

14.11: The Solubility Product

In [the section on precipitation reactions](#), we saw that there are some salts which dissolve in water to only a very limited extent. For example, if BaSO_4 crystals are shaken with water, so little dissolves that it is impossible to see that anything has happened, as you will see in the video below. Nevertheless, the few $\text{Ba}^{2+}(\text{aq})$ and $\text{SO}_4^{2-}(\text{aq})$ ions that do go into solution increase the conductivity of the water, allowing us to measure their concentration. The video below shows the creation of Barium Sulfate in a precipitation reaction between barium chloride and sodium sulfate. Notice the white precipitate that forms, which is barium sulfate.



We find that at 25°C

$$[\text{Ba}^{2+}] = 0.97 \times 10^{-5} \text{ mol L}^{-1} = [\text{SO}_4^{2-}] \quad (14.11.1)$$

that we would describe the solubility of BaSO_4 as $0.97 \times 10^{-5} \text{ mol L}^{-1}$ at this temperature. The solid salt and its ions are in dynamic equilibrium, and so we can write the equation



As in other dynamic equilibria we have discussed, a particular Ba^{2+} ion will sometimes find itself part of a crystal and at other times find itself hydrated and in solution.

Since the concentration of BaSO_4 has a constant value, it can be incorporated into K_c for Equation 14.11.2 This gives a special equilibrium constant called the **solubility product** K_{sp} :

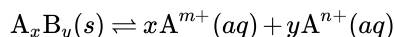
$$K_{sp} = K_c[\text{BaSO}_4] = [\text{Ba}^{2+}][\text{SO}_4^{2-}] \quad (14.11.3)$$

For BaSO_4 , K_{sp} is easily calculated from the solubility by substituting Equation 14.11.1 into 14.11.3

$$K_{sp} = (0.97 \times 10^{-5} \text{ mol L}^{-1})(0.97 \times 10^{-5} \text{ mol L}^{-1}) \quad (14.11.4)$$

$$= 0.94 \times 10^{-10} \text{ mol}^2 \text{ L}^{-2} \quad (14.11.5)$$

In the general case of an ionic compound whose formula is A_xB_y , the equilibrium can be written



The solubility product is then

$$K_{sp} = [\text{A}^{m+}]^x [\text{B}^{n+}]^y$$

Solubility products for some of the more common sparingly soluble compounds are given in the table below.

Table 14.11.1 Solubility Product Constants for Some Inorganic Compounds at 25 °C1

Substance	K_s	Substance	K_{sp}
<i>Aluminum Compounds</i>		<i>Barium Compounds</i>	
AlAsO_4	1.6×10^{-16}	$\text{Ba}_3(\text{AsO}_4)_2$	8.0×10^{-15}

