18.1: The Time-Independent Schrödinger Equation

We can start the derivation of the single-particle time-independent Schrödinger equation (TISEq) from the equation that describes the motion of a wave in classical mechanics:

$$\psi(x,t)=\exp[i(kx-\omega t)], \tag{18.1.1}$$

where x is the position, t is time, $k=\dfrac{2\pi}{\lambda}$ is the wave vector, and $\omega=2\pi\nu$ is the angular frequency of the wave. If we are not concerned with the time evolution, we can consider uniquely the derivatives of Equation 18.1.1 with respect to the location, which are:

$$\begin{aligned}\frac{\partial\psi}{\partial x}&=ik\exp[i(kx-\omega t)]=ik\psi,\\ \frac{\partial^2\psi}{\partial x^2}&=i^2k^2\exp[i(kx-\omega t)]=-k^2\psi,\end{aligned}$$

where we have used the fact that $i^2=-1$.

Assuming that particles behaves as wave—as proven by de Broglie's we can now use the first of de Broglie's equation, Equation 17.5.4, we can replace $k=\dfrac{p}{\hbar}$ to obtain:

$$\frac{\partial^2\psi}{\partial x^2}=-\frac{p^2\psi}{\hbar^2}, \tag{18.1.2}$$

which can be rearranged to:

$$p^2\psi=-\hbar^2\frac{\partial^2\psi}{\partial x^2}. \tag{18.1.3}$$

The total energy associated with a wave moving in space is simply the sum of its kinetic and potential energies:

$$E=\frac{p^2}{2m}+V(x), \tag{18.1.4}$$

from which we can obtain:

$$p^2=2m[E-V(x)], \tag{18.1.5}$$

which we can then replace into Equation 18.1.3 to obtain:

$$2m[E-V(x)]\psi=-\hbar^2\frac{\partial^2\psi}{\partial x^2}, \tag{18.1.6}$$

which can then be rearranged to the famous **time-independent Schrödinger equation (TISEq)**:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+V(x)\psi=E\psi, \tag{18.1.7}$$

A two-body problem can also be treated by this equation if the mass m is replaced with a reduced mass $\mu=\dfrac{m_1m_2}{m_1+m_2}$.

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