5.1: Spherical Polar Coordinates

The description of a rotating molecule in Cartesian coordinates would be very cumbersome. The problem is actually much easier to solve in **spherical polar coordinates**. Consider a particle that is located in space at some arbitrary point (x,y,z). In spherical polar coordinates, the position of a particle is also described by three variables, namely r, θ, and ϕ. These variables are defined according to the diagram. The distance from the origin to the point is specified by r. θ gives the angle formed by the position vector of the point and the positive z-axis. ϕ give the angle of rotation from the positive x-axis of the projection of the position vector into the xy plane. The ranges of possible values for r, θ and ϕ are given by

$$
\begin{aligned}
0 \leq r \leq \infty \\
0 \leq \theta \leq \pi \\
0 \leq \phi \leq 2\pi
\end{aligned}
$$

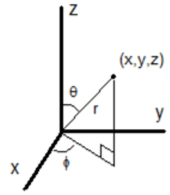

Figure 5.1.1

The coordinates of any point can be transformed from spherical polar coordinates to Cartesian coordinates using the following equations.

$$
\begin{aligned}
x &= r \sin\theta \cos\phi \\
y &= r \sin\theta \sin\phi \\
z &= r \cos\theta
\end{aligned}
$$

The coordinates can be transformed from Cartesian coordinates to spherical polar coordinates by these equations.

$$
\begin{aligned}
r &= \sqrt{x^2 + y^2 + z^2} \\
\theta &= \tan^{-1}\left(\frac{y}{x}\right) \\
\phi &= \cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)
\end{aligned}
$$