Substance	K_s	Substance	K_{sp}
Al(OH) ₃ amorphous	1.3×10^{-33}	BaCO ₃	5.1×10^{-9}
AlPO ₄	6.3×10^{-19}	BaC ₂ O ₄	1.6×10^{-7}
<i>Bismuth Compounds</i>		BaCrO ₄	1.2×10^{-10}
BiAsO ₄	4.4×10^{-10}	BaF ₂	1.0×10^{-6}
BiOCl ²	7.0×10^{-9}	Ba(OH) ₂	5×10^{-3}
BiO(OH)	4×10^{-10}	Ba ₃ (PO ₄) ₂	3.4×10^{-23}
Bi(OH) ₃	4×10^{-31}	BaSeO ₄	3.5×10^{-8}
BiI ₃	8.1×10^{-19}	BaSO ₄	1.1×10^{-10}
BiPO ₄	1.3×10^{-23}	BaSO ₃	8×10^{-7}
<i>Cadmium Compounds</i>		BaS ₂ O ₃	1.6×10^{-5}
Cd ₃ (AsO ₄) ₂	2.2×10^{-33}	<i>Calcium Compounds</i>	
CdCO ₃	5.2×10^{-12}	Ca ₃ (AsO ₄) ₂	6.8×10^{-19}
Cd(CN) ₂	1.0×10^{-8}	CaCO ₃	2.8×10^{-9}
Cd ₂ [Fe(CN) ₆]	3.2×10^{-17}	CaCrO ₄	7.1×10^{-4}
Cd(OH) ₂ fresh	2.5×10^{-14}	CaC ₂ O ₄ • H ₂ O ³	4×10^{-9}
<i>Chromium Compounds</i>		CaF ₂	5.3×10^{-9}
CrAsO ₄	7.7×10^{-21}	Ca(OH) ₂	5.5×10^{-6}
Cr(OH) ₂	2×10^{-16}	CaHPO ₄	1×10^{-7}
Cr(OH) ₃	6.3×10^{-31}	Ca ₃ (PO ₄) ₂	2.0×10^{-29}
CrPO ₄ • 4H ₂ O green	2.4×10^{-23}	CaSeO ₄	8.1×10^{-4}
CrPO ₄ • 4H ₂ O violet	1.0×10^{-17}	CaSO ₄	9.1×10^{-6}
<i>Cobalt Compounds</i>		CaSO ₃	6.8×10^{-8}
Co ₃ (AsO ₄) ₂	7.6×10^{-29}	<i>Copper Compounds</i>	
CoCO ₃	1.4×10^{-13}	CuBr	5.3×10^{-9}
Co(OH) ₂ fresh	1.6×10^{-15}	CuCl	1.2×10^{-6}
Co(OH) ₃	1.6×10^{-44}	CuCN	3.2×10^{-20}
CoHPO ₄	2×10^{-7}	CuI	1.1×10^{-12}
CO ₃ (PO ₄) ₂	2×10^{-35}	CuOH	1×10^{-14}
<i>Gold Compounds</i>		CuSCN	4.8×10^{-15}
AuCl	2.0×10^{-13}	Cu ₃ (AsO ₄) ₂	7.6×10^{-36}
AuI	1.6×10^{-23}	CuCO ₃	1.4×10^{-10}
AuCl ₃	3.2×10^{-25}	Cu ₂ [Fe(CN) ₆]	1.3×10^{-16}
Au(OH) ₃	5.5×10^{-46}	Cu(OH) ₂	2.2×10^{-20}
AuI ₃	1×10^{-46}	Cu ₃ (PO ₄) ₂	1.3×10^{-37}

