# 15.1: The Bohr Model

In 1913 Niels Bohr proposed a model for the hydrogen atom where the electron only exists in certain regions of space as it circulates around the nucleus (a single proton) as cartoonishly depicted in Figure 15.1B. He partially incorporated quantum theory by assumed that the orbiting electron can only have discrete values for the angular momentum. To model this behavior the angular momentum ($mv \cdot r$) was assumed to take integer values of $\hbar$, which is the reduced Planck constant (h/2$\pi$):

$$mv \cdot r = n \cdot \hbar$$

where n=1, 2, 3, etc. This means that velocity must be quantized: $v = \frac{n\hbar}{m \cdot r}$ . Next, Bohr conjectured that the "outward" centripetal force: $\frac{mv^2}{r}$ matches the "inward" Coulomb attraction force:

$$\frac{mv^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

According to the above $v = \sqrt{\frac{e^2}{4\pi\varepsilon_0 \cdot m \cdot r}}$ , which must be equal to our previous expression velocity:

$$\frac{n\hbar}{m \cdot r} = \sqrt{\frac{e^2}{4\pi\varepsilon_0 \cdot m \cdot r}}$$

This allows us to solve for the electron's radius: $r = \frac{4\pi\varepsilon_0 \cdot n^2\hbar^2}{m \cdot e^2}$, which is a function of the integer n. If n = 1 the radius is: $r = \frac{4\pi\varepsilon_0\hbar^2}{m \cdot e^2}$ , which is the famous Bohr unit of length $a_0$ = 0.053$\times$10$^{-9}$ m.

The energy can be calculated by adding the kinetic and potential from the electrostatic attraction: $\frac{1}{2}mv^2 - \frac{e^2}{4\pi\varepsilon_0 r}$ , where the Coulomb energy is negative since the electron and proton have opposite charges. Since both velocity and radius are quantized, Bohr was able to show that the same is true for the energy levels:

$$E = -\frac{e^4 m}{32\pi^2\varepsilon_0^2 \cdot \hbar^2} \cdot \frac{1}{n^2}$$

Inserting n = 1 gives the ground state energy: $-\frac{m \cdot e^4}{32\pi^2\varepsilon_0^2 \cdot \hbar^2} = -13.6\ eV$, which reveals how much energy has to be injected into the atom to fully remove the electron from the proton. And while this is the same value as measured experimentally, there are two problems with the model. For one, it doesn't explain the experimental observations that the spectra change under an applied magnetic field. Also, the model only works for atoms with one electron. More importantly, if an electron is circulating about a fixed point then it should emit electromagnetic waves; this is how a microwave oven heats your leftovers. If so, the electron eventually loses all its energy and crashes into the nucleus, and poof no more atom! Obviously, this doesn't happen. Consequently, the Bohr model was rejected which brings us to 1926 when Schrödinger formulated the hydrogen atom's Hamiltonian and used an eigenvalue equation approach to solve it.