

## 1.74: Normal subgroup

A subgroup  $H$  of a group  $G$  is **normal** in  $G$  ( $H \triangleleft G$ ) if  $gH = Hg$  for any  $g \in G$ . Equivalently,  $H \subset G$  is normal if and only if  $gHg^{-1} = H$  for any  $g \in G$ , i.e., if and only if each conjugacy class of  $G$  is either entirely inside  $H$  or entirely outside  $H$ . This is equivalent to say that  $H$  is invariant under all inner automorphisms of  $G$ .

The property  $gH = Hg$  means that left and right cosets of  $H$  in  $G$  coincide. From this one sees that the cosets form a group with the operation  $g_1H * g_2H = g_1g_2H$  which is called the factor group or **quotient group** of  $G$  by  $H$ , denoted by  $G/H$ .

In the special case that a subgroup  $H$  has only two cosets in  $G$  (namely  $H$  and  $gH$  for some  $g$  not contained in  $H$ ), the subgroup  $H$  is always normal in  $G$ .

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