

22.2: Essential Mathematics

22.2.1: Exponential Arithmetic

Exponential notation is used to express very large and very small numbers as a product of two numbers. The first number of the product, the *digit term*, is usually a number not less than 1 and not greater than 10. The second number of the product, the *exponential term*, is written as 10 with an exponent. Some examples of exponential notation are:

$$\begin{aligned}1000 &= 1 \times 10^3 \\100 &= 1 \times 10^2 \\10 &= 1 \times 10^1 \\1 &= 1 \times 10^0 \\0.1 &= 1 \times 10^{-1} \\0.001 &= 1 \times 10^{-3} \\2386 &= 2.386 \times 1000 = 2.386 \times 10^3 \\0.123 &= 1.23 \times 0.1 = 1.23 \times 10^{-1}\end{aligned}$$

The power (exponent) of 10 is equal to the number of places the decimal is shifted to give the digit number. The exponential method is particularly useful notation for every large and very small numbers. For example, $1,230,000,000 = 1.23 \times 10^9$, and $0.00000000036 = 3.6 \times 10^{-10}$.

22.2.1.1: Addition of Exponentials

Convert all numbers to the same power of 10, add the digit terms of the numbers, and if appropriate, convert the digit term back to a number between 1 and 10 by adjusting the exponential term.

✓ Exercise 22.2.1: Adding Exponentials

Add 5.00×10^{-5} and 3.00×10^{-3} .

Solution

$$\begin{aligned}3.00 \times 10^{-3} &= 300 \times 10^{-5} \\(5.00 \times 10^{-5}) + (300 \times 10^{-5}) &= 305 \times 10^{-5} = 3.05 \times 10^{-3}\end{aligned}$$

22.2.1.2: Subtraction of Exponentials

Convert all numbers to the same power of 10, take the difference of the digit terms, and if appropriate, convert the digit term back to a number between 1 and 10 by adjusting the exponential term.

✓ Exercise 22.2.2: Subtracting Exponentials

Subtract 4.0×10^{-7} from 5.0×10^{-6} .

Solution

$$\begin{aligned}4.0 \times 10^{-7} &= 0.40 \times 10^{-6} \\(5.0 \times 10^{-6}) - (0.40 \times 10^{-6}) &= 4.6 \times 10^{-6}\end{aligned}$$

22.2.1.3: Multiplication of Exponentials

Multiply the digit terms in the usual way and add the exponents of the exponential terms.

✓ Exercise 22.2.3: Multiplying Exponentials

Multiply 4.2×10^{-8} by 2.0×10^3 .

Solution

$$(4.2 \times 10^{-8}) \times (2.0 \times 10^3) = (4.2 \times 2.0) \times 10^{(-8)+(3)} = 8.4 \times 10^{-5}$$

22.2.1.4: Division of Exponentials

Divide the digit term of the numerator by the digit term of the denominator and subtract the exponents of the exponential terms.

✓ Exercise 22.2.4: Dividing Exponentials

Divide 3.6×10^5 by 6.0×10^{-4} .

Solution

$$\frac{3.6 \times 10^5}{6.0 \times 10^{-4}} = \left(\frac{3.6}{6.0} \right) \times 10^{(-5)-(-4)} = 0.60 \times 10^{-1} = 6.0 \times 10^{-2}$$

22.2.1.5: Squaring of Exponentials

Square the digit term in the usual way and multiply the exponent of the exponential term by 2.

✓ Exercise 22.2.5: Squaring Exponentials

Square the number 4.0×10^{-6} .

Solution

$$(4.0 \times 10^{-6})^2 = 4 \times 4 \times 10^{2 \times (-6)} = 16 \times 10^{-12} = 1.6 \times 10^{-11}$$

22.2.1.6: Cubing of Exponentials

Cube the digit term in the usual way and multiply the exponent of the exponential term by 3.

✓ Exercise 22.2.6: Cubing Exponentials

Cube the number 2×10^4 .

Solution

$$(2 \times 10^4)^3 = 2 \times 2 \times 2 \times 10^{3 \times 4} = 8 \times 10^{12}$$

22.2.1.7: Taking Square Roots of Exponentials

If necessary, decrease or increase the exponential term so that the power of 10 is evenly divisible by 2. Extract the square root of the digit term and divide the exponential term by 2.

✓ Exercise 22.2.7: Finding the Square Root of Exponentials

Find the square root of 1.6×10^{-7} .

