

## 17.2: Determining the Solubility of Ionic Compounds

### Learning Objectives

- To calculate the solubility of an ionic compound from its  $K_{sp}$ .

We begin our discussion of solubility and *complexation equilibria*—those associated with the formation of complex ions—by developing quantitative methods for describing dissolution and precipitation reactions of ionic compounds in aqueous solution. Just as with acid–base equilibria, we can describe the concentrations of ions in equilibrium with an ionic solid using an equilibrium constant expression.

### The Solubility Product

When a slightly soluble ionic compound is added to water, some of it dissolves to form a solution, establishing an equilibrium between the pure solid and a solution of its ions. For the dissolution of calcium phosphate, one of the two main components of kidney stones, the equilibrium can be written as follows, with the solid salt on the left: As you will discover in [Section 17.4](#) and in more advanced chemistry courses, basic anions, such as  $S^{2-}$ ,  $PO_4^{3-}$ , and  $CO_3^{2-}$ , react with water to produce  $OH^-$  and the corresponding protonated anion. Consequently, their calculated molarities, assuming no protonation in aqueous solution, are only approximate.



The equilibrium constant for the dissolution of a sparingly soluble salt is the solubility product ( $K_{sp}$ ). The equilibrium constant expression for the dissolution of a sparingly soluble salt that includes the concentration of a pure solid, which is a constant, of the salt. Because the concentration of a pure solid such as  $Ca_3(PO_4)_2$  is a constant, it does not appear explicitly in the equilibrium constant expression. (For more information on the equilibrium constant expression, see [Section 15.2](#).) The equilibrium constant expression for the dissolution of calcium phosphate is therefore

$$K = \frac{[Ca^{2+}]^3 [PO_4^{3-}]^2}{[Ca_3(PO_4)_2]} \quad (17.1.2a)$$

$$[Ca_3(PO_4)_2] K = K_{sp} = [Ca^{2+}]^3 [PO_4^{3-}]^2 \quad (17.1.2b)$$

At 25°C and pH 7.00,  $K_{sp}$  for calcium phosphate is  $2.07 \times 10^{-33}$ , indicating that the concentrations of  $Ca^{2+}$  and  $PO_4^{3-}$  ions in solution that are in equilibrium with solid calcium phosphate are very low. The values of  $K_{sp}$  for some common salts are listed in [Table 17.1.1](#); they show that the magnitude of  $K_{sp}$  varies dramatically for different compounds. Although  $K_{sp}$  is not a function of pH in [Equation 17.1](#), changes in pH can affect the solubility of a compound, as you will discover in [Section 17.4](#).

### Note the Pattern

As with  $K$ , the concentration of a pure solid does not appear explicitly in  $K_{sp}$ .

**Table 17.1.1 Solubility Products for Selected Ionic Substances at 25°C**

Solid	Color	$K_{sp}$	Solid	Color	$K_{sp}$
<b>Acetates</b>			<b>Iodides</b>		
$Ca(O_2CCH_3)_2 \cdot 3H_2O$	white	$4 \times 10^{-3}$	$Hg_2I_2^*$	yellow	$5.2 \times 10^{-29}$
<b>Bromides</b>			$PbI_2$	yellow	$9.8 \times 10^{-9}$
$AgBr$	off-white	$5.35 \times 10^{-13}$	<b>Oxalates</b>		
$Hg_2Br_2^*$	yellow	$6.40 \times 10^{-23}$	$Ag_2C_2O_4$	white	$5.40 \times 10^{-12}$
<b>Carbonates</b>			$MgC_2O_4 \cdot 2H_2O$	white	$4.83 \times 10^{-6}$
*These contain the $Hg_2^{2+}$ ion.					

Solid	Color	$K_{sp}$	Solid	Color	$K_{sp}$
$\text{CaCO}_3$	white	$3.36 \times 10^{-9}$	$\text{PbC}_2\text{O}_4$	white	$4.8 \times 10^{-10}$
$\text{PbCO}_3$	white	$7.40 \times 10^{-14}$	<b>Phosphates</b>		
<b>Chlorides</b>			$\text{Ag}_3\text{PO}_4$	white	$8.89 \times 10^{-17}$
$\text{AgCl}$	white	$1.77 \times 10^{-10}$	$\text{Sr}_3(\text{PO}_4)_2$	white	$4.0 \times 10^{-28}$
$\text{Hg}_2\text{Cl}_2^*$	white	$1.43 \times 10^{-18}$	$\text{FePO}_4 \cdot 2\text{H}_2\text{O}$	pink	$9.91 \times 10^{-16}$
$\text{PbCl}_2$	white	$1.70 \times 10^{-5}$	<b>Sulfates</b>		
<b>Chromates</b>			$\text{Ag}_2\text{SO}_4$	white	$1.20 \times 10^{-5}$
$\text{CaCrO}_4$	yellow	$7.1 \times 10^{-4}$	$\text{BaSO}_4$	white	$1.08 \times 10^{-10}$
$\text{PbCrO}_4$	yellow	$2.8 \times 10^{-13}$	$\text{PbSO}_4$	white	$2.53 \times 10^{-8}$
<b>Fluorides</b>			<b>Sulfides</b>		
$\text{BaF}_2$	white	$1.84 \times 10^{-7}$	$\text{Ag}_2\text{S}$	black	$6.3 \times 10^{-50}$
$\text{PbF}_2$	white	$3.3 \times 10^{-8}$	$\text{CdS}$	yellow	$8.0 \times 10^{-27}$
<b>Hydroxides</b>			$\text{PbS}$	black	$8.0 \times 10^{-28}$
$\text{Ca}(\text{OH})_2$	white	$5.02 \times 10^{-6}$	$\text{ZnS}$	white	$1.6 \times 10^{-24}$
$\text{Cu}(\text{OH})_2$	pale blue	$1 \times 10^{-14}$			
$\text{Mn}(\text{OH})_2$	light pink	$1.9 \times 10^{-13}$			
$\text{Cr}(\text{OH})_3$	gray-green	$6.3 \times 10^{-31}$			
$\text{Fe}(\text{OH})_3$	rust red	$2.79 \times 10^{-39}$			
*These contain the $\text{Hg}_2^{2+}$ ion.					

