10.18: Measuring the Composition of a Solution

When speaking of solubility or miscibility, or when doing quantitative experiments involving solutions, it is necessary to know the exact composition of a solution. This is invariably given in terms of a *ratio* telling us how much solute is dissolved in a unit quantity of solvent or solution. The ratio can be a ratio of masses, of amounts of substances, or of volumes, or it can be some **combination** of these. For example, **concentration** was defined before as the *amount of solute per unit volume of solution*:

$$c_{\text{solute}} = \frac{n_{\text{solute}}}{V_{\text{solute}}}$$

The two simplest measures of the composition are the **mass fraction** *w*, which is the *ratio of the mass of solute to the total mass of solution*, and the **mole fraction** *x*, which is the *ratio of the amount of solute to the total amount of substance in the solution*. If we indicate the solute by *A* and the solvent by *B*, the mass fraction and the mole fraction are defined by

$$w_A = \frac{m_A}{m_A + m_B} \text{ and } x_A = \frac{n_A}{n_A + n_B}$$

Example 10.18.1: Fractions

A solution is prepared by dissolving 18.65 g naphthalene, $C_{10}H_8$ in 89.32 g benzene, C_6H_6. Find (a) the mass fraction and (b) the mole fraction of the naphthalene.

Solution

a) The mass fraction is easily calculated from the masses:

$$w_{\text{C}_{10}\text{H}_8} = \frac{m_{\text{C}_{10}\text{H}_8}}{m_{\text{C}_{10}\text{H}_8} + m_{\text{C}_6\text{H}_6}} = \frac{18.65 \text{ g}}{18.65 \text{ g} + 89.32 \text{ g}} = 0.1727$$

It is sometimes useful to distinguish mass of solute and mass of solution for purposes of calculation. In such a case we can write

$w_{C_{10}H_8}$ = 0.1727 g $C_{10}H_8$ per g solution

b) In order to calculate the mole fraction, we must first calculate the amount of each substance. Since

$$M_{\text{C}_{10}\text{H}_8} = 128.18 \text{ g mol}^{-1} \text{ and } M_{\text{C}_6\text{H}_6} = 78.11 \text{ g mol}^{-1}$$

we find

$$n_{\text{C}_{10}\text{H}_8} = \frac{18.65 \text{ g}}{128.18 \text{ g mol}^{-1}} = 0.1455$$

$$n_{\text{C}_6\text{H}_6} = \frac{89.32 \text{ g}}{78.11 \text{ g mol}^{-1}} = 1.144$$

Thus

$$x_{\text{C}_{10}\text{H}_8} = \frac{n_{\text{C}_{10}\text{H}_8}}{n_{\text{C}_{10}\text{H}_8} + n_{\text{C}_6\text{H}_6}} = \frac{0.1455 \text{ mol}}{0.1455 \text{ mol} + 0.144 \text{ mol}} = 0.1455 = 0.1128$$

or 0.1128 mol $C_{10}H_8$ per mol solution

The mass fraction is useful because it does not require that we know the exact chemical nature of both solute and solvent. Thus if we dissolve 10 g crude oil in 10 g gasoline, we can calculate $w_{\text{crude oil}}$= 0.5 even though the solute and solvent are both mixtures of alkanes and have no definite molar mass. By contrast, the mole fraction is useful when we want to know the nature of the solution on the microscopic level. In the above example, for instance, we know that for every 100 mol of solution, 11.28 mol is naphthalene. On the molecular level this means that out of each 100 molecules in the solution, 11.28 will, on the average, be naphthalene molecules.

The mass fraction of a solution is often encountered in other disguises. The **weight percentage** (strictly speaking, the *mass* percentage) of a solution is often defined by the formula:

$$\text{Weight percentage of } A = \frac{m_A}{m_A + m_B} \times 100 \tag{10.18.1}$$

This definition is really the same as that of the mass fraction because the percent sign means “divided by 100.” Thus 100% is merely a synonym for 100/100, that is, the number 1, and we can write:

$$w_{C_{10}H_8} = 0.1727 \times 1 = 0.1727 \times 100\% = 17.27\%$$

When the mass fraction is very small, it is often expressed in **parts per million** (ppm) or parts **per billion (**ppb**)**. These symbols can be handled in much the same way as a percentage if you remember how they are related to unity: 1 = 100% = 10^6 ppm = 10^9 ppb

In other words:

$$1\% = \frac{1}{100} = 10^{-2} \tag{10.18.2}$$

$$1 \text{ ppm} = 10^{-6} \tag{10.18.3}$$

$$1 \text{ ppb} = 10^{-9} \tag{10.18.4}$$

Example 10.18.2: Mass Fraction

A 1-kg sample of water from Lake Powell, Utah, is found to contain 10 ng mercury. Walleyed pike caught in the lake contain 0.427 ppm mercury. (a) What is the mass fraction of mercury (in ppb) in water from Lake Powell? (b) If you ate 2 lb of walleyed pike caught from the lake, what mass of mercury would you ingest?

Solution

a) The mass fraction of mercury is by definition

$$w_{\text{Hg}} = \frac{m_{\text{Hg}}}{m_{\text{solution}}}$$

Therefore

$$w_{\text{Hg}} = \frac{10 \text{ ng}}{1 \text{ kg}} = \frac{10 \times 10^{-9} \text{ g}}{1 \times 10^{3} \text{ g}} = 1 \times 10^{-11}$$

In ppb

$$w_{\text{Hg}} = 1 \times 10^{-11} \times 1 = 1 \times 10^{-11} \times 10^{9} \text{ ppb} = 00.01 \text{ ppb}$$

b) Assuming the mercury to be uniformly distributed throughout the walleyed pike, we have

$$w_{\text{Hg}} = 0.427 \text{ ppm} = 0.427 \times 10^{-7}$$

By definition and

$$w_{\text{Hg}} = \frac{m_{\text{Hg}}}{m_{\text{walleyed pike}}}$$

and

$$m_{\text{Hg}} = w_{\text{Hg}} \times m_{\text{walleyed pike}} = 40.27 \times 10^{-7} \times 2.0 \text{ lb}$$

$$= 4.27 \times 10^{-7} \times 2.0 \text{ lb} \times \frac{1 \text{ kg}}{2.2 \text{ lb}} \times \frac{10^{3} \text{ g}}{1 \text{ kg}} = 3.9 \times 10 \text{ g} = 390 \ \mu\text{g}$$

From this example you can see that from nearly the same mass of fish, almost 40 000 times as much mercury would be obtained as from the water. Indeed, if you ate 2 lb of walleyed pike every day, you would exceed the minimum dosage (300 μg for a 70-kg human) at which symptoms of mercury poisoning can appear. Fortunately, most of us do not eat fish every day, nor are we as gluttonous as the example suggests. Nevertheless, the much higher mass fraction of mercury in fish than in water shows that very small quantities of mercury in the environment can be magnified many times in living systems. This process of bioamplification will be discussed more thoroughly in the sections on Biochemistry.