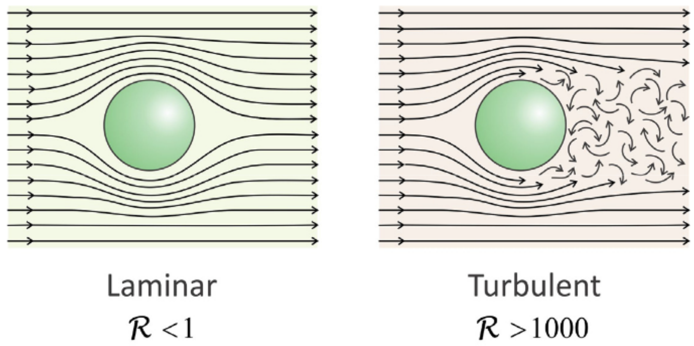
14.3: Laminar and Turbulent Flow

- Laminar flow: Fluid travels in smooth parallel lines without lateral mixing.
- Turbulent flow: Flow velocity field is unstable, with vortices that dissipate kinetic energy of fluid more rapidly than laminar regime.



Reynolds Number

The Reynolds number is a dimensionless number is used to indicate whether flow conditions are in the laminar or turbulent regimes. It indicates whether the motion of a particle in a fluid is dominated by inertial or viscous forces.1

$$\mathcal{R} = \frac{inertial\ forces}{viscous\ forces}$$

When $\mathcal{R} > 1$, the particle moves freely, experiencing only weak resistance to its motion by the fluid. If $\mathcal{R} < 1$, it is dominated by the resistance and internal forces of the fluid. For the latter case, we can consider the limit m → 0 in eq. **Error! Reference source not found.**, and find that the velocity of the particle is proportional to the random fluctuations: $v(t) = f_r(t)/\zeta$.

We can also express the Reynolds number in other forms:

- In terms of the fluid velocity flow properties: $\mathcal{R} = \dfrac{v\rho(d\overline{v}/dz)}{\eta(d^2\overline{v}/dz^2)}$
- In terms of the Langevin variables: $\mathcal{R} = f_{in}/f_d$.

Hydrodynamically, for a sphere of radius r moving through a fluid with dynamic viscosity η and density ρ at velocity v,

$$\mathcal{R} = \frac{rv\rho}{\eta}$$

Consider for an object with radius 1 cm moving at 10 cm/s through water: $\mathcal{R} = 10^3$. Now compare to a protein with radius 1 nm moving at 10 m/s: $\mathcal{R} = 10^{-2}$.

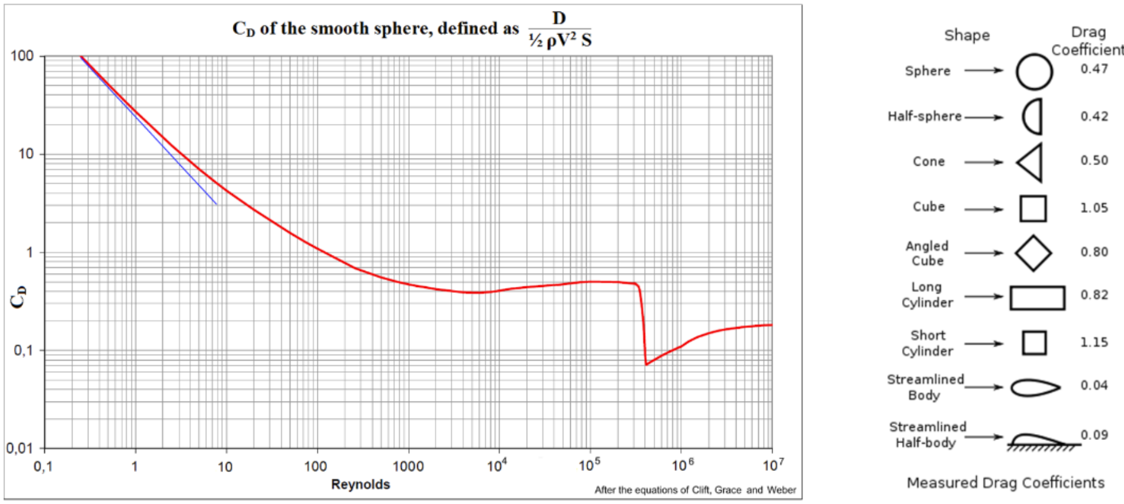
Drag Force in Hydrodynamics

The drag force on an object is determined by the force required to displace the fluid against the direction of flow. A sphere, rod, or cube with the same mass and surface area will respond differently to flow. Empirically, the drag force on an object can be expressed as

$$f_d = \left[\frac{1}{2}\rho C_d v^2\right] a$$

This expression takes the form of a pressure (term in brackets) exerted on the cross-sectional area of the object along the direction of flow, a. C_d is the drag coefficient, a dimensionless proportionality constant that depends on the shape of the object. In the case of a sphere of radius r: a = πr2 in the turbulent flow regime ($\mathcal{R} > 1000$) C_d = 0.44–0.47. Determination of C_d is somewhat empirical since it depends on $\mathcal{R}$ and the type of flow around the sphere.

The drag coefficient for a sphere in the viscous/laminar/Stokes flow regimes ($\mathcal{R} < 1$) is $C_d = 24/\mathcal{R}$. This comes from using the Stokes Law for the drag force on a sphere $f_d = 6\pi\eta v r$ and the Reynolds number $\mathcal{R} = \rho v d/\eta$.



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1. E. M. Purcell, Life at low Reynolds number, Am. J. Phys. 45, 3–11 (1977).