Solubility products are determined experimentally by directly measuring either the concentration of one of the component ions or the solubility of the compound in a given amount of water. However, whereas solubility is usually expressed in terms of  $m$

mass of solute per 100 mL of solvent,  $K_{sp}$ , like  $K$ , is defined in terms of the *molar* concentrations of the component ions.

**A kidney stone.** Kidney stones form from sparingly soluble calcium salts and are largely composed of  $\text{Ca}(\text{O}_2\text{CCO}_2) \cdot \text{H}_2\text{O}$  and  $\text{Ca}_3(\text{PO}_4)_2$ .

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### Example 17.1.1

Calcium oxalate monohydrate [ $\text{Ca}(\text{O}_2\text{CCO}_2) \cdot \text{H}_2\text{O}$ , also written as  $\text{CaC}_2\text{O}_4 \cdot \text{H}_2\text{O}$ ] is a sparingly soluble salt that is the other major component of kidney stones [along with  $\text{Ca}_3(\text{PO}_4)_2$ ]. Its solubility in water at  $25^\circ\text{C}$  is  $7.36 \times 10^{-4}$  g/100 mL. Calculate its  $K_{sp}$ .

**Given:** solubility in g/100 mL

**Asked for:**  $K_{sp}$

**Strategy:**

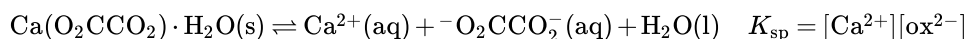
**A** Write the balanced dissolution equilibrium and the corresponding solubility product expression.

**B** Convert the solubility of the salt to moles per liter. From the balanced dissolution equilibrium, determine the equilibrium concentrations of the dissolved solute ions. Substitute these values into the solubility product expression to calculate  $K_{sp}$ .

**Solution:**



**A** We need to write the solubility product expression in terms of the concentrations of the component ions. For calcium oxalate monohydrate, the balanced dissolution equilibrium and the solubility product expression (abbreviating oxalate as  $\text{ox}^{2-}$ ) are as follows:



Neither solid calcium oxalate monohydrate nor water appears in the solubility product expression because their concentrations are essentially constant.

**B** Next we need to determine  $[\text{Ca}^{2+}]$  and  $[\text{ox}^{2-}]$  at equilibrium. We can use the mass of calcium oxalate monohydrate that dissolves in 100 mL of water to calculate the number of moles that dissolve in 100 mL of water. From this we can determine the number of moles that dissolve in 1.00 L of water. For dilute solutions, the density of the solution is nearly the same as that of water, so dissolving the salt in 1.00 L of water gives essentially 1.00 L of solution. Because each 1 mol of dissolved calcium oxalate monohydrate dissociates to produce 1 mol of calcium ions and 1 mol of oxalate ions, we can obtain the equilibrium concentrations that must be inserted into the solubility product expression. The number of moles of calcium oxalate monohydrate that dissolve in 100 mL of water is as follows:

$$\frac{7.36 \times 10^{-4} \text{ g}}{146.1 \text{ g/mol}} = 5.04 \times 10^{-6} \text{ mol Ca}(\text{O}_2\text{CCO}_2) \cdot \text{H}_2\text{O}$$

The number of moles of calcium oxalate monohydrate that dissolve in 1.00 L of the saturated solution is as follows:

$$\left( \frac{5.04 \times 10^{-6} \text{ mol Ca}(\text{O}_2\text{CCO}_2) \cdot \text{H}_2\text{O}}{100 \text{ mL}} \right) \left( \frac{1000 \text{ mL}}{1.00 \text{ L}} \right) = 5.04 \times 10^{-5} \text{ mol/L} = 5.04 \times 10^{-5} \text{ M}$$

Because of the stoichiometry of the reaction, the concentration of  $\text{Ca}^{2+}$  and  $\text{ox}^{2-}$  ions are both  $5.04 \times 10^{-5} \text{ M}$ . Inserting these values into the solubility product expression,

$$K_{\text{sp}} = [\text{Ca}^{2+}][\text{ox}^{2-}] = (5.04 \times 10^{-5})(5.04 \times 10^{-5}) = 2.54 \times 10^{-9}$$

In our calculation, we have ignored the reaction of the weakly basic anion with water, which tends to make the actual solubility of many salts greater than the calculated value.

