

## 24.5: Thermodynamic Stability of the Atomic Nucleus

### Learning Objectives

- To calculate a mass-energy balance and a nuclear binding energy.
- To understand the differences between nuclear fission and fusion.

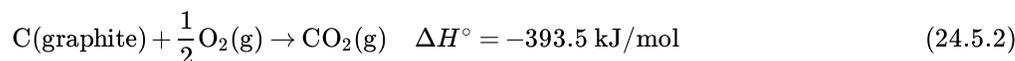
Nuclear reactions, like chemical reactions, are accompanied by changes in energy. The energy changes in nuclear reactions, however, are enormous compared with those of even the most energetic chemical reactions. In fact, the energy changes in a typical nuclear reaction are so large that they result in a measurable change of mass. In this section, we describe the relationship between mass and energy in nuclear reactions and show how the seemingly small changes in mass that accompany nuclear reactions result in the release of enormous amounts of energy.

### Mass–Energy Balance

The relationship between mass ( $m$ ) and energy ( $E$ ) is expressed in the following equation:

$$E = mc^2 \quad (24.5.1)$$

where  $c$  is the speed of light ( $2.998 \times 10^8$  m/s), and  $E$  and  $m$  are expressed in units of joules and kilograms, respectively. Albert Einstein first derived this relationship in 1905 as part of his special theory of relativity: the mass of a particle is directly proportional to its energy. Thus according to Equation 20.27, every mass has an associated energy, and similarly, any reaction that involves a change in energy must be accompanied by a change in mass. This implies that all exothermic reactions should be accompanied by a decrease in mass, and all endothermic reactions should be accompanied by an increase in mass. Given the law of conservation of mass, how can this be true? The solution to this apparent contradiction is that chemical reactions are indeed accompanied by changes in mass, but these changes are simply too small to be detected. As you may recall, all particles exhibit wavelike behavior, but the wavelength is inversely proportional to the mass of the particle (actually, to its momentum, the product of its mass and velocity). Consequently, wavelike behavior is detectable only for particles with very small masses, such as electrons. For example, the chemical equation for the combustion of graphite to produce carbon dioxide is as follows:



Combustion reactions are typically carried out at constant pressure, and under these conditions, the heat released or absorbed is equal to  $\Delta H$ . When a reaction is carried out at constant volume, the heat released or absorbed is equal to  $\Delta E$ . For most chemical reactions, however,  $\Delta E \approx \Delta H$ . If we rewrite Einstein's equation as

$$\Delta E = (\Delta m)c^2 \quad (24.5.3)$$

we can rearrange the equation to obtain the following relationship between the change in mass and the change in energy:

$$\Delta m = \frac{\Delta E}{c^2} \quad (24.5.4)$$

Because  $1 \text{ J} = 1 (\text{kg} \cdot \text{m}^2)/\text{s}^2$ , the change in mass is as follows:

$$\Delta m = \frac{-393.5 \text{ kJ/mol}}{(2.998 \times 10^8 \text{ m/s})^2} = \frac{-3.935 \times 10^5 (\text{kg} \cdot \text{m}^2)/(\text{s}^2 \cdot \text{mol})}{(2.998 \times 10^8 \text{ m/s})^2} = -4.38 \times 10^{-12} \text{ kg/mol} \quad (24.5.5)$$

This is a mass change of about  $3.6 \times 10^{-10}$  g/g carbon that is burned, or about 100-millionths of the mass of an electron per atom of carbon. In practice, this mass change is much too small to be measured experimentally and is negligible.

In contrast, for a typical nuclear reaction, such as the radioactive decay of  $^{14}\text{C}$  to  $^{14}\text{N}$  and an electron (a  $\beta$  particle), there is a much larger change in mass:



We can use the experimentally measured masses of subatomic particles and common isotopes to calculate the change in mass directly. The reaction involves the conversion of a neutral  $^{14}\text{C}$  atom to a positively charged  $^{14}\text{N}$  ion (with six, not seven, electrons) and a negatively charged  $\beta$  particle (an electron), so the mass of the products is identical to the mass of a neutral  $^{14}\text{N}$  atom. The

total change in mass during the reaction is therefore the difference between the mass of a neutral  $^{14}\text{N}$  atom (14.003074 amu) and the mass of a  $^{14}\text{C}$  atom (14.003242 amu):

$$\Delta m = \text{mass}_{\text{products}} - \text{mass}_{\text{reactants}} \quad (24.5.7)$$

$$= 14.003074 \text{ amu} - 14.003242 \text{ amu} = -0.000168 \text{ amu} \quad (24.5.8)$$