Substance	K_s	Substance	K_{sp}
<i>Iron Compounds</i>		<i>Lead Compounds</i>	
FeCO ₃	3.2×10^{-11}	Pb ₃ (AsO ₄) ₂	4.0×10^{-36}
Fe(OH) ₂	8.0×10^{-16}	PbBr ₂	4.0×10^{-5}
FeC ₂ O ₄ • 2H ₂ O ³	3.2×10^{-7}	PbCO ₃	7.4×10^{-14}
FeAsO ₄	5.7×10^{-21}	PbCl ₂	1.6×10^{-5}
Fe ₄ [Fe(CN) ₆] ₃	3.3×10^{-41}	PbCrO ₄	2.8×10^{-13}
Fe(OH) ₃	4×10^{-38}	PbF ₂	2.7×10^{-8}
FePO ₄	1.3×10^{-22}	Pb(OH) ₂	1.2×10^{-15}
<i>Magnesium Compounds</i>		PbI ₂	7.1×10^{-9}
Mg ₃ (AsO ₄) ₂	2.1×10^{-20}	PbC ₂ O ₄	4.8×10^{-10}
MgCO ₃	3.5×10^{-8}	PbHPO ₄	1.3×10^{-10}
MgCO ₃ • 3H ₂ O ³	2.1×10^{-5}	Pb ₃ (PO ₄) ₂	8.0×10^{-43}
MgC ₂ O ₄ • 2H ₂ O ³	1×10^{-8}	PbSeO ₄	1.4×10^{-7}
MgF ₂	6.5×10^{-9}	PbSO ₄	1.6×10^{-8}
Mg(OH) ₂	1.8×10^{-11}	Pb(SCN) ₂	2.0×10^{-5}
Mg ₃ (PO ₄) ₂	10^{-23} to 10^{-27}	<i>Manganese Compounds</i>	
MgSeO ₃	1.3×10^{-5}	Mn ₃ (AsO ₄) ₂	1.9×10^{-29}
MgSO ₃	3.2×10^{-3}	MnCO ₃	1.8×10^{-11}
MgNH ₄ PO ₄	2.5×10^{-13}	Mn ₂ [Fe(CN) ₆]	8.0×10^{-13}
<i>Mercury Compounds</i>		Mn(OH) ₂	1.9×10^{-13}
Hg ₂ Br ₂	5.6×10^{-23}	MnC ₂ O ₄ • 2H ₂ O ³	1.1×10^{-15}
Hg ₂ CO ₃	8.9×10^{-17}	<i>Nickel Compounds</i>	
Hg ₂ (CN) ₂	5×10^{-40}	Ni ₃ (AsO ₄) ₂	3.1×10^{-26}
Hg ₂ Cl ₂	1.3×10^{-18}	NiCO ₃	6.6×10^{-9}
Hg ₂ CrO ₄	2.0×10^{-9}	2 Ni(CN) ₂ → Ni ²⁺ + Ni(CN) ₄ ²⁰	1.7×10^{-9}
Hg ₂ (OH) ₂	2.0×10^{-24}	Ni ₂ [Fe(CN) ₆]	1.3×10^{-15}
Hg ₂ I ₂	4.5×10^{-29}	Ni(OH) ₂ fresh	2.0×10^{-15}
Hg ₂ SO ₄	7.4×10^{-7}	NiC ₂ O ₄	4×10^{-10}
Hg ₂ SO ₃	1.0×10^{-27}	Ni ₃ (PO ₄) ₂	5×10^{-31}
Hg(OH) ₂	3.0×10^{-26}	<i>Silver Compounds</i>	
<i>Strontium Compounds</i>		Ag ₃ AsO ₄	1.0×10^{-22}
Sr ₃ (AsO ₄) ₂	8.1×10^{-19}	AgBr	5.0×10^{-13}
SrCO ₃	1.1×10^{-10}	Ag ₂ CO ₃	8.1×10^{-12}
SrCrO ₄	2.2×10^{-5}	AgCl	1.8×10^{-10}

Substance	K_s	Substance	K_{sp}
$\text{SrC}_2\text{O}_4 \cdot \text{H}_2\text{O}^3$	1.6×10^{-7}	Ag_2CrO_4	1.1×10^{-12}
$\text{Sr}_3(\text{PO}_4)_2$	4.0×10^{-28}	AgCN	1.2×10^{-16}
SrSO_3	4×10^{-8}	$\text{Ag}_2\text{Cr}_2\text{O}_7$	2.0×10^{-7}
SrSO_4	3.2×10^{-7}	$\text{Ag}_4[\text{Fe}(\text{CN})_6]$	1.6×10^{-41}
<i>Tin Compounds</i>		AgOH	2.0×10^{-8}
$\text{Sn}(\text{OH})_2$	1.4×10^{-28}	AgI	8.3×10^{-17}
$\text{Sn}(\text{OH})_4$	1×10^{-56}	Ag_3PO_4	1.4×10^{-16}
<i>Zinc Compounds</i>		Ag_2SO_4	1.4×10^{-5}
$\text{Zn}_3(\text{AsO}_4)_2$	1.3×10^{-28}	Ag_2SO_3	1.5×10^{-14}
ZnCO_3	1.4×10^{-11}	AgSCN	1.0×10^{-12}
$\text{Zn}_2[\text{Fe}(\text{CN})_6]$	4.0×10^{-16}		
$\text{Zn}(\text{OH})_2$	1.2×10^{-17}		
ZnC_2O_4	2.7×10^{-8}		
$\text{Zn}_3(\text{PO}_4)_2$	9.0×10^{-33}		

1. Taken from Patnaik, Pradyot, Dean's Analytical Chemistry Handbook, 2nd ed., New York: McGraw-Hill, 2004, Table 4.2 (published on the Web by Knovel, <http://www.knovel.com>).

