2.6: Problem-Solving Strategies

We know the conversion factor is correct when units cancel appropriately, but a conversion factor is not unity, however. Rather it is a physical quantity (or the reciprocal of a physical quantity) which is related to the two other quantities we are interconverting. The conversion factor works because of the relationship, *not* because it is has a value of one. Once we have established that a relationship exists, it is no longer necessary to memorize a mathematical formula. The units tell us whether to use the conversion factor or its reciprocal. Without such a relationship, however, mere cancellation of units does not guarantee that we are doing the right thing.

A simple way to remember relationships among quantities and conversion factors is a "road map"of the type shown below:

$$\text{Mass} \overset{density}{\longleftrightarrow} \text{volume or } m \overset{\rho}{\longleftrightarrow} V \tag{2.6.1}$$

This indicates that the mass of a particular sample of matter is related to its volume (and the volume to its mass) through the conversion factor, density. The double arrow indicates that a conversion may be made in either direction, provided the units of the conversion factor cancel those of the quantity which was known initially. In general the road map can be written

$$\text{First quantity} \overset{\text{conversion factor}}{\longleftrightarrow} \text{second quantity} \tag{2.6.2}$$

General Steps in Performing Dimensional Analysis

1. Identify the "**given**" information in the problem. Look for a number with units to start this problem with.
2. What is the problem asking you to "**find**"? In other words, what unit will your answer have?
3. Use **ratios** and conversion factors to cancel out the units that aren't part of your answer, and leave you with units that are part of your answer.
4. When your units cancel out correctly, you are ready to do the **math**. You are multiplying fractions, so you multiply the top numbers and divide by the bottom numbers in the fractions.

As we come to more complicated problems, where several steps are required to obtain a final result, such road maps will become more useful in charting a path to the solution.

Example 2.6.1: Volume to Mass Conversion

Black ironwood has a density of 67.24 lb/ft^3. If you had a sample whose volume was 47.3 ml, how many grams would it weigh? (1 lb = 454 g; 1 ft = 30.5 cm).

Solution

The road map

$$V \overset{\rho}{\rightarrow} m$$

tells us that the mass of the sample may be obtained from its volume using the conversion factor, density. Since milliliters and cubic centimeters are the same, we use the SI units for our calculation:

$$\text{Mass} = m = 47.3\text{cm}^3 \times \frac{67.24 \text{ lb}}{1 \text{ ft}^3}$$

Since the volume units are different, we need a unity factor to get them to cancel:

$$m = 47.3 \text{ cm}^3 \times \left(\frac{1 \text{ ft}}{30.5 \text{ cm}}\right)^3 \times \frac{67.24 \text{ lb}}{1 \text{ ft}^3} = 47.3 \text{ cm}^3 \times \frac{1 \text{ ft}^3}{30.5^3 \text{ cm}^3} \times \frac{67.24 \text{ lb}}{1 \text{ ft}^3}$$

We now have the mass in pounds, but we want it in grams, so another unity factor is needed:

$$m = 47.3 \text{ cm}^3 \times \frac{1 \text{ ft}^3}{30.5^3 \text{ cm}^3} \times \frac{67.24 \text{ lb}}{1 \text{ ft}^3} \times \frac{454 \text{ g}}{1 \text{ lb}} = 500.9 \text{ g}$$

In subsequent chapters we will establish a number of relationships among physical quantities. Formulas will be given which define these relationships, but we do not advocate slavish memorization and manipulation of those formulas. Instead we recommend that you remember that a relationship exists, perhaps in terms of a road map, and then adjust the quantities involved so that the units cancel appropriately. Such an approach has the advantage that you can solve a wide variety of problems by using the same technique.

Contributors and Attributions

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