1.5: Unit conversions

Conversion of values in one unit to the same value in another unit, as illustrated in Figure 1.5.1 is often needed in scientific calculations. The unit conversion includes the following.

- Conversion of the same type of measurement in the same system of measurement, e.g., conversion of a measured value of the length in meters to kilometers in SI;
- conversion of the same type of measurement in different systems of measurements, e.g., conversion of a measured value of the length in kilometers from SI to miles in the English system; and
- conversion of one type of measurement to another type of measurement, e.g., conversion of a measured value of the mass in g to volume in mL of a substance using the density of the substance.

Figure 1.5.1: Illustration of unit conversions. Source: https://www.hiclipart.com/free-trans...fksbi/download

Conversion factors

The conversion factors are derived from equality between the given unit and the desired unit. For example, 1 cm = 10^{-2} m is equality between centimeter and meter. The conversion factors are derived from the equality by the following steps.

Both sides of the equality are divided by one side to get one conversion factor. For example, $1=\frac{10^{-2}\ m}{1\ cm}$, which is a conversion factor for cm to m.

Then both sides are divided by the other side of the equality to get the second conversion factor. For example, $1=\frac{1\ cm}{10^{-2}\ m}$, which is a conversion factor for m to cm.

Since both the conversion factors are equal to one, multiplying a value with a conversion factor changes the number and the unit, such that the new number and the new unit together represent the same value. The unit of the given number should be opposite to the same unit in the conversion factor, i.e., numerator versus denominator or denominator versus numerator, to cancel them out and leave the desired unit in the answer. For example, 1.83m is converted to cm unit using the conversion factor $\frac{1\ cm}{10^{-2}\ m}$ as:

$$1.83\ \cancel{\mathrm{m}}\times\frac{1\ \mathrm{cm}}{10^{-2}\ \cancel{\mathrm{m}}}=183\ \mathrm{cm}$$

Keep track of the units that cancel out and the unit left in the answer. If all the units cancel out, leaving only the desired unit means the chosen conversion factor is correct. An incorrect conversion factor leads to an unwanted unit in the answer, e.g., in the above calculation if incorrect conversion factor is chose, it will lead to:

$$1.83\ m\times\frac{10^{-2}\ m}{1\ \mathrm{cm}}=0.0183\ \frac{\mathrm{m}^2}{\mathrm{cm}}$$

, where no unit is canceled and the answer has units that are not the desired. It means an incorrect conversion factor was employed.

Converting the same type of measurement in the same system

The prefixes in the SI, listed in section 1.3 Table 2, establish equality between the base and the prefixed units. For example, centi (c) means 10^{-2}. Therefore

$$1\ \mathrm{cm}=10^{-2}\ \mathrm{m}$$

1cm = 10^{-2} m is an equality that gives two conversion factors:

$$\frac{1\ cm}{10^{-2}\ m} \quad \text{and} \quad \frac{10^{-2}\ m}{1\ cm}$$

Some of the common qualities in SI are listed in Table 1 and the English system in Table 2.

Note

The prefixes are exact numbers. The equalities within the same system of measurement are exact numbers. Therefore the equalities and the conversion factors derived from them are exact numbers. Significant figures in the answers involving exact and inexact numbers are dictated by inexact numbers only.

Example 1.5.1

Convert 325 cm to m units?

Solution

$$325\ \cancel{cm} \times \frac{10^{-2}\ m}{1\ \cancel{cm}} = 3.25\ m$$

Note that in this example, the answer has the significant figures the same as in the given number because the conversion factor within the same system of measurement is numbers.

Table 1: Some of the common equalities in SI

Length	Mass	Volume
1 km = 1,000 m	1 kg = 1,000 g	1 L = 10 dL
1 m = 100 cm	1 g = 1,000 mg	1 L = 1,000 mL
1 m = 1,000 mm	1 mg = 1,000 µg	1 L = 1,000,000 µL
1 cm = 10 mm	10-1	1 dL = 100 mL
		1 mL = 1 cm^3 = 1 cc

Table 2: Some of the common equalities in the English System

Length	Mass	Volume
ft = 12 in.	1 lb = 16 oz	1 qt = 4 cups
1 yd = 3 ft	1 ton = 2,000 lb	1 qt = 2 pints
1 mi = 5,280 ft		1 qt = 32 fl oz
		1 gal = 4 qt

Conversion of the same type of measurement in different systems

Table 3 lists some common qualities between SI and the English systems.

