15.3: Other Gas Relationships

Learning Objectives

- Review other simple gas laws.
- Learn and apply the combined gas law.

You may notice in Boyle's law and Charles's law that we actually refer to four physical properties of a gas: pressure (*P*), volume (*V*), temperature (*T*), and amount (in moles—*n*). We do this because these are the only four independent physical properties of a gas. There are other physical properties, but they are all related to one (or more) of these four properties.

Boyle's law is written in terms of two of these properties, with the other two being held constant. Charles's law is written in terms of two different properties, with the other two being held constant. It may not be surprising to learn that there are other gas laws that relate other pairs of properties—as long as the other two are held constant. In this section, we will mention a few.

Gay-Lussac's law relates pressure with absolute temperature. In terms of two sets of data, Gay-Lussac's law is

$$\frac{P_1}{T_1}=\frac{P_2}{T_2}$$

at constant V and n.

Note that it has a structure very similar to that of Charles's law, only with different variables—pressure instead of volume. **Avogadro's law** introduces the last variable for amount. The original statement of Avogadro's law states that equal volumes of different gases at the same temperature and pressure contain the same number of particles of gas. Because the number of particles is related to the number of moles (1 mol = 6.022 × 10^{23} particles), Avogadro's law essentially states that equal volumes of different gases, at the same temperature and pressure, contain the same *amount* (moles, particles) of gas. Put mathematically into a gas law, Avogadro's law is

$$\frac{V_1}{n_1}=\frac{V_2}{n_2}$$

at constant V and T.

(First announced in 1811, it was Avogadro's proposal that volume is related to the number of particles that eventually led to naming the number of things in a mole as Avogadro's number.) Avogadro's law is useful because for the first time we are seeing amount, in terms of the number of moles, as a variable in a gas law.

Example 15.3.1

A 2.45 L volume of gas contains 4.5 × 10^{21} gas particles. How many gas particles are there in 3.87 L if the gas is at constant pressure and temperature?

Solution

We can set up Avogadro's law as follows:

$$\frac{2.45\,L}{4.5\times10^{21}\text{ particles}}=\frac{3.87\,L}{n_2}$$

We algebraically rearrange to solve for n_2:

$$n_2=\frac{(3.87\,\cancel{L})(4.5\times10^{21}\text{ particles})}{2.45\,\cancel{L}}$$

The L units cancel, so we solve for n_2:

$$n_2=7.1\times10^{21}\text{ particles}$$

Exercise 15.3.1

A 12.8 L volume of gas contains 3.00 × 10^{20} gas particles. At constant temperature and pressure, what volume does 8.22 × 10^{18} gas particles fill?

Answer

0.351 L

The variable *n* in Avogadro's law can also stand for the number of moles of gas, in addition to number of particles.

One thing we notice about all gas laws, collectively, is that volume and pressure are always in the numerator, and temperature is always in the denominator. This suggests that we can propose a gas law that combines pressure, volume, and temperature. This gas law is known as the **combined gas law**, and its mathematical form is

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \; at\ constant\ n$$

This allows us to follow changes in all three major properties of a gas. Again, the usual warnings apply about how to solve for an unknown algebraically (isolate it on one side of the equation in the numerator), units (they must be the same for the two similar variables of each type), and units of temperature must be in kelvins.

Example 15.3.2

A sample of gas at an initial volume of 8.33 L, an initial pressure of 1.82 atm, and an initial temperature of 286 K simultaneously changes its temperature to 355 K and its volume to 5.72 L. What is the final pressure of the gas?

Solution

We can use the combined gas law directly; all the units are consistent with each other, and the temperatures are given in Kelvin. Substituting,

$$\frac{(1.82\, atm)(8.33\, L)}{286\, K} = \frac{P_2(5.72\, L)}{355\, K}$$

We rearrange this to isolate the P_2 variable all by itself. When we do so, certain units cancel:

$$\frac{(1.82\, atm)(8.33\ \cancel{L})(355\ \cancel{K})}{(286\ \cancel{K})(5.72\ \cancel{L})} = P_2$$

Multiplying and dividing all the numbers, we get

$$P_2 = 3.29\, atm$$

Ultimately, the pressure increased, which would have been difficult to predict because two properties of the gas were changing.

Exercise 15.3.2

If P_1 = 662 torr, V_1 = 46.7 mL, T_1 = 266 K, P_2 = 409 torr, and T_2 = 371 K, what is V_2?

Answer

105 mL

As with other gas laws, if you need to determine the value of a variable in the denominator of the combined gas law, you can either cross-multiply all the terms, or just take the reciprocal of the combined gas law. Remember, the variable you are solving for must be in the numerator and all by itself on one side of the equation.

Summary

- There are gas laws that relate any two physical properties of a gas.
- The combined gas law relates pressure, volume, and temperature of a gas.