11.4: Orientational Diffusion

The concepts we developed for translation diffusion and Brownian motion are readily extended to rotational diffusion. For continuum diffusion, if one often assumes that one can separate the particle probability density into a radial and angular part: $P(r, \theta, \phi) = P(r) P(\theta, \phi)$. Then one also separate the diffusion equation into two parts for which the orientational diffusion follows a small-angle diffusion equation

$$\frac{\partial P(\Omega, t)}{\partial t} = D_{or} \nabla^2 P(\Omega, t) \tag{11.4.1}$$

where Ω refers to the spherical coordinates (θ,φ). D_{or} is the **orientational diffusion constant** with units of $rad^2\ s^{-1}$. Microscopically, one can consider orientational diffusion as a random walk on the surface of a sphere, with steps being small angular displacements in θ and φ. Equation 11.4.1 allows us to obtain the time-dependent probability distribution function $P(\Omega, t | \Omega_0)$ that describes the distribution of directions Ω at time t, given that the vector had the orientation Ω_0 at time $t = 0$. This can be expressed as an expansion in spherical harmonics

$$P(\Omega, t | \Omega_0) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} c_\ell^m(t) [Y_\ell^m(\Omega_0)]^* Y_\ell^m(\Omega)$$

The expansion coefficients are given by

$$c_\ell^m(t) = \exp[-\ell(\ell+1) D_{or} t]$$

Readings

- H. C. Berg, Random Walks in Biology. (Princeton University Press, Princeton, N.J., 1993).
- R. Phillips, J. Kondev, J. Theriot and H. Garcia, Physical Biology of the Cell, 2nd ed. (Taylor & Francis Group, New York, 2012).