# 1.32: Direct Lattice

The direct lattice represents the triple periodicity of the ideal infinite perfect periodic structure that can be associated to the structure of a finite real crystal. To express this periodicity one calls *crystal pattern* an object in point space $E^n$ (direct space) that is invariant with respect to three linearly independent translations, $\mathbf{t_1}$, $\mathbf{t_2}$ and $\mathbf{t_3}$. One distinguishes two kinds of lattices, the *vector lattices* and the *point lattices*.

Any translation $\mathbf{t} = u^i\,\mathbf{t_i}$ ($u^i$ arbitrary integers) is also a translation of the pattern and the infinite set of all translation vectors of a crystal pattern is the **vector lattice L** of this crystal pattern.

Given an arbitrary point $P$ in point space, the set of all the points $P_i$ deduced from one of them by a translation $\mathbf{PP_i} = \mathbf{t_i}$ of the vector lattice **L** is called the **point lattice**.

A basis **a**, **b**, **c** of the vector space $\mathbf{V^n}$ is a *crystallographic basis* of the vector lattice **L** if *every* integral linear combination $\mathbf{t} = u\,\mathbf{a} + v\,\mathbf{b} + w\,\mathbf{c}$ is a lattice vector of **L**. It is called a *primitive basis* if *every* lattice vector **t** of **L** may be obtained as an integral linear combination of the basis vectors, **a**, **b**, **c**. Referred to any crystallographic basis the coefficients of each lattice vector are either integral or rational, while in the case of a primitive basis they are integral. *Non-primitive* bases are used conventionally to describe *centered lattices*.

The parallelepiped built on the basis vectors is the *unit cell*. Its volume is given by the triple scalar product, $V = (\mathbf{a}, \mathbf{b}, \mathbf{c})$.

If the basis is primitive, the unit cell is called the *primitive cell*. It contains only one lattice point. If the basis is non-primitive, the unit cell is a *multiple cell* and it contains more than one lattice point. The multiplicity of the cell is given by the ratio of its volume to the volume of a primitive cell.

The generalization of the notion of point and vector lattices to $n$-dimensional space is given in Section 8.1 of *International Tables of Crystallography, Volume A*