14.4: Spin Angular Momentum

You should recall from your many other classes that electrons are “spin one-half” and can either be spin up or spin down. You applied these facts when creating the electron configuration of transition metals in your inorganic chemistry class; an example for the 16-electron tetrahedral compound nickel carbonyl is shown in Figure 14.9. Why only spin up or down? Because, if the angular momentum of an electron is $l = \frac{1}{2}$, then the allowed m_l values are $+\frac{1}{2}$ and $-\frac{1}{2}$. More important is to understand what spin angular momentum is. While it is tempting to state that the electron is spinning on its axis like the earth, akin to the type of angular motion that was discussed in the previous sections, unfortunately this is not the case. The reason is that there are many other subatomic particles that have the same half spin momentum, including the proton, neutron, and quarks. Given that “normal” angular momentum is proportional to mass, and a proton is $\sim 2000\times$ heavier than an electron, how could they possibly have the exact same spin from classical rotational motion? The conclusion is that spin is a type of angular momemtnum, but it doesn’t originate from the type of rotational motion that we are accumtomed to.

Unfortunately no one really knows what spin angular momentum is other than its existence can be demonstrated by integrating Einstein’s theory of relativity with quantum mechanics. As a result, it is considered important to distinguish spin angular momentum from “normal” orbital angular momentum discussed here. To do so, spin angular momentum is designated with “s” as the quantum number (similar to l for orbital angular momentum). There is a spin angular momentum operator $\hat{S}$ that, when applied to the spin wavefunction $\psi = \alpha$ or $\psi = \beta$ yields:

$$\hat{S}^2 \alpha = s(s+1)\hbar^2 \cdot \alpha$$

likewise $\hat{S}^2 \beta = s(s+1)\hbar^2 \cdot \beta$. Thus, the total angular momentum is $\sqrt{s(s+1)}\,\hbar = \sqrt{\frac{3}{4}}\hbar$ for either the α or β state. There are sub-m_S states designated with the s_z quantum number, which is known as the “magnetic spin”. This name comes about because the magnetic properties of electrons are due to spin, which is the source of the magnetism used in power plant turbine electrical generators. The magnetic spin operator $\hat{S}_z$ has the following properties when applied to the spin α and β wavefunctions:

$$\hat{S}_z \alpha = s_z \cdot \alpha = ½\hbar \cdot \alpha and\, \hat{S}_z \beta = s_z \cdot \beta = -½\hbar \cdot \beta$$

which allows us to distinguish them. The names “up” for α and “down” for β came about because, if you accelerate an electron in a magnetic field, you will see the electrons either deflect upwards or downwards depending on the magnetic spin quantum number. Spin is also the source of magnetism of protons and neutrons that makes certain elements and isotopes NMR active. This type of nuclear magnetism is however much weaker than the fields created by electrons.

The wavefunctions are orthonormal: $\int \alpha^* \alpha = \int \beta^* \beta = 1$ while $\int \alpha \beta = 0$, which is important when we discuss the hydrogen atom next chapter. At this point you are probably expecting to see some kind of function for the α and β states, maybe a complex exponential or a sine or cosine; however, we don’t really know what spin is so there is no mathematical definition for it as a result. What the spin wavefunctions allow us to do is to satisfy the Pauli principle, which you may recall is why you fill atomic orbitals with spin up electrons first and then spin down. The Pauli principle states that no two electrons can be in the same state at the same time, and the fact that electrons have spin is why you can have two occupy the same atomic orbital but not violate that rule at the same time.