#### Exercise

One crystalline form of calcium carbonate ( $\text{CaCO}_3$ ) is the mineral sold as “calcite” in mineral and gem shops. The solubility of calcite in water is 0.67 mg/100 mL. Calculate its  $K_{\text{sp}}$ .

**Answer:**  $4.5 \times 10^{-9}$

#### Note the Pattern

The reaction of weakly basic anions with  $\text{H}_2\text{O}$  tends to make the actual solubility of many salts higher than predicted.



**A crystal of calcite ( $\text{CaCO}_3$ ), illustrating the phenomenon of double refraction.** When a transparent crystal of calcite is placed over a page, we see two images of the letters.

Calcite, a structural material for many organisms, is found in the teeth of sea urchins. The urchins create depressions in limestone that they can settle in by grinding the rock with their teeth. Limestone, however, also consists of calcite, so how can the urchins grind the rock without also grinding their teeth? Researchers have discovered that the teeth are shaped like needles and plates and contain magnesium. The concentration of magnesium increases toward the tip, which contributes to the hardness. Moreover, each tooth is composed of two blocks of the polycrystalline calcite matrix that are interleaved near the tip. This creates a corrugated surface that presumably increases grinding efficiency. Toolmakers are particularly interested in this approach to grinding.

Tabulated values of  $K_{\text{sp}}$  can also be used to estimate the solubility of a salt with a procedure that is essentially the reverse of the one used in Example 1. In this case, we treat the problem as a typical equilibrium problem and set up a table of initial concentrations, changes in concentration, and final concentrations as we did in [Chapter 15](#), remembering that the concentration of the pure solid is essentially constant.

### Example 17.1.2

We saw that the  $K_{sp}$  for  $\text{Ca}_3(\text{PO}_4)_2$  is  $2.07 \times 10^{-33}$  at  $25^\circ\text{C}$ . Calculate the aqueous solubility of  $\text{Ca}_3(\text{PO}_4)_2$  in terms of the following:

1. the molarity of ions produced in solution
2. the mass of salt that dissolves in 100 mL of water at  $25^\circ\text{C}$

**Given:**  $K_{sp}$

**Asked for:** molar concentration and mass of salt that dissolves in 100 mL of water

**Strategy:**

**A** Write the balanced equilibrium equation for the dissolution reaction and construct a table showing the concentrations of the species produced in solution. Insert the appropriate values into the solubility product expression and calculate the molar solubility at  $25^\circ\text{C}$ .

**B** Calculate the mass of solute in 100 mL of solution from the molar solubility of the salt. Assume that the volume of the solution is the same as the volume of the solvent.

**Solution:**

1. **A** The dissolution equilibrium for  $\text{Ca}_3(\text{PO}_4)_2$  (Equation 17.1) is shown in the following table. Because we are starting with distilled water, the initial concentration of both calcium and phosphate ions is zero. For every 1 mol of  $\text{Ca}_3(\text{PO}_4)_2$  that dissolves, 3 mol of  $\text{Ca}^{2+}$  and 2 mol of  $\text{PO}_4^{3-}$  ions are produced in solution. If we let  $x$  equal the solubility of  $\text{Ca}_3(\text{PO}_4)_2$  in moles per liter, then the change in  $[\text{Ca}^{2+}]$  will be  $+3x$ , and the change in  $[\text{PO}_4^{3-}]$  will be  $+2x$ . We can insert these values into the table.

$\text{Ca}_3(\text{PO}_4)_2(s) \rightleftharpoons 3\text{Ca}^{2+}(aq) + 2\text{PO}_4^{3-}(aq)$			
	$\text{Ca}_3(\text{PO}_4)_2$	$[\text{Ca}^{2+}]$	$[\text{PO}_4^{3-}]$
initial	pure solid	0	0
change	—	$+3x$	$+2x$
final	pure solid	$3x$	$2x$

Although the *amount* of solid  $\text{Ca}_3(\text{PO}_4)_2$  changes as some of it dissolves, its *molar concentration* does not change. We now insert the expressions for the equilibrium concentrations of the ions into the solubility product expression (Equation 17.1.2):

$$\begin{aligned}
 K_{sp} &= [\text{Ca}^{2+}]^3 [\text{PO}_4^{3-}]^2 = (3x)^3 (2x)^2 \\
 2.07 \times 10^{-33} &= 108x^5 \\
 1.92 \times 10^{-35} &= x^5 \\
 1.14 \times 10^{-7} \text{ M} &= x
 \end{aligned}$$

This is the molar solubility of calcium phosphate at  $25^\circ\text{C}$ . However, the molarity of the ions is  $2x$  and  $3x$ , which means that  $[\text{PO}_4^{3-}] = 2.28 \times 10^{-7}$  and  $[\text{Ca}^{2+}] = 3.42 \times 10^{-7}$ .

