11.5: Applications of the Ideal Gas Law- Molar Volume, Density and Molar Mass of a Gas

Learning Objectives

- Apply gas laws to solve stoichiometry problems.
- Apply principles of stoichiometry to calculate properties of gases.

The quantitative relationship of reactants and products is called **stoichiometry**. Stoichiometric problems require you to calculate the amounts of reactants required for certain amounts of products, or amounts of products produced from certain amounts of reactants. If, in a chemical reaction, one or more reactants or products are gases, gas laws must be considered for the calculation. Usually, the applications of the ideal gas law give results within 5% precision. Below, we review several important concepts that are helpful for solving Stoichiometry Problems Involving Gases.

The Mole Concept and Molar Volume

The **mole** concept is the key to both stoichiometry and gas laws. A mole is a definite amount of substance. Mole is a unit based on the number of identities (i.e. atoms, molecules, ions, or particles). A mole of anything has the same number of identities as the number of atoms in exactly 12 grams of carbon-12, the most abundant isotope of carbon.

Molar volume is defined as the volume occupied by one mole of a gas. Using the ideal gas law and assuming standard pressure and temperature (STP), the volume of one mole of gas can be calculated:

$$PV = nRT \tag{11.5.1}$$

$$V = \frac{nRT}{P} \tag{11.5.2}$$

$$V = \frac{1.00\text{mol} \cdot 0.08206 \dfrac{\text{Latm}}{\text{molK}} \cdot 273\text{K}}{1.00\text{atm}} \tag{11.5.3}$$

$$V = 22.4\text{L} \tag{11.5.4}$$

In other words, 1 mole of a gas will occupy 22.4 L at STP, assuming ideal gas behavior.

At STP, the volume of a gas is only dependent on number of moles of that gas and is independent of molar mass. With this information we can calculate the density (ρ) of a gas using only its molar mass. First, starting with the definition of density

$$\rho = \frac{m}{V} \tag{11.5.5}$$

we rearrange for volume:

$$V = \frac{m}{\rho} \tag{11.5.6}$$

We then substitute V into the ideal gas equation and rearrange for density:

$$PV = nRT \tag{11.5.7}$$

$$P\frac{m}{\rho} = nRT \tag{11.5.8}$$

$$\rho = \frac{mP}{nRT} \tag{11.5.9}$$

Finally, we remember that molar mass is equal to mass divided by number of moles:

$$MM = \frac{m}{n} \tag{11.5.10}$$

and substitute this into our expression for density to give:

$$\rho = \frac{MM \cdot P}{RT} \tag{11.5.11}$$

This equation can further be simplified if we assume STP:

$$\rho = \frac{MM \cdot 1\text{atm}}{(0.08206\frac{\text{Latm}}{\text{molK}})273\text{K}} \tag{11.5.12}$$

$$\rho = \frac{MM}{22.4\frac{\text{L}}{\text{mol}}} \tag{11.5.13}$$

Using this information, we can calculate the density of a gas using the gas's molar mass.

Example 1

Calculate the density of N_2 gas at STP.

What we know: Pressure (1 atm),temperature (273 K), the identity of the gas (N_2).

Asked for: Density of N_2

Strategy:

A. Calculate the molar mass of N_2
B. Solve for the density of using the equation relating density and molar mass at STP

Solution:

A The molar mass of N_2:

$$MM_{N2} = 2 \cdot 14.0\text{g/mol} = 28.0\text{g/mol} \tag{11.5.14}$$

B Calculate the density of N_2

$$\rho = \frac{MM}{22.4\frac{\text{L}}{\text{mol}}} \tag{11.5.15}$$

$$\rho = \frac{28.0\text{g/mol}}{22.4\frac{\text{L}}{\text{mol}}} \tag{11.5.16}$$

$$\rho = 1.25\text{g/L} \tag{11.5.17}$$

Example 2

Calculate the density of Ne gas at 143 ºC and 4.3 atm.

What we know: Pressure (4.3 atm), temperature (143 ºC), the identity of the gas (Ne), the molar mass of Ne from the periodic table (20.2 g/mol).

