

2.12: Adiabatic Changes

The following discussions apply to a sample of ideal gas in an isolated system (an adiabatic, or thermally insulated system) that undergoes a reversible, adiabatic expansion or compression against a constant external pressure. Because there is no heat exchange during these volume changes, the internal energy will decrease during an expansion and increase during a compression. These internal energy changes occur because the gas molecules lose kinetic energy as they do work to move the piston, or gain kinetic energy as work is done on the by the piston.

Changes in Temperature

In this example, we want to find the change in temperature of a gas sample as it undergoes a reversible adiabatic expansion. This expansion will lead to a decrease in temperature and an increase in volume. It would be difficult to monitor the point-by-point changes in temperature as the volume slowly increases. However, because ΔU is a state function, we can separate the overall process into two theoretical steps:

- Step 1: an isothermal expansion from V_1 to V_2
- Step 2: a decrease in temperature at constant volume.

In [Topic 2A](#) we noted that the internal energy of an ideal gas is independent of the volume of its container. Because of this fact, the isothermal expansion in Step 1 will not affect the internal energy of the gas. Thus, the change in internal energy during the adiabatic expansion can be assigned to Step 2, the change in temperature at constant volume. Once again using a concept developed in [Topic 2A](#), we know that at constant volume

$$\Delta U = C_V \Delta T = C_V (T_2 - T_1) \quad (2.12.1)$$

As noted above, in an adiabatic process $\Delta U = w_{ad}$ so that

$$w_{ad} = C_V \Delta T \quad (2.12.2)$$

This relationship makes sense because the energy needed to carry out the work of the expansion must come from the gas particles, which will lose energy as they do work, resulting in a drop in the temperature of the system. We assume that the value of C_V is independent of temperature.

To determine the relationship between the volume change and the temperature change, we can start with the premise that the work done by an ideal gas as it expands adiabatically against an external pressure, P , is

$$dw = -PdV \quad (2.12.3)$$

For an adiabatic change $dU = dw$ and for an ideal gas $dU = C_V dT$, thus in this case

$$dU = C_V dT = dw = -PdV \quad (2.12.4)$$

and

$$C_V dT = -PdV \quad (2.12.5)$$

Because we are using an ideal gas, $P = \frac{nRT}{V}$, thus

$$\frac{C_V dT}{T} = -\frac{nR dV}{V} \quad (2.12.6)$$

The limits of integration are set by the initial conditions (T_1, V_1) and final conditions (T_2, V_2):

$$C_V = \int \frac{dT}{T} = -nR \int \frac{dV}{V} \quad (2.12.7)$$

resulting in

$$C_V \ln \frac{T_2}{T_1} = -nR \ln \frac{V_2}{V_1} \quad (2.12.8)$$

which can be rearranged to

$$\frac{C_V}{nR} \ln \frac{T_2}{T_1} = \ln \frac{V_1}{V_2} \quad (2.12.9)$$

To solve for T_2 , we need to define $\frac{C_V}{nR} = c$ so that

$$\ln \left(\frac{T_2}{T_1} \right)^c = \ln \frac{V_1}{V_2} \quad (2.12.10)$$

This equation allows us to assume that

$$\left(\frac{T_2}{T_1} \right)^c = \frac{V_1}{V_2} \quad (2.12.11)$$

and so

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{1/c} \quad (2.12.12)$$

Example 2.12.1:

Calculate the final temperature, the work done, and the change in internal energy when 0.0500 moles of He at 25.0°C undergoes a reversible adiabatic expansion from 250. L to 750. L. The constant volume molar heat capacity of He at 25°C is

$$12.48 \frac{\text{J}}{\text{mole} \cdot \text{K}}.$$

Asked For: T_f , w_{ad} , and ΔU

Solution: If $C_V = 12.48 \frac{\text{J}}{\text{mole} \cdot \text{K}}$, then $c = \frac{12.48 \frac{\text{J}}{\text{mole} \cdot \text{K}}}{8.314 \frac{\text{J}}{\text{mole} \cdot \text{K}}} = 1.501$.

Use equation 2.12.12 to find that $T_2 = 298.15 \text{ K} \times \left(\frac{250. \text{ L}}{750. \text{ L}} \right)^{1/1.501} = 143 \text{ K}$.

Use equation 2.12.2 to find that $w_{ad} = (0.0500 \text{ moles He}) \times \left(12.48 \frac{\text{J}}{\text{mole} \cdot \text{K}} \right) \times (143 \text{ K} - 298 \text{ K}) = -96.7 \text{ J}$.

