# 10.1: Exact Differentials

In general, if a differential can be expressed as

$$df(x,y) = X\,dx + Y\,dy \tag{10.1.1}$$

the differential will be an **exact differential** if it follows the **Euler relation**

$$\left(\frac{\partial X}{\partial y}\right)_x = \left(\frac{\partial Y}{\partial x}\right)_y \tag{10.1.2}$$

In order to illustrate this concept, consider $P(\overline{V}, T)$ using the ideal gas law.

$$P = \frac{RT}{\overline{V}} \tag{10.1.3}$$

The total differential of $P$ can be written

$$dP = \left(-\frac{RT}{\overline{V}^2}\right) dV + \left(\frac{R}{\overline{V}}\right) dT \tag{10.1.4}$$

## Example 10.1.1: Euler Relation

Does Equation 10.1.4 follow the Euler relation (Equation 10.1.2)?

**Solution**

Let's confirm!

$$\left[\frac{1}{\partial T}\left(-\frac{RT}{\overline{V}^2}\right)\right]_{\overline{V}} \stackrel{?}{=} \left[\frac{1}{\partial \overline{V}}\left(\frac{R}{\overline{V}}\right)\right]_T$$

$$\left(-\frac{R}{\overline{V}^2}\right) \stackrel{\checkmark}{=} \left(-\frac{R}{\overline{V}^2}\right)$$

$dP$ is, in fact, an exact differential.

The differentials of all of the thermodynamic functions that are **state functions** will be ***exact***. Heat and work, which are **path functions**, are not exact differential and $dw$ and $dq$ are called ***inexact*** differentials instead.

## Contributors and Attributions

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