6.3: Bravais Lattices

Crystal lattices can be classified by their **translational** and **rotational symmetry**. In three-dimensional crytals, these symmetry operations yield 14 distinct lattice types which are called **Bravais lattices**. In these lattice diagrams (shown below) the dots represent lattice points, which are places where the whole structure repeats by translation. For example, in the body-centered cubic (**bcc**) structure of sodium metal, which is discussed below, we put one atom at the corner lattice points and another in the center of the unit cell. In the NaCl structure, which is discussed in Chapter 8, we place one NaCl formula unit on each lattice point in the face-centered cubic (**fcc**) lattice. That is, one atom (Na or Cl) would be placed on the lattice point and the other one would be placed halfway between. Similarly, in the cubic diamond structure, we place one C_2 unit around each lattice point in the fcc lattice.

The fourteen Bravais lattices fall into seven **crystal systems** that are defined by their rotational symmetry. In the lowest symmetry system (**triclinic**), there is no rotational symmetry. This results in a unit cell in which none of the edges are constrained to have equal lengths, and none of the angles are 90º. In the **monoclinic** system, there is one two-fold rotation axis (by convention, the b-axis), which constrains two of the angles to be 90º. In the **orthorhombic** system, there are three mutually perpendicular two-fold axes along the three unit cell directions. **Orthorhombic** unit cells have three unequal unit cell edges that are mutually perpendicular. **Tetragonal** unit cells have a four-fold rotation axis which constrains all the angles to be 90º and makes the a and b axes equivalent. The **rhombohedral** system has a three-fold axis, which constrains all the unit cell edges and angles to be equal, and the **hexagonal** system has a six-fold axis, which constrains the a and b lattice dimensions to be equal and the angle between them to be 120º. The **cubic** system has a three-fold axis along the body diagonal of the cube, as well as two-fold axes along the three perpendicular unit cell directions. In the cubic system, all unit cell edges are equal and the angles between them are 90º.

The translational symmetry of the Bravais lattices (the lattice **centerings**) are classified as follows:

- Primitive (P): lattice points on the cell corners only (sometimes called simple)
- Body-Centered (I): lattice points on the cell corners with one additional point at the center of the cell
- Face-Centered (F): lattice points on the cell corners with one additional point at the center of each of the faces of the cell
- Base-Centered (A, B, or C): lattice points on the cell corners with one additional point at the center of each face of one pair of parallel faces of the cell (sometimes called end-centered)

Not all combinations of the crystal systems and lattice centerings are unique. There are in total 7 × 6 = 42 combinations, but it can be shown that several of these are in fact equivalent to each other. For example, the monoclinic I lattice can be described by a monoclinic C lattice by different choice of crystal axes. Similarly, all A- or B-centred lattices can be described either by a C- or P-centering. This reduces the number of combinations to 14 conventional Bravais lattices, shown in the table below.

When the fourteen Bravais lattices are combined with the **32 crystallographic point groups**, we obtain the **230 space groups**. These space groups describe all the combinations of symmetry operations that can exist in unit cells in three dimensions. For two-dimensional lattices there are only 17 possible plane groups, which are also known as wallpaper groups.

Crystal family	Lattice system	Schönflies	14 Bravais Lattices			
			Primitive	Base-centered	Body-centered	Face-centered
Triclinic		C_i	 Triclinic			
Monoclinic		C_{2h}	Monoclinic, simple	Monoclinic, centered		
Orthorhombic		D_{2h}	Orthorhombic, simple	Orthorhombic, base-centered	Orthorhombic, body-centered	Orthorhombic, face-centered
Tetragonal		D_{4h}	Tetragonal, simple		Tetragonal, body-centered	
Hexagonal	rhombohedral	D_{3d}	Rhombohedral			
	hexagonal	D_{6h}	Hexagonal			
Cubic		O_h	Cubic, simple		Cubic, body-centered	Cubic, face-centered