7.1: Perturbation Theory

Often times, a system represents only a small difference from an exactly solvable system. In these instances, **perturbation theory** can be used to describe the system. To use perturbation theory, one must separate the Hamiltonian into two parts: one for which the solution is known ($\hat{H}^{(0)}$) and the other part which will represent the perturbation to the system ($\hat{H}^{(1)}$).

$$\hat{H} = \hat{H}^{(0)} + \hat{H}^{(1)}$$

The solution for the unperturbed system is known.

$$\hat{H}^{(0)}\psi_n^{(0)} = E_n^{(0)}\psi_n^{(0)}$$

The energy levels and wavefunctions for the perturbed system are determined by applying a series of corrections (referred to as first order, second order, etc.)

$$E_n = E_n^{(o)} + E_n^{(1)} + E_n^{(2)} + \dots$$

$$\psi_n = \psi_n^{(o)} + \psi_n^{(1)} + \psi_n^{(2)} + \dots$$

Oftentimes only the first and second order corrections are needed to give a reasonable description of the system. The first order correction to the energy is given by

$$E_n^{(1)} = \int \psi_n^{(0)}\hat{H}^{(1)}\psi_n^{(0)} d\tau$$

The second order correction to the energy depends on the first order correction to the wavefunctions.

$$E_n^{(2)} = \int \psi_n^{(0)}\hat{H}^{(1)}\psi_n^{(1)} d\tau$$

The formula for generating the first order corrections to the wavefunctions is given by

$$\psi_n^{(1)} = \sum_{i\neq n} \psi_i^{(0)} \frac{\int \psi_i^{(0)}\hat{H}^{(0)}\psi_n^{(0)} d\tau}{E_n^{(0)} - E_i^{(0)}}$$

Substitution into the expression for $E_n^{(2)}$ yields

$$E_n^{(2)} = \sum_{i\neq n} \frac{\left|\int \psi_n^{(0)} H^{(1)}\psi_n^{(0)} d\tau\right|^2}{E_n^{(0)} - E_1^{(0)}}$$