

1.74: Normal subgroup

A subgroup H of a group G is **normal** in G ($H \triangleleft G$) if $gH = Hg$ for any $g \in G$. Equivalently, $H \subset G$ is normal if and only if $gHg^{-1} = H$ for any $g \in G$, i.e., if and only if each conjugacy class of G is either entirely inside H or entirely outside H . This is equivalent to say that H is invariant under all inner automorphisms of G .

The property $gH = Hg$ means that left and right cosets of H in G coincide. From this one sees that the cosets form a group with the operation $g_1H * g_2H = g_1g_2H$ which is called the factor group or **quotient group** of G by H , denoted by G/H .

In the special case that a subgroup H has only two cosets in G (namely H and gH for some g not contained in H), the subgroup H is always normal in G .

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