3.1: Overview

Group Theory is the mathematical theory associated with the mathematical properties of groups. In chemistry, group theory is the mathematics of symmetry. A **group** (G) is a set of elements (A, B, etc.) that can be associated through a mathematical operation (sometimes referred to as a **multiplication operation**, eg. $A * B$) and satisfying the following criteria:

1. The group must have an **identity element** (E) such that for each element A in the group, $A * E = E * A = A$. (It can be proven that for a given group and multiplication operation, the identity element is unique.)
2. Each element A in the group must have an **inverse** (A^{-1}) that is also a member of the group and that satisfies the criterion $A * A^{-1} = A^{-1} * A = E$. (It can be proven that each element has one and only one inverse.)
3. The group must be **closed** under multiplication. That means that for any pair of elements in the group A and B for which $A * B = C$, C must also be a member of the group.

Note that the multiplication operation need not be **commutative**. The order of multiplication may matter. There is no guarantee that $A * B = B * A$. Many groups that satisfy this property are called **abelian** groups.

The set of numbers 1 and –1 form an abelian group under the normal operation of simple multiplication. A simple **group multiplication table** table can be constructed for this group.

	1	-1
1	1	-1
-1	-1	1

Clearly, the identity element in this group is 1 since multiplication by 1 gives the same number back. Also, both members happen to be their own inverse since

$$1 * 1 = 1 \text{ and } (-1) * (-1) = 1$$