1.5: Aperiodic crystal

A *periodic crystal* is a structure with, ideally, sharp diffraction peaks on the positions of a *reciprocal lattice*. The structure then is invariant under the translations of the *direct lattice*. Periodicity here means *lattice periodicity*. Any structure without this property is *aperiodic*. For example, an amorphous system is aperiodic. An *aperiodic crystal* is a structure with sharp diffraction peaks, but without lattice periodicity. Therefore, amorphous systems are not aperiodic crystals. The positions of the sharp diffraction peaks of an aperiodic crystal belong to a *vector module* of finite rank. This means that the diffraction wave vectors are of the form

$$\mathbf{k} = \sum_{i-1}^{n} h_i \mathbf{a}_i^*, (integer\, h_i)$$

The basis vectors a_i^* are supposed to be independent over the rational numbers, i.e. when a linear combination of them with rational coefficients is zero, all coefficients are zero. The minimum number of basis vectors is the *rank* of the vector module. If the rank *n* is larger than the space dimension, the structure is not periodic, but aperiodic.