1.8: Problems

1. Consider a sphere with a mass of 1.00 kg rolling on a frictionless parabolic surface where the relationship between the height (h) and the position (x) is given by

$$h = x^2$$

a. At what point on the surface (what value of x) will the sphere have the maximum kinetic energy?
b. What will the potential energy be at the point you specified in a?
c. If the sphere begins at rest at position x = -1.00 m, what is its potential energy?
d. Given that the sum of potential and kinetic energy is a constant, derive an expression for kinetic energy as a function of position for the system.

2. Consider the vectors **u** and **v** given by

$$\mathbf{u} = 3\mathbf{i} + 2\mathbf{j}$$
$$\mathbf{v} = 2\mathbf{i} - \mathbf{j}$$

where i and j are unit vectors in the x and y directions respectively.

a. Calculate the magnitudes of vectors **u** and **v**.
b. Find expressions for vectors $\mathbf{e_1}$ and $\mathbf{e_2}$ which are unit vectors parallel to u and v respectively.
c. Are the vectors u and v orthogonal? Demonstrate this mathematically.
d. Consider a vector $\mathbf{w} = 3\mathbf{i} - 6\mathbf{j}$. find values for c_1 and c_2 in order to express w as a linear combination of $\mathbf{e_1}$ and $\mathbf{e_2}$.
e. $\mathbf{w} = c_1\mathbf{e}_1 + c_2\mathbf{e}_2$

3. Consider a string that is distorted from equilibrium at time $t = 0$ such that its wavefunction is given by

$$\Psi(x) = \frac{1}{\sqrt{5}}\phi_1(x) + \frac{2}{\sqrt{5}}\phi_2(x)$$

where $\phi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$.

a. Show that the functions $\phi_n(x)$ form an orthogonal set of functions. To do this, show that

$\int_0^a \phi_n(x)\cdot\phi_m(x)dx = 0$ for $n \neq m$

b. Show that

$$\int_0^a \Psi(x)\cdot\Psi(x)dx = 1$$

c. Show that

$$\int_0^a \Psi(x)\cdot\phi_1(x)dx = \frac{1}{\sqrt{5}} \text{ and } \int_0^a \Psi(x)\cdot\phi_2(x)dx = \frac{2}{\sqrt{5}}$$

4. Calculate the kinetic energy and de Broglie wavelength for the following particles traveling at a velocity of 500 m/s.

a. an electron
b. a nitrogen molecule
c. a ball bearing with mass = 0.500 g

5. The wavelength of light from one line of an argon ion laser is 488 nm.

Metal	Work Function (eV)
Al	4.08
Fe	4.5
Co	5.0
Cu	4.7

Metal	Work Function (eV)
Ag	4.73
Au	5.1
Na	2.28
K	2.3
Cs	2.1

a. Calculate the energy of a photon of this energy in
   i. J
   ii. kJ/mol
   iii. eV
b. Of the elements in the table to the left, which (if any) would produce photoelectrons if light of $\lambda = 488\ nm$ is focused on the surface?
c. What would be the kinetic energy of a photoelectron ejected from the surface of cesium produced by light of $\lambda = 488\ nm$?
d. What is the longest wavelength of light that will produce photoelectrons from the surface of silver?