

## 13.2: Entropy

In addition to learning that the efficiency of a Carnot engine depends only on the high and low temperatures, more interesting things can be derived through the exploration of this system. For example, consider the total heat transferred in the cycle:

$$q_{tot} = nRT_h \ln\left(\frac{V_2}{V_1}\right) - nRT_l \ln\left(\frac{V_4}{V_3}\right)$$

Making the substitution

$$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

the total heat flow can be seen to be given by

$$q_{tot} = nRT_h \ln\left(\frac{V_4}{V_3}\right) - nRT_l \ln\left(\frac{V_4}{V_3}\right) \quad (13.2.1)$$

It is clear that the two terms do not have the same magnitude, unless  $T_h = T_l$ . This is sufficient to show that  $q$  is **not a state function**, since its net change around a closed cycle is not zero (as any value of a state function must be.) However, consider what happens when the sum of  $q/T$  is considered:

$$\begin{aligned} \sum \frac{q}{T} &= \frac{nR T_h \ln\left(\frac{V_4}{V_3}\right)}{T_h} - \frac{nR T_l \ln\left(\frac{V_4}{V_3}\right)}{T_l} \\ &= nR \ln\left(\frac{V_4}{V_3}\right) - nR \ln\left(\frac{V_4}{V_3}\right) \\ &= 0 \end{aligned}$$

This is the behavior expected for a state function! It leads to the definition of entropy in differential form,

$$dS \equiv \frac{dq_{rev}}{T} \quad (13.2.2)$$

In general,  $dq_{rev}$  will be larger than  $dq$  (since the reversible pathway defines the maximum heat flow.) So, it is easy to calculate entropy changes, as one needs only to define a reversible pathway that connects the initial and final states, and then integrate  $dq/T$  over that pathway. And since  $\Delta S$  is defined using  $q$  for a reversible pathway,  $\Delta S$  is *independent* of the actual path a system follows to undergo a change.

### Contributors

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