1.74: Normal subgroup

A subgroup H of a group G is **normal** in G ($H \triangleleft G$) if gH = Hg for any $g \in G$. Equivalently, $H \subset G$ is normal if and only if $gHg^{-1} = H$ for any $g \in G$, i.e., if and only if each conjugacy class of G is either entirely inside H or entirely outside H. This is equivalent to say that H is invariant under all inner automorphisms of G.

The property gH = Hg means that left and rights cosets of H in G coincide. From this one sees that the cosets form a group with the operation $g_1H * g_2H = g_1g_2H$ which is called the factor group or **quotient group** of *G* by *H*, denoted by *G/H*.

In the special case that a subgroup *H* has only two cosets in *G* (namely *H* and *gH* for some *g* not contained in *H*), the subgroup *H* is always normal in *G*.