Solution

$$\begin{aligned} 1.6 \times 10^{-7} &= 16 \times 10^{-8} \\ \sqrt{16 \times 10^{-8}} &= \sqrt{16} \times \sqrt{10^{-8}} = \sqrt{16} \times 10^{-\frac{8}{2}} = 4.0 \times 10^{-4} \end{aligned}$$

22.2.2: Significant Figures

A beekeeper reports that he has 525,341 bees. The last three figures of the number are obviously inaccurate, for during the time the keeper was counting the bees, some of them died and others hatched; this makes it quite difficult to determine the exact number of bees. It would have been more accurate if the beekeeper had reported the number 525,000. In other words, the last three figures are not significant, except to set the position of the decimal point. Their exact values have no meaning useful in this situation. In reporting any information as numbers, use only as many significant figures as the accuracy of the measurement warrants.

The importance of significant figures lies in their application to fundamental computation. In addition and subtraction, the sum or difference should contain as many digits to the right of the decimal as that in the least certain of the numbers used in the

computation (indicated by underscoring in the following example).

✓ Exercise 22.2.8: Addition and Subtraction with Significant Figures

Add 4.383 g and 0.0023 g.

Solution

$$\begin{array}{r} 4.383 \text{ g} \\ 0.0023 \text{ g} \\ \hline 4.385 \text{ g} \end{array}$$

In multiplication and division, the product or quotient should contain no more digits than that in the factor containing the least number of significant figures.

✓ Exercise 22.2.9: Multiplication and Division with Significant Figures

Multiply 0.6238 by 6.6.

Solution

$$0.6238 \times 6.6 = 4.1$$

When rounding numbers, increase the retained digit by 1 if it is followed by a number larger than 5 (“round up”). Do not change the retained digit if the digits that follow are less than 5 (“round down”). If the retained digit is followed by 5, round up if the retained digit is odd, or round down if it is even (after rounding, the retained digit will thus always be even).

22.2.3: The Use of Logarithms and Exponential Numbers

The common logarithm of a number (log) is the power to which 10 must be raised to equal that number. For example, the common logarithm of 100 is 2, because 10 must be raised to the second power to equal 100. Additional examples follow.

Logarithms and Exponential Numbers

Number	Number Expressed Exponentially	Common Logarithm
1000	10^3	3
10	10^1	1
1	10^0	0
0.1	10^{-1}	-1
0.001	10^{-3}	-3

What is the common logarithm of 60? Because 60 lies between 10 and 100, which have logarithms of 1 and 2, respectively, the logarithm of 60 is 1.7782; that is,

$$60 = 10^{1.7782}$$

The common logarithm of a number less than 1 has a negative value. The logarithm of 0.03918 is -1.4069, or

$$0.03918 = 10^{-1.4069} = \frac{1}{10^{1.4069}}$$

To obtain the common logarithm of a number, use the *log* button on your calculator. To calculate a number from its logarithm, take the inverse log of the logarithm, or calculate 10^x (where x is the logarithm of the number).

The natural logarithm of a number (ln) is the power to which e must be raised to equal the number; e is the constant 2.7182818. For example, the natural logarithm of 10 is 2.303; that is,

$$10 = e^{2.303} = 2.7182818^{2.303}$$

To obtain the natural logarithm of a number, use the \ln button on your calculator. To calculate a number from its natural logarithm, enter the natural logarithm and take the inverse \ln of the natural logarithm, or calculate e^x (where x is the natural logarithm of the number).

Logarithms are exponents; thus, operations involving logarithms follow the same rules as operations involving exponents.

1. The logarithm of a product of two numbers is the sum of the logarithms of the two numbers. $\log xy = \log x + \log y$ and $\ln xy = \ln x + \ln y$
2. The logarithm of the number resulting from the division of two numbers is the difference between the logarithms of the two numbers. $\log \frac{x}{y} = \log x - \log y$ and $\ln \frac{x}{y} = \ln x - \ln y$
3. The logarithm of a number raised to an exponent is the product of the exponent and the logarithm of the number.

$$\log x^n = n \log x \text{ and } \ln x^n = n \ln x$$

22.2.4: The Solution of Quadratic Equations

Mathematical functions of this form are known as second-order polynomials or, more commonly, quadratic functions.

$$ax^2 + bx + c = 0$$

The solution or roots for any quadratic equation can be calculated using the following formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solving Quadratic Equations Solve the quadratic equation $3x^2 + 13x - 10 = 0$.