These equalities between different systems usually have one side in the equality, which is the number 1, as exact, while the other side is considered an inexact number.

For example, 1 kg = 2.205 pounds (lb) has an exact number (1 kg) on the left side but an inexact number (2.205 lb) with 4 SFs on the right. Remember that only the inexact numbers dictate the significant figures in the answer.

There are some exceptions to the above general rule. Some equalities between units in different systems are defined and considered exact. They are stated to be exact in the reference tables.

For example, 1 inch = 2.54 cm is defined, which means both sides are the exact number.

Table 3: some of the common SI-to-English system equalities

Length	**Mass**	**Volume**
2.54 cm = 1 in. **defined and exact**	1 kg = 2.20 lb	946 mL = 1 qt
1 m = 39.4 in.	454 g = 1 lb	1 L = 1.06 qt
1 km = 0.621 mi	28.4 g = 1 oz	29.6 mL = 1 fl oz

Conversion of one type of measurement to another

Sometimes, equality between two different units is known under specific conditions. For example, **density (d)** which is mass (m) per unit volume (V), is a relationship between mass and volume of a given substance, i.e., $d = \frac{m}{v}$. The density is a conversion factor used to convert the volume to the mass of a substance. Reciprocal density, i.e., $\frac{1}{d} = \frac{v}{m}$, is the second conversion factor that converts the mass to the volume of the substance.

Example 1.5.2

The density of ethanol at 20 ^{o}C is 0.7893 g/mL; what is the mass of 10.0 mL of ethanol?

Solution

Multiply the given volume with the conversion factor that has volume in the denumerator, i.e., \(frac{m}{v} to get the mass desired.

$$10.0 \ \cancel{\text{mL}} \times \frac{0.7893 \text{ g}}{1 \ \cancel{\text{mL}}} = 7.89 \text{ g}$$

Note that 1mL is exact, and 10.0 and 0.7893 are inexact numbers with 3SF and 4 SF, respectively. The answer has 3 SFs.

Example 1.5.3

The density of gold is 19.30 g/mL; what is the volume of 10.123 g of gold?

Solution

Multiply the given mass with the conversion factor that has a mass in the denumerator, i.e., $fracvm$ to get the mass desired.

$$10.123 \ \cancel{\text{g}} \times \frac{1 \text{ mL}}{19.30 \ \cancel{\text{g}}} = 0.5245 \text{ mL} \tag{1.5.1}$$

Conversion factors derived from chemical equations

Chemical equations show relationships or equalities between reactants and products measured in **moles**. Mole is a SI unit for the amount of substance. Mole equals 6.02 x 10^{23} particles (atoms or molecules) of the substance. Like dozen means 12 of something, mole means 6.02 x 10^{23} atoms of element or molecules/formula units for compounds in a chemical equation.

> For calculations in chemistry, the number of moles of a substance is considered equal to its coefficient in a balanced chemical equation.

For example:

$$2\,H_2 + O_2 \longrightarrow 2\,H_2O$$

gives the following equalities and their corresponding conversion factors:

1. equality: 1 mole O_2 = 2 moles H_2, conversion factors: $\frac{1 \text{ mol } O_2}{2 \text{ mol } H_2}$ and $\frac{2 \text{ mol } H_2}{1 \text{ mol } O_2}$
2. equality: 1 mole O_2 = 2 mole H_2O, conversion factors: $\frac{1 \text{ mol } O_2}{2 \text{ mol } H_2O}$ and $\frac{2 \text{ mol } H_2O}{1 \text{ mol } O_2}$ and

3. equality: 2 moles H_2 = 2 moles H_2O, conversion factors: $\frac{2\text{ mol }H_2}{2\text{ mol }H_2O}$ and $\frac{2\text{ mol }H_2O}{2\text{ mol }H_2}$.

