1.92: Semidirect product

In group theory, a **semidirect product** describes a particular way in which a group can be put together from two subgroups, one of which is normal.

Let G be a group, N a normal subgroup of G (i.e., $N \lhd G$) and H a subgroup of G. G is a **semidirect product** of N and H if there exists a homomorphism $G \to H$ which is the identity on H and whose kernel is N. This is equivalent to say that:

- $G = NH$ and $N \cap H = \{1\}$ (where "1" is identity element of G)
- $G = HN$ and $N \cap H = \{1\}$
- Every element of G can be written as a unique product of an element of N and an element of H
- Every element of G can be written as a unique product of an element of H and an element of N

One also says that "G *splits* over N".