According to the definition of an adiabatic process, $\Delta U = w_{ad}$. Therefore, $\Delta U = -96.7 \text{ J}$.

Exercise 2.12.1

Calculate the final temperature, the work done, and the change in internal energy when 0.0400 moles of CO at 25.0°C undergoes a reversible adiabatic expansion from 200. L to 800. L. The constant volume molar heat capacity of CO at 25°C is

$$20.16 \frac{\text{J}}{\text{mole} \cdot \text{K}}.$$

Answers: $T_f = 168 \text{ K}$; $w_{ad} = -105 \text{ J}$; $\Delta U = -105 \text{ J}$

Changes in Pressure

In this example, we want to find the change in pressure of a gas sample as it undergoes a reversible adiabatic expansion. This expansion will lead to a decrease in pressure and an increase in volume. Because we are working with an ideal gas

$$\frac{P_1 V_1}{P_2 V_2} = \frac{T_1}{T_2} \quad (2.12.13)$$

Substituting in a rearranged version of Equation 2.12.12 we get

$$\frac{P_1 V_1}{P_2 V_2} = \left(\frac{V_2}{V_1} \right)^{\frac{1}{c}} \quad (2.12.14)$$

which rearranges to

$$\frac{P_1}{P_2} \left(\frac{V_1}{V_2} \right)^{\frac{1}{\gamma} + 1} = 1 \quad (2.12.15)$$

In [Topic 2B](#), it was stated that $C_{P,m} - C_{V,m} = R$. This relationship can be used to develop the term, γ :

$$\frac{1}{\gamma} + 1 = \frac{1 + c}{c} = \frac{R + C_{V,m}}{C_{V,m}} = \frac{C_{P,m}}{C_{V,m}} = \gamma \quad (2.12.16)$$

Thus, Equation [2.12.15](#) can be written as

$$\frac{P_1}{P_2} \left(\frac{V_1}{V_2} \right)^{\gamma} = 1 \quad (2.12.17)$$

Solving for P_2 , we get

$$P_2 = P_1 \times \left(\frac{V_1}{V_2} \right)^{\gamma} \quad (2.12.18)$$

recall that:

- for a monatomic ideal gas, $C_{V,m} = \frac{3}{2}R$ and $C_{P,m} = \frac{5}{2}R$, so $\gamma = \frac{5}{3}$.
- for nonlinear, polyatomic molecules, which can rotate as well as translate, $C_{V,m} = 3R$ and $C_{P,m} = 4R$, so $\gamma = \frac{4}{3}$.

Example 2.12.2:

Calculate the final pressure when a sample of He at 250. kPa undergoes a reversible adiabatic expansion from 2.00 L to 4.00 L.

Solution: Use equation [2.12.18](#) and the fact that $\gamma = \frac{5}{3}$ for He to find that $P_f = (250 \text{ kPa}) \left(\frac{2.00 \text{ L}}{4.00 \text{ L}} \right)^{5/3} = 78.7 \text{ kPa}$.

Exercise 2.12.2

Calculate the final pressure when a sample of Xe at 100. kPa undergoes a reversible adiabatic expansion from 1.00 L to 5.00 L.

Answer $P_f = 6.84 \text{ kPa}$.

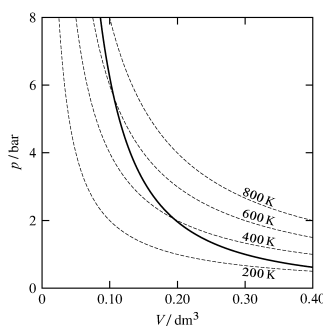


Figure 2.5.1 An adiabat (solid curve) and four isotherms (dashed curves) for an ideal gas ($n = 0.0120 \text{ mole}$, $CV_m = 1.5R$).
Image by Howard DeVoe.

The solid curve in Fig. 2.5.1 shows how the pressure of an ideal gas varies with volume during a reversible adiabatic expansion or compression. This curve is an *adiabat*. The dashed curves in the figure are *isotherms* showing how pressure changes with volume at constant temperature according to the equation of state $p = nRT/V$. In the direction of increasing V (expansion), the adiabat crosses isotherms of progressively lower temperatures. This cooling effect, of course, is due to the loss of energy by the gas as it does work on the surroundings without a compensating flow of heat into the system.

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