

## 1.51: Groupoid

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A **groupoid**  $(G, *)$  is a set  $G$  with a law of composition  $*$  mapping of a subset of  $G \times G$  into  $G$ . The properties of a groupoid are:

- if  $x, y, z \in G$  and if one of the compositions  $(x*y)*z$  or  $x*(y*z)$  is defined, so is the other and they are equal; (associativity);
- if  $x, x'$  and  $y \in G$  are such that  $x*y$  and  $x'*y$  are defined and equal, then  $x = x'$ ; (cancellation property)
- for all  $x \in G$  there exist elements  $e_x$  (left unit of  $x$ ),  $e_{x'}$  (right unit of  $x$ ) and  $x^{-1}$  ("inverse" of  $x$ ) such that:
  - $e_x * x = x$
  - $x * e_{x'} = x$
  - $x^{-1} * x = e_{x'}$ .

From these properties it follows that:

- $x * x^{-1} = e_x$ , i.e. that  $e_x$  is right unit for  $x^{-1}$ ,
- $e_{x'}$  is left unit for  $x^{-1}$
- $e_x$  and  $e_{x'}$  are idempotents, i.e.  $e_x * e_x = e_x$  and  $e_{x'} * e_{x'} = e_{x'}$ .

The concept of groupoid as defined here was introduced by Brandt (1927). An alternative meaning of groupoid was introduced by Hausmann & Ore (1937) as a set on which binary operations act but neither the identity nor the inversion are included. For this second meaning nowadays the term **magma** is used instead (Bourbaki, 1998).

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