperature. We consider this operation further in Section 12.5.