F: Maxwell's Relations

For a well-behaved function, $\mathfrak{F}$, the order of derivatives is permutable

$$\left[\frac{\partial}{\partial y}\left(\frac{\partial \mathfrak{F}}{\partial x}\right)_{y,z}\right]_{x,z}=\left[\frac{\partial}{\partial x}\left(\frac{\partial \mathfrak{F}}{\partial y}\right)_{x,z}\right]_{y,z}$$

Corresponding relationships from the second derivatives of E are

$$\begin{aligned}
\left(\frac{\partial T}{\partial V}\right)_{S,\{n_i\}} &= -\left(\frac{\partial P}{\partial S}\right)_{V,\{n_i\}} \\
\left(\frac{\partial \tilde{\mu}_i}{\partial V}\right)_{S,\{n_i\}} &= -\left(\frac{\partial P}{\partial n_i}\right)_{S,V,\{n_{j\neq i}\}} \\
\left(\frac{\partial \tilde{\mu}_i}{\partial S}\right)_{V,\{n_i\}} &= \left(\frac{\partial T}{\partial n_i}\right)_{S,V,\{n_{j\neq i}\}} \\
\left(\frac{\partial \tilde{\mu}_{\mathrm{k}}}{\partial n_{\mathrm{j}}}\right)_{S,V,\{n_{i\neq j}\}} &= \left(\frac{\partial \tilde{\mu}_{\mathrm{j}}}{\partial n_{\mathrm{k}}}\right)_{S,V,\{n_{i\neq k}\}}
\end{aligned}$$

from the second derivatives of H are

$$\begin{aligned}
\left(\frac{\partial T}{\partial P}\right)_{S,\{n_i\}} &= \left(\frac{\partial V}{\partial S}\right)_{P,\{n_i\}} \\
\left(\frac{\partial \tilde{\mu}_i}{\partial P}\right)_{S,\{n_i\}} &= \left(\frac{\partial V}{\partial n_i}\right)_{S,P,\{n_{j\neq i}\}} \\
\left(\frac{\partial \tilde{\mu}_i}{\partial S}\right)_{P,\{n_i\}} &= \left(\frac{\partial T}{\partial n_i}\right)_{S,P,\{n_{j\neq 1}\}} \\
\left(\frac{\partial \tilde{\mu}_{\mathrm{k}}}{\partial n_{\mathrm{j}}}\right)_{S,P,\{n_{i\neq j}\}} &= \left(\frac{\partial \tilde{\mu}_{\mathrm{j}}}{\partial n_{\mathrm{k}}}\right)_{S,P,\{n_{i\neq k}\}}
\end{aligned}$$

from the second derivatives of F are

$$\begin{aligned}
-\left(\frac{\partial S}{\partial V}\right)_{T,\{n_i\}} &= -\left(\frac{\partial P}{\partial T}\right)_{V,\{n_i\}} \\
\left(\frac{\partial \tilde{\mu}_i}{\partial V}\right)_{T,\{n_i\}} &= -\left(\frac{\partial P}{\partial n_i}\right)_{T,V,\{n_{j\neq i}\}} \\
\left(\frac{\partial \tilde{\mu}_i}{\partial T}\right)_{V,\{n_i\}} &= -\left(\frac{\partial S}{\partial n_i}\right)_{T,V,\{n_{j\neq t}\}} \\
\left(\frac{\partial \tilde{\mu}_{\mathrm{k}}}{\partial n_{\mathrm{j}}}\right)_{T,V,\{n_{i\neq j}\}} &= \left(\frac{\partial \tilde{\mu}_{\mathrm{j}}}{\partial n_{\mathrm{k}}}\right)_{T,V\{n_{i\neq k}\}}
\end{aligned}$$

and from the second derivatives of G are

$$-\left(\frac{\partial S}{\partial P}\right)_{T,\{n_i\}} = \left(\frac{\partial V}{\partial T}\right)_{P,\{n_i\}}$$

$$\left(\frac{\partial \tilde{\mu}_i}{\partial P}\right)_{T,\{n_i\}} = \left(\frac{\partial V}{\partial n_i}\right)_{T,P,\{n_{j\neq 1}\}}$$

$$\left(\frac{\partial \tilde{\mu}_i}{\partial T}\right)_{P,\{n_i\}} = -\left(\frac{\partial S}{\partial n_i}\right)_{T,P,\{n_{j\neq t}\}}$$

$$\left(\frac{\partial \tilde{\mu}_\mathrm{k}}{\partial n_\mathrm{j}}\right)_{T,P,\{n_{i\neq j}\}} = \left(\frac{\partial \tilde{\mu}_\mathrm{j}}{\partial n_\mathrm{k}}\right)_{T,P,\{n_{i\neq k}\}}$$