5.3: Chemical shift in units of Hz and ppm

This Chapter introduces the other most common unit to measure and report the NMR resonance frequency: *ppm*, parts-per-million. We will consider examples when the frequency units of *Hz* (1/*second*) are the most justified choice and the opposite cases- when ppm's should be used. We will also start describing quantitatively how raw NMR signal, $S(t)$, depend on time t, initial current S_0, and two properties of the target nucleus: resonance frequency Ω (or ν) and relaxation rate R.

> ### 🧠 Learning Objectives
>
> - Understand the dependence of frequency expressed in units of *Hz* from the instrumentation used (magnet strength)
> - Define the magnet strength- independent units of *ppm*, parts-per-million
> - Grasp the benefits and limitations of both types of frequency units, *Hz* and *ppm*, and learn when each one is the most appropriate
> - Get familiar with a mathematical description for an NMR signal S as a function of time t and three parameters: initial signal S_0, angular frequency Ω and relaxation rate R.
> - Appreciate why the angular resonance frequency Ω (or oscillatory frequency ν) can be expressed in units of both *Hz* and *ppm* while rates R mostly use *Hz* as units.

NMR Chemical Shift

Formula V.2.1, $\nu_{eff} = \gamma \cdot B_{eff} = \gamma \cdot (1-\sigma) \cdot B_o$, indicates that the resonance frequency of a target spin-½ nucleus is defined by its intrinsic magnetic shielding property σ and the externally generated magnetic field B_o. Whereas the value for σ for the same nucleus in the same sample will remain the same for any recording on any NMR instrument, the numerical value of B_o changes from magnet to magnet. Therefore, the numerical value of the resonance frequency ν_{eff} will change if we perform an NMR recording of the same sample on a different instrument/magnet. This is inconvenient as the same quantity ν_{eff} may have different values depending on the instrument used.

In order to avoid such a complication, resonance frequency is often expressed in NMR in relative term as a so-called chemical shift value δ with respect the to the resonance frequency of some standard chemical. Such a "standard" can be either added to the NMR sample as a separate chemical (e.g. DSS) or an existing chemical can be used (e.g. a solvent like H_2O, common in biochemistry). The numerical value for the chemical shift of a specific spin-½ nucleus (sample) δ_{sample} is expressed via ν_{ref} and ν_{sample} in units of parts-per-million (*ppm*) as follows:

Equation V.3.5.3.1

$$\delta_{sample} = \frac{\nu_{ref} - \nu_{samle}}{\nu_{ref}} \cdot 10^6 \, (ppm) \qquad (5.3.1)$$

Key Result: Because both values, ν_{ref} and ν_{sample}, are directly proportional to the magnet strength B_o, the chemical shift value δ_{sample} expressed as the ratio above in units of *ppm* is independent of B_o. In other words, the chemical shift expressed in units of *ppm* will have the *same numerical value* whether the sample is measured on a weaker or stronger magnet!

Please see how the chemical shift value can be calculate in **Example 1** below.

NMR Raw Signal: Free Induction Decay (FID), its Key Parameters and their Units

As figure **Figure V.2.1** shows in Chapter V.2, the NMR experiment produces its signal as an echo from the relaxation of the excited spin-½ system back to thermal equilibrium. This signal is registered as electric current generated in detection coils by the magnetic field oscillations caused by the spin-½ transitions between the two Zeeman split levels during relaxation.

$$S(t) = S_0 \times e^{(-R \times t)} \times e^{(i \times \Omega \times t)}$$
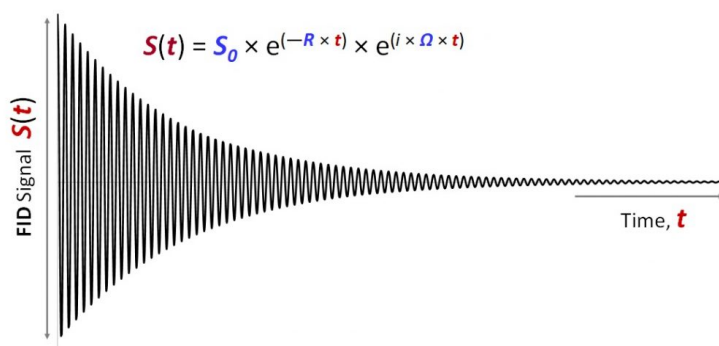
Figure V.3.1 Free induction decay (FID): the raw NMR signal detected as electric current in detection coils positioned near the sample. The electric current signal, $S(t)$, depends on time t and three parameters written in blue: S_0 (initial current, $S(t=0)$), relaxation rate R and angular resonance frequency Ω.

