# 5.4: Quantitative Relationships Based on Chemical Equations

**Learning Objectives**

- To calculate the amount of one substance that will react with or be produced from a given amount of another substance.

A balanced chemical equation not only describes some of the chemical properties of substances—by showing us what substances react with what other substances to make what products—but also shows numerical relationships between the reactants and the products. The study of these numerical relationships is called stoichiometry. The stoichiometry of chemical equations revolves around the coefficients in the balanced chemical equation because these coefficients determine the molecular ratio in which reactants react and products are made.

> *The word stoichiometry is pronounced "stow-eh-key-OM-et-tree." It is of mixed Greek and English origins, meaning roughly "measure of an element."*

**Looking Closer: Stoichiometry in Cooking**

Let us consider a stoichiometry analogy from the kitchen. A recipe that makes 1 dozen biscuits needs 2 cups of flour, 1 egg, 4 tablespoons of shortening, 1 teaspoon of salt, 1 teaspoon of baking soda, and 1 cup of milk. If we were to write this as a chemical equation, we would write

2 c flour + 1 egg + 4 tbsp shortening + 1 tsp salt + 1 tsp baking soda + 1 c milk → 12 biscuits

(Unlike true chemical reactions, this one has all 1 coefficients written explicitly—partly because of the many different units here.) This equation gives us ratios of how much of what reactants are needed to make how much of what product. Two cups of flour, when combined with the proper amounts of the other ingredients, will yield 12 biscuits. One teaspoon of baking soda (when also combined with the right amounts of the other ingredients) will make 12 biscuits. One egg must be combined with 1 cup of milk to yield the product food. Other relationships can also be expressed.

We can use the ratios we derive from the equation for predictive purposes. For instance, if we have 4 cups of flour, how many biscuits can we make if we have enough of the other ingredients? It should be apparent that we can make a double recipe of 24 biscuits.

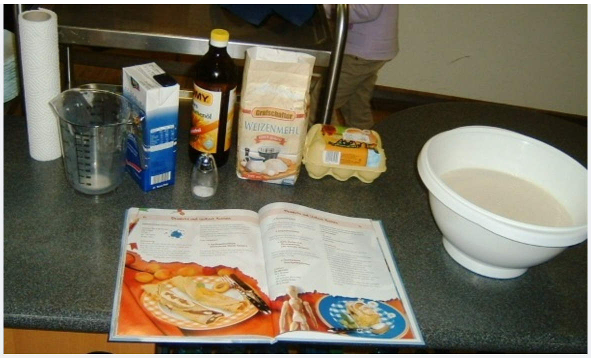
But how would we find this answer formally, that is, mathematically? We would set up a conversion factor, much like we did in Chapter 1. Because 2 cups of flour make 12 biscuits, we can set up an equivalency ratio:

$$\frac{12\text{ biscuits}}{2\text{ c flour}}$$

We then can use this ratio in a formal conversion of flour to biscuits:

$$4\text{ c flour}\times\frac{12\text{ biscuits}}{2\text{ c flour}}=24\text{ biscuits}$$

Similarly, by constructing similar ratios, we can determine how many biscuits we can make from any amount of ingredient. When you are doubling or halving a recipe, you are doing a type of stoichiometry. Applying these ideas to chemical reactions should not be difficult if you use recipes when you cook.



A recipe shows how much of each ingredient is needed for the proper reaction to take place. from Wikipedia.

Consider the following balanced chemical equation:

$$2\,C_2H_2 + 5\,O_2 \rightarrow 4\,CO_2 + 2\,H_2O \tag{5.4.1}$$

The coefficients on the chemical formulas give the ratios in which the reactants combine and the products form. Thus, we can make the following statements and construct the following ratios:

Table uses a statement from the balanced chemical reaction to show ratios and inverse ratios.

| Statement from the Balanced Chemical Reaction | Ratio | Inverse Ratio |
|---|---|---|
| two $C_2H_2$ molecules react with five $O_2$ molecules | $\frac{2C_2H_2}{5O_2}$ | $\frac{5O_2}{2C_2H_2}$ |
| two $C_2H_2$ molecules react to make four $CO_2$ molecules | $\frac{2C_2H_2}{4CO_2}$ | $\frac{4CO_2}{2C_2H_2}$ |
| five $O_2$ molecules react to make two $H_2O$ molecules | $\frac{5O_2}{2H_2O}$ | $\frac{2H_2O}{5O_2}$ |
| four $CO_2$ molecules are made at the same time as two $H_2O$ molecules | $\frac{2H_2O}{4CO_2}$ | $\frac{4CO_2}{2H_2O}$ |

Other relationships are possible; in fact, 12 different conversion factors can be constructed from this balanced chemical equation. In each ratio, the unit is assumed to be molecules because that is how we are interpreting the chemical equation.

