9.26: Twin obliquity

The concept of obliquity was introduced by Friedel in 1920 as a measure of the overlap of the lattices on the individuals forming a twin.

Let us indicate with [*u* ' *v* ' *w* '] the direction exactly perpendicular to a twin plane (*hkl*), and with (*h*' *k* ' *l* ') the plane perpendicular to a twin axis [uvw]. [*u* ' *v* ' *w* '] is parallel to the reciprocal lattice vector [*hkl*]* and (*h* ' *k* ' *l* ') is parallel to the reciprocal lattice plane (*uvw*)*. The angle between [*uvw*] and [*u* ' *v* ' *w* '] or, which is the same, between (*hkl*) and (*h* ' *k* ' *l* '), is called the **obliquity ω**.

The vector in direct space [*uvw*] has length L(*uvw*); the reciprocal lattice vector [*hkl*]* has length L*(*hkl*). The obliquity ω is thus the angle between the vectors [*uvw*] and [*hkl*]*; the scalar product between these two vectors is

$L(uvw)\, L^*(hkl) \cos\omega = \langle uvw|hkl\rangle = uh + vk + wl$

where <| stands for a 1x3 row matrix and |> for a 3x1 column matrix.

It follows that

$\cos\omega = (uh + vk + wl)/L(uvw)L^*(hkl)$

where $L(uvw) = \langle uvw|\mathbf{G}|uvw\rangle^{1/2}$ and $L^*(hkl) = \langle hkl|\mathbf{G}^*|hkl\rangle^{1/2}$, **G** and **G*** being the metric tensors in direct and reciprocal space, respectively.

Notice that $\mathbf{G}^* = \mathbf{G}^{-1}$ (and thus $\mathbf{G} = \mathbf{G}^{*-1}$) and that the matrix representation of the metric tensor is symmetric and coincides thus with its transpose ($\mathbf{G} = \mathbf{G}^T$, $\mathbf{G}^* = \mathbf{G}^{*T}$).