15.7: Essential Skills 7

Learning Objectives

- The quadratic formula

Previous Essential Skills sections introduced many of the mathematical operations you need to solve chemical problems. We now introduce the quadratic formula, a mathematical relationship involving sums of powers in a single variable that you will need to apply to solve some of the problems in this chapter.

The Quadratic Formula

Mathematical expressions that involve a sum of powers in one or more variables (e.g., *x*) multiplied by coefficients (such as *a*) are called *polynomials*. Polynomials of a single variable have the general form

$$a_n x^n + \ldots + a_2 x^2 + a_1 x + a_0 \tag{15.7.1}$$

The highest power to which the variable in a polynomial is raised is called its *order*. Thus the polynomial shown here is of the *nth* order. For example, if *n* were 3, the polynomial would be third order.

A *quadratic equation* is a second-order polynomial equation in a single variable *x*:

$$ax^2 + bx + c = 0 \tag{15.7.2}$$

According to the fundamental theorem of algebra, a second-order polynomial equation has two solutions—called *roots*—that can be found using a method called *completing the square*. In this method, we solve for *x* by first adding −*c* to both sides of the quadratic equation and then divide both sides by *a*:

$$x^2 + \frac{bx}{a} = -\frac{c}{a} \tag{15.7.3}$$

We can convert the left side of this equation to a perfect square by adding $b^2/4a^2$, which is equal to $(b/2a)^2$:

Left side:

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = (x + \frac{b}{2a})^2 \tag{15.7.4}$$

Having added a value to the left side, we must now add that same value, $b^2/4a^2$, to the right side:

$$(x + \frac{b}{2a})^2 = -\frac{c}{a} + \frac{b^2}{4a^2} \tag{15.7.5}$$

The common denominator on the right side is $4a^2$. Rearranging the right side, we obtain the following:

$$(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2} \tag{15.7.6}$$

Taking the square root of both sides and solving for *x*,

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a} \tag{15.7.7}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{15.7.8}$$

This equation, known as the *quadratic formula*, has two roots:

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{15.7.9}$$

and

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \tag{15.7.10}$$

Thus we can obtain the solutions to a quadratic equation by substituting the values of the coefficients (a, b, c) into the quadratic formula.

When you apply the quadratic formula to obtain solutions to a quadratic equation, it is important to remember that one of the two solutions may not make sense or neither may make sense. There may be times, for example, when a negative solution is not reasonable or when both solutions require that a square root be taken of a negative number. In such cases, we simply discard any solution that is unreasonable and only report a solution that is reasonable. Skill Builder ES1 gives you practice using the quadratic formula.

Skill Builder ES1

Use the quadratic formula to solve for x in each equation. Report your answers to three significant figures.

1. $x^2 + 8x - 5 = 0$
2. $2x^2 - 6x + 3 = 0$
3. $3x^2 - 5x - 4 = 6$
4. $2x(-x + 2) + 1 = 0$
5. $3x(2x + 1) - 4 = 5$

Solution:

1. $9x = -8 + 82 - 4(1)(-5)\sqrt{}2(1) = 0.583$ and $x = -8 - 82 - 4(1)(-5)\sqrt{}2(1) = -8.58$
2. $x = -(-6) + (-62) - 4(2)(3)\sqrt{}2(2) = 2.37$ and $x = -(-6) - (-62) - 4(2)(3)\sqrt{}2(2) = 0.634$
3. $x = -(-5) + (-52) - 4(3)(-10)\sqrt{}2(3) = 2.84$ and $x = -(-5) - (-52) - 4(3)(-10)\sqrt{}2(3) = -1.17$
4. $x = -4 + 42 - 4(-2)(1)\sqrt{}2(-2) = -0.225$ and $x = -4 - 42 - 4(-2)(1)\sqrt{}2((-2)) = 2.22$
5. $x = -1 + 12 - 4(2)(-3)\sqrt{}2(2) = 1.00$ and $x = -1 - 12 - 4(2)(-3)\sqrt{}2(2) = 1.50$

Contributors

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