6.2: The Bohr Model

Learning Objectives

- Describe the Bohr model of the hydrogen atom
- Use the Rydberg equation to calculate energies of light emitted or absorbed by hydrogen atoms

Following the work of Ernest Rutherford and his colleagues in the early twentieth century, the picture of atoms consisting of tiny dense nuclei surrounded by lighter and even tinier electrons continually moving about the nucleus was well established. This picture was called the planetary model, since it pictured the atom as a miniature "solar system" with the electrons orbiting the nucleus like planets orbiting the sun. The simplest atom is hydrogen, consisting of a single proton as the nucleus about which a single electron moves. The electrostatic force attracting the electron to the proton depends only on the distance between the two particles.

$$F_{gravity} = G\frac{m_1 m_2}{r^2}$$

with

- G is a gravitational constant
- m_1 and m_2 are the masses of particle 1 and 2, respectively
- r is the distance between the two particles

The electrostatic force has the same form as the gravitational force between two mass particles except that the electrostatic force depends on the magnitudes of the charges on the particles (+1 for the proton and −1 for the electron) instead of the magnitudes of the particle masses that govern the gravitational force.

$$F_{electrostatic} = k\frac{m_1 m_2}{r^2}$$

with

- k is a constant
- m_1 and m_2 are the masses of particle 1 and 2, respectively
- r is the distance between the two particles

Since forces can be derived from potentials, it is convenient to work with potentials instead, since they are forms of energy. The electrostatic potential is also called the *Coulomb potential*. Because the electrostatic potential has the same form as the gravitational potential, according to classical mechanics, the equations of motion should be similar, with the electron moving around the nucleus in circular or elliptical orbits (hence the label "planetary" model of the atom). Potentials of the form $V(r)$ that depend only on the radial distance r are known as central potentials. Central potentials have spherical symmetry, and so rather than specifying the position of the electron in the usual Cartesian coordinates (x, y, z), it is more convenient to use polar spherical coordinates centered at the nucleus, consisting of a linear coordinate r and two angular coordinates, usually specified by the Greek letters theta (θ) and phi (Φ). These coordinates are similar to the ones used in GPS devices and most smart phones that track positions on our (nearly) spherical earth, with the two angular coordinates specified by the latitude and longitude, and the linear coordinate specified by sea-level elevation. Because of the spherical symmetry of central potentials, the energy and angular momentum of the classical hydrogen atom are constants, and the orbits are constrained to lie in a plane like the planets orbiting the sun. This classical mechanics description of the atom is incomplete, however, since an electron moving in an elliptical orbit would be accelerating (by changing direction) and, according to classical electromagnetism, it should continuously emit electromagnetic radiation. This loss in orbital energy should result in the electron's orbit getting continually smaller until it spirals into the nucleus, implying that atoms are inherently unstable.

In 1913, Niels Bohr attempted to resolve the atomic paradox by ignoring classical electromagnetism's prediction that the orbiting electron in hydrogen would continuously emit light. Instead, he incorporated into the classical mechanics description of the atom Planck's ideas of quantization and Einstein's finding that light consists of photons whose energy is proportional to their frequency. Bohr assumed that the electron orbiting the nucleus would not normally emit any radiation (the stationary state hypothesis), but it would emit or absorb a photon if it moved to a different orbit. The energy absorbed or emitted would reflect differences in the orbital energies according to this equation:

$$|\Delta E| = |E_f - E_i| = hu = \frac{hc}{\lambda} \tag{6.2.1}$$

In this equation, h is Planck's constant and E_i and E_f are the initial and final orbital energies, respectively. The absolute value of the energy difference is used, since frequencies and wavelengths are always positive. Instead of allowing for continuous values for the angular momentum, energy, and orbit radius, Bohr assumed that only discrete values for these could occur (actually, quantizing any one of these would imply that the other two are also quantized). Bohr's expression for the quantized energies is:

$$E_n = -\frac{k}{n^2} \tag{6.2.2}$$

with $n = 1, 2, 3, \ldots$

In this expression, k is a constant comprising fundamental constants such as the electron mass and charge and Planck's constant. Inserting the expression for the orbit energies into the equation for ΔE gives

$$\Delta E = k\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \frac{hc}{\lambda} \tag{6.2.3}$$

or

$$\frac{1}{\lambda} = \frac{k}{hc}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \tag{6.2.4}$$