2. **B** To find the mass of solute in 100 mL of solution, we assume that the density of this dilute solution is the same as the density of water because of the low solubility of the salt, so that 100 mL of water gives 100 mL of solution. We can then determine the amount of salt that dissolves in 100 mL of water:

$$\left( \frac{1.14 \times 10^{-7} \text{ mol}}{1 \text{ L}} \right) 100 \text{ mL} \left( \frac{1 \text{ L}}{1000 \text{ mL}} \right) \left( \frac{310.18 \text{ g Ca}_3(\text{PO}_4)_2}{1 \text{ mol}} \right) = 3.54 \times 10^{-6} \text{ g Ca}_3(\text{PO}_4)_2$$

Exercise

The solubility product of silver carbonate ( $\text{Ag}_2\text{CO}_3$ ) is  $8.46 \times 10^{-12}$  at  $25^\circ\text{C}$ . Calculate the following:

1. the molarity of a saturated solution
2. the mass of silver carbonate that will dissolve in 100 mL of water at this temperature

**Answer**

1.  $1.28 \times 10^{-4} \text{ M}$
2. 3.54 mg

## The Ion Product

The ion product ( $Q$ ) A quantity that has precisely the same form as the solubility product for the dissolution of a sparingly soluble salt, except that the concentrations used are not necessarily equilibrium concentrations. of a salt is the product of the concentrations of the ions in solution raised to the same powers as in the solubility product expression. It is analogous to the reaction quotient ( $Q$ ) discussed for gaseous equilibria in Chapter 15. Whereas  $K_{sp}$  describes equilibrium concentrations, the ion product describes concentrations that are *not* necessarily equilibrium concentrations.

### Note the Pattern

The ion product  $Q$  is analogous to the reaction quotient  $Q$  for gaseous equilibria.

As summarized in Figure 17.1.1, there are three possible conditions for an aqueous solution of an ionic solid:

1.  $Q < K_{sp}$ . The solution is unsaturated, and more of the ionic solid, if available, will dissolve.
2.  $Q = K_{sp}$ . The solution is saturated and at equilibrium.
3.  $Q > K_{sp}$ . The solution is supersaturated, and ionic solid will precipitate.

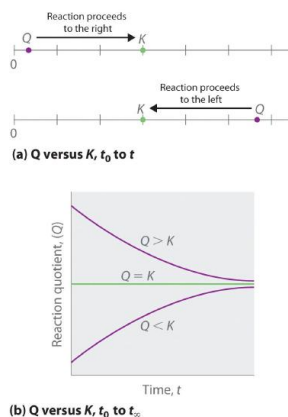


Figure 17.1.1 The Relationship between  $Q$  and  $K_{sp}$ . If  $Q$  is less than  $K_{sp}$ , the solution is unsaturated and more solid will dissolve until the system reaches equilibrium ( $Q = K_{sp}$ ). If  $Q$  is greater than  $K_{sp}$ , the solution is supersaturated and solid will precipitate until  $Q = K_{sp}$ . If  $Q = K_{sp}$ , the rate of dissolution is equal to the rate of precipitation; the solution is saturated, and no net change in the amount of dissolved solid will occur.

The process of calculating the value of the ion product and comparing it with the magnitude of the solubility product is a straightforward way to determine whether a solution is unsaturated, saturated, or supersaturated. More important, the ion product tells chemists whether a precipitate will form when solutions of two soluble salts are mixed.

### Example 17.1.3

We mentioned that barium sulfate is used in medical imaging of the gastrointestinal tract. Its solubility product is  $1.08 \times 10^{-10}$  at  $25^\circ\text{C}$ , so it is ideally suited for this purpose because of its low solubility when a “barium milkshake” is consumed by a patient. The pathway of the sparingly soluble salt can be easily monitored by x-rays. Will barium sulfate precipitate if 10.0 mL of 0.0020 M  $\text{Na}_2\text{SO}_4$  is added to 100 mL of  $3.2 \times 10^{-4}$  M  $\text{BaCl}_2$ ? Recall that  $\text{NaCl}$  is highly soluble in water.

**Given:**  $K_{sp}$  and volumes and concentrations of reactants

**Asked for:** whether precipitate will form

**Strategy:**

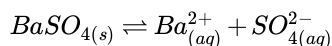
**A** Write the balanced equilibrium equation for the precipitation reaction and the expression for  $K_{sp}$ .

**B** Determine the concentrations of all ions in solution when the solutions are mixed and use them to calculate the ion product ( $Q$ ).

**C** Compare the values of  $Q$  and  $K_{sp}$  to decide whether a precipitate will form.

**Solution:**

**A** The only slightly soluble salt that can be formed when these two solutions are mixed is  $\text{BaSO}_4$  because  $\text{NaCl}$  is highly soluble. The equation for the precipitation of  $\text{BaSO}_4$  is as follows:



