1.50: Group

A set G equipped with a binary operation *: $G \times G \rightarrow G$, assigning to a pair (g,h) the product $g*h$ is called a **group** if:

1. The operation is *associative*, i.e. $(a*b)*c = a*(b*c)$.
2. G contains an *identity element* (*neutral element*) e: $g*e = e*g = g$ for all g in G
3. Every g in G has an *inverse element* h for which $g*h = h*g = e$. The inverse element of g is written asg^{-1}.

Often, the symbol for the binary operation is omitted, the product of the elements g and h is then denoted by the concatenation gh.

The binary operation need not be commutative, i.e. in general one will have $g*h \neq h*g$. In the case that $g*h = h*g$ holds for all g,h in G, the group is an Abelian group.

A group G may have a finite or infinite number of elements. In the first case, the number of elements of G is the **order** of G, in the latter case, G is called an **infinite group**. Examples of infinite groups are space groups and their translation subgroups, whereas point groups are finite groups.