Solution Substituting the values $a = 3$, $b = 13$, $c = -10$ in the formula, we obtain

$$x = \frac{-13 \pm \sqrt{(13)^2 - 4 \times 3 \times (-10)}}{2 \times 3}$$

$$x = \frac{-13 \pm \sqrt{169 + 120}}{6} = \frac{-13 \pm \sqrt{289}}{6} = \frac{-13 \pm 17}{6}$$

The two roots are therefore

$$x = \frac{-13 + 17}{6} = \frac{2}{3} \text{ and } x = \frac{-13 - 17}{6} = -5$$

Quadratic equations constructed on physical data always have real roots, and of these real roots, often only those having positive values are of any significance.

22.2.5: Two-Dimensional (x-y) Graphing

The relationship between any two properties of a system can be represented graphically by a two-dimensional data plot. Such a graph has two axes: a horizontal one corresponding to the independent variable, or the variable whose value is being controlled (x), and a vertical axis corresponding to the dependent variable, or the variable whose value is being observed or measured (y).

When the value of y is changing as a function of x (that is, different values of x correspond to different values of y), a graph of this change can be plotted or sketched. The graph can be produced by using specific values for (x, y) data pairs.

✓ Exercise 22.2.10

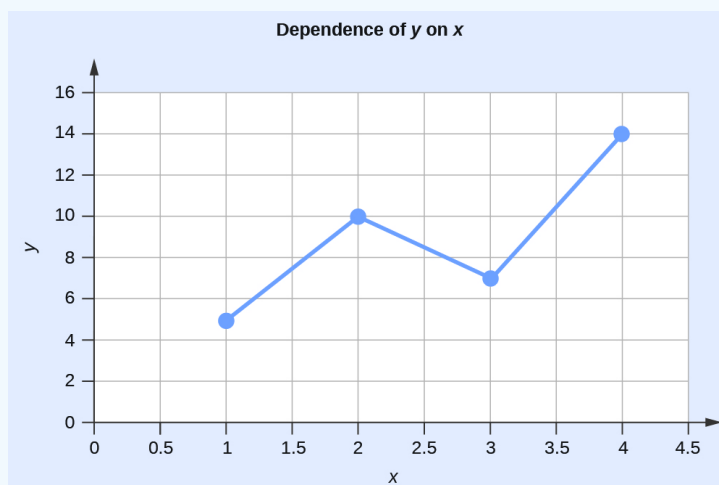
Graphing the Dependence of y on x

Table for Exercise 10

x	y
1	5
2	10
3	7

x	y
4	14

This table contains the following points: (1,5), (2,10), (3,7), and (4,14). Each of these points can be plotted on a graph and connected to produce a graphical representation of the dependence of y on x.



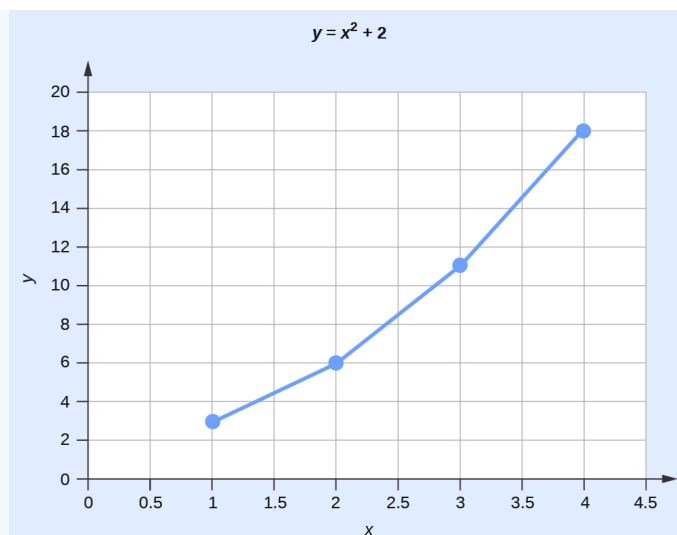
If the function that describes the dependence of y on x is known, it may be used to compute x,y data pairs that may subsequently be plotted.

✓ Exercise 22.2.11

Plotting Data Pairs If we know that $y = x^2 + 2$, we can produce a table of a few (x,y) values and then plot the line based on the data shown here.

Table for Exercise 11

x	$y = x^2 + 2$
1	3
2	6
3	11
4	18



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