6.1: Properties of a Normal Distribution

Mathematically a normal distribution is defined by the equation

$$P(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-(x-\mu)^2/(2\sigma^2)}$$

where $P(x)$ is the probability of obtaining a result, x, from a population with a known mean, μ, and a known standard deviation, σ. Figure $6.1.1$ shows the normal distribution curves for $\mu=0$ with standard deviations of 5, 10, and 20.

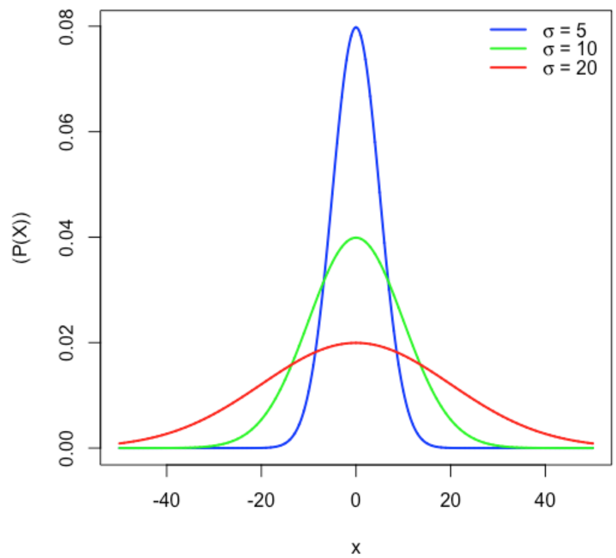

Figure 6.1.1: Three examples of normal distribution curves. Although the height and width are affected by σ, the area under each curve is the same.

Because the equation for a normal distribution depends solely on the population's mean, μ, and its standard deviation, σ, the probability that a sample drawn from a population has a value between any two arbitrary limits is the same for all populations. For example, Figure $6.1.2$ shows that 68.26% of all samples drawn from a normally distributed population have values within the range $\mu\pm1\sigma$, and only 0.14% have values greater than $\mu+3\sigma$.

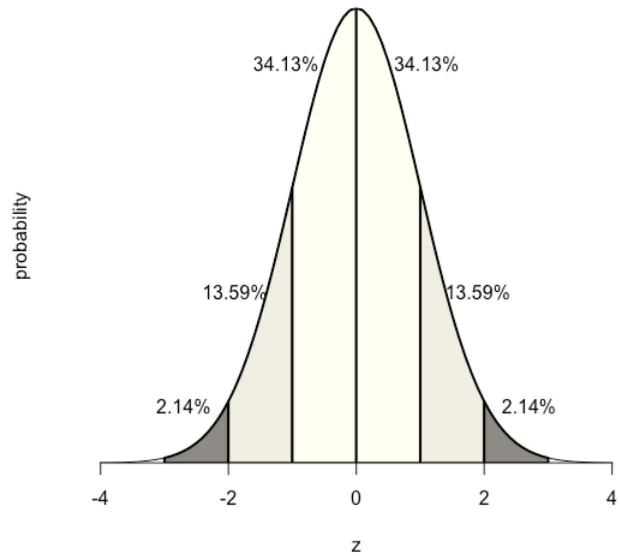

Figure 6.1.2: Normal distribution curve for $\mu = 0$ and $\sigma = 1$ showing area under the curve for various values of z in $\mu \pm z\sigma$.

This feature of a normal distribution—that the area under the curve is the same for all values of σ—allows us to create a probability table (see Appendix 1) based on the relative deviation, z, between a limit, x, and the mean, μ.

$$z = \frac{x - \mu}{\sigma}$$

The value of z gives the area under the curve between that limit and the distribution's closest tail, as shown in Figure 6.1.3.

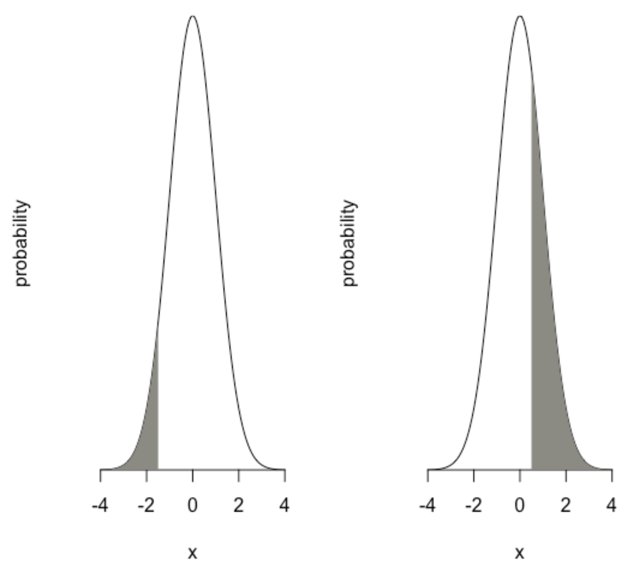

Figure 6.1.3: Normal distribution curve for $\mu = 0$ and $\sigma = 1$ showing (on the left) the area under the curve for $z = -1.5$ and (on the right for $z = +0.5$.

