

## 1.9: Automorphism

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An isomorphism from a group  $(G, *)$  to itself is called an **automorphism** of this group. It is a bijection  $f: G \rightarrow G$  such that

$$f(g) * f(h) = f(g * h)$$

An automorphism preserves the structural properties of a group, e.g.:

- The identity element of  $G$  is mapped to itself.
- Subgroups are mapped to subgroups, normal subgroups to normal subgroups.
- Conjugacy classes are mapped to conjugacy classes (the same or another).
- The image  $f(g)$  of an element  $g$  has the same order as  $g$ .

The composition of two automorphisms is again an automorphism, and with composition as binary operation the set of all automorphisms of a group  $G$ , denoted by **Aut( $G$ )**, forms itself a group, the *automorphism group* of  $G$ .

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