15.4: Ideal Gases and The Ideal Gas Law

Learning Objectives

- Explain all the quantities involved in the ideal gas law.
- Evaluate the gas constant R from experimental results.
- Calculate T, V, P, or n of the ideal gas law: $PV = nRT$.
- Describe the ideal gas law using graphics.

The Ideal Gas Law

The volume (*V*) occupied by *n* moles of any gas has a pressure (*P*) at temperature (*T*) in Kelvin. The relationship for these variables,

$$PV = nRT \tag{15.4.1}$$

where *R* is known as the gas constant, is called the **ideal gas law** or **equation of state**. Properties of the gaseous state predicted by the ideal gas law are within 5% for gases under ordinary conditions. In other words, given a set of conditions, we can predict or calculate the properties of a gas to be within 5% by applying the ideal gas law. How to apply such a law for a given set of conditions is the focus of general chemistry.

At a temperature much higher than the critical temperature and at low pressures, however, the ideal gas law is a very good model for gas behavior. When dealing with gases at low temperature and at high pressure, correction has to be made in order to calculate the properties of a gas in industrial and technological applications. One of the common corrections made to the ideal gas law is the van der Waal's equation, but there are also other methods dealing with the deviation of gas from ideality.

The Gas Constant *R*

Repeated experiments show that at standard temperature (273 K) and pressure (1 atm or 101325 N/m^2), one mole (*n* = 1) of gas occupies 22.4 L volume. Using this experimental value, you can evaluate the **gas constant *R*,**

$$R = \frac{PV}{nT} = \frac{1\text{ atm }\ 22.4\text{ L}}{1\text{ mol }\ 273\text{ K}} \tag{15.4.2}$$

$$\tag{15.4.3}$$

$$= 0.08205\ \frac{\text{L atm}}{\text{mol} \cdot \text{K}} \tag{15.4.4}$$

When SI units are desirable, *P* = 101325 N/m^2 (Pa for pascal) instead of 1 atm. The volume is 0.0224 m^3. The numerical value and units for *R* are

$$R = \frac{101325\ \frac{\text{N}}{\text{m}^2}\ 0.0224\text{ m}^3}{1\text{ mol } 273\text{ K}} \tag{15.4.5}$$

$$\tag{15.4.6}$$

$$= 8.314\ \frac{\text{J}}{\text{mol} \cdot \text{K}} \tag{15.4.7}$$

Note that $1\text{ L atm} = 0.001\text{ m}^3 \times 101325\ \frac{\text{N}}{\text{m}^2} = 101.325\text{ J (or N m)}$ Since energy can be expressed in many units, other numerical values and units for *R* are frequently in use.

The Gas Constant

For your information, the gas constant can be expressed in the following values and units.

$$R = 0.08205 \frac{\text{L atm}}{\text{mol} \cdot \text{K}} \quad \text{Notes:} \tag{15.4.8}$$

$$= 8.3145 \frac{\text{L kPa}}{\text{mol} \cdot \text{K}} \quad 1 \text{ atm} = 101.32 \text{ kPa} \tag{15.4.9}$$

$$= 8.3145 \frac{\text{J}}{\text{mol} \cdot \text{K}} \quad 1 \text{ J} = 1 \text{ L kPa} \tag{15.4.10}$$

$$= 1.987 \frac{\text{cal}}{\text{mol} \cdot \text{K}} \quad 1 \text{ cal} = 4.182 \text{ J} \tag{15.4.11}$$

$$= 62.364 \frac{\text{L torr}}{\text{mol} \cdot \text{K}} \quad 1 \text{ atm} = 760 \text{ torr} \tag{15.4.12}$$

The gas constant R is such a universal constant for all gases that its values are usually listed in the "Physical Constants" of textbooks and handbooks. It is also listed in Constants of our HandbookMenu at the left bottom. Although we try to use SI units all the time, the use of atm for pressure is still common. Thus, we often use R = 8.314 J / (mol·K) or 8.3145 J / mol·K.

The volume occupied by one mole, n = 1, of substance is called the **molar volume**, $V_{\text{molar}} = \dfrac{V}{n}$. Using the molar volume notation, the ideal gas law is:

$$PV_{\text{molar}} = RT$$

Applications of the Ideal Gas Law

The ideal gas law has four parameters and a constant, R,

$$PV = nRT,$$

and it can be rearranged to give an expression for each of P, V, n or T. For example,

$$P = \frac{nRT}{V} \text{ (Boyle's law)}$$

$$P = \left(\frac{nR}{V}\right) T \text{ (Charles's law)}$$

These equations are Boyle's law and Charles's law respectively. Similar expressions can be derived for V, n and T in terms of other variables. Thus, there are many applications. However, you must make sure that you use the proper numerical value for the gas constant R according to the units you have for the parameters.