2. Taken from Meites, L. ed., Handbook of Analytical Chemistry, 1st ed., New York: McGraw-Hill, 1963.

3. Because $[\text{H}_2\text{O}]$ does not appear in equilibrium constants for equilibria in aqueous solution in general, it does not appear in the K_{sp} expressions for hydrated solids.

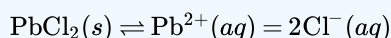
No metal sulfides are listed in this table because sulfide ion is such a strong base that the usual solubility product equilibrium equation does not apply. See Myers, R. J. *Journal of Chemical Education*, Vol. 63, 1986; pp. 687-690.

✓ Example 14.11.1: Equilibrium

When crystals of PbCl_2 are shaken with water at 25°C , it is found that 1.62×10^{-2} mol PbCl_2 dissolves per cubic decimeter of solution. Find the value of K_{sp} at this temperature.

Solution

We first write out the equation for the equilibrium:



so that

$$K_{sp}\text{PbCl}_2 = [\text{Pb}^{2+}][\text{Cl}^-]^2$$

Since 1.62×10^{-2} mol PbCl_2 dissolves per cubic decimeter, we have

$$[\text{Pb}^{2+}] = 1.62 \times 10^{-2} \text{ mol L}^{-1}$$

while

$$[\text{Cl}^-] = 2 \times 1.62 \times 10^{-2} \text{ mol L}^{-1}$$

since 2 mol Cl^- ions are produced for each mol PbCl_2 which dissolves. Thus

$$K_{sp} = (1.62 \times 10^{-2} \text{ mol L}^{-1})(2 \times 1.62 \times 10^{-2} \text{ mol L}^{-1})^2 \quad (14.11.6)$$

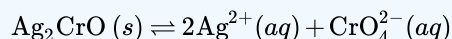
$$= 1.70 \times 10^{-5} \text{ mol}^3 \text{ L}^{-3} \quad (14.11.7)$$

✓ Example 14.11.2: Solubility

The solubility product of silver chromate, Ag_2CrO_4 , is $1.0 \times 10^{-12} \text{ mol}^3 \text{ L}^{-3}$. Find the solubility of this salt.

Solution

Again we start by writing the equation



from which

$$K_{sp}(\text{Ag}_2\text{CrO}_4) = [\text{Ag}^+]^2[\text{CrO}_4^{2-}] = 1.0 \times 10^{-12} \text{ mol}^3 \text{ L}^{-3}$$

Let the solubility be $x \text{ mol L}^{-1}$. Then

$$[\text{CrO}_4^{2-}] = x \text{ mol L}^{-1}$$

and

$$[\text{Ag}^+] = 2x \text{ mol L}^{-1}$$

Thus

$$K_{sp} = (2x \text{ mol L}^{-1})^2 x \text{ mol L}^{-1} \quad (14.11.8)$$

$$= (2x)^2 x \text{ mol}^3 \text{ L}^{-3} = 1.0 \times 10^{-12} \text{ mol}^3 \text{ L}^{-3} \quad (14.11.9)$$

or

$$4x^3 = 1.0 \times 10^{-12}$$

and

$$x^3 = \frac{1.0}{4} \times 10^{-12} = 2.5 \times 10^{-13} = 250 \times 10^{-15}$$

so that

$$x = \sqrt[3]{250} \times \sqrt[3]{10^{-15}} = 6.30 \times 10^{-5}$$

Thus the solubility is $6.30 \times 10^{-5} \text{ mol L}^{-1}$.

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