2.9: The Wave Properties of Matter

The fact that light (electromagnetic radiation) exhibited properties of particles became clear from the Compton scattering experiments where a momentum of $p = h/\lambda$ had to be associated with the x-rays to explain the experimental observations. In 1924 Louis de Broglie proposed that if light waves exhibited properties of particles, then matter particles should exhibit properties of waves, and the wavelength of these waves should be given by the same equation,

$$\lambda = \frac{\hbar}{p} \tag{2.9.1}$$

Since the wave vector k is defined as $k = \frac{2\pi}{\lambda}$, we can rewrite this equation as

$$p = \hbar k \tag{2.9.2}$$

Example 2.9.1

Calculate the de Broglie wavelength for an electron with a kinetic energy of 1000 eV. Could such electrons be used to obtain diffraction patterns of molecules?

Example 2.9.2

Calculate the de Broglie wavelength for a fast ball thrown at 100 miles per hour and weighing 4 ounces. Comment on whether the wave properties of baseballs could be observed.

The validity of de Broglie's proposal was confirmed by electron diffraction experiments of G.P. Thomson in 1926 and of C. Davisson and L. H. Germer in 1927. In these experiments it was found that electrons were scattered from atoms in a crystal and that these scattered electrons produced an interference pattern. The interference pattern was just like that produced when water waves pass through two holes in a barrier to generate separate wave fronts that combine and interfere with each other. These diffraction patterns are characteristic of wave-like behavior and are exhibited by both electrons and electromagnetic radiation. Diffraction patterns are obtained if the wavelength is comparable to the spacing between scattering centers. Immediately below in the box are two hyperlinks that show you patterns obtained by electron diffraction and by x-ray diffraction. You can find others on the Internet by searching x-ray diffraction pattern and electron diffraction pattern to see how electron and x-ray diffraction are being used in modern research.

De Broglie's proposal can be applied to Bohr's view of the hydrogen atom to show why angular momentum is quantized in units of $\hbar$. If the electron in the hydrogen atom is orbiting the nucleus in a stable orbit, then it should be described by a stable or stationary wave. Such a wave is called a standing wave. In a standing wave, the maximum and minimum amplitudes (crests and troughs) of the wave and the nodes (points where the amplitude is zero) are always found at the same position. In a traveling wave the crests, troughs, and nodes move from one position to another as a function of time. To place a standing wave in the shape of a round orbit, the circumference $2\pi r$ must be an integer multiple of the wavelength, i.e.

$$2\pi r = n\lambda \tag{2.9.3}$$

Now using the wavelength-momentum relationship $\lambda = \frac{\hbar}{p}$ to replace λ we get

$$rp = \frac{nh}{2\pi} \tag{2.9.4}$$

Since rp equals the angular momentum, we have

$$M = n\hbar \tag{2.9.5}$$

By saying the electron has the property of a standing wave around the orbit, we are led to the conclusion that angular momentum of the electron is quantized in units of ħ. The assumption of quantization thereby is replaced by the postulate that particles have wave properties characterized by a wavelength $\lambda = \frac{\hbar}{p}$, and quantization is a consequence of this new postulate.

This insight regarding the wave properties of particles led Erwin Schrödinger to build on the mathematical description of waves and develop a general theory of Quantum Mechanics, as we will see in the next chapter.

Example $2.9.3$

Draw standing waves with 2, 4, and 6 nodes on a Bohr electron orbit.