2.12: The Degeneracy of an Isolated System and Its Entropy

In Section 20.9, we find that the sum of the probabilities of the population sets of an isolated system is

$$1=\sum_{\{N_i\},E} W\left(N_i,g_i\right)\rho_{MS,N,E}.$$

By the principle of equal *a priori* probabilities, $\rho_{MS,N,E}$ is a constant, and it can be factored out of the sum. We have

$$1=\rho_{MS,N,E}\sum_{\{N_i\},E} W\left(N_i,g_i\right)$$

Moreover, the sum of the thermodynamic probabilities over all allowed population sets is just the number of microstates that have energy E. This sum is just the ***degeneracy of the system energy***, E. The symbol $\mathit{\Omega}_E$ is often given to this system-energy degeneracy. That is,

$$\mathit{\Omega}_E=\sum_{\{N_i\},E} W\left(N_i,g_i\right)$$

The sum of the probabilities of the population sets of an isolated system becomes

$$1=\rho_{MS,N,E}\mathit{\Omega}_E$$

In Section 20.9, we infer that

$$\rho_{MS,N,E}=\prod^{\infty}_{i=1}\rho(\epsilon_i)^{N_i}$$

so we have

$$1=\mathit{\Omega}_E\prod^{\infty}_{i=1}\rho(\epsilon_i)^{N_i}$$