5.6: The Uncertainty Principle

Our inability to locate an electron exactly may seem rather strange, but it arises whether we think in terms of waves or of particles. Suppose an experiment is to be done to locate a billiard ball moving across a pool table whose surface is hidden under a black cloth.

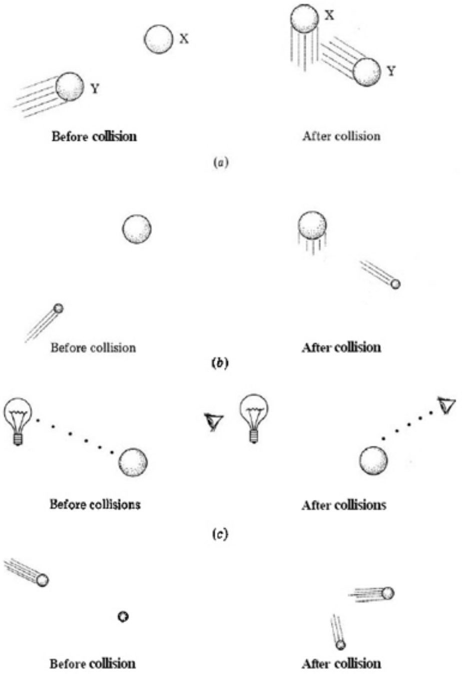

Figure 5.6.1: Determining the location and speed of a particle. (a) Locating a billiard ball (X)by bouncing a second billiard ball (Y) off it; (b) locating a billiard ball by bouncing a table-tennis ball off it; (c) locating a billiard ball by bouncing photons of visible light off it; (d) locating an electron by bouncing a photon off it.

Four different illustrations showing before and after collision. The motion of the particles after collision is a deflection from its original trajectory.

One way to do this would be to try to bounce a second billiard ball off the first one (Figure 5.6.1. When a hit was made and the second ball emerged from under the cloth, we would have a pretty good idea of where the first ball was. The only trouble with the experiment is that the position and speed of the first ball would almost certainly be changed by the collision. To lessen this effect, a table-tennis ball could be substituted for the second billiard ball—its smaller mass would produce a much smaller change in the motion of the first ball. Clearly, the lighter and more delicate the “probe” we use to try to locate the first ball, the less our measurement will affect it. The best way to locate the first billiard ball and determine its speed would be to remove the cover from the table so it could be seen. In this case, however, something is still “bouncing” off the first billiard ball (Figure 1*c*). If we are to see the ball, particles of visible light, or **photons**, must strike the ball and be reflected to our eyes. Since each photon is very small and has very little energy by comparison with that needed to change the motion of the billiard ball, looking at a ball is an excellent means of observing it without changing its position or speed.

But to observe an electron is quite another story, since the mass of an electron is far smaller than that of a billiard ball. Anything (such as a photon of light) which can be bounced off an electron in such a way as to locate it precisely would have far more energy than would be required to change the path of the electron. Hence it would be impossible to predict the electron’s future speed or position from the experiment. The idea that it is impossible to determine accurately *both* the location and the speed of any particle as small as an electron is called the **uncertainty principle**. It was first proposed in 1927 by Werner Heisenberg (1901 to 1976).

According to the uncertainty principle, even if we draw an analogy between the electron in a box and a billiard ball (Figure 1*a* in Wave Mechanics), it will be impossible to determine both the electron’s exact position in the box and its exact speed. Since kinetic energy depends on speed (½ mu^2) and Eq. (4) in Wave Mechanics assigns exact values of kinetic energy to the electron in the box, the speed can be calculated accurately. This means that determination of the electron’s position will be very inexact.

The Heisenberg Uncertainty principle is stated mathematically as

$$\Delta X \Delta P \geq \frac{\hbar}{2}$$

where ΔX is the uncertainty in the position, ΔP is the uncertainty in the momentum, and $\hbar$ is Planck's Constant divided by 2π.

