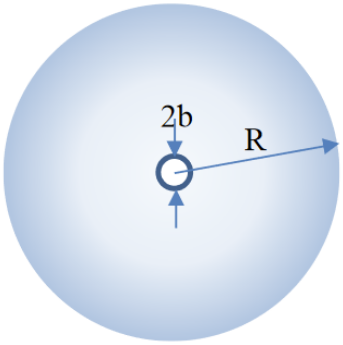
15.1: Dimensionality Reduction

One approach that does not require energy input works by recognizing that displacement is faster in systems with reduced dimensionality. Let's think about the time it takes to diffusively encounter a small fixed target in a large volume, and how this depends on the dimensionality of the search. We will look at the mean first passage time to find a small target with radius b centered in a spherical volume with radius R, where R≫b. If the molecules are initially uniformly distributed within the volume the average time it takes for them to encounter the target (i.e., MFPT) is[1]

$$\langle \tau_{3D} \rangle \simeq \frac{R^2}{3D_3}\left(\frac{R}{b}\right) \qquad R \gg b$$

$$\langle \tau_{2D} \rangle \simeq \frac{R^2}{2D_2}\ln\left(\frac{R}{b}\right) \qquad R \gg b$$

$$\langle \tau_{1D} \rangle \simeq \frac{R^2}{3D_1}$$



Here D_n is the diffusion constants in n dimensions (cm^2/sec). If we assume that the magnitude of D does not vary much with n, the leading terms in these expressions are about equal, and the big differences are in the last factor

$$\left(\frac{R}{b}\right) > \ln\left(\frac{R}{b}\right) \gg 1$$

$$\langle \tau_{3D} \rangle > \langle \tau_{2D} \rangle \gg \langle \tau_{1D} \rangle$$

Based on the volume that needs searching, there can be a tremendous advantage to lowering the dimensionality.

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1. O. G. Berg and P. H. von Hippel, Diffusion-controlled macromolecular interactions, Annu. Rev. Biophys. Biophys. Chem. 14, 131-158 (1985); H. C. Berg and E. M. Purcell, Physics of chemoreception, Biophys. J. 20, 193-219 (1977).