1.53: Group isomorphism

A **group isomorphism** is a special type of group homomorphism. It is a mapping between two groups that sets up a one-to-one correspondence between the elements of the groups in a way that respects the respective group operations. If there exists an isomorphism between two groups, then the groups are called **isomorphic**. Isomorphic groups have the same properties and the same structure of their multiplication table.

Let $(G, *)$ and $(H, \#)$ be two groups, where "*" and "#" are the binary operations in G and H, respectively. A *group isomorphism* from $(G, *)$ to $(H, \#)$ is a bijection from G to H, *i.e.* a bijective mapping $f: G \rightarrow H$ such that for all u and v in G one has

$$f(u * v) = f(u) \# f(v).$$

Two groups $(G, *)$ and $(H, \#)$ are isomorphic if an isomorphism between them exists. This is written:

$$(G, *) \cong (H, \#)$$

If $H = G$ and the binary operations # and * coincide, the bijection is an automorphism.