5.3: Efficiency of a Carnot Cycle

The efficiency (ε) of a cycle is defined as the ratio between the absolute value of the work extracted from the cycle ($|W_{\text{TOT}}|$) and the heat that gets into the system ($|Q_h|$):

$$\varepsilon = \frac{|W_{\text{TOT}}|}{|Q_h|} = \frac{-W_{\text{TOT}}}{Q_1} \tag{5.3.1}$$

where the minus sign in front of the work is necessary because the efficiency is defined as a positive number. Replacing Equation 5.2.5 into eq. 5.3.1, we obtain:

$$\varepsilon = \frac{Q_3 + Q_1}{Q_1} = 1 + \frac{Q_3}{Q_1}.$$

If we go back to Equation 5.3.1 and we replace Equation 5.2.3 for W_{TOT} and Equation 5.1.3 for Q_1, we obtain:

$$\varepsilon = \frac{nR\,(T_h - T_l)\ln V_B/V_A}{nRT_h \ln V_B/V_A} = \frac{T_h - T_l}{T_h} = 1 - \frac{T_l}{T_h} < 1, \tag{5.3.2}$$

which proves that the efficiency of a Carnot cycle is strictly smaller than 1.[1] In other words, no cycle can convert 100% of the heat into work it extracts from a hot reservoir. This finding had remarkable consequences on the entire thermodynamics field and set the foundation for the introduction of entropy. We will use eqs. 5.3.1 and 5.3.2 for this purpose in the next chapter, but we conclude the discussion on Carnot cycles by returning back to Lord Kelvin. In 1851 he used this finding to state his statement "It is impossible for a self-acting machine, unaided by any external agency, to convey heat from one body to another at a higher temperature. It is impossible, by means of inanimate material agency, to derive mechanical effect from any portion of matter by cooling it below the temperature of the coldest of the surrounding objects."[2] This statement conclusively disproved Joule's original theories and demonstrated that there is some fundamental principle to govern the flow of heat beyond the first law of thermodynamics.

1. Equation 5.3.2 can be equal to 1 only if $T_l = 0$ K or $T_h = \infty$, two conditions that are both equally impossible.
2. Thomson W. Transactions of the Royal Society of Edinburgh. 1851 XX 261–268, 289–298..