The lowest few energy levels are shown in Figure $6.2.1$. One of the fundamental laws of physics is that matter is most stable with the lowest possible energy. Thus, the electron in a hydrogen atom usually moves in the $n = 1$ orbit, the orbit in which it has the lowest energy. When the electron is in this lowest energy orbit, the atom is said to be in its ground electronic state (or simply ground state). If the atom receives energy from an outside source, it is possible for the electron to move to an orbit with a higher n value and the atom is now in an excited electronic state (or simply an excited state) with a higher energy. When an electron transitions from an excited state (higher energy orbit) to a less excited state, or ground state, the difference in energy is emitted as a photon. Similarly, if a photon is absorbed by an atom, the energy of the photon moves an electron from a lower energy orbit up to a more excited one.

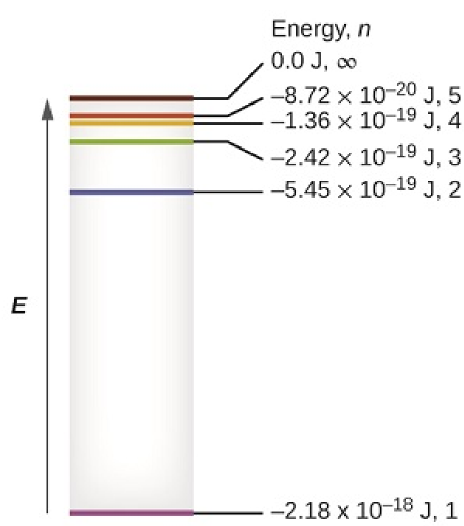

Figure 6.2.1: Quantum numbers and energy levels in a hydrogen atom. The more negative the calculated value, the lower the energy.

We can relate the energy of electrons in atoms to what we learned previously about energy. The law of conservation of energy says that we can neither create nor destroy energy. Thus, if a certain amount of external energy is required to excite an electron from one energy level to another, that same amount of energy will be liberated when the electron returns to its initial state (Figure $6.2.2$). In effect, an atom can "store" energy by using it to promote an electron to a state with a higher energy and release it when the electron returns to a lower state. The energy can be released as one quantum of energy, as the electron returns to its ground state (say, from

$n = 5$ to $n = 1$), or it can be released as two or more smaller quanta as the electron falls to an intermediate state, then to the ground state (say, from $n = 5$ to $n = 4$, emitting one quantum, then to $n = 1$, emitting a second quantum).

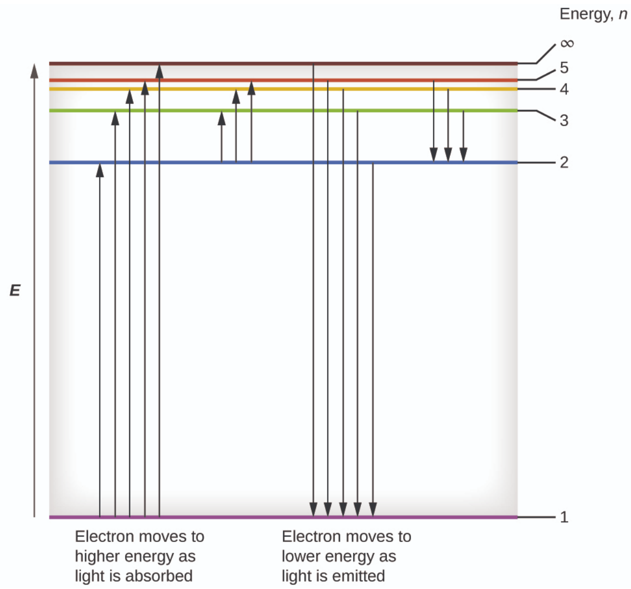

Figure 6.2.2: The horizontal lines show the relative energy of orbits in the Bohr model of the hydrogen atom, and the vertical arrows depict the energy of photons absorbed (left) or emitted (right) as electrons move between these orbits.

