

1.51: Groupoid

A **groupoid** $(G, *)$ is a set G with a law of composition $*$ mapping of a subset of $G \times G$ into G . The properties of a groupoid are:

- if $x, y, z \in G$ and if one of the compositions $(x*y)*z$ or $x*(y*z)$ is defined, so is the other and they are equal; (associativity);
- if x, x' and $y \in G$ are such that $x*y$ and $x'*y$ are defined and equal, then $x = x'$; (cancellation property)
- for all $x \in G$ there exist elements e_x (left unit of x), $e_{x'}$ (right unit of x) and x^{-1} ("inverse" of x) such that:
 - $e_x * x = x$
 - $x * e_{x'} = x$
 - $x^{-1} * x = e_{x'}$.

From these properties it follows that:

- $x * x^{-1} = e_{x'}$, *i.e.* that $e_{x'}$ is right unit for x^{-1} ,
- e_x is left unit for x^{-1}
- e_x and $e_{x'}$ are idempotents, *i.e.* $e_x * e_x = e_x$ and $e_{x'} * e_{x'} = e_{x'}$.

The concept of groupoid as defined here was introduced by Brandt (1927). An alternative meaning of groupoid was introduced by Hausmann & Ore (1937) as a set on which binary operations act but neither the identity nor the inversion are included. For this second meaning nowadays the term **magma** is used instead (Bourbaki, 1998).

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