2.6: Dimensional Analysis

Learning Objectives

- To convert a value reported in one unit to a corresponding value in a different unit using conversion factors.

During your study of chemistry, you will notice the frequent use of mathematical equations. These equations often use multiple variables and usually require measurements with specific units. Algebra skills are very important! However, conversions may often be accomplished through the use of a technique called dimensional analysis without using equations or algebraic manipulation.

Converting Between Units with Conversion Factors

A **conversion factor** is a factor used to convert one unit of measurement into another. A simple conversion factor can be used to convert meters into centimeters, or a more complex one may be used to convert miles per hour into meters per second. Since most calculations require measurements to be in certain units, you will find many uses for conversion factors. A conversion always represents a fact and may either be simple or complex. For instance, you already know that 12 eggs equal 1 dozen. A more complex fact is that the speed of light is 1.86×10^5 miles/s. Either one of these can be used as a conversion factor depending on the type of calculation (Table 2.6.1).

Table 2.6.1: Conversion Factors from SI units to English Units

English Units	Metric Units	Quantity
1 ounce (oz)	28.35 grams (g)	*mass
1 fluid ounce (fl oz)	29.57 milliliters (mL)	volume
2.205 pounds (lb)	1 kilogram (kg)	*mass
1 inch (in)	2.540 centimeters (cm)	length
1 mile (mi)	1.609 kilometers (km)	length
1.057 quarts (qt)	1 liter (L)	volume

*Pounds and ounces are technically units of force, not mass, but this fact is often ignored by the non-scientific community.

Common Chemistry Conversions

A more complete list of conversions may be found in Appendix 2: Conversions.

Of course, there are other ratios which are not listed in either Table 2.6.1 or Appendix 2: Conversions. They may include:

- Ratios embedded in the text of the problem (using words such as *per* or *in each*, or using symbols such as / or %).
- Conversions using metric units, as covered in 2.5: The Metric System.
- Common knowledge ratios (such as 60 seconds in 1 minute or 60 minutes in 1 hour).

If you learned the SI units and prefixes described, then you know that 1 cm is $\frac{1}{100}$ of a meter.

$$1 \text{ cm} = \frac{1}{100} \text{ m} = 10^{-2} \text{ m}$$

or

$$100 \text{ cm} = 1 \text{ m}$$

Suppose we divide both sides of the equation by 1 m (both the number *and* the unit):

$$\frac{100 \text{ cm}}{1 \text{ m}} = \frac{1 \text{ m}}{1 \text{ m}}$$

As long as we perform the same operation on both sides of the equals sign, the expression remains an equality. Look at the right side of the equation; it now has the same quantity in the numerator (the top) as it has in the denominator (the bottom). Any fraction that has the same quantity in the numerator and the denominator has a value of 1:

$$\frac{100\text{ cm}}{1\text{ m}} = \frac{1000\text{ mm}}{1\text{ m}} = \frac{1 \times 10^6 \mu\text{m}}{1\text{ m}} = 1$$

We know that 100 cm *is* 1 m, so we have the same quantity on the top and the bottom of our fraction, although they are expressed in different units. The fraction can then be used for calculations converting m to cm.

Performing Dimensional Analysis

The use of units in a calculation to ensure that we obtain the final proper units is called *dimensional analysis* and it is one of the most valuable tools that physical scientists use. Simply put, dimensional analysis is the conversion between an amount in one unit to the corresponding amount in a desired unit using various conversion factors. This is valuable because certain measurements are more accurate or easier to find than others.

Here is a simple example. How many centimeters are there in 3.55 m? Perhaps you can determine the answer in your head. If there are 100 cm in every meter, then 3.55 m equals 355 cm. To solve the problem more formally with a conversion factor, we first write the quantity we are given, 3.55 m. Then we multiply this quantity by a conversion factor, which is the same as multiplying it by 1. We can write 1 as $\frac{100\text{ cm}}{1\text{ m}}$ and multiply:

$$3.55\text{ m} \times \frac{100\text{ cm}}{1\text{ m}}$$

The 3.55 m may be thought of as a fraction with a 1 in the denominator. Because m, the abbreviation for meters, occurs in both the numerator *and* the denominator of our expression, they cancel out:

$$\frac{3.55\ \cancel{\text{m}}}{1} \times \frac{100\text{ cm}}{1\ \cancel{\text{m}}}$$

The final step is to perform the calculation that remains once the units have been canceled:

$$\frac{3.55}{1} \times \frac{100\text{ cm}}{1} = 355\text{ cm}$$

In the final answer, we omit the 1 in the denominator. Thus, by a more formal procedure, we find that 3.55 m equals 355 cm. A generalized description of this process is as follows:

quantity (in old units) × conversion factor = quantity (in new units)

You may be wondering why we use a seemingly complicated procedure for a straightforward conversion. In later studies, the conversion problems you will encounter *will not always be so simple*. If you can master the technique of applying conversion factors, you will be able to solve a large variety of problems.

