3.7: Essential Skills 2

Topics

- Proportions
- Percentages
- Unit Conversions

In Essential Skills 1 in Chapter 1 Section 1.8 , we introduced you to some of the fundamental mathematical operations you need to successfully manipulate mathematical equations in chemistry. Before proceeding to the problems in Chapter 7 you should become familiar with the additional skills described in this section on proportions, percentages, and unit conversions.

Proportions

We can solve many problems in general chemistry by using ratios, or proportions. For example, if the ratio of some quantity A to some quantity B is known, and the relationship between these quantities is known to be constant, then any change in A (from A_1 to A_2) produces a proportional change in B (from B_1 to B_2) and vice versa. The relationship between A_1, B_1, A_2, and B_2 can be written as follows:

$$\frac{A_1}{B_1} = \frac{A_2}{B_2} = constant$$

To solve this equation for A_2, we multiply both sides of the equality by B_2, thus canceling B_2 from the denominator:

$$(B_2)\,\frac{A_1}{B_1} = \left(\cancel{B_2}\right)\frac{A_2}{\cancel{B_2}}$$

$$\frac{B_2 A_1}{B_1} = A_2$$

Similarly, we can solve for B_2 by multiplying both sides of the equality by $1/A_2$, thus canceling A_2 from the numerator:

$$\left(\frac{1}{A_2}\right)\frac{A_1}{A_1} = \left(\frac{1}{\cancel{A_2}}\right)\frac{\cancel{A_2}}{B_2} \tag{3.7.1}$$

$$\frac{A_1}{A_2 B_1} = \frac{1}{B_2}$$

If the values of A_1, A_2, and B_1 are known, then we can solve the left side of the equation and invert the answer to obtain B_2:

$$B_2 = numerical\ value$$

$$B_2 = \frac{1}{numerical\ value}$$

If the value of A_1, A_2, or B_1 is unknown, however, we can solve for B_2 by inverting both sides of the equality:

$$B_2 = \frac{A_2 B_1}{A_1}$$

When you manipulate equations, remember that *any operation carried out on one side of the equality must be carried out on the other*.

Skill Builder ES1 illustrates how to find the value of an unknown by using proportions.

Skill Builder ES1

If 38.4 g of element A are needed to combine with 17.8 g of element B, then how many grams of element A are needed to combine with 52.3 g of element B?

Solution

We set up the proportions as follows:

$$\begin{aligned} A_1 &= 38.4\ g \\ B_1 &= 17.8\ g \\ A_2 &= \ ? \\ B_2 &= 52.3\ g \\ \tfrac{A_1}{B_1} &= \frac{A_2}{B_2} \\ \tfrac{38.4\ g}{17.8\ g} &= \frac{A_2}{52.3\ g} \end{aligned}$$

Multiplying both sides of the equation by 52.3 g gives

$$\begin{aligned} \frac{(38.4\ g)\,(52.3\ g)}{17.8\ g} &= \frac{A_2\left(\cancel{52.3}\right)}{\cancel{52.3}\ g} \\ A_2 &= 113\ g \end{aligned}$$

Notice that grams cancel to leave us with an answer that is in the correct units. *Always check to make sure that your answer has the correct units.*

Skill Builder ES2

Solve to find the indicated variable.

1. $\frac{(16.4\ g)}{41.2\ g} = \frac{x}{18.3\ g}$
2. $\frac{(2.65\ m)}{4.02\ m} = \frac{3.28\ m}{y}$
3. $\frac{3.27\times10^{-3}\ g}{x} = \frac{5.0\times10^{-1}\ g}{3.2\ g}$
4. Solve for V_1: $\frac{P_1}{P_2} = \frac{V_2}{V_1}$
5. Solve for T_1: $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

