2.9: Density

After trees are cut, logging companies often move the raw material down a river to a sawmill where it can be shaped into building materials or other products. The logs float on the water because they are less dense than the water they are in. Knowledge of density is important in the characterization and separation of materials. Information about density allows us to make predictions about the behavior of matter.

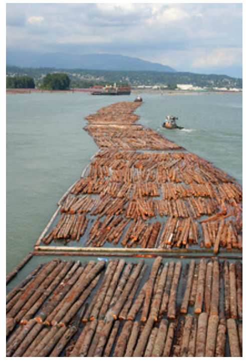

Figure $2.9.1$: Driving logs on the Fraser river near Vancouver, BC (Tony Hisgett via Flickr)

Density

A golf ball and a table tennis ball are about the same size. However, the golf ball is much heavier than the table tennis ball. Now imagine a similar size ball made out of lead. That would be very heavy indeed! What are we comparing? By comparing the mass of an object relative to its size, we are studying a property called **density**. Density is the ratio of the mass of an object to its volume.

$$\text{density} = \frac{\text{mass}}{\text{volume}} \tag{2.9.1}$$

or simply

$$d = \frac{m}{V} \tag{2.9.2}$$

Density is an intensive property, meaning that it does not depend on the amount of material present in the sample. Water has a density of 1.0 g/mL. That density is the same whether you have a small glass of water or a swimming pool full of water. Density is a property that is constant for the particular identity of the matter being studied.

The SI units of density are kilograms per cubic meter $\left(\text{kg/m}^3\right)$, since the kg and the m are the SI units for mass and length respectively. In everyday usage in a laboratory, this unit is awkwardly large. Most solids and liquids have densities that are conveniently expressed in grams per cubic centimeter $\left(\text{g/cm}^3\right)$. Since a cubic centimeter is equal to a milliliter, density units can also be expressed as g/mL. Gases are much less dense than solids and liquids, so their densities are often reported in g/L. Densities of some common substances at 20°C (unless otherwise noted) are listed in Table $2.9.1$ below.

Table $2.9.1$: Densities of Some Common Substances

Liquids and Solids	Density at 20°C (g/mL)	Gases	Density at 20°C (g/L)
Ethanol	0.79	Hydrogen	0.084
Ice @ 0°C	0.917	Helium	0.166
Corn oil	0.922	Air	1.20
Water	0.998	Oxygen	1.33
Water @ 4°C	1.000	Carbon dioxide	1.83
Corn syrup	1.36	Radon	9.23
Aluminum	2.70		
Copper	8.96		

Liquids and Solids	Density at 20°C (g/mL)	Gases	Density at 20°C (g/L)
Lead	11.35		
Mercury	13.6		
Gold	19.3		

The density of a substance is temperature dependent and usually decreases as temperature increases due to the fact that most materials expand as temperature increases. Since we know ice floats in water, it should make sense that it is less dense. Likewise, we would expect corn syrup to sink in water since it is more dense.

✅ Example $2.9.1$

A block of wood measures 16.6 cm by 8.7 cm by 3.6 cm and has a mass of 234.1 g. What is the density of the wood block?

Solution

Steps for Problem Solving	
List the known quantities.	Mass = 234.1 g Dimensions = 16.6 cm × 8.7 cm × 3.6 cm
Plan the problem.	$d=\frac{m}{V}$ $V=l\times w\times h$
Calculate the answer.	$d=\frac{m}{V}=\frac{m}{l\times w\times h}=\frac{234.1\text{ g}}{16.6\text{ cm}\times 8.7\text{ cm}\times 3.6\text{ cm}}=0.4\underline{5}03\text{ g/cm}^3=\boxed{0.45\text{g/}}$
Think about your result.	Since the density of water is 1 g/cm^3 and we know that almost all wood floats on water, a density of 0.45 g/cm^3 makes sense for the block of wood. Also, the answer is rounded to two significant figures, since 3.6 cm and 8.7 cm both have two significant figures and all of the measurements and all of the operations involve multiplication or division.

Since density values are known for many substances, density can be used to determine an unknown mass or an unknown volume. Dimensional analysis will be used to ensure that units cancel appropriately.

✅ Example $2.9.2$

If 55.8 mL of water at 22°C is added to an empty graduated cylinder that has a mass of 100.62 g, what is the combined mass of the cylinder and the water? The density of water at 22°C is 0.998 g/mL.

