2.4: Significant Figures in Calculations

Learning Objectives

- Use significant figures correctly in arithmetical operations.

Calculators do just what is asked of them – no more and no less. However, they can sometimes get a little out of hand. If you multiply 2.49 by 6.3, you get an answer of 15.687, a value that ignores the number of significant figures in either number. Division with a calculator is even worse. When you divide 12.2 by 1.7, the answer you obtain is 7.176470588. Neither piece of data is accurate to nine decimal places, but the calculator does not know that. The human being operating the instrument has to make the decision about how the answer should be reported.

Figure 2.4.1: A TI-84 Plus graphing calculator. (Asimzb via Wikimedia Commons)

Rounding

Before dealing with the specifics of the rules for determining the significant figures in a calculated result, we need to be able to round numbers correctly. To **round** a number, first decide how many significant figures the number should have. Once you know that, round to that many digits, starting from the left. If the number immediately to the right of the last significant digit is less than 5, it is dropped and the value of the last significant digit remains the same. If the number immediately to the right of the last significant digit is greater than or equal to 5, the last significant digit is increased by 1.

Consider the measurement 207.518 m. As it stands, the measurement contains six significant figures. How would this number be rounded to a fewer number of significant figures? Follow the process as outlined in Table 2.4.1.

Table 2.4.1: Rounding Examples

Number of Significant Figures	Rounded Value	Reasoning
6	$207.51\underline{8}$ m ⇒ 207.518 m	All digits are significant.
5	$207.5\underline{1}8$ m ⇒ 207.52 m	Round up, since the 6^{th} digit is an "8" (≥5).
4	$207.\underline{5}18$ m ⇒ 207.5 m	The last two digits are simply dropped, since the 5^{th} digit is a "1" (<5).

Number of Significant Figures	Rounded Value	Reasoning
3	$20\underline{7}.518$ m ⇒ 208 m	Round up, since the 4th digit is a "5" (≥5).
2	$2\underline{0}7.518$ m ⇒ $2\underline{1}0$ m ⇒ 2.1×10^2 m	Round up, since the 3rd digit is a "7" (≥5). However, trailing zeros in a number that has no decimal point are ambiguous, so express in scientific notation.
1	$\underline{2}07.518$ m ⇒ $\underline{2}00$ m ⇒ 2×10^2 m	The last five digits are dropped, since the 2nd digit is a "0" (<5). The ones place and tens place are held with zeros. However, trailing zeros in a number that has no decimal point are ambiguous, so express in scientific notation.

Notice that the further a number is rounded, the less reliable that measurement becomes. An approximate value may be sufficient for some purposes, but scientific work requires a much higher level of detail.

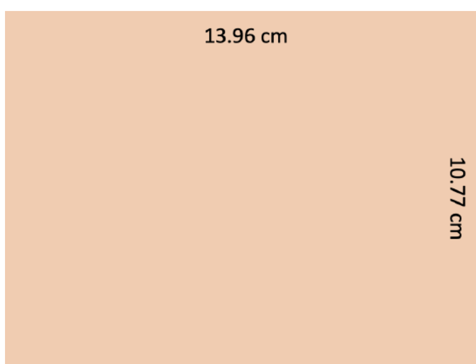
It is important to be aware of significant figures when mathematically manipulating numbers. For example, dividing 125 by 307 on a calculator gives 0.4071661238… to an infinite number of digits. But do the digits in this answer have any practical meaning, especially when you are starting with numbers that have only three significant figures each? When performing mathematical operations, there are two rules for limiting the number of significant figures in an answer – one rule is for addition and subtraction, and one rule is for multiplication and division.

⚡ Significant Figures and Rounding

In operations involving significant figures, the answer is reported in such a way that it reflects the reliability of the **least precise** operation. An answer is no more precise than the least precise number used to get the answer.

Multiplication and Division

Suppose you wanted to find the area of a rectangle and you measured the dimensions as 13.96 cm by 10.77 cm. Multiplying the two lengths together on a calculator yields an area of 150.349 cm^2. Should the answer be reported as 150.349 cm^2 or should it be rounded to 150.35 cm^2 or 150.3 cm^2 or even more?