Solution

1. Multiply both sides of the equality by 18.3 g to remove this measurement from the denominator:

$$(18.3\ g)\,\frac{16.4\ \cancel{g}}{41.2\ \cancel{g}} = \left(\cancel{18.3\ g}\right)\frac{x}{\cancel{18.3\ g}}$$

$$7.28\ g = x$$

2, Multiply both sides of the equality by 1/3.28 m, solve the left side of the equation, and then invert to solve for *y*:

$$\left(\frac{1}{3.28\ m}\right)\frac{2.65\ m}{4.02\ m} = \left(\frac{1}{\cancel{3.28\ m}}\right)\frac{\cancel{3.28\ m}}{y} = \frac{1}{y}$$

$$y = \frac{(4.02)\,(3.28)}{2.65} = 4.98\ m$$

3. Multiply both sides of the equality by 1/3.27 × 10^{-3} g, solve the right side of the equation, and then invert to find *x*:

$$\left(\frac{1}{3.27\times10^{-3}\ \cancel{g}}\right)\frac{3.27\times10^{-3}\ \cancel{g}}{x} = \left(\frac{1}{3.27\times10^{-3}\ g}\right)\frac{5.0\times10^{-1}\ \cancel{g}}{3.2\ \cancel{g}} = \frac{1}{x}$$

$$x = \frac{(3.2\ g)\left(3.27\times10^{-3}\ g\right)}{5.0\times10^{-1}\ g} = 2.1\times10^{-2}\ g$$

1. Multiply both sides of the equality by $1/V_2$, and then invert both sides to obtain V_1: $\left(\frac{1}{V_2}\right)\frac{P_1}{P_2} = \left(\frac{1}{\cancel{V_2}}\right)\frac{\cancel{V_2}}{V_1}$

   $\frac{P_2V_2}{P_1} = V_1$

2. Multiply both sides of the equality by $1/P_1V_1$ and then invert both sides to obtain T_1: $\left(\frac{1}{\cancel{P_1V_1}}\right)\frac{\cancel{P_1V_1}}{T_1} = \left(\frac{1}{P_1V_1}\right)\frac{P_2V_2}{T_2}$

   $\frac{1}{T_1} = \frac{P_2V_2}{T_2P_1V_{1]}}$

   $T_1 = \frac{T_2P_1V_{1]}}{P_2V_2}$

Percentages

Because many measurements are reported as percentages, many chemical calculations require an understanding of how to manipulate such values. You may, for example, need to calculate the mass percentage of a substance, as described in Section 11.2 , or determine the percentage of product obtained from a particular reaction mixture.

You can convert a percentage to decimal form by dividing the percentage by 100:

$$52.8\% = \frac{52.8}{100} = 0.528$$

Conversely, you can convert a decimal to a percentage by multiplying the decimal by 100:

$$0.356\times100 = 35.6\%$$

Suppose, for example, you want to determine the mass of substance *A*, one component of a sample with a mass of 27 mg, and you are told that the sample consists of 82% *A*. You begin by converting the percentage to decimal form:

$$82\% = \frac{82}{100} = 0.82$$

The mass of *A* can then be calculated from the mass of the sample:

$$0.82\times27\ mg = 22\ mg$$

Skill Builder ES3 provides practice in converting and using percentages.

Skill Builder ES3

Convert each number to a percentage or a decimal.

1. 29.4%
2. 0.390
3. 101%
4. 1.023

Solution

1. $\frac{29.4}{100} = 0.294$
2. $0.390\times\times100 = 39.0$
3. $\frac{101}{100} = 1.01$
4. $1.023\times100 = 102.3\%$

Skill Builder ES4

Use percentages to answer the following questions, being sure to use the correct number of significant figures (see Essential Skills 1 in Chapter 1 Section 1.8). Express your answer in scientific notation where appropriate.

1. What is the mass of hydrogen in 52.83 g of a compound that is 11.2% hydrogen?
2. What is the percentage of carbon in 28.4 g of a compound that contains 13.79 g of that element?
3. A compound that is 4.08% oxygen contains 194 mg of that element. What is the mass of the compound?

