2.8: Entanglement and Schrödinger's Cat

There are many elements of the quantum theory that produce bizarre results (at least compared to our intuition as residents in a classical physics world. As it turns out, some of the early pioneers of a quantum theory (such as Albert Einstein and Erin Schrödinger) found these elements of strangeness too much to handle. Both expended a great deal of energy to eliminate quantum mechanics as an accepted theory that would shape modern science. As it turns out, all of the bizarreness predicted by quantum mechanics has withstood the tests of experimentation, despite the concerns and well-thought objections of these two scientific giants.

Entanglement and Spooky Action at a Distance

One of Einstein's objections came in the form of what he named "spooky action at a distance." To understand this phenomenon, consider the decomposition of a p-meson into an electron and a positron. Since the original particle has zero spin, in order to conserve angular momentum, must be "spinning" in opposite directions. In other words, one has $m_s = +\frac{1}{2}$ and the other has $\mathrm{m_s} = -\frac{1}{2}$.

$$\beta^+ \longleftarrow \pi_0 \longrightarrow \beta^-$$

The wavefunction that describes this system prior to the measurement of the spin of either particle is given by

$$\psi_{\text{spin}} = \frac{1}{\sqrt{2}}(\alpha_+\beta_- - \beta_+\alpha_-)$$

which allows for the possibility that either particle is spin up or spin down to be equally lightly. But the spins of the two particles are intimately coupled to one another. If the electron $(\beta^-)$is spin up (α) then the positron (β^+)must be spin down (β) (and vice versa.) This property is an example of entanglement where the properties of one particle are entangled with those of the other through the wavefunction that describes the entire system.

Now suppose that the spin of the electron is measured and determined, the spin of the other is determined at the same time. As such, the measurement of the property of one particle causes the wavefunction of the other particle to change instantaneously. This is what Einstein referred to as "spooky action at a distance." This action would require information to be transferred across space at a speed faster than the speed of light, violating Einstein's theory of relativity.

This paradox has been studied extensively and remains a topic of research interest. It should be noted that whenever these sort of issues crop up, it is quantum mechanics that seems to prevail over relativity. (Sorry Einstein!)

Schrödinger's Cat

Erwin Schrödinger's involvement in trying to dissuade the scientific community from embracing quantum theory is particularly peculiar, as it was the development of the wave equation that is still used today that won him the Nobel Prize in 1933. None the less, Schrödinger found himself quite troubled by the conclusions of the quantum theory. Toward that end, in 1935, he published a paper in which he described a thought experiment that had to give the scientific world pause where quantum theory was concerned.

The problem was stated thusly. Imagine a box inside of which no observation could be made unless the box was opened. Inside, was placed a cat, a bottle of poison (prussic acid) and a radioactive atom. If the atom decays, a hammer will drop on the poison, killing the cat. The experiment was to wait one half-life of the atom. At that point, the wavefunction for the atom was given by

$$\Psi_{\text{atom}} = \frac{1}{\sqrt{2}}\psi_{\text{decayed}} + \frac{1}{\sqrt{2}}\psi_{\text{undecayed}}$$

This implies that it is equally likely that the atom has decayed as not decayed. And since the life of the cat was tied to the state of the atom, it is equally likely that the cat is dead or alive. Therefore, the "wavefunction" for the cat would be given by

$$\Psi_{\text{cat}} = \frac{1}{\sqrt{2}}\psi_{\text{dead}} + \frac{1}{\sqrt{2}}\psi_{\text{alive}}$$

This implies that the cat is neither dead nor alive, but both with equal probability! And even for the most lethargic of cats, it is very clear that animal is either alive or not. The notion that it is both is simply preposterous! This is the conclusion of which Schrödinger hoped to convince the scientific world. Alas, experimentation has failed to uphold Schrödinger's notion that quantum mechanics provides an incorrect description of the atom.

There have been numerous treatises on these topics and beyond. (The strangeness of quantum mechanics has been a very thought provoking topic indeed!) After completing a course in quantum mechanics (such as this one) a student should be well prepared to explore some of these very intriguing and perplexing predictions.