Appendix A

The Bloch Equations

The following simplified treatment of Bloch's derivation of equations which define nuclear resonance line shapes is intended primarily to show the difference between nuclear resonance absorption and dispersion modes. As such, it should make clear the reasons for certain internal adjustments in the NMR probe which influence the shape of the signal curves.

We consider first the magnetic vector M which is the resultant sum of the magnetic vectors of the individual nuclei per unit volume. For the present purposes, we shall not have M collinear with any of the axes but assume it to have components along the axes of M,, M,, and M,, as shown in Fig. A-1. The Z axis (here the vertical direction) will be taken along the magnetic field axis, while the X axis will coincide with the axis of the oscillator coil, and the Y axis will coincide with the axis of the receiver coil. The vector M is subjected to the oscillator field, which as before (page 15) will be resolved into two vectors of length HI rotating at angular velocities o and in opposite directions in the X, Y plane with phase relationships such as to give no net field along Y. These contrarotating fields have components in the X and Y directions which are given by the equations

Clockwise rotation	Counterclockwise rotation
H, = HI cos wt	H, = HI cos ot
Hy = -HI sin cot	Hy = H1 sin wt

The sum of these fields gives Hz = 2H1 cos ot and H, = 0. We shall assume henceforth that only one of these rotating fields, here arbitrarily taken to have clockwise rotation, will influence the nuclei to change their m values (cf. Sec. 1-4).