The difference in mass, which has been released as energy, corresponds to almost one-third of an electron. The change in mass for the decay of 1 mol of  $^{14}\text{C}$  is  $-0.000168 \text{ g} = -1.68 \times 10^{-4} \text{ g} = -1.68 \times 10^{-7} \text{ kg}$ . Although a mass change of this magnitude may seem small, it is about 1000 times larger than the mass change for the combustion of graphite. The energy change is as follows:

$$\Delta E = (\Delta m)c^2 = (-1.68 \times 10^{-7} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2 \quad (24.5.9)$$

$$= -1.51 \times 10^{10} (\text{kg} \cdot \text{m}^2)/\text{s}^2 = -1.51 \times 10^{10} \text{ J} = -1.51 \times 10^7 \text{ kJ} \quad (24.5.10)$$

The energy released in this nuclear reaction is more than 100,000 times greater than that of a typical chemical reaction, even though the decay of  $^{14}\text{C}$  is a relatively low-energy nuclear reaction.

Because the energy changes in nuclear reactions are so large, they are often expressed in kiloelectronvolts (1 keV =  $10^3$  eV), mega-electronvolts (1 MeV =  $10^6$  eV), and even giga-electronvolts (1 GeV =  $10^9$  eV) per atom or particle. The change in energy that accompanies a nuclear reaction can be calculated from the change in mass using the relationship 1 amu = 931 MeV. The energy released by the decay of one atom of  $^{14}\text{C}$  is thus

$$(-1.68 \times 10^{-4} \text{ amu}) \left( \frac{931 \text{ MeV}}{\text{amu}} \right) = -0.156 \text{ MeV} = -156 \text{ keV} \quad (24.5.11)$$

#### ✓ Example 24.5.1

Calculate the changes in mass (in atomic mass units) and energy (in joules per mole and electronvolts per atom) that accompany the radioactive decay of  $^{238}\text{U}$  to  $^{234}\text{Th}$  and an  $\alpha$  particle. The  $\alpha$  particle absorbs two electrons from the surrounding matter to form a helium atom.

**Given:** nuclear decay reaction

**Asked for:** changes in mass and energy

**Strategy:**

**A** Use the mass values in [Table 20.1](#) to calculate the change in mass for the decay reaction in atomic mass units.

**B** Use Equation 20.30 to calculate the change in energy in joules per mole.

**C** Use the relationship between atomic mass units and mega-electronvolts to calculate the change in energy in electronvolts per atom.

**Solution**

**A** Using particle and isotope masses from [Table 20.1](#), we can calculate the change in mass as follows:

$$\Delta m = \text{mass}_{\text{products}} - \text{mass}_{\text{reactants}} = (\text{mass } ^{234}\text{Th} + \text{mass } ^4_2\text{He}) - \text{mass } ^{238}\text{U} \quad (24.5.12)$$

$$= (234.043601 \text{ amu} + 4.002603 \text{ amu}) - 238.050788 \text{ amu} = -0.004584 \text{ amu} \quad (24.5.13)$$

**B** Thus the change in mass for 1 mol of  $^{238}\text{U}$  is  $-0.004584 \text{ g}$  or  $-4.584 \times 10^{-6} \text{ kg}$ . The change in energy in joules per mole is as follows:

$$\Delta E = (\Delta m)c^2 = (-4.584 \times 10^{-6} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2 = -4.120 \times 10^{11} \text{ J/mol}$$

**C** The change in energy in electronvolts per atom is as follows:

$$\Delta E = -4.584 \times 10^{-3} \text{ amu} \times \frac{931 \text{ MeV}}{\text{amu}} \times \frac{1 \times 10^6 \text{ eV}}{1 \text{ MeV}} = -4.27 \times 10^6 \text{ eV/atom}$$

### ? Exercise 24.5.1

Calculate the changes in mass (in atomic mass units) and energy (in kilojoules per mole and kiloelectronvolts per atom) that accompany the radioactive decay of tritium ( $^3\text{H}$ ) to  $^3\text{He}$  and a  $\beta$  particle.

