

11.9: Essential Skills 6

Learning Objectives

- Natural Logarithms
- Calculations Using Natural Logarithms

Essential Skills 3 in [Section 4.11](#), introduced the common, or base-10, logarithms and showed how to use the properties of exponents to perform logarithmic calculations. In this section, we describe natural logarithms, their relationship to common logarithms, and how to do calculations with them using the same properties of exponents.

Natural Logarithms

Many natural phenomena exhibit an exponential rate of increase or decrease. Population growth is an example of an exponential rate of increase, whereas a runner's performance may show an exponential decline if initial improvements are substantially greater than those that occur at later stages of training. Exponential changes are represented logarithmically by e^x , where e is an irrational number whose value is approximately 2.7183. The *natural logarithm*, abbreviated as \ln , is the power x to which e must be raised to obtain a particular number. The natural logarithm of e is 1 ($\ln e = 1$).

Some important relationships between base-10 logarithms and natural logarithms are as follows:

$$10^1 = 10 = e^{2.303} \quad \ln e^x = x \quad \ln 10 = \ln(e^{2.303}) = 2.303 \quad \log 10 = \ln e = 1$$

According to these relationships, $\ln 10 = 2.303$ and $\log 10 = 1$. Because multiplying by 1 does not change an equality,

$$\ln 10 = 2.303 \log 10$$

Substituting any value y for 10 gives

$$\ln y = 2.303 \log y$$

Other important relationships are as follows:

$$\log A^x = x \log A \quad \ln e^x = x \quad \ln e = x = e^{\ln x}$$

Entering a value x , such as 3.86, into your calculator and pressing the “ln” key gives the value of $\ln x$, which is 1.35 for $x = 3.86$. Conversely, entering the value 1.35 and pressing “ e^x ” key gives an answer of 3.86. On some calculators, pressing [INV] and then [ln x] is equivalent to pressing [e^x]. Hence

$$e^{\ln 3.86} = e^{1.35} = 3.86 \quad \ln(e^{3.86}) = 3.86$$

Skill Builder ES1

Calculate the natural logarithm of each number and express each as a power of the base e .

1. 0.523
2. 1.63

Solution:

1. $\ln(0.523) = -0.648$; $e^{-0.648} = 0.523$
2. $\ln(1.63) = 0.489$; $e^{0.489} = 1.63$

Skill Builder ES2

What number is each value the natural logarithm of?

1. 2.87
2. 0.030
3. -1.39

Solution:

1. $\ln x = 2.87$; $x = e^{2.87} = 17.6 = 18$ to two significant figures
2. $\ln x = 0.030$; $x = e^{0.030} = 1.03 = 1.0$ to two significant figures
3. $\ln x = -1.39$; $x = e^{-1.39} = 0.249 = 0.25$

Calculations with Natural Logarithms

Like common logarithms, natural logarithms use the properties of exponents. We can compare the properties of common and natural logarithms:

Operation	Exponential Form	Logarithm
Multiplication	$(10^a)(10^b) = 10^{a+b}$	$\log(ab) = \log a + \log b$
	$(e^x)(e^y) = e^{x+y}$	$\ln(e^x e^y) = \ln(e^x) + \ln(e^y) = x + y$
Division		$\log\left(\frac{a}{b}\right) = \log a - \log b$
	$\frac{10^a}{10^b} = 10^{a-b}$	$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$
	$\frac{e^a}{e^b} = e^{a-b}$	$\ln\left(\frac{e^x}{e^y}\right) = \ln(e^x) - \ln(e^y) = x - y$
Inverse		$\log\left(\frac{1}{a}\right) = -\log(a)$
		$\ln\left(\frac{1}{x}\right) = -\ln(x)$

The number of significant figures in a number is the same as the number of digits after the decimal point in its logarithm. For example, the natural logarithm of 18.45 is 2.9151, which means that $e^{2.9151}$ is equal to 18.45.

Skill Builder ES3

Calculate the natural logarithm of each number.

- 22×18.6
- $\frac{0.51}{2.67}$
- 0.079×1.485
- $\frac{20.5}{0.026}$

Solution:

- $\ln(22 \times 18.6) = \ln(22) + \ln(18.6) = 3.09 + 2.923 = 6.01$. Alternatively, $22 \times 18.6 = 410$; $\ln(410) = 6.02$.
- $\ln\left(\frac{0.51}{2.67}\right) = \ln(0.51) - \ln(2.67) = -0.67 - 0.982 = -1.65$ $\ln(0.19) = -1.66$.
- $\ln(0.079 \times 1.485) = \ln(0.079) + \ln(1.485) = -2.54 + 0.395 = -2.15$. Alternatively, $0.079 \times 1.485 = 0.12$; $\ln(0.12) = -2.12$.
- $\ln\left(\frac{20.5}{0.026}\right) = \ln(20.5) - \ln(0.026) = 3.0204 - (-3.65) = 6.67$ $\ln(790) = 6.67$.

The answers obtained using the two methods may differ slightly due to rounding errors.

Skill Builder ES4

Calculate the natural logarithm of each number.

- 34×16.5
- $2.10/0.052$
- 0.402×3.930
- $0.164/10.7$

Solution:

1. 6.33
2. 3.70
3. 0.457
4. -4.178

Contributors

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