The solubility product expression is as follows:

$$K_{sp} = [\text{Ba}^{2+}][\text{SO}_4^{2-}] = 1.08 \times 10^{-10}$$

**B** To solve this problem, we must first calculate the ion product— $Q = [\text{Ba}^{2+}][\text{SO}_4^{2-}]$ —using the concentrations of the ions that are present after the solutions are mixed and before any reaction occurs. The concentration of  $\text{Ba}^{2+}$  when the solutions are mixed is the total number of moles of  $\text{Ba}^{2+}$  in the original 100 mL of  $\text{BaCl}_2$  solution divided by the final volume (100 mL + 10.0 mL = 110 mL):

$$\begin{aligned} \text{moles Ba}^{2+} &= 100 \text{ mL} \left( \frac{1 \text{ L}}{1000 \text{ mL}} \right) \left( \frac{3.2 \times 10^{-4} \text{ mol}}{1 \text{ L}} \right) = 3.2 \times 10^{-5} \text{ mol Ba}^{2+} \\ [\text{Ba}^{2+}] &= \left( \frac{3.2 \times 10^{-5} \text{ mol Ba}^{2+}}{110 \text{ mL}} \right) \left( \frac{1000 \text{ mL}}{1 \text{ L}} \right) = 2.9 \times 10^{-4} \text{ M Ba}^{2+} \end{aligned}$$

Similarly, the concentration of  $\text{SO}_4^{2-}$  after mixing is the total number of moles of  $\text{SO}_4^{2-}$  in the original 10.0 mL of  $\text{Na}_2\text{SO}_4$  solution divided by the final volume (110 mL):

$$\begin{aligned} \text{moles SO}_4^{2-} &= 10.0 \text{ mL} \left( \frac{1 \text{ L}}{1000 \text{ mL}} \right) \left( \frac{0.0020 \text{ mol}}{1 \text{ L}} \right) = 2.0 \times 10^{-5} \text{ mol SO}_4^{2-} \\ [\text{SO}_4^{2-}] &= \left( \frac{2.0 \times 10^{-5} \text{ mol SO}_4^{2-}}{110 \text{ mL}} \right) \left( \frac{1000 \text{ mL}}{1 \text{ L}} \right) = 1.8 \times 10^{-4} \text{ M SO}_4^{2-} \end{aligned}$$

We can now calculate  $Q$ :

$$Q = [\text{Ba}^{2+}][\text{SO}_4^{2-}] = (2.9 \times 10^{-4})(1.8 \times 10^{-4}) = 5.2 \times 10^{-8}$$

**C** We now compare  $Q$  with the  $K_{sp}$ . If  $Q > K_{sp}$ , then  $\text{BaSO}_4$  will precipitate, but if  $Q < K_{sp}$ , it will not. Because  $Q > K_{sp}$ , we predict that  $\text{BaSO}_4$  will precipitate when the two solutions are mixed. In fact,  $\text{BaSO}_4$  will continue to precipitate until the system reaches equilibrium, which occurs when  $[\text{Ba}^{2+}][\text{SO}_4^{2-}] = K_{sp} = 1.08 \times 10^{-10}$ .

Exercise

The solubility product of calcium fluoride ( $\text{CaF}_2$ ) is  $3.45 \times 10^{-11}$ . If 2.0 mL of a 0.10 M solution of  $\text{NaF}$  is added to 128 mL of a  $2.0 \times 10^{-5}$  M solution of  $\text{Ca}(\text{NO}_3)_2$ , will  $\text{CaF}_2$  precipitate?

**Answer:** yes ( $Q = 4.7 \times 10^{-11} > K_{sp}$ )

## The Common Ion Effect and Solubility

The solubility product expression tells us that the equilibrium concentrations of the cation and the anion are inversely related. That is, as the concentration of the anion increases, the maximum concentration of the cation needed for precipitation to occur decreases—and vice versa—so that  $K_{sp}$  is constant. Consequently, the solubility of an ionic compound depends on the concentrations of other salts that contain the same ions. This dependency is another example of the common ion effect discussed in [Section 16.6](#): adding a common cation or anion shifts a solubility equilibrium in the direction predicted by Le Chatelier's principle. As a result, *the solubility of any sparingly soluble salt is almost always decreased by the presence of a soluble salt that contains a common ion.* The exceptions generally involve the formation of complex ions, which is discussed in [Section 17.3](#).

Consider, for example, the effect of adding a soluble salt, such as  $\text{CaCl}_2$ , to a saturated solution of calcium phosphate  $[\text{Ca}_3(\text{PO}_4)_2]$ . We have seen that the solubility of  $\text{Ca}_3(\text{PO}_4)_2$  in water at  $25^\circ\text{C}$  is  $1.14 \times 10^{-7}$  M ( $K_{sp} = 2.07 \times 10^{-33}$ ). Thus a saturated solution of  $\text{Ca}_3(\text{PO}_4)_2$  in water contains  $3 \times (1.14 \times 10^{-7} \text{ M}) = 3.42 \times 10^{-7}$  M  $\text{Ca}^{2+}$  and  $2 \times (1.14 \times 10^{-7} \text{ M}) = 2.28 \times 10^{-7}$  M  $\text{PO}_4^{3-}$ , according to the stoichiometry shown in [Equation 17.1.1](#) (neglecting hydrolysis to form  $\text{HPO}_4^{2-}$  as described in [Chapter 16](#)). If  $\text{CaCl}_2$  is added to a saturated solution of  $\text{Ca}_3(\text{PO}_4)_2$ , the  $\text{Ca}^{2+}$  ion concentration will increase such that  $[\text{Ca}^{2+}] > 3.42 \times 10^{-7}$  M, making  $Q > K_{sp}$ . The only way the system can return to equilibrium is for the reaction in [Equation 17.1.1](#) to proceed to the left, resulting in precipitation of  $\text{Ca}_3(\text{PO}_4)_2$ . This will decrease the concentration of both  $\text{Ca}^{2+}$  and  $\text{PO}_4^{3-}$  until  $Q = K_{sp}$ .

