11.3: The Stern-Gerlach Experiment

One of the very interesting aspects of many small particles, including electrons, is that of spin. (The original Stern-Gerlach experiment [13] was performed on a beam of silver atoms, but the result apply to electrons as well.) The property of spin creates a magnetic moment for these particles. For electrons, which have $s = \frac{1}{2}$, the component of angular momentum along an external axis can take two possible values, $m_s = \pm\frac{1}{2}$. That means that an electron traveling through an inhomogeneous magnetic field can align its magnetic moment either with or against the external field. The ramifications are very interesting.

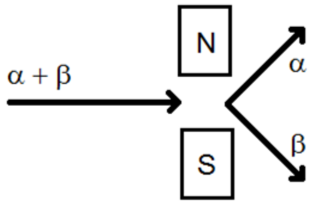

Figure 11.3.1

A beam of electrons that passes through an inhomogeneous magnetic field will be split into two beams. Those electrons whose magnetic moment aligned with the field will be deflected in one direction, and those with a magnetic field aligned against the external field will be deflected in the other. Each beam can then be considered as containing only electrons that are either “spin up” ($\alpha, m_s = +\frac{1}{2}$) or “spin down” ($\beta, m_s = -\frac{1}{2}$). As such, if one of the beams passes through another magnetic field that it oriented parallel to the first, no further splitting occurs since all of the electrons in that sub-beam have their spins aligned.

However, things get very interesting when the second magnetic field is oriented at 90° to the first. Since the magnetic moments of the electrons are aligned perpendicular to the external magnetic field, there should be no effect. What actually happens is that the beam again splits into two sub-beams, just as the original beam did!

If the second magnetic field is placed at some other angle, the beam will still split into two components, but the intensities will be determined by the magnitude of the projection of the electron magnetic moment along the external axis. That magnitude is easily calculable if one thinks of the spin wavefunction as a linear combination of two spin functions in the rotated axis system.

$$\Psi_{spin} = \frac{1}{\sqrt{2}} cos(\theta) \cdot \alpha + \frac{1}{\sqrt{2}} sin(\theta) \cdot \beta$$

where θ is the angle between the two magnetic fields. The factors of $\frac{1}{\sqrt{2}}$ are to normalize the wavefunction. The probabilities then of measuring the spin as either an α or β state is given by the squares of the corresponding Fourier coefficients.

$$P(\alpha) = \frac{1}{2}\cos^2(\theta)$$

$$P(\beta) = \frac{1}{2}\sin^2(\theta)$$

This conclusion will be useful in interpreting later results.

One very important question that the Stern-Gerlach result raises deals directly with Determinacy. The question is whether or not an individual electron “knows” that it is α or β before interacting with the detector. The results (particularly for the experiments where a beam of selected spin particles is resplit) suggests that it is the interaction with the detector that forces the particle into one state or the other.

In this manner, the Stern-Gerlach result shows is that making a measurement on a system will, in fact, alter that system. The interaction of the electrons with the external field causes an alignment of the individual magnetic moments (either with or against the external field.)

The types of experiments (and specifically spin detectors) used in the Stern-Gerlach experiment can be used to help to frame the next step in the Einstein-Bohr debates on the completeness of quantum mechanics.

13. W. Gerlach and O. Stern, "Das magnetische Moment des Silberatoms," Zeitschrift für Physik, vol. 9, p. 353–355, 1922. W430W9405