Recall that all measurements have uncertainty. Assuming the last digit is the uncertain digit and may be estimated to the nearest ±0.01 cm, a second student may measure the dimensions of the rectangle as 13.95 cm by 10.76 cm. A third student may measure it as 13.97 cm by 10.78 cm. A fourth student may come up with 13.96 cm by 10.78 cm. The measurements are tabulated in the table below, along with the calculated areas.

If the calculated areas are examined more closely, it is easy to see that the hundreds place (the 1), tens place (the 5), and ones place (the 0) are all certain. They have no variation. We begin to see variation or uncertainty in the tenths place. Since significant figures are defined as all of the certain digits in a measurement plus one uncertain digit, the calculated areas should be rounded and reported to the nearest ±0.1 cm^2.

Student	Length	Width	Calculated Area	Reported Area
1	13.96 cm	10.77 cm	150.$\underline{3}$49 cm^2	150.3 cm^2
2	13.95 cm	10.76 cm	150.$\underline{1}$02 cm^2	150.1 cm^2
3	13.97 cm	10.78 cm	150.$\underline{5}$97 cm^2	150.6 cm^2
4	13.96 cm	10.78 cm	150.$\underline{4}$89 cm^2	150.5 cm^2

This is because the length and width each have four significant figures. Multiplying them together results in the area having four significant figures, i.e. three certain digits plus one uncertain digit. When the multiplied values have a different number of significant figures, the answer should be limited to the factor that has the lowest count of significant figures. The same rule applies to division.

⚡ Rounding Rule for Multiplication and Division

The answer should be rounded so it contains the same number of significant figures as the measurement having the fewest number of significant figures.

✅ Example 2.4.1

Write the answer for each expression using the appropriate number of significant figures.

A. $\dfrac{346.1 \text{ mi}}{5.3 \text{ h}} =$
B. $14.58 \text{ ft} \times 5.73 \text{ ft}$
C. $36 \text{ in} \times 26 \text{ in} \times 16 \text{ in}$

Solution

A

Explanation	Answer
The calculated answer is 65.301887 mi/h. The reported answer should have two significant figures, since 5.3 h has the fewest significant figures (two) between the two measurements.	65 mi/h

B

Explanation	Answer
The calculated answer is 83.5434 ft^2. The reported answer should have three significant figures, since 5.73 ft has the fewest significant figures (three) between the two measurements.	83.5 ft^2

C

Explanation	Answer

Explanation	Answer
The calculated answer is 14,976 in^3. The reported answer should have two significant figures, since all three measurements have two significant figures. Since an answer of 15,000 in^3 would represent an ambiguous number of significant figures, the answer should be reported in scientific notation with two significant figures.	$1.5 \times 10^4\ in^3$

Addition and Subtraction

Now that we know how to report answers that involved multiplication and division, you may wonder if answers resulting from addition and subtraction are handled in the same manner. What if we wanted to know the combined distance of 385.17 m, 6.2 m, and 45.86 m? Notice that the first and last lengths were measured to the nearest ±0.01 m, while the second length was measured to the nearest ±0.1 m.

Original Measurements	Change 1st Measurement	Change 2nd Measurement	Change 3rd Measurement
385.17 m +6.2 m +45.86 m 437.23 m	385.18 m +6.2 m +45.86 m 437.24 m	385.17 m +6.3 m +45.86 m 437.33 m	385.17 m +6.2 m +45.87 m 437.24 m

The table above shows what happens to the sum when the uncertain digit in each measurement is changed, one by one. Notice when the second measurement is changed from 6.2 m to 6.3 m that the sum also changes in the tenths place, going from 437.23 m to 437.33 m. In other words, the second measurement has the greatest impact on the calculated result since it is the least precise of the three measurements, ±0.1 m vs. ±0.01 m.

±0.01 m
±0.1 m
±0.01 m
±0.1 m

Therefore, the sum of 385.17 m, 6.2 m, and 45.86 m should be reported as 437.2 m.

385.17 m
+6.2 m
+45.86 m
437.2 m

This leads us to the rounding rule for addition and subtraction. While different, it is worded similarly to the rule for multiplication and division. Can you spot the difference? It is likely the rule with which you are most familiar.