Solution

1. $52.83\ g \times \frac{11.2}{100} = 52.83\ g \times 0.112 = 5.92\ g$
2. $\frac{13.79\ g\ carbon}{28.4\ g} \times 100 = 48.6\%\ carbon$
3. This problem can be solved by using a proportion: $\frac{4.08\%\ oxygen}{100\%\ compound} \frac{194\ mg}{x\ mg}$

   $x = 4.75 \times 10^3\ mg\ (or\ 4.75\ g)$

Unit Conversions

As you learned in Essential Skills 1 in Chapter 1, all measurements must be expressed in the correct units to have any meaning. This sometimes requires converting between different units (Table 1.8.1). Conversions are carried out using conversion factors, which are are ratios constructed from the relationships between different units or measurements. The relationship between milligrams and grams, for example, can be expressed as either 1 g/1000 mg or 1000 mg/1 g. When making unit conversions, use arithmetic steps accompanied by unit cancellation.

Suppose you have measured a mass in milligrams but need to report the measurement in kilograms. In problems that involve SI units, you can use the definitions of the prefixes given in Table 1.8.2 to get the necessary conversion factors. For example, you can convert milligrams to grams and then convert grams to kilograms:

$$milligrams \rightarrow grams \rightarrow kilograms\ 1000mg \rightarrow 1g\ 1000g \rightarrow 1\ kilogram$$

If you have measured 928 mg of a substance, you can convert that value to kilograms as follows:

$$928\ \cancel{mg} \times \frac{1\ g}{1000\ \cancel{mg}} = 0.928\ g$$

$$928\ \cancel{g} \times \frac{1\ kg}{1000\ \cancel{g}} = 0.000928\ kg = 9.28 \times 10^{-4}\ kg$$

In each arithmetic step, the units cancel as if they were algebraic variables, leaving us with an answer in kilograms. In the conversion to grams, we begin with milligrams in the numerator. Milligrams must therefore appear in the denominator of the conversion factor to produce an answer in grams. The individual steps may be connected as follows:

$$928\ \cancel{mg} \times \frac{1\ \cancel{g}}{1000\ \cancel{mg}} \times \frac{1\ kg}{1000\ \cancel{g}} = \frac{928\ kg}{10^6} = 928 \times 10^{-6}\ kg = 9.28 \times 10^{-4}\ kg$$

Skill Builder ES5 provides practice converting between units.

Skill Builder ES5

Use the information in Table 1.8.2 to convert each measurement. Be sure that your answers contain the correct number of significant figures and are expressed in scientific notation where appropriate.

1. 59.2 cm to decimeters
2. 3.7 × 10^5 mg to kilograms
3. 270 mL to cubic decimeters
4. 2.04 × 10^3 g to tons
5. 9.024 × 10^{10} s to years

Solution

1. $59.2\ \cancel{cm} \times \frac{1\ \cancel{m}}{100\ \cancel{cm}} \times \frac{10\ dm}{1\ \cancel{m}} = 5.92\ dm$

2. $3.7 \times 10^5 \ \cancel{mg} \times \frac{1 \ \cancel{g}}{1000 \ \cancel{mg}} \times \frac{1 \ kg}{1000 \ \cancel{g}} = 3.7 \times 10^{-1} \ kg$

3. $270 \ \cancel{mL} \times \frac{1 \ \cancel{L}}{1000 \ \cancel{mL}} \times \frac{1 \ dm^3}{1 \ \cancel{L}} = 270 \times 10^{-3} \ dm^3 = 2.70 \times 10^{-1} \ dm^3$

4. $2.04 \times 10^3 \ \cancel{g} \times \frac{1 \ \cancel{lb}}{453.6 \ \cancel{g}} \times \frac{1 \ tn}{2000 \ \cancel{lb}} = 0.00225 \ tn = 2.25 \times 10^{-3} \ tn$

5. $9.024 \times 10^{10} \ \cancel{s} \times \frac{1 \ \cancel{min}}{60 \ \cancel{s}} \times \frac{1 \ \cancel{h}}{60 \ \cancel{min}} \times \frac{1 \ \cancel{d}}{24 \ \cancel{h}} \times \frac{1 \ yr}{365 \ \cancel{d}} = 2.86 \times 10^3 \ yr$

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