7.7: Ideal gas law

Combined relationships between pressure, volume, temperature, and amount of gases

The gas laws described above give the following relationships between volume V, pressure P, the temperature in kelvin T, and the amount of gas in moles n:

$V \propto 1/P$, at constant T and n,

$V \propto T$, at constant P and n, and

$V \propto n$, at constat T and P.

The three proportionalities combine to give the following ideal gas relationship:

$$V \propto \frac{nT}{P}$$

The proportionality changes to an equation by introducing a constant of proportionality:

$$V = \frac{nRT}{P}$$

, that rearranges to

$$PV = nRT$$

where R is the proportionality constant called the **gas constant**.

Ideal gas law

The equation: $PV = nRT$ is called the ideal gas law.

The value of R can be calculated by: $R = \frac{PV}{nT}$, where n is the quantity of gas in a mole, T is the temperature in kelvin, P is the pressure that can be in various units, and V is the volume that can be in various units. The value of R in different units of P, V, and PV products are given in Table 1. If values of any three among the P, V, n, and T are known, the value of the fourth one can be calculated by using the ideal gas law.

Caution

In these calculations, the units of R should be in agreement with the units of P, V, n, and T. If they are not in agreement, the given unit of P, V, n, and T must be converted to agree with the units of R.

Table 1: The numerical values of gas constant R in various units

Value	Units
0.08206	L-atm/mol-K
8.314	J/mol-K
1.987	cal/mol-K
8.314	m^3-Pa/mol-K
62.36	L-torr/mol-K

The ideal gas equation in a rearranged form is $\frac{PV}{nT} = R$ is a constant, that implies that:

$$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2} = R$$

If one or two parameters in the ideal gas equation $\frac{P_1V_1}{n_1T_1}=\frac{P_2V_2}{n_2T_2}$ are constant; they cancel out, leaving the relationship between the remaining parameters, e.g.

if n and T are constant: $P_1V_1=P_2V_2$, that is Boyle's law,

if P and n are constant: $\frac{V_1}{T_1}=\frac{V_2}{T_2}$, that is Charles's law,

if P and T are constant: $\frac{V_1}{n_1}=\frac{V_2}{n_2}$, that is Avogadro's law,

if V and n are constant: $\frac{P_1}{T_1}=\frac{P_2}{T_2}$, that is Gay Lussac's law,

if V and T are constant: $\frac{P_1}{n_1}=\frac{P_2}{n_2}$, that is pressure-mole relationship,

and if n is constant: $\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$, that is combined gas law.

Example 7.7.1

Calculate the volume of 1.000 mole of a gas in liters (L) at 0.000 °C and 1.000 atm?
Solution: Given n=1.000 mol, T= 0 °C+273.15=273.15 K, P=1.000 atm, and R=0.08206 L-atm/mol-K

Solution

Given n=1.000 mol, T= 0 °C+273.15=273.15 K, P=1.000 atm, and R=0.08206 L-atm/mol-K

Formula: $PV=nRT$, rearrange the formula to: $V=\frac{nRT}{P}$.

Plug in the values and calculate: $V=\frac{1.000\text{ mol}\times 0.08206\frac{\text{L-atm}}{\text{mol-K}}\times 273.15\text{ K}}{1.000\text{ atm}}=22.41\text{ L}$

Note

The volume of 1 mol of an ideal gas is 22.41 L at 0 °C and 1 atm pressure, as calculated in the above example

Example 7.7.2

Calculate the volume of a container that has 1.50 mol of He gas at 7.2 atm and 25 °C?

Solution

P=7.2 atm, V=? n=1.50 mol, T=25 °C}+273=298 K, R=0.08206 $\text{L}-\text{atm}/\text{mol}-\text{K}$

Formula: $PV=nRT$, rearrange the formula to: $V=\frac{nRT}{P}$

Plug in the values and calculate: $V=\frac{1.50\text{ mol}\times 0.08206\frac{\text{L-atm}}{\text{mol-}K}\times 298\text{ K}}{7.2\text{ atm}}=5.1\text{ L}$

Example 7.7.3

Calculate the pressure in a 5.1 L container that has 0.60 mol of He at 25 °C?

Solution

P = ?, V=5.1 L, n = 0.60 mol, T=25 °C+273=298 K, $R=0.08206\frac{\text{L-atm}}{\text{mol-}K}$

Formula: $PV=nRT$, rearrange the formula, plug in the values and calculate: $P=\frac{nRT}{V}=\frac{0.60\text{ mol}\times 0.08206\frac{\text{L-atm}}{\text{mol-}K}\times 298\text{ K}}{5.1\text{ L}}=2.9\text{ atm}$

Example 7.7.4

Calculate the pressure of 0.60 mol of He in example 3 mixed with 1.50 mol of He in example 2 in a container of 5.1 L volume at 25 ^{o}C?**Solution**

P_{total} = ?, n_{total} = 1.5 mol + 0.60 mol = 2.1 mol, V = 5.1 L, T = 25 ^{o}C + 273 = 298 K, and $R = 0.08206\frac{\text{L-atm}}{\text{mol}-K}$

