5.5: Wave-Particle Duality of Matter

So far we have seen that to fully understand some experimental results with light we cannot simply treat them as *waves* but sometimes we have to consider that they also have properties of *particles* (photons). It turns out that for particles like the electrons, there are some experimental results whose results can only be understood if we treat those particles as waves.

Light Waves

Many of the properties of light (e.g., surface of water or acoustics) including **reflection, refraction, diffraction, and interference** can be explained, both qualitatively and quantitatively, in terms of light viewed as a wave.

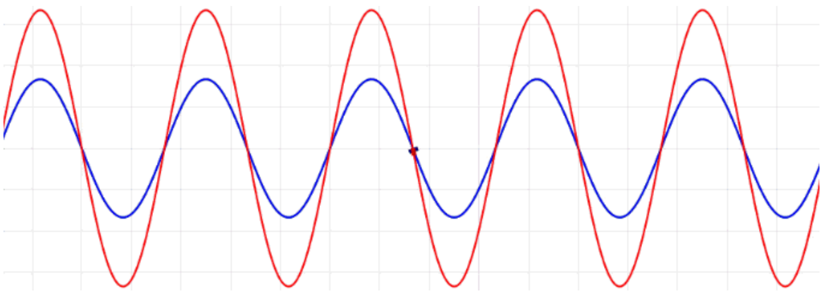


Figure: Linear Interference: The phenomenon of interference between waves is based on this idea. When two or more waves traverse the same space, the net amplitude at each point is the sum of the amplitudes of the individual waves. In some cases, such as in noise-cancelling headphones, the summed variation has a smaller amplitude than the component variations; this is called destructive interference. In other cases, such as in Line Array, the summed variation will have a bigger amplitude than any of the components individually; this is called constructive interference. (CC BY-SA 4.0 Internation; Lookangmany via Wikipedia)

The amplitudes of waves add. **Constructive interference** is obtained when identical waves are in phase and **destructive interference** occurs when identical waves are exactly out of phase, or shifted by half a wavelength.

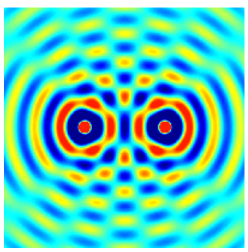


Figure: 2-D Interference: Interference of waves from two point sources. Image use with permission (Public Domain; Oleg Alexandrov)

Quantum Matter Waves

Continuing with our analysis of experiments that lead to the new quantum theory, we now look at the phenomenon of electron diffraction. It is well-known that *light* has the ability to diffract around objects in its path, leading to an interference pattern that is particular to the object. This is, in fact, how holography works (the interference pattern is created by allowing the diffracted light to interfere with the original beam so that the hologram can be viewed by shining the original beam on the image). A simple illustration of diffraction is the Young double slit experiment pictured below:

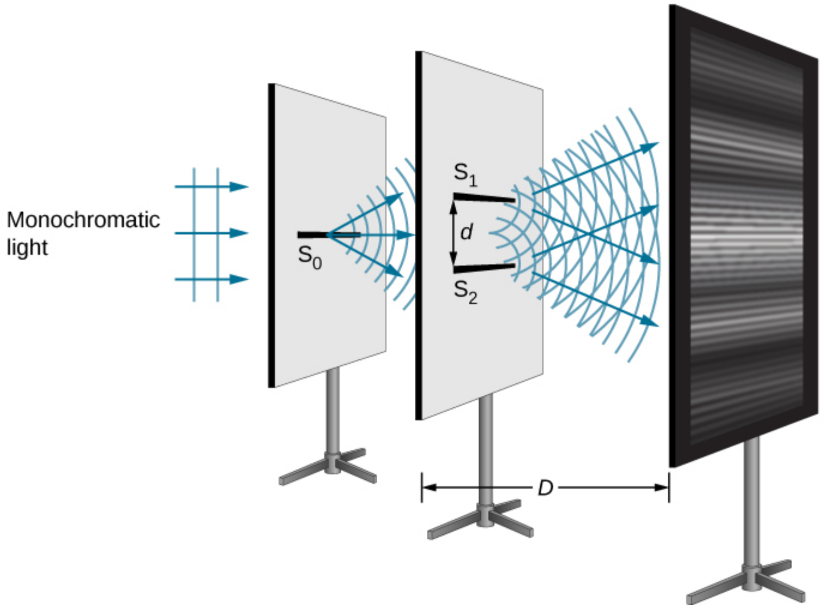


Figure: The double-slit interference experiment using monochromatic light and narrow slits. Fringes produced by interfering Huygens wavelets from slits S_1 and S_2 are observed on the screen. (CC-BY-SA; OpenStax).

