6.6: Problems

1. Calculate the finite-mass Rydberg constant (R_M) for
   a. H
   b. D
   c. $_7N$
   d. $_{11}Na$
2. The 1s orbital wavefunction for hydrogen is given by

$$\psi_{1s} = \frac{1}{\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2} e^{-\frac{r}{a_0}}$$

   a. Show that this wavefunction is normalized.
   b. Find the expectation value of r in units of a_0 (the Bohr Radius.)
3. Show that the 2s wavefunction for hydrogen is
   a. Normalized
   b. An eigenfunction of the Hamiltonian. (What is the eigenvalue?)
4. The Laguerre Polynomial $L_1(x)$ is given by

$$L_1(x) = -x + 1$$

The Associated Laguerre polynomials are generated from the relationship

$$L_n^{\alpha}(x) = \frac{d^{\alpha}}{dx^{\alpha}} L_n(x)$$

   a. Show that the Associated Laguerre polynomials $L_1^0(x) = -x + 1$, $L_1^1(x) = -1$, and $L_1^2(x) = 0$. (In fact, $L_1^{\alpha}(x) = 0$ for any choice of $\alpha > 1$.)
   b. Given that the Associated Laguerre polynomials used in the radial wavefunctions of the Hydrogen atom problem are $L_{n+l}^{2l+1}(x)$, derive a relationship between n and l that ensure that $L_{n+l}^{2l+1}(x) \neq 0$.
5. Using the Laguerre polynomials $L_2(x) = \frac{1}{2}\left(x^2 - 4x + 2\right)$ and $L_1(x) = -x + 1$, show that

$$\frac{d}{dx} L_n(x) = \frac{d}{dx} L_{n-1}(x) - L_{n-1}(x)$$

6. Sketch the radial wavefunctions for the 1s, 2s, 2p, 3s, 3p, and 3d orbital wavefunctions of Hydrogen.
7. Determine the number of nodes in each of the following hydrogen atom orbital wavefunctions:

wavefunction	Total nodes	Angular nodes	Radial nodes
2s			
3p			
5d			
6f			

8. Determine the ionization potential for $_3He^+$.
   a. Find R_{He} for the He-3 isotope.
   b. Use the relationship

$$IP = Z^2 R_M \left(\frac{1}{(1)^2} - \frac{1}{(\infty)^2}\right)$$

9. Based on the following data, find the ionization energy of Rb, using the fact that at high excitation, the quantum defect (δ) becomes constant.

n (for the $np \leftarrow 5s$ transition)	Wavenumber (cm^{-1})
5	12578.950
6	23715.081
7	27835.02
8	29834.94
9	30958.91
10	31653.85
11	32113.55
12	32433.50
13	32665.03
14	32838.02
15	32970.66
16	33074.59
17	33157.54
18	33224.83
19	33280.13
20	33326.13

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