Asked for: Density of Ne

Strategy:

A. The temperature is given in degrees Celsius. This must be converted to Kelvin
B. Solve for the density

Solution:

A Calculate temperature in Kelvin:

$$T = 143\text{C} + 273 = 416\text{K} \tag{11.5.18}$$

B Calculate the density of Ne:

$$\rho=\frac{MM\cdot P}{RT} \tag{11.5.19}$$

$$\rho=\frac{20.2\text{g/mol}\cdot 4.3\text{atm}}{(0.08206\frac{\text{Latm}}{\text{molK}})416\text{K}} \tag{11.5.20}$$

$$\rho=2.54\text{g/L} \tag{11.5.21}$$

Molar Mass of a Gas

The equations for calculating the density of a gas can rearranged to calculate the molar mass of a gas:

$$MM=\frac{\rho RT}{P} \tag{11.5.22}$$

this can be further simplified if we work at STP:

$$MM=\rho\cdot 22.4L/mol \tag{11.5.23}$$

We can use these equations to identify an unknown gas, as shown below:

Example 3

A unknown gas has density of 1.78 g/L at STP. What is the identify of this gas?

What we know: Pressure (1.00 atm), temperature (273 K), density of the gas (1.783 g/L)

Asked for: Identity of the unknown gas

Strategy:

A. First calculate the molar mass of the unknown gas
B. Determine the identity of the gas by comparing the calculated molar mass to molar masses of known gases.

Solution:

A Since we are at STP, we can use the following equation to calculate molar mass:

$$MM=\rho\cdot 22.4L/mol \tag{11.5.24}$$

$$MM=1.783\text{g/L}\cdot 22.4\text{L/mol} \tag{11.5.25}$$

$$MM=39.9\text{g/mol} \tag{11.5.26}$$

B The calculated molar mass is 33.9 g/mol. Examination of the periodic table reveals that Argon has a mass of 39.948 g/mol. Therefore, the unknown gas is most likely argon.

Stoichiometry and Gas Laws

Stoichiometry is the theme of the previous block of modules, and the ideal gas law is the theme of this block of modules. These subjects are related. Be prepared to solve problems requiring concepts or principles of stoichiometry and gases. For example, we can calculate the number of moles from a certain volume, temperature and pressure of a HCl gas. When *n* moles are dissolved in *V* L solution, its concentration is *n/V* M.

Three examples are given to illustrate some calculations of stoichiometry involving gas laws and more are given in question form for you to practice.

Example 1

If 500 mL of HCl gas at 300 K and 100 kPa dissolve in 100 mL of pure water, what is the concentration? Data required: R value 8.314 kPa L / (K mol).

Solution

$$n_{\mathrm{HCl}} = \frac{0.50\ \mathrm{L} \times 100\ \mathrm{kPa}}{8.314\ \dfrac{\mathrm{kPa\ L}}{\mathrm{K\ mol}} \times 300\ \mathrm{K}} \tag{11.5.27}$$

$$= 0.02\ \mathrm{mol} \tag{11.5.28}$$

Concentration of HCl, [HCl]

$$[\mathrm{HCl}] = \frac{0.02\ \mathrm{mol}}{0.1\ \mathrm{L}} = 0.2\ \mathrm{mol/L}$$

Discussion

Note that *R* = 0.08205 L atm /(K mol) will not be suitable in this case. If you have difficulty, review Solutions.

Example 2

If 500 mL of HCl gas at 300 K and 100 kPa dissolved in pure water requires 12.50 mL of the NaOH solution to neutralize in a titration experiment, what is the concentration of the NaOH solution?

Solution

Solution in Example 1 showed n_{HCl} = 0.02 mol. From the titration experiment, we can conclude that there were 0.02 moles of NaOH in 12.50 mL. Thus,

$$[\mathrm{NaOH}] = \frac{0.02\ \mathrm{mol}}{0.0125\ \mathrm{L}} = 1.60\ \mathrm{mol/L}$$

Discussion

Think in terms of reaction,

$$\mathrm{HCl} + \mathrm{NaOH} \rightarrow \mathrm{NaCl} + \mathrm{H_2O} \quad \Leftarrow \text{Reaction} \tag{11.5.29}$$

$$0.02\ \mathrm{mol}\quad 0.02\ \mathrm{mol} \qquad \Leftarrow \text{Quantities reacted} \tag{11.5.30}$$

Note that 0.02 mol of NaOH is in 0.0125 mL solution.