Figure V.3.A above shows a typical shape of the FID signal (current) vs. time: it is a harmonically oscillating, exponentially decaying curve. The signal intensity depends on time approximately as follows:

Equation V.3.5.3.2

$$S(t) = S_0 \cdot e^{-R \cdot t} \cdot e^{i \cdot \Omega \cdot t} \tag{5.3.2}$$

Such a signal would be produced by a single type of a spin-½ nucleus, that is by a specific proton in our sample biomolecule (e.g. by $^1H_\alpha$ of Pro108, or $^{13}C_\beta$ in Ala35 etc.). Parameter Ω is the angular resonance frequency of this nucleus $\Omega = 2\pi v$, where v is the effective oscillatory frequency of this nucleus (v_{eff}) described in Chapter V.2. Angular frequency Ω is often used in NMR theory for rather technical reasons, but essentially it is linear resonance frequency v scaled by factor 2π. Parameter R in the equation above specifies the rate of relaxation from the excited state to thermal equilibrium forthis particular spin. Just like the resonance frequency Ω, relaxation rate R is specific to each nucleus as it depends on the local covalent and non-covalent environment of the nucleus (and not so much on the external magnetic field B_o). Lastly, factor S_0 corresponds to the initial signal (detected electric current) when time $t=0$. The value of S_0 depends on the amount of the sample and the spin-up vs spin-down population polarization, which in turn depends on the magnitude of the main magnetic field B_o and gyromagnetic ratio γ (mathematical details will come a bit later). The units of S and S_0 (they are the same as can be deduced from **Equation V.3.1**) usually do not play much role in practical use of NMR spectroscopy as we will see in later chapters. For clarity and simplicity, we can postulate here that S and S_0 have units of electric current: Ampere (Amp).

It is important to appreciate that relaxation rate R is measured in units of Hz. Because R does not depend on the external magnetic field strength B_o (it is *almost* correct), expressing it in any units other than Hz (e.g. in *ppm*) really gives no benefit. Thus, we have an example of two types of quantities, R and Ω, which can both be expressed in units of Hz or *ppm*. Importantly, for only one quantity – resonance frequency Ω (or v) – it is meaningful to express it in relative units (ppm) to normalize the quantity with respect to the magnet strength B_o. This is why relaxation rates R almost always are listed in units of Hz.

> ✔ Example 5.3.1
>
> Let us consider an NMR spectrometer with an 11.7 Tesla magnet (B_o = 11.7 T). If for two protons, H_{ref} (reference) and H_a (target sample), their resonance frequency values are measured as 500,010,000 Hz and 500,000,000 Hz respectively, what are the chemical shift values in ppm (δ_{ref} and δ_a) ?
>
> **Solution**
>
> According to Equation V.3.1 above:
>
> δ_{ref} = (500,010,000 {Hz} − 500,010,000 {Hz}) / (500,010,000 {Hz}) × 10^6 ({ppm}) = 0 ppm (this makes sense: *chemical shift of a reference is zero)*
>
> δ_a = (500,010,000 {Hz} − 500,000,000 {Hz}) / (500,010,000 {Hz}) × 10^6 ({ppm}) = 20 ppm
>
> The difference between the resonance frequencies of the reference and proton H_a are 20 ppm (this value is independent of magnet strength B_o) and 10,000 Hz (this value is proportional to B_o and thus will change proportionately to B_o if a different magnet is used for recording).

✔ **Example 5.3.2**

If we have two non-interacting (e.g., remotely placed) spin-½ nuclei **a** and **b** within the sample molecule, each can generate an NMR signal described by formula 5.3.2 above, $S_a(t)$ and $S_b(t)$ respectively. How would a combined signal generated by the entire molecule can be mathematically described?

Solution

$S(t) = S_a(t) + S_b(t)$

Practice Problems

Problem 1. Consider Example 1 above. Calculate the chemical shift of proton H_A and reference proton H_{ref} as well as the chemical shift difference if the same recording is done on a magnet with: **a**) B_o= 9.4 Tesla, **b**) B_o = 23.4 Tesla. Calculate all the values in units of both *Hz* and *ppm* and compare the obtained numbers with the results of Example 1 above.

Problem 2. Example 1 above shows how to calculate chemical shift δ in units of ppm using oscillatory resonance frequency v. Will the value δ change if instead oscillatory frequency one uses angular frequency ω and if yes, what will the difference be?.

Problem 3. Let's consider two spin-½ nuclei: nucleus *a* having relaxation rate R_a=10 Hz, the other, nucleus *b*, having relaxation rate R_b = 100 Hz. Which signal, the one from nucleus *a* or the one from nucleus *b*, will be reduced faster below 10% of their initial values, S_{0a} and S_{0b} respectively? Justify quantitatively.

Problem 4. Let's consider two spin-½ nuclei described in Example 1 above. What nucleus, H_{ref} or H_a, will give more oscillations per unit of time when their NMR signals, $S_{ref}(t)$ and $S_a(t)$ respectively, are recorded and analyzed? Justify quantitatively.

Problem 5*. Imagine that both axes on Figure V.3.1 are labeled with actual numbers. Can you determine the numerical value of the resonance frequency Ω from this graph? If so, describe the algorithm for such a determination.