5.1: Total Differential of a Dependent Variable

Recall from Sec. 2.4.1 that the state of the system at each instant is defined by a certain minimum number of state functions, the independent variables. State functions not treated as independent variables are dependent variables. Infinitesimal changes in any of the independent variables will, in general, cause an infinitesimal change in each dependent variable.

A dependent variable is a function of the independent variables. The **total differential** of a dependent variable is an expression for the infinitesimal change of the variable in terms of the infinitesimal changes of the independent variables. As explained in Sec. F.2 of Appendix F, the expression can be written as a sum of terms, one for each independent variable. Each term is the product of a partial derivative with respect to one of the independent variables and the infinitesimal change of that independent variable. For example, if the system has two independent variables, and we take these to be T and V, the expression for the total differential of the pressure is

$$\mathrm{d}p = \left(\frac{\partial p}{\partial T}\right)_V \mathrm{d}T + \left(\frac{\partial p}{\partial V}\right)_T \mathrm{d}V \tag{5.1.1}$$

Thus, in the case of a fixed amount of an ideal gas with pressure given by $p = nRT/V$, the total differential of the pressure can be written

$$\mathrm{d}p = \frac{nR}{V}\,\mathrm{d}T - \frac{nRT}{V^2}\,\mathrm{d}V \tag{5.1.2}$$