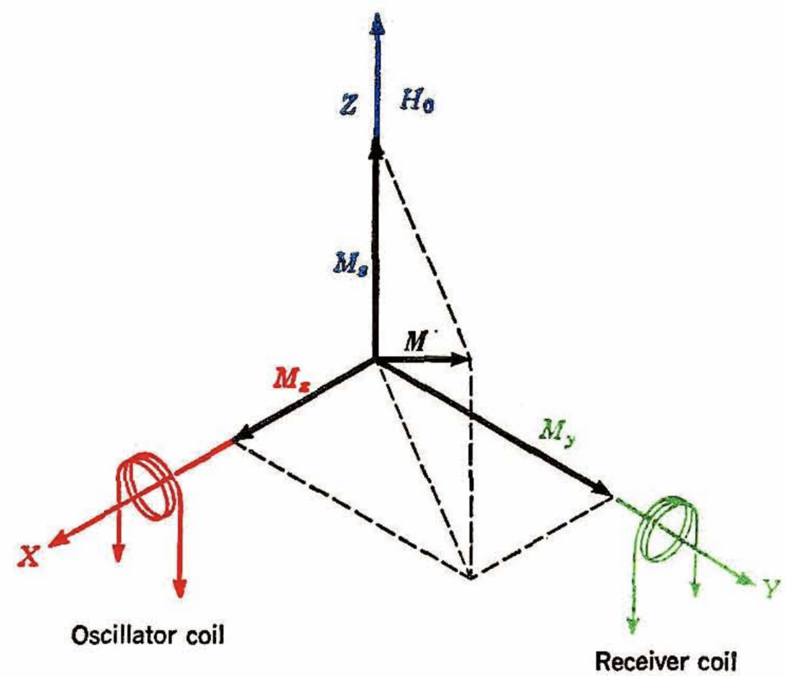


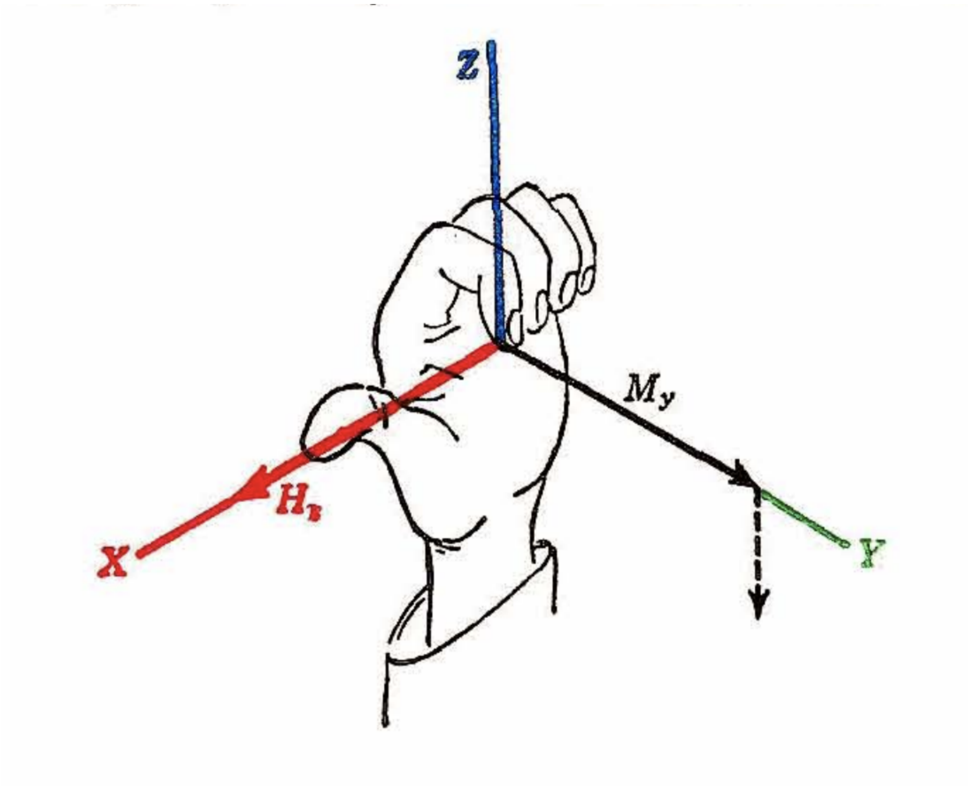
Fig. A-1. Cartesian coordinate system for consideration of interaction of oscillator field with vector of nuclear magnets. Note that the *Z* axis is taken vertically in contrast to the convention used in Chap. 1.

Let us consider possible changes of M,. In the first place, M, will tend to increase and approach its equilibrium value Mo by relaxation with the time constant T1 so that, if nothing else were to happen, we would write

$$\frac{dM_z}{dt} = \frac{1}{T_1}(M_0 - M_z)$$

latex later

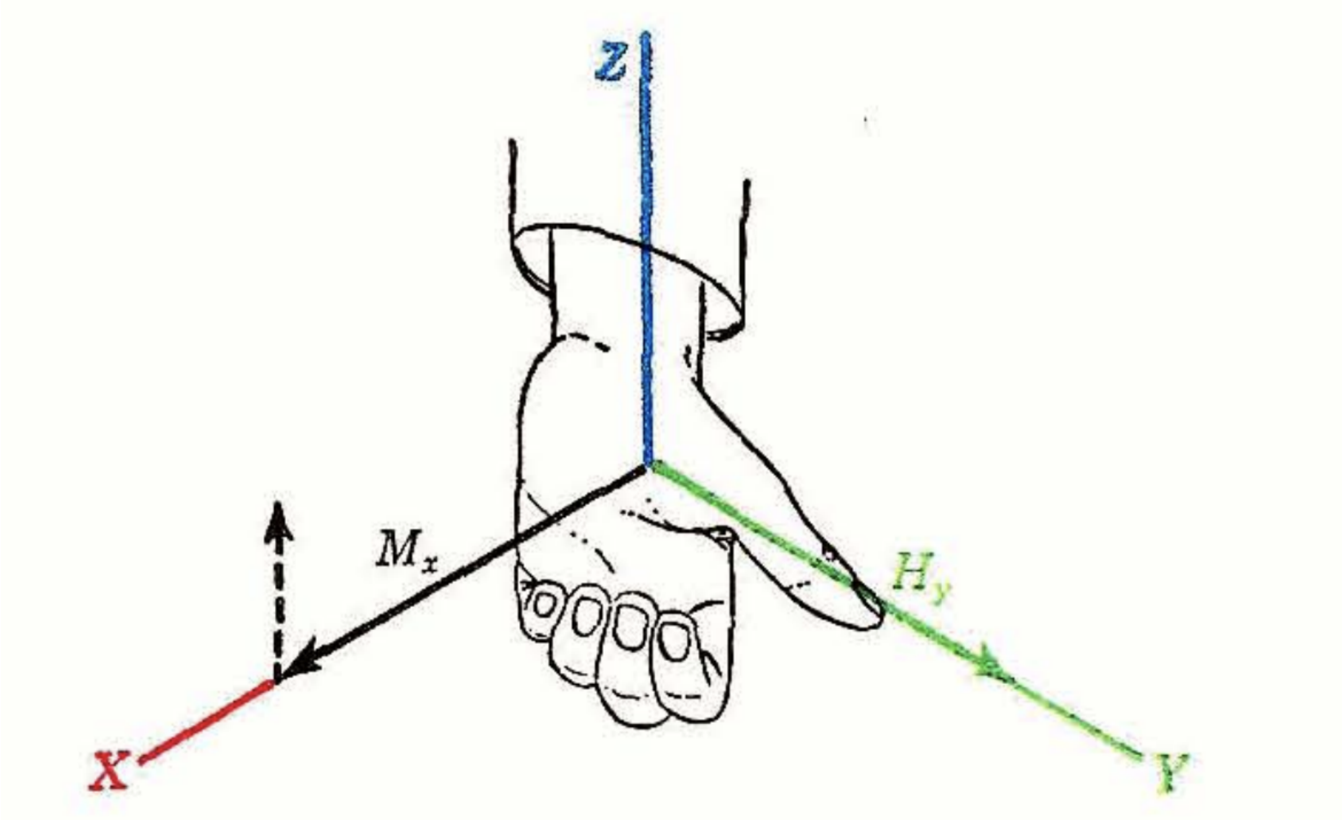
At the same time, M, will change by the action of H, and H, on the vectors M, and M, respectively. Consider first the action of H5 on M,.



