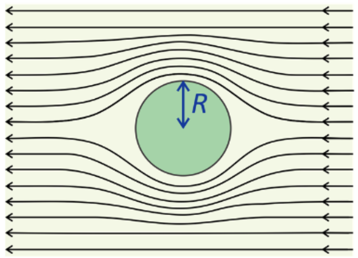
14.2: Stokes' Law

How is a fluid's macroscopic resistance to flow related to microscopic friction originating in random forces between the fluid's molecules? In discussing the Langevin equation, we noted that the friction coefficient ζ was the proportionality constant between the drag force experienced by an object and its velocity through the fluid: $f_d = -\zeta v$. Since this drag force is equal and opposite to the stress exerted on an object as it moves through a fluid, there is a relationship of the drag force to the fluid viscosity. Specifically, we can show that Einstein's friction coefficient ζ is related to the dynamic viscosity of the fluid η, as well as other factors describing the size and shape of the object (but not its mass).



This connection is possible as a result of George Stokes' description of the fluid velocity field around a sphere moving through a viscous fluid at a constant velocity. He considered asphere of radius R moving through a fluid with laminar flow: that in which the fluid exhibits smooth parallel velocity profiles without lateral mixing. Under those conditions, and no-slip boundary conditions, one finds that the drag force on a sphere is

$$f_d = 6\pi\eta R_h v$$

and viscous force per unit area is entirely uniform across the surface of the sphere. This gives us Stokes' Law

$$\zeta = 6\pi\eta R_h \tag{14.2.1}$$

Here Rh is referred to as the hydrodynamic radius of the sphere, the radius at which one can apply the no-slip boundary condition, but which on a molecular scale may include water that is strongly bound to the molecule. Combining eq. (1) with the Einstein formula for diffusion coefficient, $D = k_B T/\zeta$ gives the Stokes–Einstein relationship for the translation diffusion constant of a sphere[1]

$$D_{trans} = \frac{k_B T}{6\pi\eta R_h} \tag{14.2.2}$$

One can obtain a similar a Stokes–Einstein relationship for orientational diffusion of a sphere in a viscous fluid. Relating the orientational diffusion constant and the drag force that arises from resistance to shear, one obtains

$$D_{rot} = \frac{k_B T}{6V_h \eta}$$

1. B. J. Berne and R. Pecora, Dynamic Light Scattering: With Applications to Chemistry, Biology, and Physics. (Wiley, New York, 1976), pp. 78, 91.