

## 1.92: Semidirect product

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In group theory, a **semidirect product** describes a particular way in which a group can be put together from two subgroups, one of which is normal.

Let  $G$  be a group,  $N$  a normal subgroup of  $G$  (i.e.,  $N \triangleleft G$ ) and  $H$  a subgroup of  $G$ .  $G$  is a **semidirect product** of  $N$  and  $H$  if there exists a homomorphism  $G \rightarrow H$  which is the identity on  $H$  and whose kernel is  $N$ . This is equivalent to say that:

- $G = NH$  and  $N \cap H = \{1\}$  (where "1" is identity element of  $G$  )
- $G = HN$  and  $N \cap H = \{1\}$
- Every element of  $G$  can be written as a unique product of an element of  $N$  and an element of  $H$
- Every element of  $G$  can be written as a unique product of an element of  $H$  and an element of  $N$

One also says that " $G$  splits over  $N$ ".

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