3.7: Meaning of the Wavefunction

Since wavefunctions can in general be complex functions, the physical significance cannot be found from the function itself because the $\sqrt{-1}$ is not a property of the physical world. Rather, the physical significance is found in the product of the wavefunction and its complex conjugate, i.e. the absolute square of the wavefunction, which also is called the square of the modulus.

$$\psi^*(r,t)\psi(r,t) = |\psi(r,t)|^2 \tag{3.7.1}$$

where r is a vector (x, y, z) specifying a point in three-dimensional space. The square is used, rather than the modulus itself, just like the intensity of a light wave depends on the square of the electric field.

At one time it was thought that for an electron described by the wavefunction $\psi(r)$, the quantity $e\psi^*(ri)\psi(r_i)d\tau$ was the amount of charge to be found in the volume $d\tau$ located at r_i. However, Max Born found this interpretation to be inconsistent with the results of scattering experiments. The Born interpretation, which generally is accepted today, is that $\psi^*(r_i)\psi(r_i)\, d\tau$ is the probability that the electron is in the volume dτ located at r_i. The Born interpretation therefore calls the wavefunction the probability amplitude, the absolute square of the wavefunction is called the probability density, and the probability density times a volume element in three-dimensional space $(d\tau)$ is the probability. The idea that we can understand the world of atoms and molecules only in terms of probabilities is disturbing to some, who are seeking more satisfying descriptions through ongoing research.

Example 3.7.1

Show that the square of the modulus of $\psi(r,t) = \psi(r)e^{-i\,\omega - t}$ is time independent. What insight regarding stationary states do you gain from this proof?

Example 3.7.2

Show that the square of the modulus

Example 3.7.3

Show that the square of the modulus

Example 3.7.4

According to the Born interpretation, what is the physical significance of $e\psi^*(r_0)(r_0)d\tau$?