4.14: Average Atomic Weights

There are 21 elements with only one isotope, so all their atoms have identical masses. All other elements have two or more isotopes, so their atoms have at least two different masses. But all elements obey the law of definite proportions when they combine with other elements, so they *behave as if* they had just one kind of atom with a definite mass. In order to solve this dilemma, we define the atomic weight as the weighted average mass of all naturally occurring (occasionally radioactive) isotopes of the element.

A weighted average is defined as

$$\text{Atomic Weight} = \left(\frac{\%\text{ abundance isotope 1}}{100}\right) \times (\text{mass of isotope 1}) + \left(\frac{\%\text{ abundance isotope 2}}{100}\right) \times (\text{mass of isotope 2}) \quad + \ \ldots$$

Similar terms would be added for all the isotopes. The calculation is analogous to the method used to calculate grade point averages in most colleges:

$$\text{GPA} = \left(\frac{\text{Credit Hours Course 1}}{\text{total credit hours}}\right) \times (\text{Grade in Course 1}) + \left(\frac{\text{Credit Hours Course 2}}{\text{total credit hours}}\right) \times (\text{Grade in Course 2}) \ + \ldots$$

Some Conventions

The term ***"Average Atomic Weight"*** or simply ***"Atomic Weight"*** is commonly used to refer to what is properly called a ***"relative atomic mass"***. Atomic Weights are technically dimensionless, because they cannot be determined as absolute values. They were historically calculated from mass ratios (early chemists could say that magnesium atoms have atoms of mass 24.305/15.999 times as heavy as oxygen atoms, because that is the mass ratio of magnesium to oxygen in MgO). Now atomic weights are calculated from the position of peaks in a mass spectrum.

While the peak positions may be labeled in amu, that is only possible if the mass spectrometer is calibrated with a standard, whose mass can only be known relative to another, and it is also technically dimensionless. To solve this dilemma, we ***define*** an amu as 1/12 the mass of a $^{12}_{6}\text{C}$ atom. $^{12}_{6}\text{C}$ can then be used to calibrate a mass spectrometer. For convenience, we often use "token" dimensions of *amu/average atom* for atomic weight, or *g/mol* for molar mass.

The calculation of an atomic weight includes "naturally occurring isotopes", which are defined by the Commission on Isotopic Aundances and Atomic Weights of IUPAC (IUPAC/CIAAW) to include radioactive isotopes with half lives greater than 1 x 10^{10} years. Thus thorium, protactinium, and uranium are assigned atomic weights of 232.0, 231.0, and 238.0, but no other radioactive elements have isotopes with long enough lifetimes to be assigned atomic weights.

✔ Example 4.14.1: Isotopes

Naturally occurring lead is found to consist of four isotopes:

- 1.40% $^{204}_{82}\text{Pb}$ whose isotopic weight is 203.973.
- 24.10% $^{206}_{82}\text{Pb}$ whose isotopic weight is 205.974.
- 22.10% $^{207}_{82}\text{Pb}$ whose isotopic weight is 206.976.
- 52.40% $^{208}_{82}\text{Pb}$ whose isotopic weight is 207.977.

Calculate the atomic weight of an average naturally occurring sample of lead.

Solution

Suppose that you had 1 mol lead. This would contain 1.40% ($\frac{1.40}{100}$ × 1 mol) $^{204}_{82}\text{Pb}$ whose molar mass is 203.973 g mol^{-1}. The mass of $^{204}_{82}\text{Pb}$ would be

$$\begin{aligned} \text{m}_{204} &= n_{204} \times\ M_{204} \\ &= \left(\frac{1.40}{100} \times \text{ 1 mol}\right)\ (203.973 \text{ g mol}^{1}) \\ &= 20.86 \text{ g} \end{aligned}$$

Similarly for the other isotopes

$$\begin{aligned} m_{206} &= n_{206} \times M_{206} \\ &= \left(\frac{24.10}{100} \times 1 \text{ mol}\right)(205.974 \text{ g mol}^{-1}) \\ &= 490.64 \text{ g} \end{aligned}$$

$$\begin{aligned} m_{207} &= n_{207} \times M_{207} \\ &= \left(\frac{22.10}{100} \times 1 \text{ mol}\right)(206.976 \text{ g mol}^{-1}) \\ &= 450.74 \text{ g} \end{aligned}$$

$$\begin{aligned} m_{208} &= n_{208} \times M_{208} \\ &= \left(\frac{52.40}{100} \times 1 \text{ mol}\right)(207.977 \text{ g mol}^{-1}) \\ &= 1080.98 \text{ g} \end{aligned}$$

Upon summing all four results, the mass of 1 mol of the mixture of isotopes is to be found

$$2.86\, g + 49.64\, g + 45.74\, g + 108.98\, g = 207.22\, g$$

Thus the atomic weight of lead is 207.2 g/mol, as mentioned earlier in the discussion.