#### Answer

$$\Delta m = -2.0 \times 10^{-5} \text{ amu}; \Delta E = -1.9 \times 10^6 \text{ kJ/mol} = -19 \text{ keV/atom}$$

## Nuclear Binding Energies

We have seen that energy changes in both chemical and nuclear reactions are accompanied by changes in mass. Einstein's equation, which allows us to interconvert mass and energy, has another interesting consequence: The mass of an atom is always less than the sum of the masses of its component particles. The only exception to this rule is hydrogen-1 ( $^1\text{H}$ ), whose measured mass of 1.007825 amu is identical to the sum of the masses of a proton and an electron. In contrast, the experimentally measured mass of an atom of deuterium ( $^2\text{H}$ ) is 2.014102 amu, although its calculated mass is 2.016490 amu:

$$m_{^2\text{H}} = m_{\text{neutron}} + m_{\text{proton}} + m_{\text{electron}} \quad (24.5.14)$$

$$= 1.008665 \text{ amu} + 1.007276 \text{ amu} + 0.000549 \text{ amu} = 2.016490 \text{ amu} \quad (24.5.15)$$

The difference between the sum of the masses of the components and the measured atomic mass is called the **mass defect** of the nucleus. Just as a molecule is more stable than its isolated atoms, a nucleus is more stable (lower in energy) than its isolated components. Consequently, when isolated nucleons assemble into a stable nucleus, energy is released. According to Equation 24.5.4, this release of energy must be accompanied by a decrease in the mass of the nucleus.

The amount of energy released when a nucleus forms from its component nucleons is the **nuclear binding energy** (Figure 21.2.1). In the case of deuterium, the mass defect is 0.002388 amu, which corresponds to a nuclear binding energy of 2.22 MeV for the deuterium nucleus. Because the magnitude of the mass defect is proportional to the nuclear binding energy, both values indicate the stability of the nucleus.

Just as a molecule is more stable (lower in energy) than its isolated atoms, a nucleus is more stable than its isolated components.

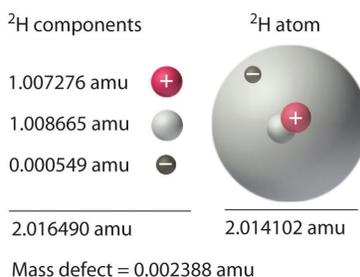


Figure 24.5.1: Nuclear Binding Energy in Deuterium. The mass of a  $^2\text{H}$  atom is less than the sum of the masses of a proton, a neutron, and an electron by 0.002388 amu; the difference in mass corresponds to the nuclear binding energy. The larger the value of the mass defect, the greater the nuclear binding energy and the more stable the nucleus.

Not all nuclei are equally stable. Chemists describe the relative stability of different nuclei by comparing the binding energy per nucleon, which is obtained by dividing the nuclear binding energy by the mass number ( $A$ ) of the nucleus. As shown in Figure 24.5.2, the binding energy per nucleon increases rapidly with increasing atomic number until about  $Z = 26$ , where it levels off to about 8–9 MeV per nucleon and then decreases slowly. The initial increase in binding energy is not a smooth curve but exhibits sharp peaks corresponding to the light nuclei that have equal numbers of protons and neutrons (e.g.,  $^4\text{He}$ ,  $^{12}\text{C}$ , and  $^{16}\text{O}$ ). As mentioned earlier, these are particularly stable combinations.

Because the maximum binding energy per nucleon is reached at  $^{56}\text{Fe}$ , all other nuclei are thermodynamically unstable with regard to the formation of  $^{56}\text{Fe}$ . Consequently, heavier nuclei (toward the right in 24.5.2) should spontaneously undergo reactions such as alpha decay, which result in a decrease in atomic number. Conversely, lighter elements (on the left in Figure 24.5.2) should spontaneously undergo reactions that result in an increase in atomic number. This is indeed the observed pattern.