### Note the Pattern

The common ion effect usually decreases the solubility of a sparingly soluble salt.

### Example 17.1.4

Calculate the solubility of calcium phosphate  $[\text{Ca}_3(\text{PO}_4)_2]$  in 0.20 M  $\text{CaCl}_2$ .

**Given:** concentration of  $\text{CaCl}_2$  solution

**Asked for:** solubility of  $\text{Ca}_3(\text{PO}_4)_2$  in  $\text{CaCl}_2$  solution

**Strategy:**

**A** Write the balanced equilibrium equation for the dissolution of  $\text{Ca}_3(\text{PO}_4)_2$ . Tabulate the concentrations of all species produced in solution.

**B** Substitute the appropriate values into the expression for the solubility product and calculate the solubility of  $\text{Ca}_3(\text{PO}_4)_2$ .

**Solution:**

**A** The balanced equilibrium equation is given in the following table. If we let  $x$  equal the solubility of  $\text{Ca}_3(\text{PO}_4)_2$  in moles per liter, then the change in  $[\text{Ca}^{2+}]$  is once again  $+3x$ , and the change in  $[\text{PO}_4^{3-}]$  is  $+2x$ . We can insert these values into the table.

$\text{Ca}_3(\text{PO}_4)_2(\text{s}) = 3\text{Ca}^{2+}(\text{aq}) + 2\text{PO}_4^{3-}(\text{aq})$			
	$\text{Ca}_3(\text{PO}_4)_2$	$[\text{Ca}^{2+}]$	$[\text{PO}_4^{3-}]$
initial	pure solid	0.20	0
change	—	$+3x$	$+2x$
final	pure solid	$0.20 + 3x$	$2x$

**B** The  $K_{\text{sp}}$  expression is as follows:

$$K_{\text{sp}} = [\text{Ca}^{2+}]^3[\text{PO}_4^{3-}]^2 = (0.20 + 3x)^3(2x)^2 = 2.07 \times 10^{-33}$$

Because  $\text{Ca}_3(\text{PO}_4)_2$  is a sparingly soluble salt, we can reasonably expect that  $x \ll 0.20$ . Thus  $(0.20 + 3x)$  M is approximately 0.20 M, which simplifies the  $K_{\text{sp}}$  expression as follows:

$$\begin{aligned} K_{\text{sp}} &= (0.20)^3(2x)^2 = 2.07 \times 10^{-33} \\ x^2 &= 6.5 \times 10^{-32} \\ x &= 2.5 \times 10^{-16} \text{ M} \end{aligned} \quad (17.2.1)$$

This value is the solubility of  $\text{Ca}_3(\text{PO}_4)_2$  in 0.20 M  $\text{CaCl}_2$  at 25°C. It is approximately nine orders of magnitude less than its solubility in pure water, as we would expect based on Le Chatelier's principle. With one exception, this example is identical to Example 2—here the initial  $[\text{Ca}^{2+}]$  was 0.20 M rather than 0.

Exercise

Calculate the solubility of silver carbonate in a 0.25 M solution of sodium carbonate. The solubility of silver carbonate in pure water is  $8.45 \times 10^{-12}$  at 25°C.

**Answer:**  $2.9 \times 10^{-6}$  M (versus  $1.3 \times 10^{-4}$  M in pure water)

### Summary

The equilibrium constant for a dissolution reaction, called the **solubility product** ( $K_{\text{sp}}$ ), is a measure of the solubility of a compound. Whereas solubility is usually expressed in terms of *mass* of solute per 100 mL of solvent,  $K_{\text{sp}}$  is defined in terms of the *molar* concentrations of the component ions. In contrast, the **ion product** ( $Q$ ) describes concentrations that are not necessarily equilibrium concentrations. Comparing  $Q$  and  $K_{\text{sp}}$  enables us to determine whether a precipitate will form when solutions of two soluble salts are mixed. Adding a common cation or common anion to a solution of a sparingly soluble salt shifts the solubility equilibrium in the direction predicted by Le Chatelier's principle. The solubility of the salt is almost always decreased by the presence of a common ion.

### Key Takeaway

- The solubility product ( $K_{\text{sp}}$ ) is used to calculate equilibrium concentrations of the ions in solution, whereas the ion product ( $Q$ ) describes concentrations that are not necessarily at equilibrium.

### Conceptual Problems

- Write an expression for  $K_{sp}$  for each salt.
  - AgI
  - CaF<sub>2</sub>
  - PbCl<sub>2</sub>
  - Ag<sub>2</sub>CrO<sub>4</sub>
- Some species are not represented in a solubility product expression. Why?
- Describe the differences between  $Q$  and  $K_{sp}$ .
- How can an ion product be used to determine whether a solution is saturated?
- When using  $K_{sp}$  to directly compare the solubilities of compounds, why is it important to compare only the  $K_{sp}$  values of salts that have the same stoichiometry?
- Describe the effect of a common ion on the solubility of a salt. Is this effect similar to the common ion effect found in buffers? Explain your answer.
- Explain why the presence of MgCl<sub>2</sub> decreases the molar solubility of the sparingly soluble salt MgCO<sub>3</sub>.