Amazingly, if electrons are used instead of light in the double-slit experiment, and a fluorescent screen is used, one finds the same kind of interference pattern! This is shown in the electron double-slit diffraction pattern below:

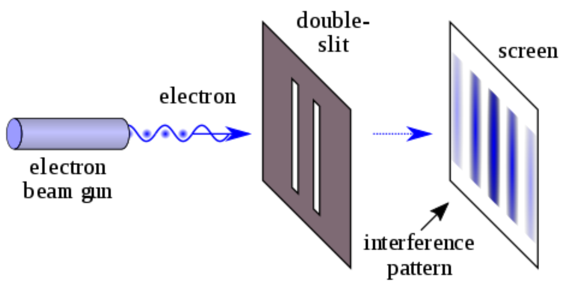


Figure: Particles of matter (like an electron) produce a wave pattern when two slits are used. (Public domain; NekoJaNekoJa).

Obviously, classical mechanics is not able to predict such a result. If the electrons are treated as classical particles, one would predict an intensity pattern corresponding to particles that can pass through one slit or the other, landing on the screen directly opposite the slit (i.e., no intensity maximum at the center of the screen):

Video: De Broglie's Theory can be seen in Young's Double Slit Experiment. https://youtu.be/O55XiriEaQI

Through such experiments, the idea that electrons can behave as waves, creating interference patterns normally associated with light, is now well-established. The fact that particles can behave as waves but also as particles, depending on which experiment you perform on them, is known as the *particle-wave duality*.

Deriving the De Broglie Wavelength

De Broglie derived his equation using well established theories through the following series of substitutions:

1. De Broglie first used Einstein's famous equation relating matter and energy:

$$E = mc^2 \tag{5.5.1}$$

with

- E = energy,
- m = mass,
- c = speed of light

2. Using Planck's theory which states every quantum of a wave has a discrete amount of energy given by Planck's equation:

$$E = h\nu \tag{5.5.2}$$

with

- E = energy,
- h = Plank's constant (6.62607 x 10^{-34} J s),
- ν= frequency

3. Since de Broglie believed particles and wave have the same traits, he hypothesized that the two energies would be equal:

$$mc^2 = h\nu \tag{5.5.3}$$

4. Because real particles do not travel at the speed of light, De Broglie submitted velocity (v) for the speed of light (c).

$$mv^2 = h\nu \tag{5.5.4}$$

5. Since the speed of a wave relates its frequency to its wavelength, de Broglie substituted v/λ for ν and arrived at the final expression that relates wavelength and particle with speed.

$$mv^2 = \frac{hv}{\lambda} \tag{5.5.5}$$

Hence:

$$\lambda = \frac{hv}{mv^2} = \frac{h}{mv} = \frac{h}{p} \tag{5.5.6}$$

At this stage, a majority of Wave-Particle Duality problems are simply "plug and chug" using Equation 5.5.6 with some variation of canceling out units

✔ Example 5.5.1: Wavelength of a Baseball in Motion

Calculate the wavelength of a baseball, which has a mass of 149 g and a speed of 100 mi/h.

Given: mass and speed of object

Asked for: wavelength

Strategy:

A. Convert the speed of the baseball to the appropriate SI units: meters per second.
B. Substitute values into Equation 5.5.6 and solve for the wavelength.