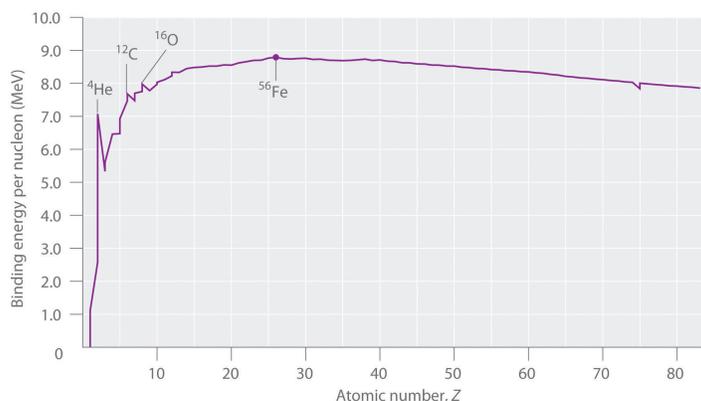


Figure 24.5.3: The Curve of Nuclear Binding Energy. This plot of the average binding energy per nucleon as a function of atomic number shows that the binding energy per nucleon increases with increasing atomic number until about  $Z = 26$ , levels off, and then decreases. The sharp peaks correspond to light nuclei that have equal numbers of protons and neutrons.

Heavier nuclei spontaneously undergo nuclear reactions that *decrease* their atomic number. Lighter nuclei spontaneously undergo nuclear reactions that *increase* their atomic number.

### ✓ Example 24.5.2

Calculate the total nuclear binding energy (in megaelectronvolts) and the binding energy per nucleon for  $^{56}\text{Fe}$ . The experimental mass of the nuclide is given in [Table A4](#).

**Given:** nuclide and mass

**Asked for:** nuclear binding energy and binding energy per nucleon

**Strategy:**

**A** Sum the masses of the protons, electrons, and neutrons or, alternatively, use the mass of the appropriate number of  $^1\text{H}$  atoms (because its mass is the same as the mass of one electron and one proton).

**B** Calculate the mass defect by subtracting the experimental mass from the calculated mass.

**C** Determine the nuclear binding energy by multiplying the mass defect by the change in energy in electronvolts per atom. Divide this value by the number of nucleons to obtain the binding energy per nucleon.

**Solution**

**A** An iron-56 atom has 26 protons, 26 electrons, and 30 neutrons. We could add the masses of these three sets of particles; however, noting that 26 protons and 26 electrons are equivalent to 26  $^1\text{H}$  atoms, we can calculate the sum of the masses more quickly as follows:

$$\text{calculated mass} = 26(\text{mass } ^1_1\text{H}) + 30(\text{mass } ^1_0\text{n}) \quad (24.5.16)$$

$$= 26(1.007825)\text{amu} + 30(1.008665)\text{amu} = 56.463400 \text{ amu} \quad (24.5.17)$$

$$\text{experimental mass} = 55.934938 \quad (24.5.18)$$

**B** We subtract to find the mass defect:

$$\text{mass defect} = \text{calculated mass} - \text{experimental mass} \quad (24.5.19)$$

$$= 56.463400 \text{ amu} - 55.934938 \text{ amu} = 0.528462 \text{ amu} \quad (24.5.20)$$

**C** The nuclear binding energy is thus  $0.528462 \text{ amu} \times 931 \text{ MeV/amu} = 492 \text{ MeV}$ . The binding energy per nucleon is  $492 \text{ MeV}/56 \text{ nucleons} = 8.79 \text{ MeV/nucleon}$ .

## ? Exercise 24.5.2

Calculate the total nuclear binding energy (in megaelectronvolts) and the binding energy per nucleon for  $^{238}\text{U}$ .

**Answer** 1800 MeV/ $^{238}\text{U}$ ; 7.57 MeV/nucleon

## Nuclear Fission and Fusion

**Nuclear fission** is the splitting of a heavy nucleus into two lighter ones. Fission was discovered in 1938 by the German scientists Otto Hahn, Lise Meitner, and Fritz Strassmann, who bombarded a sample of uranium with neutrons in an attempt to produce new elements with  $Z > 92$ . They observed that lighter elements such as barium ( $Z = 56$ ) were formed during the reaction, and they realized that such products had to originate from the neutron-induced fission of uranium-235:



This hypothesis was confirmed by detecting the krypton-92 fission product. The nucleus usually divides asymmetrically rather than into two equal parts, and the fission of a given nuclide does not give the same products every time.