⚡ Rounding Rule for Addition and Subtraction

The answer should be rounded so it contains the same number of decimal places as the measurement having the fewest number of decimal places.

✅ Example 2.4.2

A. 275.71 kg
− 8.4 kg

B. 1,027 mL + 611 mL + 363.06 mL

Solution

A

Explanation	Answer
A calculator provides an answer of 267.31 kg, but because 8.4 kg is known to the nearest ±0.1 kg, the final answer should be expressed to the nearest ±0.1 kg.	267.3 kg

B

Explanation	Answer
A calculator provides an answer of 2,001.06 mL, but because 1027 mL and 611 mL are both known to the nearest ±1 mL, the final answer should be expressed to the nearest ±1 mL.	2001 mL

✏️ Exercise 2.4.1

Write the answer for each expression using the correct number of significant figures. As a reminder, calculators do not understand significant figures. *You* are the one who must apply the rules for calculated answers to a result obtained from a calculator.

A. $\frac{165.110 \text{ g}}{8.35 \text{ mL}} =$
B. $8.6 \text{ g} + 32.06 \text{ g} + 88.7 \text{ g}$
C. $255.0 \text{ km} - 99 \text{ km}$
D. $44 \text{ cm} \times 43 \text{ cm}$

Answer A

19.8 g/mL

Answer B

129.4 g

Answer C

156 km

Answer D

$1.9 \times 10^3 \text{ cm}^2$

Calculations Involving Mixed Operations

In practice, chemists generally work with a calculator and carry all digits forward through subsequent calculations. When working on paper, however, we often want to minimize the number of digits we have to write out. Because successive rounding can compound inaccuracies, intermediate rounding needs to be handled correctly. When working on paper, always round an intermediate result so as to retain *at least* one more digit than can be justified and carry this number into the next step in the calculation. The final answer is then rounded to the correct number of significant figures at the very end.

✅ Example 2.4.3

A cylinder filled with water was combined with 5.2 cm^3 of water that had been drawn into a syringe. If the cylinder had an inner height of 16.8 cm and an inner radius of 1.86 cm, what is the combined volume of water? Report the answer with the correct number of significant figures. (Note: For a cylinder, $V = \pi r^2 h$. You may use the π function on your calculator or estimate the value of π as 3.1416.)

Solution

Volume	Explanation	Result
Syringe	The volume of water in the syringe is 5.2 cm^3, known to the nearest ±0.1 cm^3.	$5.\underline{2}\ cm^3$
Cylinder	The volume of water in the cylinder = $\pi r^2 h$ = $3.141\underline{6}(1.8\underline{6}\ \text{cm})^2(16.\underline{8}\ \text{cm}) = 18\underline{2}.594\ \text{cm}^3$. The rules for multiplication and division apply. Since 16.8 cm and 1.86 cm both have three significant figures, the calculated volume has three significant figures, or 183 cm^3. Compounding of rounding errors in the final answer may be avoided by carrying extra digits along in the intermediate results. The uncertain digit is underlined (the ones place).	$18\underline{2}.594\ cm^3$
Combined	The sum of 5.2 cm^3 and 182.594 cm^3 is 187.794 cm^3. The rules for addition and subtraction apply. Since $5.\underline{2}$ cm^3 is uncertain in the tenths place (±0.1 cm^3) and $18\underline{2}.594$ cm^3 is uncertain in the ones place (±1 cm^3), the sum is uncertain in the ones place (±1 cm^3), or $18\underline{7}.794$ cm^3.	$188\ cm^3$

The volume should be reported as 188 cm^3. Check out the video below for additional examples.

Video 2.4.1: Significant figures in mixed operations (https://www.youtube.com/watch?v=yBntMndXQWA).

Summary

- Rounding
  - If the number to be dropped is greater than or equal to 5, increase the number to its left by 1.
  - If the number to be dropped is less than 5, there is no change.
- The rule in multiplication and division is that the final answer should have the same number of significant figures as the measurement having the fewest significant figures.
- The rule in addition and subtraction is that the final answer is should have the same number of decimal places as the measurement having the fewest decimal places.