Solution:

The wavelength of a particle is given by $\lambda = h/mv$. We know that m = 0.149 kg, so all we need to find is the speed of the baseball:

$$v = \left(\frac{100\ \cancel{mi}}{\cancel{h}}\right)\left(\frac{1\ \cancel{h}}{60\ \cancel{min}}\right)\left(\frac{1.609\ \cancel{km}}{\cancel{mi}}\right)\left(\frac{1000\ m}{\cancel{km}}\right)$$

B Recall that the joule is a derived unit, whose units are (kg•m^2)/s^2. Thus the wavelength of the baseball is

$$\lambda = \frac{6.626 \times 10^{-34}\ J \cdot s}{(0.149\ kg)(44.69\ m \cdot s)} = \frac{6.626 \times 10^{-34}\ \cancel{kg} \cdot m^{\cancel{2}} \cdot \cancel{s}^{\cancel{-2}} \cdot \cancel{s}}{\left(0.149\ \cancel{kg}\right)\left(44.69\ \cancel{m} \cdot \cancel{s^{-1}}\right)} = 9.95 \times 10^{-35}\ m \tag{5.5.7}$$

(You should verify that the units cancel to give the wavelength in meters.) Given that the diameter of the nucleus of an atom is approximately 10^{-14} m, the wavelength of the baseball is almost unimaginably small.

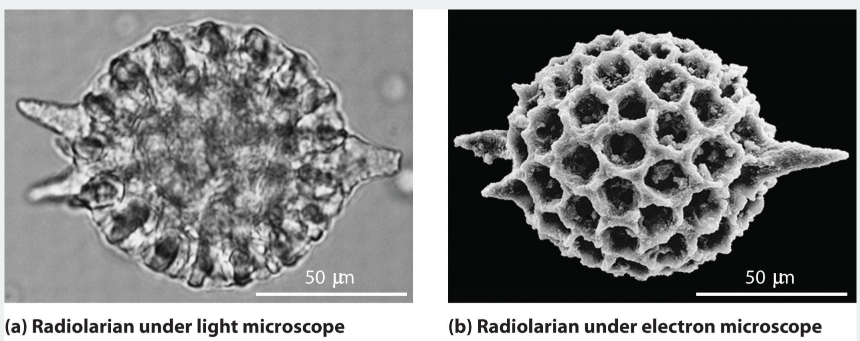
✔ Example 5.5.2: Wavelength of an Electron in Motion

Find the de Broglie wavelength for an electron moving at the speed of $5.0 \times 10^6\ m/s$ (mass of an electron is $9.1 \times 10^{-31}\ kg$).

Size matters in Quantum Mechanics

As you calculated in Example 5.5.1, objects such as a baseball have such short wavelengths that they are best regarded primarily as particles. In contrast, objects with very small masses (such as photons) have large wavelengths and can be viewed primarily as waves. Objects with intermediate masses, however, such as electrons (Example 5.5.2), exhibit the properties of both particles *and* waves.To effectively diffract a wave, the wavelength of the wave must be comparable to the dimensions of the diffraction device!

Although we still usually think of electrons as particles, the wave nature of electrons is employed in an *electron microscope*, which has revealed most of what we know about the microscopic structure of living organisms and materials. Because the wavelength of an electron beam is much shorter than the wavelength of a beam of visible light, this instrument can resolve smaller details than a light microscope can:



A Comparison of Images Obtained Using a Light Microscope and an Electron Microscope. Because of their shorter wavelength, high-energy electrons have a higher resolving power than visible light. Consequently, an electron microscope (b) is able to resolve finer details than a light microscope (a). Radiolaria, which are shown here, are unicellular planktonic organisms. (CC BY-SA-NC; Anonymous by request).

Standing Waves

De Broglie also investigated why only certain orbits were allowed in Bohr's model of the hydrogen atom. He hypothesized that the electron behaves like a **standing wave** (a wave that does not travel in space). An example of a standing wave is the motion of a string of a violin or guitar. When the string is plucked, it vibrates at certain fixed frequencies because it is fastened at both ends (Figure 5.5.3). If the length of the string is L, then the lowest-energy vibration (the fundamental) has wavelength

$$\begin{aligned}\frac{\lambda}{2} &= L \\ \lambda &= 2L\end{aligned} \tag{5.5.9}$$

Higher-energy vibrations are called *overtones* (the vibration of a standing wave that is higher in energy than the fundamental vibration) and are produced when the string is plucked more strongly; they have wavelengths given by

$$\lambda = \frac{2L}{n} \tag{5.5.10}$$

where n has any integral value. When plucked, all other frequencies die out immediately. Only the resonant frequencies survive and are heard. Thus, we can think of the resonant frequencies of the string as being quantized. Notice in Figure 5.5.3 that all overtones have one or more nodes, points where the string does not move. The amplitude of the wave at a node is zero.