Any of these fractions can be used as a conversion factor to relate an amount of one substance to an amount of another substance. For example, suppose we want to know how many $CO_2$ molecules are formed when 26 molecules of $C_2H_2$ are reacted. As usual with a conversion problem, we start with the amount we are given—$26C_2H_2$—and multiply it by a conversion factor that cancels out our original unit and introduces the unit we are converting to—in this case, $CO_2$. That conversion factor is $\frac{4CO_2}{2C_2H_2}$, which is composed of terms that come directly from the balanced chemical equation. Thus, we have

$$26C_2H_2 \times \frac{4CO_2}{2C_2H_2}$$

The molecules of $C_2H_2$ cancel, and we are left with molecules of $CO_2$. Multiplying through, we get

$$26C_2H_2 \times \frac{4CO_2}{2C_2H_2} = 52CO_2$$

Thus, 52 molecules of $CO_2$ are formed.

This application of stoichiometry is extremely powerful in its predictive ability, as long as we begin with a balanced chemical equation. Without a balanced chemical equation, the predictions made by simple stoichiometric calculations will be incorrect.

Example 5.4.1

Start with this balanced chemical equation.

$$KMnO_4 + 8\,HCl + 5\,FeCl_2 \rightarrow 5\,FeCl_3 + MnCl_2 + 4\,H_2O + KCl$$

1. Verify that the equation is indeed balanced.
2. Give 2 ratios that give the relationship between HCl and $FeCl_3$.

**Solution**

1. Each side has 1 K atom and 1 Mn atom. The 8 molecules of HCl yield 8 H atoms, and the 4 molecules of $H_2O$ also yield 8 H atoms, so the H atoms are balanced. The Fe atoms are balanced, as we count 5 Fe atoms from 5 $FeCl_2$ reactants and 5 $FeCl_3$ products. As for Cl, on the reactant side, there are 8 Cl atoms from HCl and 10 Cl atoms from the 5 $FeCl_2$ formula units, for a total of 18 Cl atoms. On the product side, there are 15 Cl atoms from the 5 $FeCl_3$ formula units, 2 from the $MnCl_2$ formula unit, and 1 from the KCl formula unit. This is a total of 18 Cl atoms in the products, so the Cl atoms are balanced. All the elements are balanced, so the entire chemical equation is balanced.
2. Because the balanced chemical equation tells us that 8 HCl molecules react to make 5 $FeCl_3$ formula units, we have the following 2 ratios: $\frac{8HCl}{5FeCl_3}$ and $\frac{5FeCl_3}{8HCl}$. There are a total of 42 possible ratios. Can you find the other 40 relationships?

Exercise 5.4.1

Start with this balanced chemical equation.

$$2KMnO_4 + 3CH_2{=}CH_2 + 4H_2O \rightarrow 2MnO_2 + 3HOCH_2CH_2OH + 2KOH$$

a. Verify that the equation is balanced.
b. Give 2 ratios that give the relationship between $KMnO_4$ and $CH_2$=$CH_2$. (A total of 30 relationships can be constructed from this chemical equation. Can you find the other 28?)

**Answer a:**

Each side has 2 K atoms and 2 Mn atoms. On the reactant side, 3 $CH_2$=$CH_2$ yield 6 C atoms and on the product side, 3 $HOCH_2CH_2OH$ also yield 6 C atoms, so the C atoms are balanced. There are 20 H atoms on the reactants side: 12 H atoms from 3 $CH_2$=$CH_2$ and 8 H atoms from 4 $H_2O$. On the product side, there are also 20 H atoms: 18 H atoms from 3 $HOCH_2CH_2OH$ and 2 H atoms from 2 KOH. So, the H atoms are balanced. As for O, on the reactant side, there are 8 O atoms from 2 $KMnO_4$ and 4 O atoms from 4 $H_2O$, for a total of 12 O atoms. On the product side, there are 4 O atoms from the 2 $MnO_2$ formula units, 6 O atoms from 3 $HOCH_2CH_2OH$, and 2 O atoms from the 2 KOH formula units. This is a total of 12 O atoms in the products, so the O atoms are balanced. All the elements are balanced, so the entire chemical equation is balanced.

**Answer b:**

Because the balanced chemical equation tells us that 2 $KMnO_4$ formula units react with 3 $CH_2$=$CH_2$ molecules, we have the following 2 ratios: $\frac{2KMnO_4}{3CH_2{=}CH_2}$ and $\frac{3CH_2{=}CH_2}{2KMnO_4}$. There are a total of 30 possible ratios. Can you find the other 28 relationships?

## Key Takeaway

The coefficients in a balanced chemical equation give the ratios in which molecules of substances react and are produced in a chemical reaction.