# 17.1: Newtonian Formulation

Classical mechanics as formulated by Isaac Newton (1652-1727) is all about forces. Newtonian mechanics works well for problems where we know the forces and have a reasonable coordinate system. In these cases, the net force acting on a system at position $q$ is simply:

$$F_{\text{net}}(q)=m\ddot{q}=m\frac{d^2q}{dt^2}. \tag{17.1.1}$$

Or, in other words, if we know the net force acting on a system of mass $m$ at position $q$ at some time $t_0$, we can use Equation 17.1.1 to calculate the position of the system at any future (or past) time. We have completely determined the dynamical evolution of the system.[1]

## Example 17.1.1

A ball of mass $m$ is at ground level and tossed straight up from an initial position $q_0$ with an initial velocity $\dot{q}_0$ and subject to gravity alone. Calculate the equation of motion for the ball (i.e. where is the ball going to be after some time $t$?).[2]

**Answer**

Since the only force acting on the ball is gravity, we can use the equation for the gravitational force to start our derivation:

$$F_{\text{gravity}}=-mG,$$

with $G$ the usual gravitational constant ($G=9.8\ \mathrm{m/s^2}$). We can then replace this expression into Equation 17.1.1, to obtain:

$$\begin{aligned}-mG&=m\ddot{q}\\ -G&=\ddot{q}\\ -G&=\frac{d\dot{q}}{dt},\end{aligned}$$

which can then be integrated with respect to time, to obtain:

$$\begin{aligned}-G\int_{t=0}^{t}dt&=\int_{\dot{q}_0}^{\dot{q}}d\dot{q}\\ \dot{q}&=\dot{q}_0-Gt\\ \frac{dq}{dt}&=\dot{q}_0-Gt,\end{aligned}$$

which can be further integrated with respect to time, to give:

$$\begin{aligned}\int_{q_0}^{q}dq&=\int_{t=0}^{t}\dot{q}_0dt-G\int_{t=0}^{t}tdt\\ q&=q_0+\dot{q}_0t-\frac{1}{2}Gt^2.\end{aligned}$$

This final equation is the equation of motion for the ball, from which we can calculate the position of the ball at any time $t$. Notice how the equation of motion does not depend on the mass of the ball!

## Exercise 17.1.2

How much time will a ball ejected from a height of $1\ \mathrm{m}$ at an initial velocity of $10\ \mathrm{m/s}$ take to hit the floor?

**Answer**

We can use the equation of motion obtained above to solve this problem, and obtain for this specific case $t\simeq 2.12\ \mathrm{s}$.[3]

The formula of Newtonian mechanics are not the only one we can use to solve a problem in classical mechanics. We have at least two other equivalent approaches to the same problem that might end up being more useful in certain situations.

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1. Notice that, in principle, $q \in \mathbb{R}^n$ is the position vector and $\dot{q} \in \mathbb{R}^n$ is the velocity vector. As such, all the equation of classical mechanics are vector equation, and not just simple numerical equation, as we present them here! For our purposes, we can restrict ourselves to a 1-dimensional space, hence forgetting the complications of vector algebra.
2. This example is based on Rhett Allain's blog post that can be found (here)[https://rhettallain.com/2018/10/31/classical-mechanics-newtonian-lagrangian-and-hamiltonian/]
3. Can you write a python program to do this calculation?

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