The use of these conversion factors in the calculation is explained in the following example.

Example 1.5.4

If 5 moles of oxygen (O_2) is consumed, how many moles of water are produced by the chemical equation mentioned above?

Solution

Given: 5 mol O_2, Desired: ? mol H_2O

Multiply the given quantity with the conversion factor that has the given unit in denumerator and the desired unit in the numerator:

$$5.0\ \text{mole}\ \cancel{O_2} \times \frac{2\ \text{mole}\ H_2O}{1\ \text{mole}\ \cancel{O_2}} = 10\ \text{mole}\ H_2O$$

Unit conversion involving more than one conversion factor

Often, there is no direct conversion factor between the given unit and the desired unit. In this situation, convert the given unit to another unit that can, later on, be linked with the desired unit, as explained in the following examples.

Example 1.5.5

How many µg are in 10.0 mg?

Solution

Problems like this can be solved in two steps: i) by converting the given unit to the base unit and then, ii) converting the base unit to the desired unit:

$$10.0\ \cancel{mg} \times \frac{10^{-3}\ g}{1\ \cancel{mg}} = 1.00 \times 10^{-2}\ g$$

$$1.00 \times 10^{-2}\ \cancel{g} \times \frac{1\ \mu g}{10^{-6}\ \cancel{g}} = 1.00 \times 10^{4}\ \mu g$$

Note that the first conversion factor converts mg to g, and then the second conversion factor converts g to the desired unit µg. The same calculation can be done using two conversion factors in a rwo:

$$10.0\ \cancel{\text{mg}} \times \frac{10^{-3}\ \cancel{\text{g}}}{10\ \cancel{\text{mg}}} \times \frac{1\ \mu g}{10^{-6}\ \cancel{\text{g}}} = 1.00 \times 10^{4}\ \mu g$$

Example 1.5.6

What is 100. km/h speed in m/s, where h represents hours?

Solution

This problem asks to convert two units, i.e., km to m and h to s. First convert one unit and then follow on to convert the second unit as:

$$\frac{100.\ \cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{10^{3}\ \text{m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\ \text{s}} = 27.8\ \text{m/s}$$

Note that the first conversion factor converts km to m, and then two conversion factors are needed to convert h to s via min.

Example 1.5.7

A prescription says a dosage of 0.225 mg of Synthroid to be taken once a day. If tablets in stock contain 75 µg of Synthroid, how many tablets are needed per day?

Solution

The given mass is in mg, while the equality "1 tablet = 75 µg" takes mass in µg. First convert mg to base unit, i.e., g, then from g to needed unit, i.e., µg, and finally take appropriate conversion factor from the two given by the equality to convert µg to tablet, i.e., three conversion factors in a row:

$$0.225\ \cancel{\text{mg}} \times \frac{10^{-3}\ \cancel{\text{g}}}{1\ \cancel{\text{mg}}} \times \frac{1\ \cancel{\mu\text{g}}}{10^{-6}\ \cancel{\text{g}}} \times \frac{1\ \text{tablet}}{75\ \cancel{\mu\text{g}}} = 3.0\ \text{tablet}$$

Example 1.5.8

A healthy person has 16% body fat by mass. Calculate the mass of fat in kg of a person who weighs 180. lb?

Solution

Given: mass of a person = 180. lb, Desired: mass of body fat in kg.

16% body fat by mass means: 16 lb body fat = 100 lb body mass, and the equality kg and lb is: 1 kg = 2.20 lb. Take one conversion factor from each equality such that the units cancel out leaving the desired unit in the answer:

$$180.\ \cancel{\text{lb body mass}} \times \frac{16\ \cancel{\text{lb body fat}}}{100\ \cancel{\text{lb body mass}}} \times \frac{1\ \text{kg body fat}}{2.20\ \cancel{\text{lb body fat}}} = 16\ \text{kg body fat}$$

Note that there are three inexact numbers in the calculation, i.e., 180, 16, and 2.20, and the answer as two significant figures in agreement with the smallest significant figure among the inexact numbers.