### Answers

- $K_{sp} = [Ag^+][I^-]$
  - $K_{sp} = [Ca^{2+}][F^-]^2$
  - $K_{sp} = [Pb^{2+}][Cl^-]^2$
  - $K_{sp} = [Ag^+]^2[CrO_4^{2-}]$
- 
- 
- 
- For a 1:1 salt, the molar solubility is simply  $\sqrt{K_{sp}}$ ; for a 2:1 salt, the molar solubility is  $\sqrt[3]{K_{sp}/4}$ . Consequently, the magnitudes of  $K_{sp}$  can be correlated with molar solubility *only* if the salts have the same stoichiometry.
- 
- Because of the common ion effect. Adding a soluble Mg<sup>2+</sup> salt increases [Mg<sup>2+</sup>] in solution, and Le Chatelier's principle predicts that this will shift the solubility equilibrium of MgCO<sub>3</sub> to the left, decreasing its solubility.

### Numerical Problems

- Predict the molar solubility of each compound using the  $K_{sp}$  values given in [Table E3](#).
  - Cd(IO<sub>3</sub>)<sub>2</sub>
  - AgCN
  - HgI<sub>2</sub>
- Predict the molar solubility of each compound using the  $K_{sp}$  values given.
  - Li<sub>3</sub>PO<sub>4</sub>:  $2.37 \times 10^{-11}$
  - Ca(IO<sub>3</sub>)<sub>2</sub>:  $6.47 \times 10^{-6}$
  - Y(IO<sub>3</sub>)<sub>3</sub>:  $1.12 \times 10^{-10}$
- A student prepared 750 mL of a saturated solution of silver sulfate (Ag<sub>2</sub>SO<sub>4</sub>). How many grams of Ag<sub>2</sub>SO<sub>4</sub> does the solution contain?  $K_{sp} = 1.20 \times 10^{-5}$ .
- Given the  $K_{sp}$  values in [Table 17.1.1](#) and [Table E3](#), predict the molar concentration of each species in a saturated aqueous solution.
  - silver bromide
  - lead oxalate
  - iron(II) carbonate
  - silver phosphate

5. copper(I) cyanide
5. Given the  $K_{sp}$  values in Table 17.1.1 and Table E3, predict the molar concentration of each species in a saturated aqueous solution.
  1. copper(I) chloride
  2. lanthanum(III) iodate
  3. magnesium phosphate
  4. silver chromate
  5. strontium sulfate
6. Silicon dioxide, the most common binary compound of silicon and oxygen, constitutes approximately 60% of Earth's crust. Under certain conditions, this compound can react with water to form silicic acid, which can be written as either  $H_4SiO_4$  or  $Si(OH)_4$ . Write a balanced chemical equation for the dissolution of  $SiO_2$  in basic solution. Write an equilibrium constant expression for the reaction.
7. The  $K_{sp}$  of  $Mg(OH)_2$  is  $5.61 \times 10^{-12}$ . If you tried to dissolve 24.0 mg of  $Mg(OH)_2$  in 250 mL of water and then filtered the solution and dried the remaining solid, what would you predict to be the mass of the undissolved solid? You discover that only 1.0 mg remains undissolved. Explain the difference between your expected value and the actual value.
8. The  $K_{sp}$  of lithium carbonate is  $8.15 \times 10^{-4}$ . If 2.34 g of the salt is stirred with 500 mL of water and any undissolved solid is filtered from the solution and dried, what do you predict to be the mass of the solid? You discover that all of your sample dissolves. Explain the difference between your predicted value and the actual value.
9. You have calculated that 24.6 mg of  $BaSO_4$  will dissolve in 1.0 L of water at 25°C. After adding your calculated amount to 1.0 L of water and stirring for several hours, you notice that the solution contains undissolved solid. After carefully filtering the solution and drying the solid, you find that 22.1 mg did not dissolve. According to your measurements, what is the  $K_{sp}$  of barium sulfate?
10. In a saturated silver chromate solution, the molar solubility of chromate is  $6.54 \times 10^{-5}$ . What is the  $K_{sp}$ ?
11. A saturated lead(II) chloride solution has a chloride concentration of  $3.24 \times 10^{-2}$  mol/L. What is the  $K_{sp}$ ?
12. From the solubility data given, calculate  $K_{sp}$  for each compound.
  1. AgI:  $2.89 \times 10^{-7}$  g/100 mL
  2.  $SrF_2$ :  $1.22 \times 10^{-2}$  g/100 mL
  3.  $Pb(OH)_2$ : 78 mg/500 mL
  4.  $BiAsO_4$ : 14.4 mg/2.0 L
13. From the solubility data given, calculate  $K_{sp}$  for each compound.
  1.  $BaCO_3$ : 10.0 mg/500 mL
  2.  $CaF_2$ : 3.50 mg/200 mL
  3.  $Mn(OH)_2$ :  $6.30 \times 10^{-4}$  g/300 mL
  4.  $Ag_2S$ :  $1.60 \times 10^{-13}$  mg/100 mL
14. Given the following solubilities, calculate  $K_{sp}$  for each compound.
  1.  $BaCO_3$ :  $7.00 \times 10^{-5}$  mol/L
  2.  $CaF_2$ : 1.70 mg/100 mL
  3.  $Pb(IO_3)_2$ : 2.30 mg/100 mL
  4.  $SrC_2O_4$ :  $1.58 \times 10^{-7}$  mol/L
15. Given the following solubilities, calculate  $K_{sp}$  for each compound.
  1.  $Ag_2SO_4$ :  $4.2 \times 10^{-1}$  g/100 mL
  2.  $SrSO_4$ :  $1.5 \times 10^{-3}$  g/100 mL
  3.  $CdC_2O_4$ :  $6.0 \times 10^{-3}$  g/100 mL
  4.  $Ba(IO_3)_2$ :  $3.96 \times 10^{-2}$  g/100 mL
16. The  $K_{sp}$  of the phosphate fertilizer  $CaHPO_4 \cdot 2H_2O$  is  $2.7 \times 10^{-7}$  at 25°C. What is the molar concentration of a saturated solution? What mass of this compound will dissolve in 3.0 L of water at this temperature?

