11.1: Nodes and Wave Nature

One of the first introductions students of the Quantum Theory receive involves the nodes in the wavefunctions of a one-dimensional particle in a box. The probability of measuring the particle to exist at any given position in the box is given by the square of the wavefunction. For the $n = 2$ level, the squared wavefunction is plotted above.

The figure shows that the probability of measuring the position of the particle at positions $x = \frac{a}{4}$ and $x = \frac{3a}{4}$ or the maxima and that there is zero probability of measuring the particle to exist at the endpoints or at $x = \frac{a}{2}$, the middle position of the box. One might wonder how the particle can travel from one side of the box to the other without ever actually being in the middle. If one models the particle as a small ball bearing traveling from end to end in an evacuated, sealed glass tube (consistent with a deterministic view in which the particle has a definite location at all times) the prediction is clearly troubling. For many, this creates a dilemma.

The reconciliation of this dilemma requires that one abandon a notion of determinism in embracing the wave-nature of the particle. Namely, if one accepts the wave description of the particle, the notion of a definite location become meaningless since the wave must be delocalized across the entire box. In fact, the wave even exists at the central node despite the value of the wavefunction being zero! This concept provides a clear challenge to the notion of determinism that is suggested by Newtonian physics. The idea of "matter waves" also lead to a proposal by Louis de Broglie that matter-wave interference should be observable.