4.11: Essential Skills 3

Learning Objectives

- Base-10 Logarithms
- Calculations Using Common Logarithm

Essential Skills 1 and Essential Skills 2 described some fundamental mathematical operations used for solving problems in chemistry. This section introduces you to base-10 logarithms, a topic with which you must be familiar to do the Questions and Problems for end of Chapter 4.

Base-10 (Common) Logarithms

Essential Skills 1 introduced exponential notation, in which a base number is multiplied by itself the number of times indicated in the exponent. The number 10^3, for example, is the base 10 multiplied by itself three times (10 × 10 × 10 = 1000). Now suppose that we do not know what the exponent is—that we are given only a base of 10 and the final number. If our answer is 1000, the problem can be expressed as

$$10^a = 1000$$

We can determine the value of *a* by using an operation called the *base-10 logarithm*, or *common logarithm*, abbreviated as *log*, that represents the power to which 10 is raised to give the number to the right of the equals sign. This relationship is stated as log 10^a = *a*. In this case, the logarithm is 3 because 10^3 = 1000:

$$log\, 10^3 = 3$$

$$log\, 1000 = 3$$

Now suppose you are asked to find *a* when the final number is 659. The problem can be solved as follows (remember that any operation applied to one side of an equality must also be applied to the other side):

$$10^a = 659$$

$$log\, 10^a = log\, 659$$

$$a = log\, 659$$

If you enter 659 into your calculator and press the “log” key, you get 2.819, which means that *a* = 2.819 and $10^{2.819}$ = 659. Conversely, if you enter the value 2.819 into your calculator and press the “10^x” key, you get 659.

You can decide whether your answer is reasonable by comparing it with the results you get when *a* = 2 and *a* = 3:

$$a = 2\text{: } 10^2 = 100$$

$$a = 2.819\text{: } 10^{2.819} = 659$$

$$a = 3\text{: } 10^3 = 1000$$

Because the number 659 is between 100 and 1000, *a* must be between 2 and 3, which is indeed the case. Table 4.11.1 lists some base-10 logarithms, their numerical values, and their exponential forms.

Table 4.11.1 Relationships in Base-10 Logarithms

Numerical Value	Exponential Form	Logarithm (*a*)
1000	10^3	3
100	10^2	2
10	10^1	1
1	10^0	0
0.1	10^{-1}	−1
0.01	10^{-2}	−2

Numerical Value	Exponential Form	Logarithm (a)
0.001	10^{-3}	−3

Base-10 logarithms may also be expressed as $\log_{10}$, in which the base is indicated as a subscript. We can write $\log 10^a = a$ in either of two ways:

$$log\,10^a = a$$

$$log_{10} = (10^a) = a$$

The second equation explicitly indicates that we are solving for the base-10 logarithm of 10^a.

The number of significant figures in a logarithmic value is the same as the number of digits *after* the decimal point in its logarithm, so log 62.2, a number with three significant figures, is 1.794, with three significant figures after the decimal point; that is, $10^{1.794}$ = 62.2, *not* 62.23. Skill Builder ES1 provides practice converting a value to its exponential form and then calculating its logarithm.

Skill Builder ES1

Express each number as a power of 10 and then find the common logarithm.

1. 10,000
2. 0.00001
3. 10.01
4. 2.87
5. 0.134

Solution

1. 10,000 = 1×10^4; log 1×10^4 = 4.0
2. 0.00001 = 1×10^{-5}; log 1×10^{-5} = −5.0
3. 10.01 = 1.001 × 10; log 10.01 = 1.0004 (enter 10.01 into your calculator and press the “log” key); $10^{1.0004}$ = 10.01
4. 2.87 = 2.87×10^0; log 2.87 = 0.458 (enter 2.87 into your calculator and press the “log” key); $10^{0.458}$ = 2.87
5. 0.134 = 1.34×10^{-1}; log 0.134 = −0.873 (enter 0.134 into your calculator and press the “log” key); $10^{-0.873}$ = 0.134

Skill Builder ES2

Convert each base-10 logarithm to its numerical value.

1. 3
2. −2.0
3. 1.62
4. −0.23
5. −4.872

Solution

1. 10^3
2. 10^{-2}
3. $10^{1.62}$ = 42
4. $10^{-0.23}$ = 0.59
5. $10^{-4.872}$ = 1.34×10^{-5}

Calculations Using Common Logarithms

Because logarithms are exponents, the properties of exponents that you learned in Essential Skills 1 apply to logarithms as well, which are summarized in Table 4.11.1. The logarithm of (4.08 × 20.67), for example, can be computed as follows:

$$log(4.08 \times 20.67) = log\,4.08 + log\,20.67 = 0.611 + 1.3153 = 1.926$$

We can be sure that this answer is correct by checking that $10^{1.926}$ is equal to 4.08 × 20.67, and it is.

