7.5: The combined gas law

The laws relating to pressure P, volume V, and temperature T for a constant amount n of a gas are the following:

1. If n and T are constant: $P_1V_1 = P_2V_2$, that is Boyle's law.
2. If P and n are constant: $\frac{V_1}{T_1} = \frac{V_2}{T_2}$, that is Charles's law.
3. If V and n are constant: $\frac{P_1}{T_1} = \frac{P_2}{T_2}$, that is Gay Lussac's law.

All three relationships are combined in the following law.

Combined gas law

If n is constant:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

, that is the combined gas law.

The combined gas law allows calculating the effect of varying two parameters on the third.

Example 7.5.1

A weather balloon contains $212\ \mathrm{L}$ of helium at $25^\circ\mathrm{C}$ and $750\ \mathrm{mmHg}$. What is the volume of the balloon when it ascends to an altitude where the temperature is $-40^\circ\mathrm{C}$ and $540\ \mathrm{mmHg}$, assuming the quantity of gas remains the same?

Solution

Given and desired parameters (temperatures must be converted to Kelvin scale):

$$\begin{aligned} &\mathrm{P_1} = 750\ \mathrm{mmHg}, \quad &&\mathrm{V_1} = 212\ \mathrm{L}, \quad &&\mathrm{T_1} = 25^\circ\mathrm{C} + 273.15 = 298.15\ \mathrm{K} \\ &\mathrm{P_2} = 540\ \mathrm{mmHg}, \quad &&\mathrm{V_2} = ? \quad &&\mathrm{T_2} = -40^\circ\mathrm{C} + 273.15 = 233.15\ \mathrm{K} \end{aligned}$$

Formula:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2},$$

rearrange the formula to isolate the desired parameter:

$$V_2 = \frac{P_1V_1T_2}{T_1P_2}.$$

Calculations:

$$V_2 = \frac{750\ \cancel{\mathrm{mmHg}} \times 212\ \mathrm{L} \times 233.15\ \cancel{\mathrm{K}}}{298.15\ \cancel{\mathrm{K}} \times 540\ \cancel{\mathrm{mmHg}}} = 230\ \mathrm{L}.$$