The figure includes a diagram representing the relative energy levels of the quantum numbers of the hydrogen atom. An upward pointing arrow at the left of the diagram is labeled, “E.” A grey shaded vertically oriented rectangle is placed just right of the arrow. The rectangle height matches the arrow length. Colored, horizontal line segments are placed inside the rectangle and labels are placed to the right of the box, arranged in a column with the heading, “Energy, n.” At the very base of the rectangle, a purple horizontal line segment is drawn. A black line extends to the right to the label, “1.” At a level approximately three-quarters of the distance to the top of the rectangle, a blue horizontal line segment is drawn. A black line extends to the right to the label, “2.” At a level approximately seven-eighths the distance from the base of the rectangle, a green horizontal line segment is drawn. A black line extends to the right to the label, “3.” Just a short distance above this segment, an orange horizontal line segment is drawn. A black line segment extends to the right to the label, “4.” Just above this segment, a red horizontal line segment is drawn. A black line extends to the right to the label, “5.” Just a short distance above this segment, a brown horizontal line segment is drawn. A black line extends to the right to the label, “infinity.” Arrows are drawn to depict energies of photons absorbed, as shown by upward pointing arrows on the left, or released as shown by downward pointing arrows on the right side of the diagram between the colored line segments. The label, “Electron moves to higher energy as light is absorbed,” is placed beneath the upward pointing arrows. Similarly, the label, “Electron moves to lower energy as light is emitted,” appears beneath the downward pointing arrows. Moving left to right across the diagram, arrows extend from one colored line segment to the next in the following order: purple to blue, purple to green, purple to orange, purple to red, purple to brown, blue to green, blue to orange, and blue to red. The arrows originating from the same colored segment are grouped together by close placement of the arrows. Similarly, the downward arrows follow in this sequence; brown to purple, red to purple, orange to purple, green to purple, blue to purple, red to blue, orange to blue, and green to blue. Arrows are again grouped by close placement according to the color at which the arrows end.

Since Bohr’s model involved only a single electron, it could also be applied to the single electron ions He^+, Li^{2+}, Be^{3+}, and so forth, which differ from hydrogen only in their nuclear charges, and so one-electron atoms and ions are collectively referred to as *hydrogen-like* or *hydrogenic* atoms. The energy expression for hydrogen-like atoms is a generalization of the hydrogen atom energy, in which Z is the nuclear charge (+1 for hydrogen, +2 for He, +3 for Li, and so on) and k has a value of $2.179 \times 10^{-18}\ J$.

$$E_n = -\frac{kZ^2}{n^2} \tag{6.2.5}$$

The sizes of the circular orbits for hydrogen-like atoms are given in terms of their radii by the following expression, in which a_o is a constant called the Bohr radius, with a value of $5.292 \times 10^{-11}\ m$:

$$r = \frac{n^2}{Z} a_0 \tag{6.2.6}$$

The equation also shows us that as the electron’s energy increases (as n increases), the electron is found at greater distances from the nucleus. This is implied by the inverse dependence on r in the Coulomb potential, since, as the electron moves away from the nucleus, the electrostatic attraction between it and the nucleus decreases, and it is held less tightly in the atom. Note that as n gets larger and the orbits get larger, their energies get closer to zero, and so the limits $n \longrightarrow \infty$ and $r \longrightarrow \infty$ imply that $E = 0$

corresponds to the ionization limit where the electron is completely removed from the nucleus. Thus, for hydrogen in the ground state $n = 1$, the ionization energy would be:

$$\Delta E = E_{n \longrightarrow \infty} - E_1 = 0 + k = k \tag{6.2.7}$$

With three extremely puzzling paradoxes now solved (blackbody radiation, the photoelectric effect, and the hydrogen atom), and all involving Planck's constant in a fundamental manner, it became clear to most physicists at that time that the classical theories that worked so well in the macroscopic world were fundamentally flawed and could not be extended down into the microscopic domain of atoms and molecules. Unfortunately, despite Bohr's remarkable achievement in deriving a theoretical expression for the Rydberg constant, he was unable to extend his theory to the next simplest atom, He, which only has two electrons. Bohr's model was severely flawed, since it was still based on the classical mechanics notion of precise orbits, a concept that was later found to be untenable in the microscopic domain, when a proper model of quantum mechanics was developed to supersede classical mechanics.

✓ Example 6.2.1: Calculating the Energy of an Electron in a Bohr Orbit

Early researchers were very excited when they were able to predict the energy of an electron at a particular distance from the nucleus in a hydrogen atom. If a spark promotes the electron in a hydrogen atom into an orbit with $n = 3$, what is the calculated energy, in joules, of the electron?