Example 3

A 5.0-L air sample containing H_2S at STP is treated with a catalyst to promote the reaction

$$\mathrm{H_2S} + \mathrm{O_2} \rightarrow \mathrm{H_2O} + \mathrm{S_{(solid)}}\ .$$

If 3.2 g of solid S was collected, calculate the volume percentage of H_2S in the original sample.

Solution

$$3.2\ \mathrm{g\ S} \times \frac{1\ \mathrm{mol\ H_2S}}{32\ \mathrm{g\ S}} = 0.10\ \mathrm{mol\ H_2S}$$

$$V_{\mathrm{H_2S}} = 0.10\ \mathrm{mol} \times 22.4\ \mathrm{L/mol} \tag{11.5.31}$$

$$= 2.24\ \mathrm{L} \tag{11.5.32}$$

$$\text{Volume}\ \% = \frac{2.25\ \mathrm{L}}{5.0\ \mathrm{L}} \tag{11.5.33}$$

$$= 0.45 \tag{11.5.34}$$

$$= 45\% \tag{11.5.35}$$

Discussion

Data required: Atomic mass: $\mathrm{H} = 1$; $\mathrm{O} = 16$; $\mathrm{S} = 32$. *R* = 0.08205 L atm /(K mol) is now suitable *R* values or molar volume at STP (22.4 L/mol)

The volume percentage is also the mole percentage, but not the weight percentage.

Example 4

Hydrogen sulfide reacts with sulfur dioxide to give H_2O and S,

$$H_2S + SO_2 \rightarrow H_2O + S_{(solid)} \text{ , unbalanced.}$$

If 6.0 L of H_2S gas at 750 torr produced 3.2 g of sulfur, calculate the temperature in C.

Solution
Balanced reaction:

$$\begin{array}{ll} 2H_2S + SO_2 \rightarrow 2H_2O + 3S_{(solid)} & \\ 2\text{ mol} & 3 \times 32 = 96\text{ g} \end{array}$$

$$3.2\text{ g S} \times \frac{2\text{ mol } H_2S}{96\text{ g S}} = 0.067\text{ mol } H_2S$$

$$P = \frac{750}{760} = 0.987\text{ atm}$$

$$T = \frac{PV}{nR} = \frac{0.987\text{ atm} \times 6\text{ L}}{0.067\text{ mol} \times 0.08205\,\frac{\text{atm L}}{\text{mol K}}} \tag{11.5.36}$$

$$= 1085\text{ K} \tag{11.5.37}$$

$$= 812^{\circ}\text{C} \tag{11.5.38}$$

Discussion
Atomic mass: $H = 1.0$; $O = 16.0$; $S = 32.0$. R = 0.08205 L atm /(K mol) is OK but watch units used for pressure.

Example 5

When 50.0 mL of $AgNO_3$ solution is treated with an excess amount of HI gas to give 2.35 g of AgI, what is the concentration of the $AgNO_3$ solution?

Solution

$$2.35\text{ g AgI} \times \frac{1\text{ mol Ag}^+}{234.8\text{ g AgI}} \times \frac{1\text{ mol AgNO}_3}{1\text{ mol Ag}^+} = 0.010\text{ mol AgNO}_3$$

$$[\text{AgNO}_3] = \frac{0.01\text{ mol AgNO}_3}{0.050\text{ L}} \tag{11.5.39}$$

$$= 0.20\text{ M AgNO}_3 \tag{11.5.40}$$

Discussion

A gas is involved, but there is no need to consider the gas law. At. mass: $Ag = 107.9$; $N = 14.0$; $O = 16.0$; $I = 126.9$

Example 6

What volume (L) will 0.20 mol HI occupy at 300 K and 100.0 kPa? $R = 8.314\,\frac{\text{kPa L}}{\text{K mol}} = 0.08205\,\frac{\text{atm L}}{\text{mol K}}$

Solution

$$V = \frac{nRT}{P} \tag{11.5.41}$$

$$= \frac{0.20\text{ mol} \times 8.314\,\frac{\text{kPa L}}{\text{mol K}} \times 300\text{ K}}{100\text{ kPa}} \tag{11.5.42}$$

$$= 5\text{ L} \tag{11.5.43}$$

Example 7

A 3.66-g sample containing Zn (at.wt. 65.4) and Mg (24.3) reacted with a dilute acid to produce 2.470 L H_2 gas at 101.0 kPa and 300 K. Calculate the percentage of Zn in the sample.