Formula: $P_{\text{total}}V = n_{\text{total}}RT$, rearrange, plug in the values and calculate: $P_{\text{total}} = \frac{n_{\text{total}}RT}{V} = \frac{2.1\text{ mol}\times 0.08206\frac{L-atm}{mol-K}\times 298\text{ K}}{5.1\text{ L}} = 10.1\text{ atm}$

Note

The P_{total} of 2.1 mol He in example 4 is equal to P of 1.50 mole He in example 2+P of 0.6 mol He in example 3, i.e., 7.2 atm +2.9 atm =10.1 atm. This calculation demonstrates that when gases are mixed, the total pressure is the sum of the pressures that each fraction will exert if it was alone in that space. It is demonstrated by mixing the same gas, i.e., He with He, but it remains true when different gases are mixed, as long as all the gases involved obey the ideal gas law.

Molar volume of gases at standard temperature and pressure

The temperature of 0^{o}C and pressure of 1 atm is called **standard temperature and pressure (STP)** for gases.

Standard temperature and pressure

Currently accepted STP is 0^{o}C and 1 bar. The molar volume of an ideal gas at 0^{o}C and 1 bar is 22.71 L, but for most practical purposes, the older definition of STP of 0^{o}C and 1 atm is used.

The calculations in example 1 of the previous section show that the molar volume of an ideal gas is 22.41 L at STP. Fig. 7.7.1 illustrates the molar volume of an ideal gas at STP.

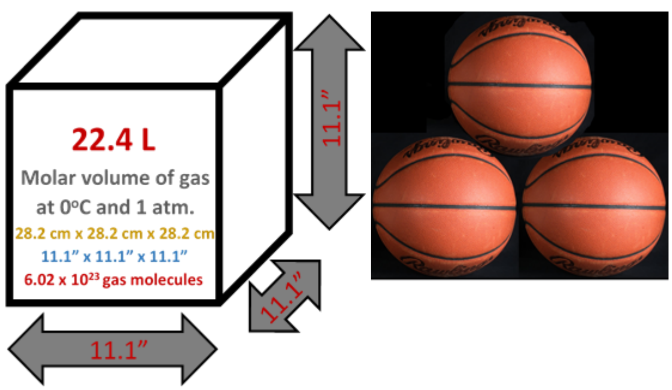

Figure 7.7.1: The molar volume of gases at STP is 22.4 L which is the volume of 11.1” x 11.1” x 11.1”, i.e., about the volume of three basketballs.

Fig. 7.7.2 shows that the molar volume of real gases is very close to that of the ideal gas. The small differences between the molar volume of real gases and the ideal gas are because ideal gas molecules are assumed to have negligible volume and negligible intermolecular interactions. The real gas molecules do have some volume and some intermolecular interactions that cause deviations of real gases from the ideal behavior. However, for practical purposes, the calculations based on ideal gas law remain applicable for the majority of real gases under ambient conditions.

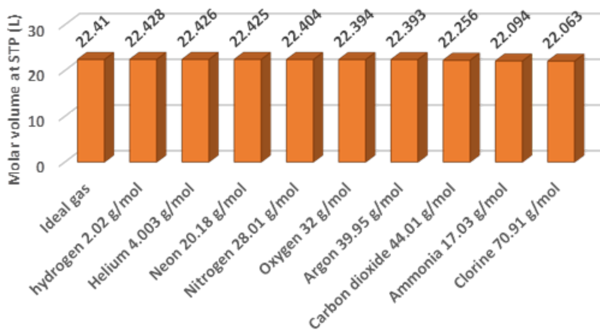

Figure 7.7.2: Comparison of molar volume of some real gases with an ideal gas at STP.

The molar volume of gases at STP is an equality between the number of moles and the volume of gas at STP, i.e.:

$$1 \text{ mol gas } = 22.4 \text{ L gas at STP}$$

The equality gives two conversion factors, i.e.,

$$\frac{1 \text{ mol gas}}{22.4 \text{ L gas}}, \text{ and } \frac{22.4 \text{ L gas}}{1 \text{ mol gas}}.$$

The conversion factors are used to convert volume to moles and mol to volume of gas, respectively, at STP.

Example 7.7.5

Calculate the volume of 64.0 g of oxygen at STP?

Solution

Given: mass of oxygen = 55.2 g. Required: volume of the oxygen =?

Strategy: 1st convert the grams of oxygen to moles of oxygen by using reciprocal of molar mass as a conversion factor and then convert the moles of oxygen to volume of oxygen by using 2nd conversion factor described above:

$$\text{volume of oxygen } = 64.0 \text{ g } \cancel{\text{oxygen}} \times \frac{1 \text{ mol } \cancel{\text{oxygen}}}{32 \text{ g of } \cancel{\text{oxygen}}} \times \frac{22.41 \text{ L}}{1 \text{ mol } \cancel{\text{oxygen}}} = 44.82 \text{ L oxygen}$$