Solution

The energy of the electron is given by Equation 6.2.5:

$$E = \frac{-kZ^2}{n^2}$$

The atomic number, Z, of hydrogen is 1; $k = 2.179 \times 10^{-18}\ J$; and the electron is characterized by an n value of 3. Thus,

$$E = \frac{-(2.179 \times 10^{-18}\ J) \times (1)^2}{(3)^2} = -2.421 \times 10^{-19}\ J$$

? Exercise 6.2.1

The electron in Example 6.2.1 in the $n = 3$ state is promoted even further to an orbit with $n = 6$. What is its new energy?

Answer

TBD

✓ Example 6.2.2: Calculating Electron Transitions in a One–electron System

What is the energy (in joules) and the wavelength (in meters) of the line in the spectrum of hydrogen that represents the movement of an electron from Bohr orbit with n = 4 to the orbit with n = 6? In what part of the electromagnetic spectrum do we find this radiation?

Solution

In this case, the electron starts out with $n = 4$, so $n_1 = 4$. It comes to rest in the $n = 6$ orbit, so $n_2 = 6$. The difference in energy between the two states is given by this expression:

$$\Delta E = E_1 - E_2 = 2.179 \times 10^{-18} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$\Delta E = 2.179 \times 10^{-18} \left(\frac{1}{4^2} - \frac{1}{6^2} \right)\ J$$

$$\Delta E = 2.179 \times 10^{-18} \left(\frac{1}{16} - \frac{1}{36} \right)\ J$$

$$\Delta E = 7.566 \times 10^{-20}\ J$$

This energy difference is *positive*, indicating a photon enters the system (is absorbed) to excite the electron from the n = 4 orbit up to the $n = 6$ orbit. The wavelength of a photon with this energy is found by the expression $E = hc\lambda$. Rearrangement gives:

$$\lambda = \frac{hc}{E}$$

From the figure of electromagnetic radiation, we can see that this wavelength is found in the infrared portion of the electromagnetic spectrum.

Exercise 6.2.2

What is the energy in joules and the wavelength in meters of the photon produced when an electron falls from the $n = 5$ to the $n = 3$ level in a He^+ ion ($Z = 2$ for He^+)?

Answer

$6.198 \times 10^{-19}\ J$ and $3.205 \times 10^{-7}\ m$

Bohr's model of the hydrogen atom provides insight into the behavior of matter at the microscopic level, but it is does not account for electron–electron interactions in atoms with more than one electron. It does introduce several important features of all models used to describe the distribution of electrons in an atom. These features include the following:

- The energies of electrons (energy levels) in an atom are quantized, described by quantum numbers: integer numbers having only specific allowed value and used to characterize the arrangement of electrons in an atom.
- An electron's energy increases with increasing distance from the nucleus.
- The discrete energies (lines) in the spectra of the elements result from quantized electronic energies.

Of these features, the most important is the postulate of quantized energy levels for an electron in an atom. As a consequence, the model laid the foundation for the quantum mechanical model of the atom. Bohr won a Nobel Prize in Physics for his contributions to our understanding of the structure of atoms and how that is related to line spectra emissions.

Summary

Bohr incorporated Planck's and Einstein's quantization ideas into a model of the hydrogen atom that resolved the paradox of atom stability and discrete spectra. The Bohr model of the hydrogen atom explains the connection between the quantization of photons and the quantized emission from atoms. Bohr described the hydrogen atom in terms of an electron moving in a circular orbit about a nucleus. He postulated that the electron was restricted to certain orbits characterized by discrete energies. Transitions between these allowed orbits result in the absorption or emission of photons. When an electron moves from a higher-energy orbit to a more stable one, energy is emitted in the form of a photon. To move an electron from a stable orbit to a more excited one, a photon of energy must be absorbed. Using the Bohr model, we can calculate the energy of an electron and the radius of its orbit in any one-electron system.

Glossary

Bohr's model of the hydrogen atom

structural model in which an electron moves around the nucleus only in circular orbits, each with a specific allowed radius; the orbiting electron does not normally emit electromagnetic radiation, but does so when changing from one orbit to another.

excited state

state having an energy greater than the ground-state energy

ground state

state in which the electrons in an atom, ion, or molecule have the lowest energy possible

quantum number

integer number having only specific allowed values and used to characterize the arrangement of electrons in an atom

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