In a typical nuclear fission reaction, more than one neutron is released by each dividing nucleus. When these neutrons collide with and induce fission in other neighboring nuclei, a self-sustaining series of nuclear fission reactions known as a **nuclear chain reaction** can result (Figure 24.5.3). For example, the fission of  $^{235}\text{U}$  releases two to three neutrons per fission event. If absorbed by other  $^{235}\text{U}$  nuclei, those neutrons induce additional fission events, and the rate of the fission reaction increases geometrically. Each series of events is called a generation. Experimentally, it is found that some minimum mass of a fissile isotope is required to sustain a nuclear chain reaction; if the mass is too low, too many neutrons are able to escape without being captured and inducing a fission reaction. The minimum mass capable of supporting sustained fission is called the **critical mass**. This amount depends on the purity of the material and the shape of the mass, which corresponds to the amount of surface area available from which neutrons can escape, and on the identity of the isotope. If the mass of the fissile isotope is greater than the critical mass, then under the right conditions, the resulting supercritical mass can release energy explosively. The enormous energy released from nuclear chain reactions is responsible for the massive destruction caused by the detonation of nuclear weapons such as fission bombs, but it also forms the basis of the nuclear power industry.

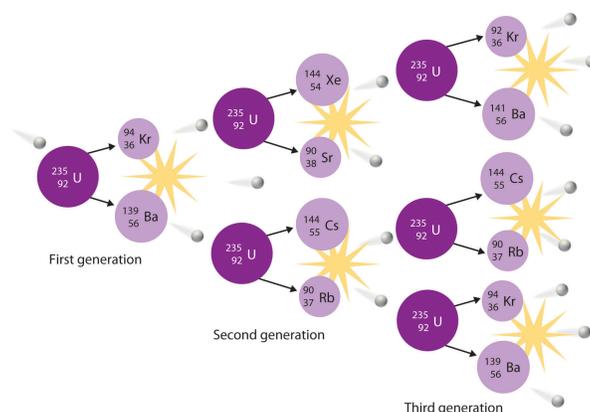


Figure 24.5.3: A Nuclear Chain Reaction. The process is initiated by the collision of a single neutron with a  $^{235}\text{U}$  nucleus, which undergoes fission, as shown in Figure 20.6. Because each neutron released can cause the fission of another  $^{235}\text{U}$  nucleus, the rate of a fission reaction accelerates geometrically. Each series of events is a generation.

**Nuclear fusion**, in which two light nuclei combine to produce a heavier, more stable nucleus, is the opposite of nuclear fission. As in the nuclear transmutation reactions discussed in Section 20.2, the positive charge on both nuclei results in a large electrostatic energy barrier to fusion. This barrier can be overcome if one or both particles have sufficient kinetic energy to overcome the electrostatic repulsions, allowing the two nuclei to approach close enough for a fusion reaction to occur. The principle is similar to adding heat to increase the rate of a chemical reaction. As shown in the plot of nuclear binding energy per nucleon versus atomic number in Figure 21.2.3, fusion reactions are most exothermic for the lightest element. For example, in a typical fusion reaction, two deuterium atoms combine to produce helium-3, a process known as deuterium–deuterium fusion (D–D fusion):



In another reaction, a deuterium atom and a tritium atom fuse to produce helium-4 (Figure 24.5.4), a process known as deuterium–tritium fusion (D–T fusion):

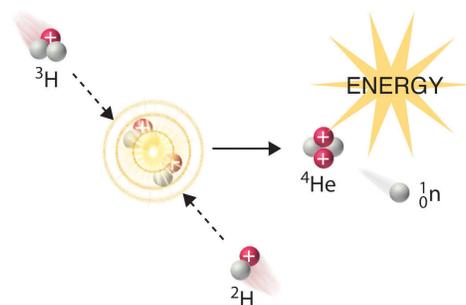


Figure 24.5.4: Nuclear Fusion. In a nuclear fusion reaction, lighter nuclei combine to produce a heavier nucleus. As shown, fusion of  ${}^3\text{H}$  and  ${}^2\text{H}$  to give  ${}^4\text{He}$  and a neutron releases an enormous amount of energy. In principle, nuclear fusion can produce much more energy than fission, but very high kinetic energy is required to overcome electrostatic repulsions between the positively charged nuclei and initiate the fusion reaction.

