5.2: Energy, Heat, and Work in the Carnot Cycle

Summarizing the results of the previous sections, the total amount of energy for a Carnot cycle is:

$$\begin{aligned}
\Delta U_{\text{TOT}} &= \Delta U_1 + \Delta U_2 + \Delta U_3 + \Delta U_4 \\
&= 0 + n\int_{T_h}^{T_l} C_V dT + 0 + n\int_{T_l}^{T_h} C_V dT \\
&= n\int_{T_h}^{T_l} C_V dT - n\int_{T_h}^{T_l} C_V dT = 0
\end{aligned}$$

which is obviously zero, since $\oint dU = 0$. The amounts of work and heat, however, are not zero, since Q and W are path functions. Therefore:

$$\begin{aligned}
W_{\text{TOT}} &= W_1 + W_2 + W_3 + W_4 \\
&= -nRT_h \ln\frac{V_B}{V_A} + n\int_{T_h}^{T_l} C_V dT - nRT_l \ln\frac{V_D}{V_C} + n\int_{T_l}^{T_h} C_V dT \\
&= -nRT_h \ln\frac{V_B}{V_A} - nRT_l \ln\frac{V_D}{V_C},
\end{aligned}$$

which, considering that $V_C/V_D = V_B/V_A$, reduces to:

$$W_{\text{TOT}} = -nR\left(T_h - T_l\right)\ln\frac{V_B}{V_A} < 0,$$

which is negative, because $T_h > T_l$ and $V_B > V_A$. Negative work means that the work is done by the system. In other words, the system is performing PV-work by transferring heat from a hot reservoir to a cold one via a Carnot cycle. On the other hand, for the heat:

$$\begin{aligned}
Q_{\text{TOT}} &= Q_1 + Q_2 + Q_3 + Q_4 \\
&= Q_h + 0 + Q_l + 0 \\
&= nRT_h \ln\frac{V_B}{V_A} + nRT_l \ln\frac{V_D}{V_C} \\
&= nR\left(T_h - T_l\right)\ln\frac{V_B}{V_A} = -W_{\text{TOT}},
\end{aligned}$$

which, simplifies to:

$$W_{\text{TOT}} = -(Q_1 + Q_3),$$

and, replacing Q_1 and Q_3 with the absolute values of the heats drawn from the hot and cold reservoirs, $|Q_h|$, and $|Q_l|$ respectively:

$$|W_{\text{TOT}}| = |Q_h| - |Q_l|,$$

or, in other words, more heat is extracted from the hot reservoir than it is put into the cold one. The difference between the absolute value of these amounts of heat gives the total work of the cycle. This process is depicted in Figure $5.2.1$.

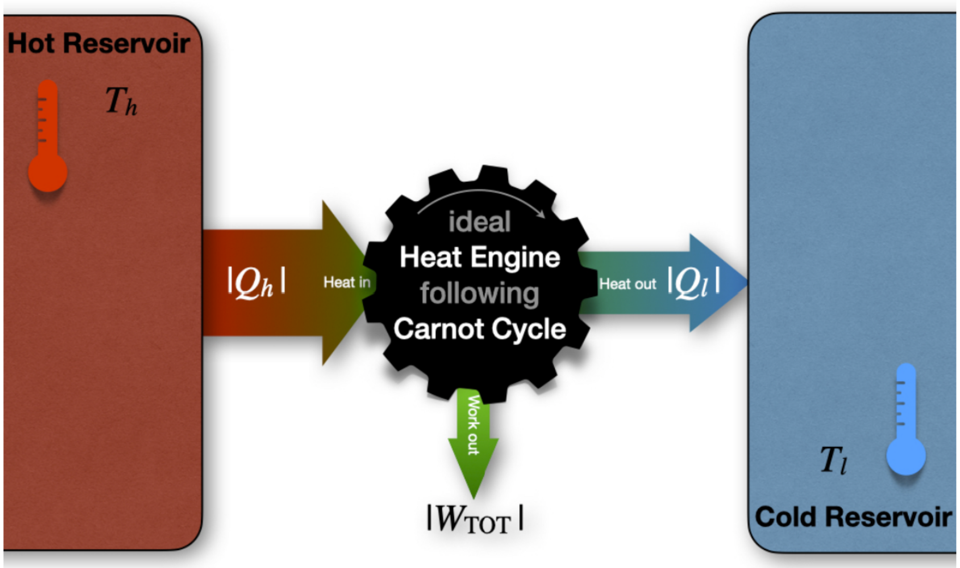

Figure $5.2.1$: Carnot Cycle Diagram.

? Exercise $5.2.1$

Up to this point, we have discussed Carnot cycles working in the hot $\rightarrow$ cold direction ($A \rightarrow B \rightarrow C \rightarrow D \rightarrow A$), since this is the primary mode of operation of heat engines that produce work. However, a heat engine could also—in principle—work in the reversed cold $rightarrow$ hot direction ($A \rightarrow D \rightarrow C \rightarrow B \rightarrow A$). Write the equations for heat, work, and energy of each stage of a Carnot cycle going the opposite direction than the one discussed in sections 5.1 and 5.2.

Answer

When the heat engine works in reverse order, the formulas remain the same, but all the signs in front of Q, W, and U will be reversed. In this case, the total work would get into the systems, and heat would be transferred from the cold reservoir to the hot one. Figure $5.2.1$would be modified as:

This reversed mode of operation is the basic principle behind refrigerators and air conditioning.