Solution

The number of moles of gas produced is the number of moles of metals in the sample. Once you know the number of moles, set up an equation to give the number of moles of metal in the sample.

$$n = \frac{101 \text{ kPa} \times 2.470 \text{ L}}{8.3145 \frac{\text{kPa L}}{\text{mol K}} \times 300 \text{ K}} \tag{11.5.44}$$

$$= 0.100 \text{ mol} \tag{11.5.45}$$

Let x be the mass of Zn, then the mass of Mg is 3.66 - x g. Thus, we have

$$\frac{x}{65.4} + \frac{3.66 - x}{24.3} = 0.100 \text{ mole}$$

Solving for x gives x = 1.96 g Zn,

and the weight percent $= 100 \times \frac{1.96}{3.66} = 53.6\%$

Discussion

Find the mole percent of Zn in the sample.

$$\#\text{ mol of Zn} = \frac{1.96}{65.4} = 0.03 \text{ mol}$$

$$\#\text{ mol of Mg} = \frac{1.70}{24.3} = 0.07 \text{ mol}$$

$$\text{mole percent} = 100 \times \frac{0.03}{0.03 + 0.07} = 30\%$$

Example 8

When a 2.00 g mixture of Na and Ca reacted with water, 1.164 L hydrogen was produced at 300.0 K and 100.0 kPa. What is the percentage of Na in the sample?

Solution

$$2\,\text{Na} + 2\,\text{H}_2\text{O} \rightarrow 2\,\text{Na(OH)} + \text{H}_{2(g)}$$

$$\text{Ca} + \text{H}_2\text{O} \rightarrow \text{Ca(OH)} + \text{H}_{2(g)}$$

Let x be the mass of Na, then (2.00-x) is the mass of Ca.

We have the following relationship

$$\frac{\text{x g}}{23.0 \text{ g/mol}} \times \frac{1 \text{ mol H}_2}{2 \text{ mol Na}} + \frac{(2.0 - \text{x}) \text{ g Ca}}{40.1 \text{ g Ca/mol}} \times \frac{1 \text{ mol H}_2}{1 \text{ mol Ca}} = \frac{1.164 \text{ L H}_2 \times 100.0 \text{ kPa}}{8.3145 \text{ kPa L mol}^{-1} \text{ K}^{-1}\ 300.0 \text{ K}}$$

Simplify to give

$$\frac{\text{x}}{46.0} + \frac{2}{40.1} - \frac{\text{x}}{40.1} = 0.0467 \text{ all in mol}$$

Multiply all terms by (40.1 * 46.0)

$$40.1\,\text{x} + 2 \times 46.0 - 46.0\,\text{x} = 86.1$$

Simplify

$$-5.9\,\text{x} = 86.1 - 92.0 = -5.91$$

Thus,

$$\text{Mass of Na} = \text{x} = 1.0\ \text{g}$$

$$\text{Mass of Ca} = 2.0 - \text{x} = 1.0\ \text{g}$$

$$\text{Mass Percentage of Na} = 100 \times \frac{1}{2.0} = 50\%$$

Discussion

$$\text{Mole of Na} = \frac{1}{23} = 0.0435\ \text{mol}$$

$$\text{Mole percentage} = \frac{\frac{1}{23}}{\frac{1}{23} + \frac{1}{40.1}} = 0.635 = 63.5\%$$

Compare this example with gravimetric analyses using the reaction

$$\text{Ag}^{+}_{(aq)} + \text{Cl}^{-}_{(aq)} \rightarrow \text{AgCl}_{(s)}$$

where $\text{Cl}^{-}_{(aq)}$ comes from the disolution of two salts such as NaCl and MgCl_2.

Also compare with analyses making use of the reaction

$$\text{Ba}^{2+}_{(aq)} + \text{SO}^{2-}_{4(aq)} \rightarrow \text{BaSO}_{4(s)}$$

where the anion $\text{SO}^{2-}_{4(aq)}$ comes from the dissolution of two sulfate salts.

This example is very similar to Example 7.

Contributors and Attributions

- Chung (Peter) Chieh (Professor Emeritus, Chemistry @ University of Waterloo)