3.E: The Schrödinger Equation (Exercises)

Q3.1

Prove **Euler's formula** is correct by expanding $e^{\pm i\theta}$, $\cos\theta$, and $\sin\theta$ each in terms of a **Maclaurin series** and showing that corresponding terms are identical.

Q3.2

The following table gives the results of many measurements of the length of a laser cavity. Complete the table by calculating the probability for each value. Use the probabilities that you calculated to compute the average value for the length, the average of the length squared, the variance, and the standard deviation in the measurements.

length (cm)	number of times the value was obtained	probability
100.05	4	
100.04	3	
100.03	6	
100.02	9	
100.01	8	
100.00	9	
99.99	9	
99.98	8	
99.97	2	
99.96	3	

Q3.3

Consider an electron trapped by a positively charged point defect in a one-dimensional world. The following wavefunction with α = 20/nm describes the distance x of the electron from the point defect located at x=0. Note that in 1, 2, and 3 dimensions, r = |x|, $(x^2+y^2)^{1/2}$, and $(x^2+y^2+z^2)^{1/2}$, respectively.

$$\psi(r) = Ne^{-\alpha|x|} \tag{3.E.1}$$

1. Evaluate the normalization constant N.
2. Graph the probability density for this electron.
3. Calculate the expectation value for x and |x|.
4. If the electron were in a two or three-dimensional world, such as on the surface of a crystal or in a free atom, would the average distance of the electron from the origin <r> be less, the same, or larger than the value you found for one dimension?
5. Determine whether the expectation value for r depends upon the dimensionality of the world (1, 2, or 3) in which the atom lives.

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