Using a "left-hand rule," we can say that H, will cause the magnetization vector along Y to move downward and thus increase M, in the negative direction. This contribution to M, will be given by

$$\frac{dM_z}{dt} = -\gamma M_y H_x$$

Through proper choice of units the proportionality constant y can have the same numerical value as the nuclear gyromagnetic ratio because, in effect, H, causes the components of the individual nuclear vectors (which add to give the Y magnetization) to tend to precess around the X axis with an angular velocity of yH,.



H, produces a similar contribution with opposite sign to M, by making the Ma vector tip. We may then write

$$\frac{dM_z}{dt} = \frac{1}{T_1}(M_0 - M_z) - \gamma M_y H_x + \gamma M_x H_y$$

with opposite sign to M, by making the Ma vector tip. We may then write

$$\frac{dM_z}{dt} = \frac{1}{T_1}(M_0 - M_z) - \gamma(M_y H_1 \cos \omega t + M_x H_1 \sin \omega t)$$

Operating in the same way on Ma, we have

$$\frac{dM_x}{dt} = \gamma M_y H_0 - \gamma M_z H_y - \frac{M_x}{T_2}$$

where the first and second terms on the right correspond to the tipping of M, by Ho and M, by H,, respectively. The last term represents the first-order decay of M, with the time constant Tz. Similar treatment of M, and substitution of the values for H, and H, as a function of time afford the following equations, which in combination with the expression above for dM,ldt are called the Bloch equations:

$$\frac{dM_x}{dt} = \gamma M_y H_0 + \gamma M_z H_1 \sin \omega t - \frac{M_x}{T_2}$$

$$\frac{dM_y}{dt} = -\gamma M_x H_0 + \gamma M_z H_1 \cos \omega t - \frac{M_y}{T_2}$$

Consider now the projection of M on the X, Y plane M,,. Movement of M,, so as to produce a change in My will cause a current to be induced in the receiver coil mounted with its axis along Y. It is particularly useful to consider M,, to be made up of two magnetic components u and v which are in phase with HI and 90" out of phase with HI, respectively, so that M,, = u f iv. The components u and v can be defined by the equations

u = M, cos ot - M, sin ot

v = - (M, sin ot + My cos at)

and thence, in combination with the Bloch equations (and remembering that yHo = coo),

$$\frac{du}{dt} = -(\omega_0 - \omega)v - \frac{u}{T_2}$$

$$\frac{dv}{dt} = (\omega_0 - \omega)u - \frac{v}{T_2} - \gamma H_1 M_z$$

$$\frac{dM_z}{dt} = \frac{(M_0 - M_z)}{T_1} + \gamma H_1 v$$

The last equation is particularly significant, since it shows that the energy absorbed by the nuclei through changes in their magnetic quantum numbers with respect to Ho (cf. Sec. 1-4) is a function of -v and not of u. This means that one must measure -v if one desires a measure of the energy absorbed by the nuclei as a function of Ho at constant HI. However, the receiver responds to M, which is made up of both u and v, and our problem will be to show how a measure of v can be obtained independently of u.

We shall be particularly interested in the case where Ho is held constant and a steady signal is picked up in the receiver such as if the magnetic field sweep were stopped on the side or peak of a resonance signal. In these circumstances, M,, has a constant length and rotates around the Z axis at the frequency w. The steady-state condition requires that

$$\frac{du}{dt} = \frac{dv}{dt} = \frac{dM_z}{dt} = 0$$

With these conditions, it is easy to show that

$$u = -T_2(\omega_0 - \omega)v$$

(coo - o is a measure of how far we are off the peak of resonance.)

$$v = -\frac{\gamma T_2 H_1 M_z}{1 + T_2^2(\omega_0 - \omega)^2}$$

1 F. Bloch, Phys. Rev., 70, 460 (1946).