

22.2: Sequential Stern-Gerlach Experiments

An interesting result can be obtained if we link multiple Stern–Gerlach apparatuses into one experiment and we perform the measurement along two orthogonal directions in space. As we showed in the previous section, all particles leaving the first Stern–Gerlach apparatus are in an eigenstate of the \hat{S}_z operator (i.e., their spin is either “up” or “down” with respect to the z -direction). We can then take either one of the two resulting beams (for simplicity let’s take the “spin up” output), and perform another spin measurement on it. If the second measurement is also aligned along the z -direction then only one outcome will be measured, since all particles are already in the “spin up” eigenstate of \hat{S}_z . In other words, the measurement of a particle being in an eigenstate of the corresponding operator leaves the state unchanged.

If, however, we perform the spin measurement along a direction perpendicular to the original z -axis (i.e., the x -axis) then the output will equally distribute among “spin up” or “spin down” in the x -direction, which in order to avoid confusion, we can call “spin left” and “spin right”. Thus, even though we knew the state of the particles beforehand, in this case the measurement resulted in a random spin flip in either of the measurement directions. Mathematically, this property is expressed by the nonvanishing of the commutator of the spin operators:

$$[\hat{S}_z, \hat{S}_x] \neq 0. \quad (22.2.1)$$

We can finally repeat the measurement a third time, with the magnet aligned along the original z -direction. According to classical physics, after the second apparatus, we would expect to have one beam with characteristic “spin up” and “spin left”, and another with characteristic “spin up” and “spin right”. The outcome of the third measurement along the original z -axis should be one output with characteristic “spin up”, regardless to which beam the magnet is applied (since the “spin down” component should have been “filtered out” by the first experiment, and the “spin left” and “spin right” component should be filtered out by the third magnet). This is not what is observed. The output of the third measurement is—once again—two beams in the z direction, one with “spin up” characteristic and the other with “spin down”.

This experiment shows that spin is not a classical property. The Stern–Gerlach apparatus does not behave as a simple filter, selecting beams with one specific pre-determined characteristic. The second measurement along the x axis destroys the previous determination of the angular momentum in the z direction. This means that this property cannot be measured on two perpendicular directions at the same time.

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