1.113: Wigner-Seitz cell

The Wigner-Seitz cell is a a polyhedron obtained by connecting a lattice point *P* to all other lattice points and drawing the planes perpendicular to these connecting lines and passing through their midpoints (Figure 1). The polyhedron enclosed by these planes is the Wigner-Seitz cell. This construction is called the *Dirichlet* construction. The cell thus obtained is a primitive cell and it is possible to fill up the whole space by translation of that cell.

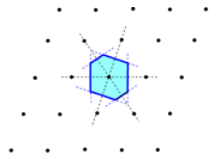

Figure 1 Wigner-Seitz cell.
Shaded: domain of influence.

The Wigner-Seitz cell of a body-centered cubic lattice *I* is a cuboctahedron (Figure 2) and the Wigner-Seitz cell of a face-centered cubic lattice *F* is a rhomb-dodecahedron (Figure 3). In reciprocal space this cell is the first Brillouin zone. Since the reciprocal lattice of body-centered lattice is a face-centered lattice and reciprocally, the first Brillouin zone of a body-centered cubic lattice is a rhomb-dodecahedron and that of a face-centered cubic lattice is a cuboctahedron.

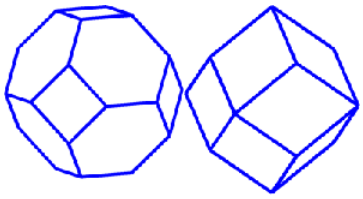

Figure 2. Figure 3.

The inside of the Wigner-Seitz cell has been called domain of influence by Delaunay (1933). It is also called Dirichlet domain or Voronoi domain. The domain of influence of lattice point *P* thus consists of all points *Q* in space that are closer to this lattice point than to any other lattice point or at most equidistant to it (such that $\mathbf{OP} \leq |\mathbf{t} - \mathbf{OP}|$ for any vector $\mathbf{t} \in L$).