7.4: The Ideal Gas Equation

Learning Outcomes

- State the ideal gas law and identify the variables.
- Use the ideal gas law to solve for an unknown.
- State the combined gas law.
- Simplify the combined gas law for any values that are constant.
- Use the combined gas law to solve for an unknown value.

Individual Gas Laws

Properties of gases such as pressure (P), volume (V), temperature (T), and moles(n) are relatively easy to measure. Unlike with liquids and solids, the particles (molecules or atoms) in a gas phase sample are very far apart from one another. As a result, their behavior is much more predictable because intermolecular forces become insignificant for most samples in the gas phase even over a wide range of conditions. The presence of intermolecular forces in liquid and solid samples makes their behavior harder to predict.

Experiments with gas phase samples over time showed the relationship between pairs of variables (P, V, T, and n) and individual gas laws (equations) show the quantitative relationship between those variables. Avogadro's law tells us that at constant P and T, the volume of a gas is directly proportional to the amount of gas. Boyle's law says that volume is inversely proportional to pressure at constant T and n. Charles' indicates that volume is directly proportional to temperature at constant P and n.

The video below shows a situation where 3 variables, pressure, volume, and amount of substance (moles) are all interrelated: inside our lungs.

As you can see in the video, when the pink balloon on the bottom (the "diaphragm") is pulled down, the balloon inside expands. This expansion causes a decrease in pressure (Boyle's Law). The pressure decrease causes a pressure differential, drawing air in through the straw, an increase in the amount of air (moles). So in your lungs, volume, pressure, and amount of air are all related. But none of the current laws explain the relation between 2 variables. How can this be resolved?

The Ideal Gas Law

These three laws may all be applied at once if we write ($\propto$ means "proportional to"):

$$V \propto n \times \frac{1}{P} \times T \tag{7.4.1}$$

or, introducing a constant of proportionality R,

$$V = R\,\frac{nT}{P} \tag{7.4.2}$$

This is known as the ideal gas law which results from the combination of the individual gas laws. Equation 7.4.2 applies to all gases at low pressures and high temperatures and is a very good approximation under nearly all conditions. The value of R, the **gas constant**, is independent of the kind of gas, the temperature, or the pressure and has a value of $\frac{0.08206\ L\cdot atm}{mol\cdot K}$.

Equation 7.4.2 is usually rearranged by multiplying both sides by P, so that it reads

$$PV = nRT \tag{7.4.3}$$

This is called the ideal gas equation or the ideal gas law. With the ideal gas equation we can convert from volume of a gas to amount of substance (provided that P and T are known). This is very useful since the volume, pressure, and temperature of a gas are easier to measure than mass, and because knowledge of the molar mass is unnecessary.

Note that for any gas law calculations, the temperature must be in units of Kelvin. The relationship between °C and K is K = °C + 273.15.

Example 7.4.1 : MOLES of Gas

Calculate the moles of gas in a 0.100 L sample at a temperature of 300 K and a pressure of 0.987 atm.

Solution

$$PV = nRT \tag{7.4.4}$$

$$(0.987\ atm)(0.100\ L) = n\left(\frac{0.08206\ L\cdot atm}{mol\cdot K}\right)(300\ K) \tag{7.4.5}$$

$$n = 0.00401\ mol \tag{7.4.6}$$

Example 7.4.2 : unit considerations

A sample of benzene (C_6H_6) was heated to 100.°C in an evacuated flask whose volume was 247.2 ml, a sample of benzene vaporized. When the benzene was condensed to a liquid, its mass was found to be 0.616 g. What was the pressure in the flask?

Solution

The problem gives values for temperature, volume, and mass of the sample. Since R has units of \(\left (\frac{L\cdot atm}{mol\cdot K} \right)), we need to have the temperature in units of Kelvin, the volume in liters, and the amount of sample in moles.

Temperature:

K = °C + 273.15

K = 100.°C + 273.15

K = 373 K

Volume: $247.2\ mL\left(\frac{10^{-3}\ L}{1\ mL}\right) = 0.2472\ L$

Moles: $0.616\ g\left(\frac{1\ mol}{78.11\ g}\right) = 7.89\times10^{-3}\ mol$

Now that all of the values are in the correct units, the value for the unknown pressure can be determined.

$$PV = nRT \tag{7.4.7}$$

$$P(0.2472\ L) = (7.89\times10^{-3}\ mol)\left(\frac{0.08206\ L\cdot atm}{mol\cdot K}\right)(373\ K) \tag{7.4.8}$$

$$P = .977 atm \tag{7.4.9}$$

Combined Gas law

While the ideal gas law is useful in solving for a single unknown when the other values are known, the combined gas law is useful when comparing initial and final situations. The ideal gas law can be rearranged to solve for R, the gas constant.

$$R = \frac{PV}{nT}$$

Under the initial conditions, $R = \frac{P_i V_i}{n_i T_i}$ and under final conditions, $R = \frac{P_f V_f}{n_f T_f}$. Since both expressions are equal to R, they are equal to each other.

$$\frac{P_i V_i}{n_i T_i} = \frac{P_f V_f}{n_f T_f}$$

This equation is typically used when one or more of the variables is constant. As a result, that variable is canceled from the equation. For example, the equation $2x^2 = 2y$ can be simplified to $x^2 = y$ since the 2 is on both sides of the equation.

What happens to the combined gas law equation when the initial and final pressures are equal ($P_i = P_f$)? Since they are equal, P_i can replace P_f.

$$\frac{P_i V_i}{n_i T_i} = \frac{P_i V_f}{n_f T_f}$$

which simplifies to

$$\frac{V_i}{n_i T_i} = \frac{V_f}{n_f T_f}$$

If two variables are constant, the equation can be simplified even more. If temperature and volume are constant, then $T_i = T_f$ and $V_i = V_f$. Then,

$$\frac{P_i V_i}{n_i T_i} = \frac{P_f V_i}{n_f T_i}$$

simplifies to

$$\frac{P_i}{n_i} = \frac{P_f}{n_f}$$

Example 7.4.3

Imagine a 1855 L balloon initially at 30°C and 745 mmHg. The balloon rises to an altitude of 23,000 ft and that the pressure and temperature at that altitude were 312 mmHg and −30°C, respectively. To what volume would the balloon have to expand to hold the same amount of hydrogen gas at the higher altitude?

Solution:

Begin by setting up a table of the two sets of conditions (note that some values will need to be converted to different units):

Initial	Final
$P_i = 745 \text{ mmHg} = 0.980 \text{ atm}$	$P_f = 312 \text{ mmHg} = 0.411 \text{ atm}$
$T_i = 30\ ^\circ\text{C} = 303 \text{ K}$	$T_f = -30\ ^\circ\text{C} = 243 \text{ K}$
$V_i = 1855 \text{ L}$	$V_f = ?$

By eliminating the constant property (n) of the gas, the combined gas law is simplified to

$$\frac{P_i V_i}{T_i} = \frac{P_f V_f}{T_f} \tag{7.4.10}$$

By solving the equation for V_f, we get:

$$\frac{P_i V_i}{T_i} = \frac{P_f V_f}{T_f} \tag{7.4.11}$$

$$\frac{0.980\ atm \cdot 1855\ L}{303\ K} = \frac{0.411\ atm \cdot V_f}{243\ K} \tag{7.4.12}$$

$$V_f = 3.55 \times 10^3\ L \tag{7.4.13}$$

Contributors and Attributions

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