17. The  $K_{sp}$  of zinc carbonate monohydrate is  $5.5 \times 10^{-11}$  at  $25^\circ\text{C}$ . What is the molar concentration of a saturated solution? What mass of this compound will dissolve in 2.0 L of water at this temperature?
18. Silver nitrate eye drops were formerly administered to newborn infants to guard against eye infections contracted during birth. Although silver nitrate is highly water soluble, silver sulfate has a  $K_{sp}$  of  $1.20 \times 10^{-5}$  at  $25^\circ\text{C}$ . If you add 25.0 mL of 0.015 M  $\text{AgNO}_3$  to 150 mL of  $2.8 \times 10^{-3}$  M  $\text{Na}_2\text{SO}_4$ , will you get a precipitate? If so, what will its mass be?
19. Use the data in Table E3 to predict whether precipitation will occur when each pair of solutions is mixed.
  1. 150 mL of 0.142 M  $\text{Ba}(\text{NO}_3)_2$  with 200 mL of 0.089 M  $\text{NaF}$
  2. 250 mL of 0.079 M  $\text{K}_2\text{CrO}_4$  with 175 mL of 0.087 M  $\text{CaCl}_2$
  3. 300 mL of 0.109 M  $\text{MgCl}_2$  with 230 mL of 0.073 M  $\text{Na}_2(\text{C}_2\text{O}_4)$
20. What is the maximum volume of 0.048 M  $\text{Pb}(\text{NO}_3)_2$  that can be added to 250 mL of 0.10 M  $\text{NaSCN}$  before precipitation occurs?  $K_{sp} = 2.0 \times 10^{-5}$  for  $\text{Pb}(\text{SCN})_2$ .
21. Given 300 mL of a solution that is 0.056 M in lithium nitrate, what mass of solid sodium carbonate can be added before precipitation occurs (assuming that the volume of solution does not change after adding the solid)?  $K_{sp} = 8.15 \times 10^{-4}$  for  $\text{Li}_2\text{CO}_3$ .
22. Given the information in the following table, calculate the molar solubility of each sparingly soluble salt in 0.95 M  $\text{MgCl}_2$ .

Saturated Solution	$K_{sp}$
$\text{MgCO}_3 \cdot 3\text{H}_2\text{O}$	$2.4 \times 10^{-6}$
$\text{Mg}(\text{OH})_2$	$5.6 \times 10^{-12}$
$\text{Mg}_3(\text{PO}_4)_2$	$1.04 \times 10^{-24}$

### Answers

1. 1.  $1.84 \times 10^{-3}$  M  
2.  $7.73 \times 10^{-9}$  M  
3.  $1.9 \times 10^{-10}$  M
- 2.
3. 3.37 g
- 4.
5. 1.  $4.15 \times 10^{-4}$  M  
2.  $7.26 \times 10^{-4}$  M  
3.  $6.26 \times 10^{-6}$  M  
4.  $6.54 \times 10^{-5}$  M  
5.  $5.86 \times 10^{-4}$  M
- 6.
7. 22.4 mg; a secondary reaction occurs, where  $\text{OH}^-$  from the dissociation of the salt reacts with  $\text{H}^+$  from the dissociation of water. This reaction causes further dissociation of the salt (Le Chatelier's principle).
- 8.
9.  $1.2 \times 10^{-10}$
- 10.
11.  $1.70 \times 10^{-5}$
- 12.
- 13.
- 14.
15. 1.  $8.8 \times 10^{-6}$   
2.  $6.7 \times 10^{-9}$

- 3.  $9.0 \times 10^{-8}$
- 4.  $2.16 \times 10^{-9}$

16.

17.  $7.4 \times 10^{-6}$  M; 2.1 mg

18.

19. Precipitation will occur in all cases.

20.

21. 8.27 g

22.

### Contributors

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