It will be possible to talk about the *probability* that the electron is at a specific location, but there will also be some probability of finding it somewhere else in the box. Since it is impossible to know precisely where the electron is at a given instant, the question, “How does it get from one place to another?” is pointless. There is a finite probability that it was at the other place to begin with!

It is possible to be quantitative about the *probability* of finding a “billiard-ball” electron at a given location, however. Shortly after the uncertainty principle was proposed, the German physicist Max Born (1882 to 1969) suggested that the intensity of the electron wave at any position in the box was proportional to the probability of finding the electron (as a particle) at that same position. Thus if we can determine the shapes of the waves to be associated with an electron, we can also determine the relative probability of its being located at one point as opposed to another. The wave and particle models for the electron are thus connected to and reinforce each other. Niels Bohr suggested the term *complementary* to describe their relationship. It does no good to ask, “Is the electron a wave or a particle?” Both are ways of drawing an analogy between the microscopic world and macroscopic things whose behavior we understand. Both are useful in our thinking, and they are complementary rather than mutually exclusive.

A graphic way of indicating the probability of finding the electron at a particular location is by the density of shading or stippling along the length of the box. This has been done in Figure 2 for the same three electron waves previously illustrated in Figure 1 from Wave Mechanics. Notice that the density of dots is large wherever the electron wave is large.(This would correspond in the guitar-string analogy to places where the string was vibrating quite far from its rest position.)

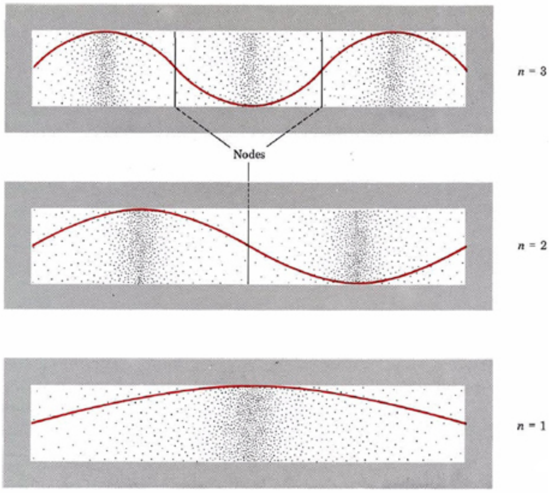

Figure 5.6.2: Indicating electron density or electron probability by the density of dots for the electron in a box. Nodes. where the probability of finding the electron is zero, are indicated. In the guitar-string analogy a node corresponds to a point on the string which is not vibrating up and down.

First box has four equally spaced nodes between its end and a wavelength of one and a half. Dots are dispersed throughout the entire box with three concentrated vertical areas corresponding to the regions occupied by the oscillating motion of the three wave maximum. The second box has a wavelength of one and three nodes between its end. There are two vertical areas of electron concentrated regions. The third and final box has only one vertical concentrated region corresponding to the one peak at the center.

Where the electron wave is small (near the ends of the box in all three cases and at the nodes indicated in the figure), there are only a small number of dots. (A **node** is a place where the intensity of the wave is zero, that is, in the guitar-string analogy, where the string has not moved from its rest position.)

If the electron is thought of as a wave occupying all parts of the box at once, we can speak of an **electron cloud** which has greater or lesser density in various parts of the box. There will be a greater quantity of negative charge in a region of high density (a region where there is a greater concentration of dots) than in one of low density. In an atom or molecule, according to the uncertainty principle, the best we can do is indicate **electron density** in various regions—we cannot locate the precise position of the electron. Therefore electron dot-density diagrams, such as the ones shown in Figure 2, give a realistic and useful picture of the behavior of electrons in atoms. In such a diagram the *electron density or probability of finding the electron is indicated by the number of dots per unit area*. We will encounter electron dot-density diagrams quite often throughout this book. These have all been generated by a computer from accurate mathematical descriptions of the atom or molecule under discussion.

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