Example 6.1.1

Suppose we know that μ is 5.5833 ppb Pb and that σ is 0.0558 ppb Pb for a particular standard reference material (SRM). What is the probability that we will obtain a result that is greater than 5.650 ppb if we analyze a single, random sample drawn from the SRM?

Solution

Figure $6.1.4$ shows the normal distribution curve given values of 5.5833 ppb Pb for μ and of 0.0558 ppb Pb σ. The shaded area in the figures is the probability of obtaining a sample with a concentration of Pb greater than 5.650 ppm. To determine the probability, we first calculate z

$$z=\frac{x-\mu}{\sigma}=\frac{5.650-5.5833}{0.0558}=1.195$$

Next, we look up the probability in Appendix 1 for this value of z, which is the average of 0.1170 (for $z=1.19$) and 0.1151 (for $z=1.20$), or a probability of 0.1160; thus, we expect that 11.60% of samples will provide a result greater than 5.650 ppb Pb.

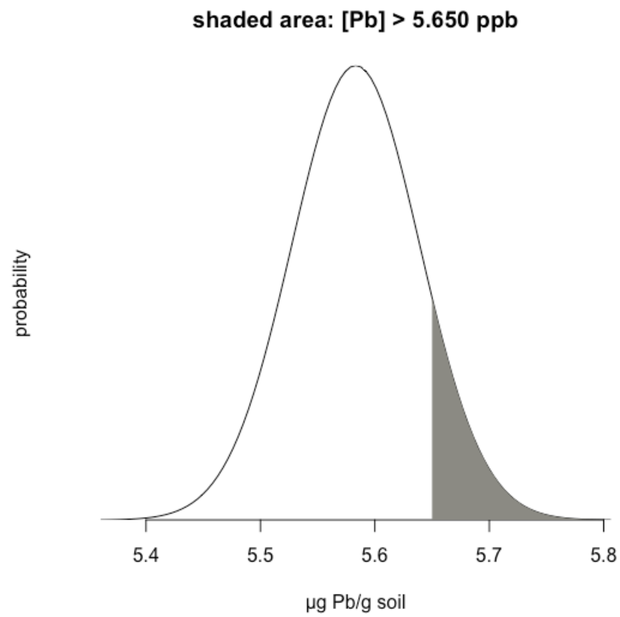

Figure $6.1.4$: Normal distribution curve for the amount of lead in a standard reference with $\mu=5.5833$ ppb and $\sigma=0.0558$ ppb. The shaded area shows those results for which the concentration of lead exceeds 5.650 ppb.

Example 6.1.2

Example $6.1.1$ considers a single limit—the probability that a result exceeds a single value. But what if we want to determine the probability that a sample has between 5.580 g Pb and 5.625 g Pb?

Solution

In this case we are interested in the shaded area shown in Figure $6.1.5$. First, we calculate z for the upper limit

$$z=\frac{5.625-5.5833}{0.0558}=0.747$$

and then we calculate z for the lower limit

$$z=\frac{5.580-5.5833}{0.0558}=-0.059$$

Then, we look up the probability in Appendix 1 that a result will exceed our upper limit of 5.625, which is 0.2275, or 22.75%, and the probability that a result will be less than our lower limit of 5.580, which is 0.4765, or 47.65%. The total unshaded area is 71.4% of the total area, so the shaded area corresponds to a probability of

$$100.00 - 22.75 - 47.65 = 100.00 - 71.40 = 29.6\%$$

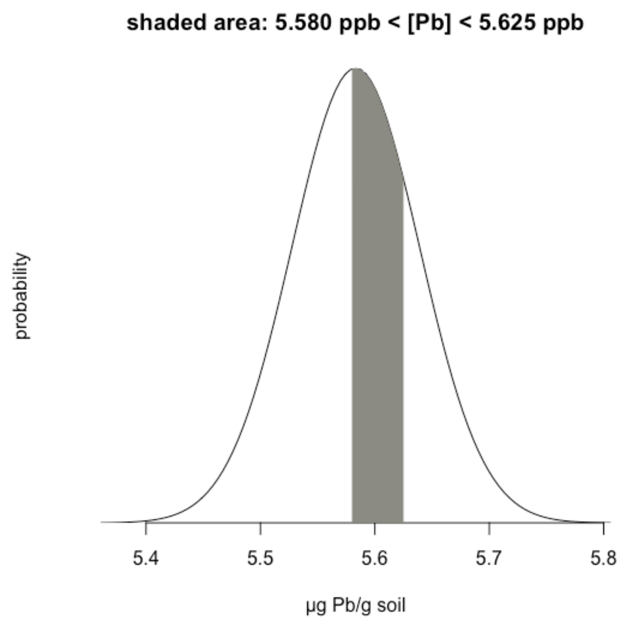

Figure 6.1.5: Normal distribution curve for the amount of lead in a standard reference with $\mu = 5.5833$ ppb and $\sigma = 0.0558$ ppb. The shaded area shows those results for which the concentration of lead is more than 5.580 ppb and less than 5.625 ppb.