Furthermore, $\dfrac{n}{V}$ is number of moles per unit volume, and this quantity has the same units as the concentration (C). Thus, the concentration is a function of pressure and temperature,

$$C = \frac{P}{RT}$$

At 1.0 atm pressure and room temperature of 298 K, the concentration of an ideal gas is 0.041 mol/L.

Avogadro's law can be further applied to correlate gas density ρ (weight per unit volume or $n\,M\,/\,V$) and molecular mass M of a gas. The following equation is easily derived from the ideal gas law:

$$PM = \frac{nM}{V} RT \tag{15.4.13}$$

Thus, we have

$$PM = \frac{dRT}{M} \tag{15.4.14}$$

$$\rho = \frac{nM}{V} \leftarrow \text{definition, and} \tag{15.4.15}$$

$$\rho = \frac{PM}{RT} \tag{15.4.16}$$

$$M = \frac{dRT}{P} \tag{15.4.17}$$

Example 1

An air sample containing only nitrogen and oxygen gases has a density of 1.3393 g / L at STP. Find the weight and mole percentages of nitrogen and oxygen in the sample.

Solution

From the density ρ, we can evaluate an average molecular weight (also called molar mass).

$$\begin{align} PM &= dRT \tag{15.4.18} \\ M &= 22.4 \times d \tag{15.4.19} \\ &= 22.4 \text{ L/mol} \times 1.3393 \text{ g/L} \tag{15.4.20} \\ &= 30.0 \text{ g/mol} \tag{15.4.21} \end{align}$$

Assume that we have 1.0 mol of gas, and *x* mol of which is nitrogen, then (1 - *x*) is the amount of oxygen. The average molar mass is the mole weighted average, and thus,

$$28.0\,\mathrm{x} + 32.0(1-\mathrm{x}) = 30.0$$

$$-4\,\mathrm{x} = -2$$

$$\mathrm{x} = 0.50 \text{ mol of } \mathrm{N_2}, \text{ and } 1.0 - 0.50 = 0.50 \text{ mol } \mathrm{O_2}$$

Now, to find the weight percentage, find the amounts of nitrogen and oxygen in 1.0 mol (30.0 g) of the mixture.

$$\text{Mass of } 0.5 \text{ mol nitrogen} = 0.5 \times 28.0 = 14.0 \text{ g}$$

$$\text{Mass of } 0.5 \text{ mol oxygen} = 0.5 \times 32.0 = 16.0 \text{ g}$$

$$\text{Percentage of nitrogen} = 100 \times \frac{14.0}{30.0} = 46.7\%$$

$$\text{Percentage of oxygen} = 100 \times \frac{16.0}{30.0} = 100 - 46.7 = 53.3\%$$

DISCUSSION

We can find the density of pure nitrogen and oxygen first and evaluate the fraction from the density.

$$\rho \text{ of } \mathrm{N_2} = \frac{28.0}{22.4} = 1.2500 \text{ g/L}$$

$$\rho \text{ of } \mathrm{O_2} = \frac{32.0}{22.4} = 1.4286 \text{ g/L}$$

$$1.2500\,\mathrm{x} + 1.4286(1-\mathrm{x}) = 1.3393$$

Solving for x gives

$$\mathrm{x} = 0.50 \text{ (same result as above)}$$

Exercise 1

Now, repeat the calculations for a mixture whose density is 1.400 g/L.

Example 2

What is the density of acetone (C_3H_6O) vapor at 1.0 atm and 400 K?

Solution

The molar mass of acetone = 3*12.0 + 6*1.0 + 16.0 = 58.0. Thus,

$$\rho = \frac{PM}{RT} \tag{15.4.22}$$

$$\tag{15.4.23}$$

$$= \frac{1.0 \times 58.0 \text{ atm } \frac{\text{g}}{\text{mol}}}{0.08205 \frac{\text{L atm}}{\text{mol K}} \times 400 \text{ K}} \tag{15.4.24}$$

$$\tag{15.4.25}$$

$$= 1.767 \text{ g/L} \tag{15.4.26}$$

? Exercise 2

The density of acetone is 1.767 g/L; calculate its molar mass.

