12.1: Appendix I

Some Useful Mathematical Identities

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^{n+1}\frac{x^{2n-1}}{(2n-1)!}$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n\frac{x^{2n}}{(2n)!}$$

$$e^{\pm i\theta} = \cos(\theta) \pm i\sin(\theta) \qquad \cos(\theta) = \frac{e^{i\theta}+e^{-i\theta}}{2} \qquad \sin(\theta) = \frac{e^{i\theta}-e-i\theta}{2}$$

Some Useful Integrals

$$\int \sin(\alpha x)\sin(\beta x)dx = \frac{\sin[(\alpha-\beta)x]}{2(\alpha-\beta)} - \frac{\sin[(\alpha+\beta)x]}{2(\alpha+\beta)} \qquad \alpha \neq \beta$$

$$\int \sin^2(\alpha x)dx = \frac{x}{2} - \frac{\sin(2\alpha x)}{4\alpha} \qquad \int \sin^3(\alpha x)\ dx = \frac{\cos(3\alpha x) - 9\cos(\alpha x)}{12\alpha}$$

$$\int x\sin^2(\alpha x)dx = \frac{x^2}{4} - \frac{x\sin(2\alpha x)}{4\alpha} - \frac{\cos(2\alpha x)}{8\alpha^2}$$

$$\int x^2\sin^2(\alpha x)dx = \frac{x^3}{6} - \left(\frac{x^2}{4\alpha} - \frac{1}{8\alpha^3}\right)\sin(2\alpha x) - \frac{x\cos(2\alpha x)}{4\alpha^2}$$

$$\int \cos(\alpha x)\cos(\beta x)dx = \frac{\sin[(\alpha-\beta)x]}{2(\alpha-\beta)} + \frac{\sin[(\alpha+\beta)x]}{2(\alpha+\beta)}$$

$$\int \cos^2(\alpha x)dx = \frac{x}{2} + \frac{1}{4\alpha}\sin(2\alpha x) \qquad \int x\ \cos^2(\alpha x)dx = \frac{x^2}{4} + \frac{x\sin(2\alpha x)}{4\alpha} + \frac{\cos(2\alpha x)}{8\alpha^2}$$

$$\int x^2\ \cos^2(\alpha x)dx = \frac{x^3}{6} + \left(\frac{x^2}{4\alpha} - \frac{1}{8\alpha^3}\right)\sin(2\alpha x) + \frac{x\cos(2\alpha x)}{4\alpha^2}$$

$$\int_0^\infty x^n e^{-\alpha x}dx = \frac{n!}{\alpha^{n+1}} \text{ (n a positive integer)} \qquad \int_0^\infty e^{-\alpha x^2}dx = \sqrt{\frac{\pi}{4\alpha}}$$

$$\int_0^\infty x^{2n}e^{-\alpha x^2}dx = \frac{1\cdot 3\cdot 5\cdots(2n-1)}{2^{n+1}\alpha^n}\sqrt{\frac{\pi}{\alpha}} \qquad \int_0^\infty x^{2n+1}e^{-\alpha x^2}dx = \frac{n!}{2\alpha^{n+1}}$$

Some Useful Coordinate Transformations

Plane Polar Coordinates: $0 \leq r \leq \infty$; $0 \leq \theta \leq 2\pi$

$$x = r\cos\theta \qquad y = r\sin\theta \qquad \theta = \arctan\left(\frac{y}{x}\right) \qquad r = \sqrt{x^2+y^2}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$$

Spherical Polar Coordinates: $0 \leq r \leq \infty$; $0 \leq \theta \leq \pi$; $0 \leq \phi \leq 2\pi$

$$x = r\sin\theta\cos\phi \qquad y = r\sin\theta\sin\phi \qquad z = r\cos\theta$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial \phi^2}$$

$$dx\ dy\ dz = r^2 \sin\theta\, dr\, d\theta\, d\phi$$