1.9: Density

The terms *heavy* and *light* are commonly used in two different ways. We refer to weight when we say that an adult is heavier than a child. On the other hand, something else is alluded to when we say that oak is heavier than balsa wood. A small shaving of oak would obviously weigh less than a roomful of balsa wood, but oak is heavier in the sense that a piece of given size weighs more than the same-size piece of balsa.

What we are actually comparing is the *mass per unit volume*, that is, the **density**. In order to determine these densities, we might weigh a cubic centimeter of each type of wood. If the oak sample weighed 0.71 g and the balsa 0.15 g, we could describe the density of oak as 0.71 g cm^{-3} and that of balsa as 0.15 g cm^{-3}. (Note that the negative exponent in the units cubic centimeters indicates a reciprocal. Thus 1 cm^{-3} = 1/cm^3 and the units for our densities could be written as $\frac{g}{cm^3}$, g/cm^3, or g cm^{-3}. In each case the units are read as grams per cubic centimeter, the *per* indicating division.) We often abbreviate "cm^3" as "cc", and 1 cm^3 = 1 mL exactly by definition.

In general it is not necessary to weigh exactly 1 cm^3 of a material in order to determine its density. We simply measure mass and volume and divide volume into mass:

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

or

$$\rho = \frac{m}{V} \tag{1.9.1}$$

where ρ is the density, m is the mass, and V volume.

Example 1.9.1: Density of Aluminum

Calculate the density of

a. a piece of aluminum whose mass is 37.42 g and which, when submerged, increases the water level in a graduated cylinder by 13.9 ml;
b. an aluminum cylinder of mass 25.07 g, radius 0.750 cm, and height 5.25 cm.

Solution

a

Since the submerged metal displaces its own volume,

$$\begin{aligned} \text{Density } &= \rho = \frac{m}{V} \\ &= \frac{37.42 \text{ g}}{13.9 \text{ ml}} \\ &= 2.69 \text{ g/ml or } 2.69 \text{ g ml}^{-1} \end{aligned}$$

b

The volume of the cylinder must be calculated first, using the formula

$$\begin{aligned} \text{V} &= \pi r^2 h \\ &= 3.142 \times (0.750 \text{ cm})^2 \times 5.25 \text{ cm} \\ &= 9.278\ 718\ 8 \text{ cm}^3 \end{aligned}$$

Then

$$\rho = \frac{m}{V} = \frac{25.07 \text{ g}}{9.278\ 718\ 8 \text{ cm}^3} = \begin{cases} 2.70 \dfrac{\text{g}}{\text{cm}^3} \\ 2.70 \text{ g cm}^{-3} \\ 2.70 \text{ g/cm}^3 \end{cases}$$

which are all acceptable alternatives.

Note that unlike mass or volume (**extensive properties**), the density of a substance is independent of the size of the sample (**intensive property**). Thus density is a property by which one substance can be distinguished from another. A sample of pure aluminum can be trimmed to any desired volume or adjusted to have any mass we choose, but its density will always be 2.70 g/cm^3 at 20°C. The densities of some common pure substances are listed below.

Tables and graphs are designed to provide a maximum of information in a minimum of space. When a physical quantity (number × units) is involved, it is wasteful to keep repeating the same units. Therefore it is conventional to use pure numbers in a table or along the axes of a graph. A pure number can be obtained from a quantity if we divide by appropriate units. For example, when divided by the units gram per cubic centimeter, the density of aluminum becomes a pure number 2.70:

$$\frac{\text{Density of aluminum}}{1 \text{ g cm}^{-3}} = \frac{2.70 \text{ g cm}^{-3}}{1 \text{ g cm}^{-3}} = 2.70$$

Therefore, a column in a table or the axis of a graph is conveniently labeled in the following form:

Quantity/units

This indicates the units that must be divided into the quantity to yield the pure number in the table or on the axis. This has been done in the second column of the table.

Table 1.9.1: Density of Several Substances at 20°C.Anchor

Substance	**Density / g cm^{-3}**
Helium gas	0.000 16
Dry air	0.001 185
Gasoline	0.66-0.69 (varies)
Kerosene	0.82
Benzene	0.880
Water	1.000
Carbon tetrachloride	1.595
Magnesium	1.74
Salt	2.16
Aluminum	2.70
Iron	7.87
Copper	8.96
Silver	10.5
Lead	11.34
Uranium	19.05
Gold	19.32

Converting Densities

In our exploration of density, notice that chemists may express densities differently depending on the subject. The density of pure substances may be expressed in kg/m^3 in some journals which insist on strict compliance with SI units; densities of soils may be expressed in lb/ft^3 in some agricultural or geological tables; the density of a cell may be expressed in mg/µL; and other units are in common use. It is easy to transform densities from one set of units to another, by multiplying the original quantity by one or more ***unity factors***:

Example 1.9.2: Density of Water

Convert the density of water, 1 g/cm^3 to (a) lb/cm^3 and (b) lb/ft^3

Solution

a

The equality $454\text{ g} = 1\text{ lb}$ can be used to write two unity factors,

$$\frac{454\text{ g}}{1\text{ lb}}$$

or

$$\frac{1\text{ lb}}{454\text{ g}}$$

The given density can be multiplied by one of the unity factors to get the desired result. The correct conversion factor is chosen so that the units cancel:

$$1\frac{\text{g}}{\text{cm}^3} \times \frac{1\text{ lb}}{454\text{ g}} = 0.002203\frac{\text{lb}}{\text{cm}^3}$$

b

Similarly, the equalities $2.54\text{ cm} = 1\text{ inch}$, and $12\text{ inches} = 1\text{ ft}$ can be use to write the unity factors:

$$\frac{2.54\text{ cm}}{1\text{ in}}, \frac{1\text{ in}}{2.54\text{ cm}}, \frac{12\text{ in}}{1\text{ ft}} \text{ and } \frac{1\text{ ft}}{12\text{ in}}$$

In order to convert the cm^3 in the denominator of 0.002203 to in^3, we need to multiply by the appropriate unity factor three times, or by the cube of the unity factor:

$$0.002203\frac{\text{g}}{\text{cm}^3} \times \frac{2.54\text{ cm}}{1\text{ in}} \times \frac{2.54\text{ cm}}{1\text{ in}} \times \frac{2.54\text{ cm}}{1\text{ in}}$$

or

$$0.002203\frac{\text{g}}{\text{cm}^3} \times \left(\frac{2.54\text{ cm}}{1\text{ in}}\right)^3 = 0.0361\text{ lb/in}^3$$

This can then be converted to lb/ft^3:

$$0.0361\text{ lb/in}^3 \times \left(\frac{12\text{ in}}{1\text{ ft}}\right)^3 = 62.4\text{ lb/ft}^3$$

It is important to notice that we have used conversion factors to convert from one unit to another unit of the **same parameter**