# 1.5: Combining Symmetry Operations - 'Group Multiplication'

Now we will investigate what happens when we apply two symmetry operations in sequence. As an example, consider the NH_3 molecule, which belongs to the C_{3v} point group. Consider what happens if we apply a C_3 rotation followed by a σ_v reflection. We write this combined operation $\sigma_v C_3$ (when written, symmetry operations operate on the thing directly to their right, just as operators do in quantum mechanics – we therefore have to work backwards from right to left from the notation to get the correct order in which the operators are applied). As we shall soon see, the order in which the operations are applied is important.
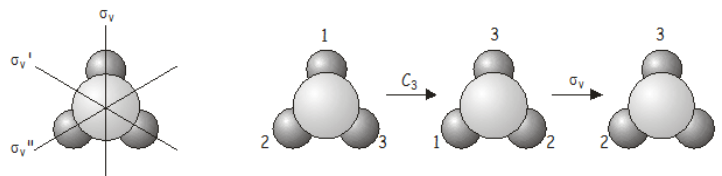
The combined operation $\sigma_v C_3$ is equivalent to σ_v'', which is also a symmetry operation of the C_{3v} point group. Now let's see what happens if we apply the operators in the reverse order i.e. $C_3 \sigma_v$ (σ_v followed by C_3).
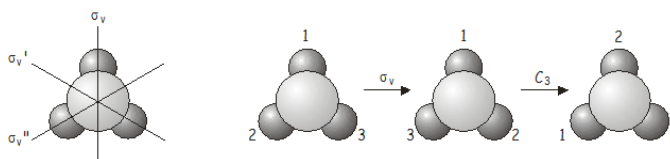
Again, the combined operation $C_3 \sigma_v$ is equivalent to another operation of the point group, this time σ_v'.

There are two important points that are illustrated by this example:

1. The order in which two operations are applied is important. For two symmetry operations A and B, AB is not necessarily the same as BA, i.e. symmetry operations do not in general commute. In some groups the symmetry elements do commute; such groups are said to be Abelian.

2. If two operations from the same point group are applied in sequence, the result will be equivalent to another operation from the point group. Symmetry operations that are related to each other by other symmetry operations of the group are said to belong to the same *class*. In NH_3, the three mirror planes σ_v, σ_v' and σ_v'' belong to the same class (related to each other through a C_3 rotation), as do the rotations C_3^+ and C_3^- (anticlockwise and clockwise rotations about the principal axis, related to each other by a vertical mirror plane

The effects of applying two symmetry operations in sequence within a given point group are summarized in *group multiplication tables*. As an example, the complete group multiplication table for C_{3v} using the symmetry operations as defined in the figures above is shown below. The operations written along the first row of the table are carried out first, followed by those written in the first column (note that the table would change if we chose to name σ_v, σ_v' and σ_v'' in some different order).

$$
\begin{array}{c|cccccc}
C_{3v} & E & C_3^+ & C_3^- & \sigma_v & \sigma_v' & \sigma_v'' \\
\hline
E & E & C_3^+ & C_3^- & \sigma_v & \sigma_v' & \sigma_v'' \\
C_3^+ & C_3^+ & C_3^- & E & \sigma_v' & \sigma_v'' & \sigma_v \\
C_3^- & C_3^- & E & C_3^+ & \sigma_v'' & \sigma_v & \sigma_v' \\
\sigma_v & \sigma_v & \sigma_v'' & \sigma_v' & E & C_3^- & C_3^+ \\
\sigma_v' & \sigma_v' & \sigma_v & \sigma_v'' & C_3^+ & E & C_3^- \\
\sigma_v'' & \sigma_v'' & \sigma_v' & \sigma_v & C_3^- & C_3^+ & E \\
\end{array}
\qquad (1.5.1)
$$