An important corollary to the existence of isotopes should be emphasized at this point. When highly accurate results are obtained, atomic weights may vary slightly depending on where a sample of an element was obtained. For this reason, the IUPAC CIAAW has recently redefined the atomic weights of 10 elements having two or more isotopes [1]. The percentages of different isotopes often depends on the source of the element.

For example, oxygen in Antarctic precipitation has an atomic weight of 15.99903, but oxygen in marine N_2O has an atomic weight of 15.9997. *"Fractionation"* of the isotopes results from slightly different rates of chemical and physical processes caused by small differences in their masses. The difference can be more dramatic when an isotope is derived from transmutation.

For example, lead produced by decay of uranium contains a much larger percentage of $^{206}_{82}\text{Pb}$ than the 24.1 percent given in the example for the average sample. Consequently the atomic weight of lead found in uranium ores is less than 207.2 and is much closer to 205.974, the isotopic weight of $^{206}_{82}\text{Pb}$.

After the discovery of isotopes of the elements by J.J. Thompson in 1913 [2], it was suggested that the scale of relative masses of the atoms (the atomic weights) should use as a reference the mass of an atom of a particular isotope of one of the elements. The standard that was eventually chosen was $^{12}_{6}\text{C}$, and it was assigned an atomic-weight value of exactly 12.000 000.

Thus the atomic weights given in the Table of Atomic Weights are the ratios of weighted averages (calculated as in the Example) of the masses of atoms of all isotopes of each naturally occurring element to the mass of a single $^{12}_{6}\text{C}$ atom. Since carbon consists of two isotopes, 98.99% $^{12}_{6}\text{C}$ isotopic weight 12.000 and 1.11% $^{13}_{6}\text{C}$ of isotopic weight 13.003, the average atomic weight of carbon is

$$\frac{98.89}{100.00} \times 12.000 + \frac{1.11}{100.00} \times 13.003 = 12.011$$

for example.

Conventional Atomic Weights and "Intervals"

Deviations from average isotopic composition are usually not large, and so the ***Conventional Atomic Weight Values*** were defined by the IUPAC/CIAAW for the elements showing the most variation in abundance. They can be used for nearly all chemical calculations. But at the same time, ***Atomic Weights*** were redefined for those elements as ranges, or ***"intervals"***, for any work where small differences may be important[3]. The table below gives typical values.

Table 4.14.1 Atomic Weights

Element Name	Symbol	Conventional Atomic Weight	Atomic Weight
Boron	B	10.81	[10.806; 10.821]

Element Name	Symbol	Conventional Atomic Weight	Atomic Weight
Carbon	C	12.011	[12.0096; 12.0116]
Chlorine	Cl	35.45	[35.446; 35.457]
Hydrogen	H	1.008	[1.00784; 1.00811]
Lithium	Li	6.94	[6.938; 6.997]
Nitrogen	N	14.007	[14.00643; 14.00728]
Oxygen	O	15.999	[15.99903; 15.99971]
Silicon	Si	28.085	[28.084; 28.086]
Sulfur	S	32.06	[32.059; 32.076]
Thallium	Tl	204.38	[204.382; 204.385]

In the study of nuclear reactions, however, one must be concerned about isotopic weights. This is discussed further in the section on Nuclear Chemistry.

SI Definition of the Mole

The SI definition of the mole also depends on the isotope ${}^{12}_{6}C$ and can now be stated. One mole is defined as the amount of substance of a system which contains as many elementary entities as there are atoms in exactly 0.012 kg of ${}^{12}_{6}C$. The *elementary entities* may be atoms, molecules, ions, electrons, or other microscopic particles.

This definition of the mole makes the mass of 1 mole of an element in grams numerically equal to the average mass of the atoms in grams. This official definition of the mole makes possible a more accurate determination of the Avogadro constant than was reported earlier. The currently accepted value is N_A = 6.02214179 × 10^{23} mol^{-1}. This is accurate to 0.00000001 percent and contains five more significant figures than 6.022 × 10^{23} mol^{-1}, the number used to define the mole previously. It is very seldom, however, that more than four significant digits are needed in the Avogadro constant. The value 6.022 × 10^{23} mol^{-1} will certainly suffice for most calculations needed.