1.9: Automorphism

An isomorphism from a group $(G,*)$ to itself is called an **automorphism** of this group. It is a bijection $f: G \to G$ such that

$$f(g) * f(h) = f(g * h)$$

An automorphism preserves the structural properties of a group, e.g.:

- The identity element of G is mapped to itself.
- Subgroups are mapped to subgroups, normal subgroups to normal subgroups.
- Conjugacy classes are mapped to conjugacy classes (the same or another).
- The image $f(g)$ of an element g has the same order as g.

The composition of two automorphisms is again an automorphism, and with composition as binary operation the set of all automorphisms of a group G, denoted by **Aut(*G*)**, forms itself a group, the *automorphism group* of G.