Confidence Building Questions

1. **What does the variable *n* stand for in the ideal gas law,**

   $PV = nRT$?

   Hint: number of moles of gas in a closed system.

   Skill:
   Describe the ideal gas law.

2. **A closed system means no energy or mass flow into or out of a system. In a closed system, how many independent variables are there among *n*, *T*, *V* and *P* for a gas?** Note: an independent variable can be of any arbitrary values.

   Hint: one

   Skill:
   The ideal gas equation shows the interdependence of the variables. Only one of them can be varied independently.

3. **What is the molar volume of an ideal gas at 2 atm and 1000 K?**

   Hint: 41.0

   Skill:
   Evaluate molar volume at any condition.

4. **A certain amount of a gas is enclosed in a container of fixed volume. If you let heat (energy) flow into it, what will increase?**
   (In a multiple choice, you may have volume, pressure, temperature, and any combination of these to choose from.)

   Hint: Both pressure and temperature will increase.

   Skill:
   Explain a closed system and apply ideal gas law.

5. **For a certain amount (*n* = constant) of gas in a closed system, how does volume *V* vary with the temperature? In the following, *k* is a constant depending on *n* and *P*.**

   a. $V = kT$
   b. $V = \frac{k}{T}$
   c. $TV = k$
   d. $V = kT^2$
   e. $V = k$

   Hint: a

   Skill:
   Explain Charles's law.

6. **Boyle's law is *P V* = constant. A sketch of *P* vs *V* on graph paper is similar to a sketch of the equation *x y* = 5. What curve(s) does this equation represent?**
   a. a parabola
   b. an ellipse
   c. a hyperbola
   d. a pair of hyperbolas
   e. a straight line
   f. a surface

   Hint: d

   Skill:
   Apply the skills acquired in math courses to chemical problem solving.

7. **For a certain amount of gas in a closed system, which one of the following equations is valid? Subscripts 1 and 2 refer to specific conditions 1 and 2 respectively.**
   a. $P_1V_1T_1 = P_2V_2T_2$
   b. $P_1V_1T_2 = P_2V_2T_1$
   c. $P_1V_2T_1 = P_2V_1T_2$
   d. $P_2V_1T_1 = P_1V_2T_2$
   e. $\dfrac{P_1V_2}{T_1} = \dfrac{P_2V_1}{T_2}$

   Hint: b

   Skill:
   Rearrange a mathematical equation.

8. **The gas constant *R* is 8.314 J / mol·K. Convert the numerical value of *R* so that its units are cal / (mol·K). A unit conversion table will tell you that 1 cal = 4.184 J. Make sure you know where to find it. During the exam, the conversion factor is given, but you should know how to use it.**

   Hint: 1.987 cal/(mol K).

   Skill:
   Use conversion factors, for example:

   $$8.314 \text{ J} \times \frac{1 \text{ cal}}{4.184 \text{ J}} = ?\text{ cal}$$

9. **At standard temperature and pressure, how many moles of H_2 are contained in a 1.0 L container?**

   Hint: 0.045 mol/L

   Discussion:
   There are many methods for calculating this value.

10. **At standard temperature and pressure, how many grams of CO_2 are contained in a 3.0 L container? Molar mass of $CO_2 = 44$.**

    Hint: 5.89 g in 3 L

    One method:
    It contains $n = \dfrac{1 \text{ atm} \times 3 \text{ L}}{0.08205 \dfrac{\text{L atm}}{\text{mol} \cdot \text{K}} \times 273 \text{ K}}$

11. **What is the pressure if 1 mole of N_2 occupies 1 L of volume at 1000 K?**

    Hint: 82.1 atm

    Discussion:
    Depending on the numerical value and units of *R* you use, you will get the pressure in various units.
    At 1000 K, some of the N_2 molecules may dissociate. If that is true, the pressure will be higher!

12. **What is the temperature if 1 mole of N_2 occupying 100 L of volume has a pressure of 20 Pa (1 Pa = 1 Nm^{-2})?**

    Hint: 240 K

    Discussion:
    At T = 240 K, ideal gas law may not apply to CO_2, because this gas liquifies at a rather high temperature. The ideal gas law is still good for N_2, H_2, O_2 etc, because these gases liquify at much lower temperatures.
