1.24: Coset

If G is a group, H a subgroup of G, and g an element of G, then

> gH = { gh : h ∈ H } is a **left coset of H** in *G*
> Hg = { hg : h ∈ H } is a **right coset of H** in *G*.

The decomposition of a group into cosets is unique. Left coset and right cosets however in general do not coincide, unless H is a normal subgroup of G.

Any two left cosets are either identical or disjoint: the left cosets form a partition of G, because every element of G belongs to one and only one left coset. In particular the identity is only in one coset, and that coset is H itself; this is also the only coset that is a subgroup. The same holds for right cosets.

All left cosets and all right cosets have the same order (number of elements, or cardinality), equal to the order of H, because H is itself a coset. Furthermore, the number of left cosets is equal to the number of right cosets and is known as the **index** of H in G, written as [G : H] and given by Lagrange's theorem:

> $|G|/|H| = [G : H]$.

Cosets are also sometimes called *associate complexes*.

Example

The coset decomposition of the twin lattice point group with respect to the point group of the individual gives the different possible twin laws. Each element in a coset is a possible twin operation.

Contributors