In the previous example, we used the fraction $\frac{100\text{ cm}}{1\text{ m}}$ as a conversion factor, but the conversion factor $\frac{1\text{ m}}{100\text{ cm}}$ is also equal to 1. $\frac{1\text{ m}}{100\text{ cm}}$ has the same quantity in the numerator as in the denominator, except that they are expressed in different units. However, had we used the second conversion factor, the original unit would not have canceled, and the result would have been meaningless. Here is what it would look like:

$$3.55\text{ m} \times \frac{1\text{ m}}{100\text{ cm}} = 0.0355\ \frac{\text{m}^2}{\text{cm}}$$

For the answer to be meaningful, we have to *construct the conversion factor in a form that causes the original unit to cancel out*. Here is a concept map for constructing a proper conversion.

$$\boxed{\text{meters (m)}} \xrightarrow[1\text{ m}]{100\text{ cm}} \boxed{\text{centimeters (cm)}}$$

Notice that the units from which you are converting go on the bottom, while the units to which you are converting go on the top.

⚖️ General Steps in Performing Dimensional Analysis

1. Identify the "**given**" information in the problem. Look for a number with units to start this problem with.
2. What is the problem asking you to "**find**"? In other words, what unit will the answer have?
3. Use **ratios** and conversion factors to cancel out the units that aren't part of the answer, and leave you with units that are part of the answer.
4. When units cancel out correctly, you are ready to do the **math**. You are multiplying fractions, so multiply by each of the numbers in the numerator and divide by each of the numbers in the denominator.

Significant Figures in Conversions

How do conversion factors affect the determination of significant figures?

- Metric-to-metric conversions are *not* considered in the determination of significant figures in a calculation since they are exact.
- Other defined relationships, not measured quantities, are also exact and are considered to have an infinite number of significant figures. In other words, there are exactly 12 inches in 1 foot or there are exactly 60 minutes in 1 hour.
- Counted numbers are also exact. If one counts 16 students in a classroom, the number 16 is exact.
- In contrast, conversion factors that come from measurements (such as density, as we will see shortly) or that are approximations have a limited number of significant figures and should be considered in determining the significant figures of the final answer.

✅ Examples 2.6.1

A. The average volume of blood in an adult male is 4.7 L. What is this volume in milliliters?
B. A table is 42 inches long. What is this length in feet?

Solution

Steps for Problem Solving	Part A	Part B
Identify the "given" information and what the problem is asking you to "find."	Given: 4.7 L Find: mL	Given: 42 inches Find: feet
List other known quantities. Metric prefixes are found in 2.5: The Metric System. Other conversions may be found in Appendix 2: Conversions.	1 mL = 10^{-3} L	1 ft = 12 in
Prepare a concept map and use the proper conversion factor(s).	$\boxed{\text{L}} \xrightarrow[1\ \text{L}]{1000\ \text{mL}} \boxed{\text{mL}}$	$\boxed{\text{in}} \xrightarrow[12\ \text{in}]{1\ \text{ft}} \boxed{\text{ft}}$
Cancel units and calculate.	$4.7\ \cancel{\text{L}} \times \frac{1\ \text{mL}}{10^{-3}\ \cancel{\text{L}}} = \boxed{4700\ \text{mL}}$ or $4.7\ \cancel{\text{L}} \times \frac{1000\ \text{mL}}{1\ \cancel{\text{L}}} = \boxed{4700\ \text{mL}}$	$42\ \cancel{\text{in}} \times \frac{1\ \text{ft}}{12\ \cancel{\text{in}}} = \boxed{3.5\ \text{ft}}$
Think about your result.	The amount in mL should be 1000 times larger than the given amount in L.	The amount in feet should be less than the given amount in inches.

Exercise 2.6.1

Perform each conversion.

A. 101,000 ns to seconds
B. 3.208 kg to grams
C. 215 lb to kg

Answer A

1.01×10^{-4} s

Answer B

3208 g

Answer C

97.5 kg

Summary

- Conversion factors are used to convert one unit of measurement into another.
- Dimensional analysis (unit conversions) involves the use of conversion factors that will cancel unwanted units and produce the appropriate units.

This page is shared under a CK-12 license and was authored, remixed, and/or curated by Lance S. Lund (Anoka-Ramsey Community College), Melissa Alviar-Agnew, and Henry Agnew. Original source: https://www.ck12.org/c/chemistry/.

CK-12 Foundation
LICENSED UNDER
CK-12 Foundation is licensed under CK-12 Curriculum Materials License

2.6: Dimensional Analysis is shared under a CC BY-NC license and was authored, remixed, and/or curated by LibreTexts.