Solution

Steps for Problem Solving	

Steps for Problem Solving	
List the known quantities.	$d_{water} = 0.998 \text{ g/mL}$ $V_{water} = 55.8 \text{ mL}$ $m_{cylinder} = 100.62 \text{ g}$
Plan the problem.	$m_{total} = m_{cylinder} + m_{water} \Rightarrow m_{total} = 100.62 \text{ g} + m_{water}$ $d = \frac{m}{V}$ Solve for m.
Calculate the answer.	Rearrange the equation to solve for the mass of the water: $V \cdot d = \frac{m}{\cancel{V}} \cdot \cancel{V}$ $m = V \cdot d \;\Rightarrow\; m_{water} = V_{water} \cdot d_{water}$ Plug in the volume and density of the water to find the mass of the water. $m_{total} = (55.8 \; \cancel{\text{mL}}) \left(0.998 \; \frac{\text{g}}{\cancel{\text{mL}}} \right) = 55.\underline{6}9 \text{ g} \; = \; 55.7 \text{ g}$ The mass of the water is known to three significant figures, or 55.7 g. The total mass may be found by summing the mass of the cylinder and the mass of the water. To avoid potential rounding issues, we should carry through at least one extra digit in the mass of the water. $m_{total} = 100.62 \text{ g} \; + \; 55.\underline{6}9 \text{ g} \; = \; 156.\underline{3}1 \text{ g} \; = \; \boxed{156.3 \text{ g}}$ The combined mass is 156.3 g. The answer is rounded to the nearest tenth, since the mass of the water was known to the nearest tenth (the last operation involves addition, so we pay attention to decimal places when rounding rather than significant figures).
Think about your result.	Since the density of water at 22°C is about 1 g/mL, its mass (in grams) is numerically close to its volume (in mL). Therefore, a mass of 55.7 g for the water makes sense. Adding the mass of the water to the mass of the cylinder, which was already known, is very straightforward.

✏️ Exercise 2.9.1

A. Find the density (in kg/L) of a sample that has a volume of 36.5 L and a mass of 10.0 kg.
B. What is the volume of 100.0 g of air at 20°C? Consult Table 2.9.1 for the density.

Answer A

0.274 kg/L

Answer B

83.3 L

Density as a Conversion Factor

Conversion factors can also be constructed for converting between different kinds of units. For example, density can be used to convert between the mass and the volume of a substance. Consider mercury, which is a liquid at room temperature and has a density of 13.6 g/mL. The density tells us that 13.6 g of mercury have a volume of 1 mL. We can write that relationship as follows:

$$13.6 \text{ g mercury} = 1 \text{ mL mercury}$$

This relationship can be used to construct two conversion factors:

$$\frac{13.6 \text{ g}}{1 \text{ mL}} = 1$$

and

$$\frac{1 \text{ mL}}{13.6 \text{ g}} = 1$$

Which one do we use? It depends, as usual, on the units we need to cancel and introduce. For example, suppose we want to know the mass of 2.0 mL of mercury. We would use the conversion factor that has milliliters on the bottom (so that the milliliter unit cancels) and grams on top, so that our final answer has a unit of mass:

$$2.0 \cancel{\text{ mL}} \times \frac{13.6 \text{ g}}{1 \cancel{\text{ mL}}} = 27.2 \text{ g} = \boxed{27 \text{ g}}$$

In the last step, we limit our final answer to two significant figures because the volume quantity has only two significant figures; the 1 in the volume unit is considered an exact number, so it does not affect the number of significant figures. The other conversion factor would be useful if we were given a mass and asked to find volume, as the following example illustrates.

✅ Example 2.9.3

A mercury thermometer for measuring a patient's temperature contains 0.750 g of mercury. What is the volume of this mass of mercury?

Solution

Steps for Problem Solving	Unit Conversion
Identify the "given" information and what the problem is asking you to "find."	Given: 0.750 g Find: mL
List other known quantities (see Table 2.9.1).	density of mercury = 13.6 g/mL
Prepare a concept map and use the proper conversion(s).	$\boxed{\text{g}} \xrightarrow[13.6 \text{ g}]{1 \text{ mL}} \boxed{\text{mL}}$
Calculate the answer.	$0.750 \cancel{\text{ g}} \times \frac{1 \text{ mL}}{13.6 \cancel{\text{ g}}} = 0.055\underline{1}47... \text{ mL} = \boxed{0.0551 \text{ mL}}$ The final answer is rounded to three significant figures.

✏️ Exercise 2.9.2

Several copper coins were placed dropped into a graduated cylinder, displacing 18.1 mL of water. Using Table 2.9.1, what is the mass of the coins?

Answer

162 g

Summary

- Density is the ratio of the mass of an object to its volume.
- Gases are less dense than either solids or liquids.

- Both liquid and solid materials can have a variety of densities.
- For liquids and gases, the temperature will affect the density to some extent.
- Density can be used as a conversion factor between mass and volume.