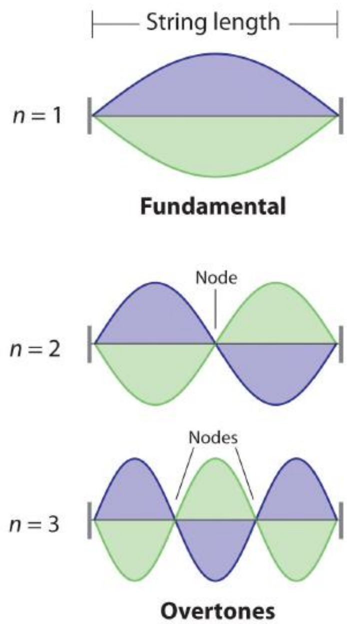


Figure 5.5.3: Standing Waves on a Vibrating String. The vibration with \(*n* = 1\) is the fundamental and contains no nodes. Vibrations with higher values of *n* are called overtones; they contain \(*n* − 1\) nodes.

Quantized vibrations and overtones containing nodes are not restricted to one-dimensional systems, such as strings. A two-dimensional surface, such as a drumhead, also has quantized vibrations. Similarly, when the ends of a string are joined to form a circle, the only allowed vibrations are those with wavelength

$$2\pi r = n\lambda \tag{5.5.11}$$

where r is the radius of the circle. De Broglie argued that Bohr's allowed orbits could be understood if the electron behaved like a *standing circular wave* (Figure 5.5.4). The standing wave could exist only if the circumference of the circle was an integral multiple of the wavelength such that the propagated waves were all in phase, thereby increasing the net amplitudes and causing *constructive interference*. Otherwise, the propagated waves would be out of phase, resulting in a net decrease in amplitude and causing *destructive interference*. The nonresonant waves interfere with themselves! De Broglie's idea explained Bohr's allowed orbits and energy levels nicely: in the lowest energy level, corresponding to $n = 1$ in Equation 5.5.11, one complete wavelength would close the circle. Higher energy levels would have successively higher values of *n* with a corresponding number of nodes.

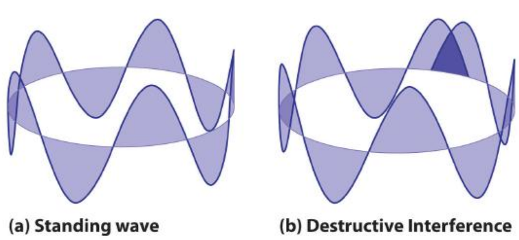


Figure 5.5.4: Standing Circular Wave and Destructive Interference. (a) In a standing circular wave with \(*n* = 5\), the circumference of the circle corresponds to exactly five wavelengths, which results in constructive interference of the wave with itself when overlapping occurs. (b) If the circumference of the circle is not equal to an integral multiple of wavelengths, then the wave does not overlap exactly with itself, and the resulting destructive interference will result in cancellation of the wave. Consequently, a standing wave cannot exist under these conditions.

Like all analogies, although the standing wave model helps us understand much about why Bohr's theory worked, it also, if pushed too far, can mislead. As you will see, some of de Broglie's ideas are retained in the modern theory of the electronic structure of the atom: the wave behavior of the electron and the presence of nodes that increase in number as the energy level increases. Unfortunately, his (and Bohr's) explanation also contains one major feature that we now know to be incorrect: in the currently accepted model, the electron in a given orbit is *not* at the same distance from the nucleus.

Summary

An electron possesses both particle and wave properties. The modern model for the electronic structure of the atom is based on recognizing that an electron possesses particle and wave properties, the so-called **wave–particle duality**. Louis de Broglie showed that the wavelength of a particle is equal to Planck’s constant divided by the mass times the velocity of the particle.

$$\lambda = \frac{h}{mv}$$

The electron in Bohr’s circular orbits could thus be described as a **standing wave**, one that does not move through space. Standing waves are familiar from music: the lowest-energy standing wave is the **fundamental** vibration, and higher-energy vibrations are **overtones** and have successively more **nodes**, points where the amplitude of the wave is always zero.