1.10: Binary Operation

A **binary operation** on a set S is a mapping f from the Cartesian product $S \times S$ to S. A mapping from K x S to S, where K need not be S, is called an **external binary operation**.

Many binary operations are commutative (i.e. $f(a,b) = f(b,a)$ holds for all a, b in S) or associative (i.e. $f(f(a,b), c) = f(a, f(b,c))$ holds for all a,b,c in S). Many also have identity elements and inverse elements. Typical examples of binary operations are the addition (+) and multiplication (*) of numbers and matrices as well as composition of functions or symmetry operations.

Examples of binary operations that are not commutative are subtraction (-), division (/), exponentiation(^), super-exponentiation(@), and composition.

Binary operations are often written using infix notation such as $a * b$, $a + b$, or $a \cdot b$ rather than by functional notation of the form $f(a,b)$. Sometimes they are even written just by concatenation: ab.