In an alternative approach, we multiply the two values before computing the logarithm:

$$4.08 \times 20.67 = 84.3$$

$$log\, 84.3 = 1.926$$

We could also have expressed 84.3 as a power of 10 and then calculated the logarithm:

$$log\, 84.3 = log(8.43 \times 10) = log\, 8.43 + log\, 10 = 0.926 + 1 = 1.926$$

As you can see, *there may be more than one way to correctly solve a problem*.

We can use the properties of exponentials and logarithms to show that the logarithm of the inverse of a number (1/*B*) is the negative logarithm of that number (−log *B*):

$$log\left(\frac{1}{B}\right) = -log\,(B)$$

If we use the formula for division given Table 8.6 and recognize that log 1 = 0, then the logarithm of 1/*B* is

$$log\left(\frac{1}{B}\right) = log\,(1) - log\,(B) = -log\,(B)$$

Table 8.11.2 Properties of Logarithms

Operation	Exponential Form	Logarithm
multiplication	$(10^a)(10^b) = 10^{a+b}$	$log(ab) = log\, a + log\, b$
division	$\frac{10^a}{10^b} = 10^{a-b}$	$log\left(\frac{a}{b}\right) = log\, a - log\, b$

Skill Builder ES3

Convert each number to exponential form and then calculate the logarithm (assume all trailing zeros on whole numbers are not significant).

1. 100 × 1000
2. 0.100 ÷ 100
3. 1000 × 0.010
4. 200 × 3000
5. 20.5 ÷ 0.026

Solution

1. 100 × 1000 = (1 × 10^2)(1 × 10^3)

   log[(1 × 10^2)(1 × 10^3)] = 2.0 + 3.0 = 5.0

   Alternatively, (1 × 10^2)(1 × 10^3) = 1 × $10^{2\,+\,3}$ = 1 × 10^5

   log(1 × 10^5) = 5.0
2. 0.100 ÷ 100 = (1.00 × 10^{-1}) ÷ (1 × 10^2)

   log[(1.00 × 10^{-1}) ÷ (1 × 10^2)] = 1 × 10^{-1-2} = 1 × 10^{-3}

   Alternatively, (1.00 × 10^{-1}) ÷ (1 × 10^2) = 1 × $10^{[(-1)\,-\,2]}$ = 1 × 10^{-3}

   log(1 × 10^{-3}) = −3.0
3. 1000 × 0.010 = (1 × 10^3)(1.0 × 10^{-2})

   log[(1 × 10^3)(1 × 10^{-2})] = 3.0 + (−2.0) = 1.0

   Alternatively, (1 × 10^3)(1.0 × 10^{-2}) = 1 × $10^{[3\,+\,(-2)]}$ = 1 × 10^1

   log(1 × 10^1) = 1.0
4. 200 × 3000 = (2 × 10^2)(3 × 10^3)

   log[(2 × 10^2)(3 × 10^3)] = log(2 × 10^2) + log(3 × 10^3)

   = (log 2 + log 10^2) + (log 3 + log 10^3)

   = 0.30 + 2 + 0.48 + 3 = 5.8

Alternatively, $(2 \times 10^2)(3 \times 10^3) = 6 \times 10^{2+3} = 6 \times 10^5$

$\log(6 \times 10^5) = \log 6 + \log 10^5 = 0.78 + 5 = 5.8$

5. $20.5 \div 0.026 = (2.05 \times 10) \div (2.6 \times 10^{-2})$

   $\log[(2.05 \times 10) \div (2.6 \times 10^{-2})] = (\log 2.05 + \log 10) - (\log 2.6 + \log 10^{-2})$

   $= (0.3118 + 1) - [0.415 + (-2)]$

   $= 1.3118 + 1.585 = 2.90$

   Alternatively, $(2.05 \times 10) \div (2.6 \times 10^{-2}) = 0.788 \times 10^{[1-(-2)]} = 0.788 \times 10^3$

   $\log(0.79 \times 10^3) = \log 0.79 + \log 10^3 = -0.102 + 3 = 2.90$

Skill Builder ES4

Convert each number to exponential form and then calculate its logarithm (assume all trailing zeros on whole numbers are not significant).

1. $10 \times 100{,}000$
2. $1000 \div 0.10$
3. $25{,}000 \times 150$
4. $658 \div 17$

Solution

1. $(1 \times 10)(1 \times 10^5)$; logarithm = 6.0
2. $(1 \times 10^3) \div (1.0 \times 10^{-1})$; logarithm = 4.00
3. $(2.5 \times 10^4)(1.50 \times 10^2)$; logarithm = 6.57
4. $(6.58 \times 10^2) \div (1.7 \times 10)$; logarithm = 1.59

Contributors

- Anonymous

Modified by Joshua Halpern