3.3: Reversible and Irreversible Processes

Let's now consider the cycle in Figure $3.3.1$. The process in this case starts from state 1 (system at P_1V_1), expands to state 2 (system at P_2V_2), and compresses back to state 1 (system back to P_1V_1).

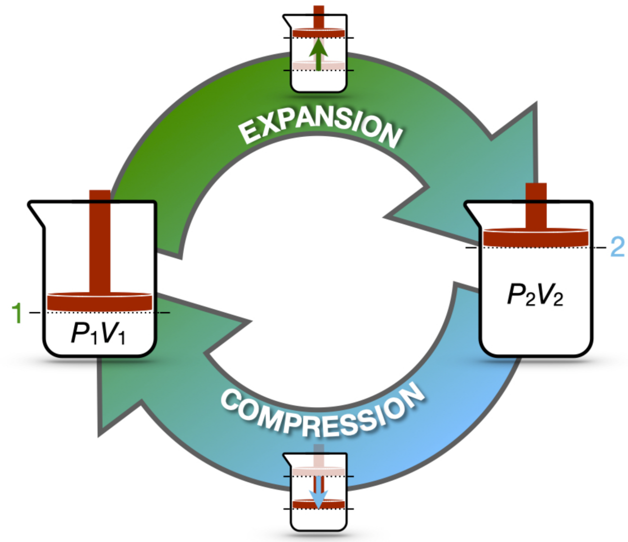

Figure $3.3.1$: Expansion/Compression Cycle of an Ideal Gas.

Since the process starts and finishes at the same state, the value of the internal energy at the end of the process will be the same as its value at the beginning, regardless of the path:[1]

$$\oint dU = 0, \tag{3.3.1}$$

where the symbol $\oint$ indicates an integral around a cycle. Considering the work associated with the cycle, however, the situation is radically different because it depends on the path that the system is taking, and in general

$$\oint_{\text{path}} đW \neq 0. \tag{3.3.2}$$

For instance, if we perform the expansion in one step, the work associated with it will be (using Equation $3.3.8$):[2]

$$W_{\text{1-step}}^{\text{expansion}} = -P_2 \underbrace{(V_2 - V_1)}_{>0} < 0, \tag{3.3.3}$$

and if we also perform the compression in 1-step: [3]

$$W_{\text{1-step}}^{\text{compression}} = -P_1 \underbrace{(V_1 - V_2)}_{<0} > 0. \tag{3.3.4}$$

With a little bit of math, it is easy to prove that the total work for the entire cycle is:

$$\begin{aligned} W_{\text{1-step}}^{\text{cycle}} &= W_{\text{1-step}}^{\text{expansion}} + W_{\text{1-step}}^{\text{compression}} \\ &= -P_2(V_2 - V_1) - P_1(V_1 - V_2) \\ &= -P_2(V_2 - V_1) + P_1(V_2 - V_1) \\ &= \underbrace{(V_2 - V_1)}_{>0}\underbrace{(P_1 - P_2)}_{>0} > 0, \end{aligned} \tag{3.3.5}$$

or, in other words, net work is destroyed.

Note

In practice, if we want to manually perform this cycle by pushing on the piston by hand, we will notice that it requires more energy to push down than the amount it gives back when we release it, and it moves back up.

In contrast, if both the expansion and the compression happen in a slow ∞-step manner, the work associated with them will be W_{max} and W_{min}, respectively, which are calculated using Equation $3.3.14$. The total work related with the cycle will be in this case:

$$\begin{aligned} W_{\infty\text{-step}}^{\text{cycle}} &= W_{\text{max}}^{\text{expansion}} + W_{\text{min}}^{\text{compression}} \\ &= -nRT \ln \frac{V_f}{V_i} - nRT \ln \frac{V_i}{V_f} \\ &= -nRT \underbrace{\left(\ln \frac{V_f}{V_i} - \ln \frac{V_f}{V_i} \right)}_{=0} = 0, \end{aligned} \tag{3.3.6}$$

which means that, in this case, work is not destroyed nor created.

Note

In practice, if we were able to perform this cycle manually by pushing on the piston down by hand, we will notice that it requires the same amount of energy to push down than the amount it gives back when it moves up.

This process can happen both ways without losses, and is called *reversible*:

Definition: Reversible Process

Reversible Process: a process whose direction can be returned to its original position by inducing infinitesimal changes to some property of the system via its surroundings.[4]

Reversible processes are ideal processes that are hard to realize in practice since they require transformations that happen in an infinite amount of steps (infinitely slowly).

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1. recall that the internal energy is a state function, so its value depends exclusively from the conditions at the beginning and at the end. In a cycle, we're going back to the same point, so the conditions at the beginning and at the end are equal by definition.
2. notice that the work for the expansion is negative, as it should be.
3. notice that the work for the compression is positive, as it should be.
4. Definition from: Sears, F.W. and Salinger, G.L. (1986), Thermodynamics, Kinetic Theory, and Statistical Thermodynamics, 3rd edition (Addison-Wesley.)
