1.51: Groupoid

A **groupoid** (G,*) is a set G with a law of composition * mapping of a subset of G x G into G. The properties of a groupoid are:

- if x, y, z ∈ G and if one of the compositions (x*y)*z or x*(y*z) is defined, so is the other and they are equal; (associativity);
- if x, x' and y ∈ G are such that x*y and x'*y are defined and equal, then x = x'; (cancellation property)
- for all x ∈ G there exist elements e_x (left unit of x), e_x' (right unit of x) and x^{-1} ("inverse" of x) such that:
  - $e_x*x = x$
  - $x* e_x' = x$
  - $x^{-1}*x = e_x'$.

From these properties it follows that:

- $x* x^{-1} = e_x$, *i.e.* that that e_x is right unit for x^{-1},
- e_x' is left unit for x^{-1}
- e_x and e_x' are idempotents, *i.e.* $e_x* e_x = e_x$ and $e_x'* e_x' = e_x'$.

The concept of groupoid as defined here was introduced by Brandt (1927). An alternative meaning of groupoid was introduced by Hausmann & Ore (1937) as a set on which binary operations act but neither the identity nor the inversion are included. For this second meaning nowadays the term **magma** is used instead (Bourbaki, 1998).