Initiating these reactions, however, requires a temperature comparable to that in the interior of the sun (approximately  $1.5 \times 10^7$  K). Currently, the only method available on Earth to achieve such a temperature is the detonation of a fission bomb. For example, the so-called hydrogen bomb (or H bomb) is actually a deuterium–tritium bomb (a D–T bomb), which uses a nuclear fission reaction to create the very high temperatures needed to initiate fusion of solid lithium deuteride ( ${}^6\text{LiD}$ ), which releases neutrons that then react with  ${}^6\text{Li}$ , producing tritium. The deuterium–tritium reaction releases energy explosively. Example 9 and its corresponding exercise demonstrate the enormous amounts of energy produced by nuclear fission and fusion reactions. In fact, fusion reactions are the power sources for all stars, including our sun.

### ✓ Example 24.5.3

Calculate the amount of energy (in electronvolts per atom and kilojoules per mole) released when the neutron-induced fission of  ${}^{235}\text{U}$  produces  ${}^{144}\text{Cs}$ ,  ${}^{90}\text{Rb}$ , and two neutrons:



**Given:** balanced nuclear reaction

**Asked for:** energy released in electronvolts per atom and kilojoules per mole

**Strategy:**

**A** Following the method used in Example 24.5.1, calculate the change in mass that accompanies the reaction. Convert this value to the change in energy in electronvolts per atom.

**B** Calculate the change in mass per mole of  ${}^{235}\text{U}$ . Then use Equation 24.5.3 to calculate the change in energy in kilojoules per mole.

**Solution**

**A** The change in mass that accompanies the reaction is as follows:

$$\Delta m = \text{mass}_{\text{products}} - \text{mass}_{\text{reactants}} = \text{mass}({}^{144}_{55}\text{Cs} + {}^{90}_{37}\text{Rb} + {}^1_0\text{n}) - \text{mass}({}^{235}_{92}\text{U}) \quad (24.5.24)$$

$$= (143.932077 \text{ amu} + 89.914802 \text{ amu} + 1.008665 \text{ amu}) - 235.043930 \text{ amu} \quad (24.5.25)$$

$$= -0.188386 \text{ amu} \quad (24.5.26)$$

The change in energy in electronvolts per atom is as follows:

$$\Delta E = (-0.188386 \text{ amu})(931 \text{ MeV/amu}) = -175 \text{ MeV}$$

**B** The change in mass per mole of  ${}^{235}\text{U}$  is  $-0.188386 \text{ g} = -1.88386 \times 10^{-4} \text{ kg}$ , so the change in energy in kilojoules per mole is as follows:

$$\Delta E = (\Delta m)c^2 = (-1.88386 \times 10^{-4} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2 \quad (24.5.27)$$

$$= -1.693 \times 10^{13} \text{ J/mol} = -1.693 \times 10^{10} \text{ kJ/mol} \quad (24.5.28)$$

### ? Exercise 24.5.3

Calculate the amount of energy (in electronvolts per atom and kilojoules per mole) released when deuterium and tritium fuse to give helium-4 and a neutron:



#### Answer

$$\Delta E = -17.6 \text{ MeV/atom} = -1.697 \times 10^9 \text{ kJ/mol}$$

### Summary

Unlike a chemical reaction, a nuclear reaction results in a significant change in mass and an associated change of energy, as described by Einstein's equation. Nuclear reactions are accompanied by large changes in energy, which result in detectable changes in mass. The change in mass is related to the change in energy according to Einstein's equation:  $\Delta E = (\Delta m)c^2$ . Large changes in energy are usually reported in kiloelectronvolts or megaelectronvolts (thousands or millions of electronvolts). With the exception of  ${}^1_1\text{H}$ , the experimentally determined mass of an atom is always less than the sum of the masses of the component particles (protons, neutrons, and electrons) by an amount called the mass defect of the nucleus. The energy corresponding to the mass defect is the nuclear binding energy, the amount of energy released when a nucleus forms from its component particles. In nuclear fission, nuclei split into lighter nuclei with an accompanying release of multiple neutrons and large amounts of energy. The critical mass is the minimum mass required to support a self-sustaining nuclear chain reaction. Nuclear fusion is a process in which two light nuclei combine to produce a heavier nucleus plus a great deal of energy.

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