22.1: Stern-Gerlach Experiment

In 1920, Otto Stern and Walter Gerlach designed an experiment that unintentionally led to the discovery that electrons have their own individual, continuous spin even as they move along their orbital of an atom. The experiment was done by putting a silver foil in an oven to vaporize its atoms. The silver atoms were collected into a beam that passed through an inhomogeneous magnetic field. The result was that the magnetic beam split the beam into two (and only two) separate ones. The Stern–Gerlach experiment demonstrated that the spatial orientation of angular momentum is quantized into two components (up and down). Thus an atomic-scale system was shown to have intrinsically quantum properties. The experiment is normally conducted using electrically neutral particles such as silver atoms. This avoids the large deflection in the path of a charged particle moving through a magnetic field and allows spin-dependent effects to dominate.

If the particle is treated as a classical spinning magnetic dipole, it will precess in a magnetic field because of the torque that the magnetic field exerts on the dipole. If it moves through a homogeneous magnetic field, the forces exerted on opposite ends of the dipole cancel each other out and the trajectory of the particle is unaffected. However, if the magnetic field is inhomogeneous then the force on one end of the dipole will be slightly greater than the opposing force on the other end, so that there is a net force which deflects the particle's trajectory. If the particles were classical spinning objects, one would expect the distribution of their spin angular momentum vectors to be random and continuous. Each particle would be deflected by an amount proportional to its magnetic moment, producing some density distribution on the detector screen. Instead, the particles passing through the Stern–Gerlach apparatus are equally distributed among two possible values, with half of them ending up at an upper spot ("spin up"), and the other half at the lower spot ("spin down"). Since the particles are deflected by a magnetic field, spin is a magnetic property that is associated to some intrinsic form of angular momentum. As we saw in chapter 6, the quantization of the angular momentum gives energy levels that are $(2\ell+1)$-fold degenerate. Since along the direction of the magnet we observe only two possible eigenvalues for the spin, we conclude the following value for s:

$$2s+1=2 \quad \Rightarrow \quad s=\frac{1}{2}. \tag{22.1.1}$$

The Stern-Gerlach experiment proves that electrons are spin-$\frac{1}{2}$ particles. These have only two possible spin angular momentum values measured along any axis, $+\frac{\hbar}{2}$ or $-\frac{\hbar}{2}$, a purely quantum mechanical phenomenon. Because its value is always the same, it is regarded as an intrinsic property of electrons, and is sometimes known as "intrinsic angular momentum" (to distinguish it from orbital angular momentum, which can vary and depends on the presence of other particles).

The act of observing (measuring) the momentum along the z direction corresponds to the operator $\hat{S}_z$, which project the value of the total spin operator $\hat{S}^2$ along the z axis. The eigenvalues of the projector operator are:

$$\hat{S}_z\phi=\hbar m_s\phi, \tag{22.1.2}$$

where $m_s=\{-s,+s\}=\left\{-\frac{1}{2},+\frac{1}{2}\right\}$ is the spin quantum number along the z component. The eigenvalues for the total spin operator $\hat{S}^2$—similarly to the angular momentum operator $\hat{L}^2$ seen in Equation 22.3.6—are:

$$\hat{S}^2\phi=\hbar^2 s(s+1)\phi, \tag{22.1.3}$$

The initial state of the particles in the Stern-Gerlach experiment is given by the following wave function:

$$\phi=c_1\,\phi_\uparrow+c_2\,\phi_\downarrow, \tag{22.1.4}$$

where $\uparrow=+\frac{\hbar}{2}, \downarrow=-\frac{\hbar}{2}$, and the coefficients c_1 and c_2 are complex numbers. In this initial state, spin can point in any direction. The expectation value of the operator $\hat{S}_z$ (the quantity that the Stern-Gerlach experiment measures), can be obtained using Equation 22.2.6:

$$\begin{aligned} <S_z> &= \int \phi^* \hat{S}_z \phi \, d\mathbf{s} \\ &= +\frac{\hbar}{2}|c_1|^2-\frac{\hbar}{2}|c_2|^2, \end{aligned}$$

where the integration is performed along a special coordinate $\mathbf{s}$ composed of only two values, and the coefficient c_1 and c_2 are complex numbers. Applying the normalization condition, Equation 6.2.4 we can obtain:

$$|c_1|^2 + |c_2|^2 = 1 \quad \longrightarrow \quad |c_1|^2 = |c_2|^2 = \frac{1}{2}. \tag{22.1.5}$$

This equation is not sufficient to determine the values of the coefficients since they are complex numbers. Equation 22.1.5, however, tells us that the squared magnitudes of the coefficients can be interpreted as probabilities of outcome from the experiment. This is true because their values are obtained from the normalization condition, and the normalization condition guarantees that the system is observed with probability equal to one. Summarizing, since we started with random initial directions, each of the two states, $\phi_{\uparrow}$ and $\phi_{\downarrow}$, will be observed with equal probability of $\frac{1}{2}$.