

## 15.5: Appendix E- Calculus Review

### E.1 Derivatives

Let  $f$  be a function of the variable  $x$ , and let  $\Delta f$  be the change in  $f$  when  $x$  changes by  $\Delta x$ . Then the **derivative**  $df/dx$  is the ratio  $\Delta f/\Delta x$  in the limit as  $\Delta x$  approaches zero. The derivative  $df/dx$  can also be described as the rate at which  $f$  changes with  $x$ , and as the slope of a curve of  $f$  plotted as a function of  $x$ .

The following is a short list of formulas likely to be needed. In these formulas,  $u$  and  $v$  are arbitrary functions of  $x$ , and  $a$  is a constant.

$$\begin{aligned}\frac{d(u^a)}{dx} &= au^{a-1} \frac{du}{dx} \\ \frac{d(uv)}{dx} &= u \frac{dv}{dx} + v \frac{du}{dx} \\ \frac{d(u/v)}{dx} &= \left(\frac{1}{v^2}\right) \left(v \frac{du}{dx} - u \frac{dv}{dx}\right) \\ \frac{d \ln(ax)}{dx} &= \frac{1}{x} \\ \frac{d(e^{ax})}{dx} &= ae^{ax} \\ \frac{df(u)}{dx} &= \frac{df(u)}{du} \cdot \frac{du}{dx}\end{aligned}$$

### E.2 Partial Derivatives

If  $f$  is a function of the independent variables  $x$ ,  $y$ , and  $z$ , the **partial derivative**  $(\partial f/\partial x)_{y,z}$  is the derivative  $df/dx$  with  $y$  and  $z$  held constant. It is important in thermodynamics to indicate the variables that are held constant, as  $(\partial f/\partial x)_{y,z}$  is not necessarily equal to  $(\partial f/\partial x)_{a,b}$  where  $a$  and  $b$  are variables different from  $y$  and  $z$ .

The variables shown at the bottom of a partial derivative should tell you which variables are being used as the independent variables. For example, if the partial derivative is  $\left(\frac{\partial f}{\partial y}\right)_{a,b}$  then  $f$  is being treated as a function of  $y$ ,  $a$ , and  $b$ .

### E.3 Integrals

Let  $f$  be a function of the variable  $x$ . Imagine the range of  $x$  between the limits  $x'$  and  $x''$  to be divided into many small increments of size  $\Delta x_i$  ( $i = 1, 2, \dots$ ). Let  $f_i$  be the value of  $f$  when  $x$  is in the middle of the range of the  $i$ th increment. Then the **integral**

$$\int_{x'}^{x''} f dx \quad (15.5.1)$$

is the sum  $\sum_i f_i \Delta x_i$  in the limit as each  $\Delta x_i$  approaches zero and the number of terms in the sum approaches infinity. The integral is also the area under a curve of  $f$  plotted as a function of  $x$ , measured from  $x = x'$  to  $x = x''$ . The function  $f$  is the **integrand**, which is integrated over the integration variable  $x$ .

This e-book uses the following integrals:

$$\begin{aligned}\int_{x'}^{x''} dx &= x'' - x' \\ \int_{x'}^{x''} \frac{dx}{x} &= \ln \left| \frac{x''}{x'} \right| \\ \int_{x'}^{x''} x^a dx &= \frac{1}{a+1} [(x'')^{a+1} - (x')^{a+1}] \quad (a \text{ is a constant other than } -1) \\ \int_{x'}^{x''} \frac{dx}{ax+b} &= \frac{1}{a} \ln \left| \frac{ax''+b}{ax'+b} \right| \quad (a \text{ is a constant})\end{aligned}$$

Here are examples of the use of the expression for the third integral with  $a$  set equal to 1 and to  $-2$ :

$$\int_{x'}^{x''} x \, dx = \frac{1}{2} [(x'')^2 - (x')^2]$$
$$\int_{x'}^{x''} \frac{dx}{x^2} = -\left(\frac{1}{x''} - \frac{1}{x'}\right)$$

## E.4 Line Integrals

A **line integral** is an integral with an implicit single integration variable that constraints the integration to a path.

The most frequently-seen line integral in this e-book,  $\int p \, dV$ , will serve as an example. The integral can be evaluated in three different ways:

1. The integrand  $p$  can be expressed as a function of the integration variable  $V$ , so that there is only one variable. For example, if  $p$  equals  $c/V$  where  $c$  is a constant, the line integral is given by  $\int p \, dV = c \int_{V_1}^{V_2} (1/V) \, dV = c \ln(V_2/V_1)$ .
2. If  $p$  and  $V$  can be written as functions of another variable, such as time, that coordinates their values so that they follow the desired path, this new variable becomes the integration variable.
3. The desired path can be drawn as a curve on a plot of  $p$  versus  $V$ ; then  $\int p \, dV$  is equal in value to the area under the curve.

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