9.10: The Ideal Gas Equation

Earlier, we learned quantitatively the effects of pressure and thermodynamic temperature on gas volume. We also learned about the relation between volume and amount of substance. To recap what we've learned thus far, let's review each of the gas laws. Avogadro's law tells us that at constant *P* and *T*, the volume of a gas is directly proportional to the amount of gas. Boyle's law says that volume is inversely proportional to pressure and Charles' indicates that volume is directly proportional to temperature.

These laws ares simple, limited to relating 1 quantity to another. The video below shows a situation where 3 variables, pressure, volume, and amount of substance (moles) are all interrelated: inside our lungs.

As you can see in the video, when the pink balloon on the bottom (the "diaphragm") is pulled down, the balloon inside expands. This expansion causes a decrease in pressure (Boyle's Law). The pressure decrease causes a pressure differential, drawing air in through the straw, an increase in the amount of air (moles). So in your lungs, volume, pressure, and amount of air are all related. But none of the current laws explain the relation between 2 variables... How can this be resolved?

Solution: The Ideal Gas Law

These three laws may all be applied at once if we write:

$$V \propto n \times \frac{1}{P} \times T \tag{9.10.1}$$

or, introducing a constant of proportionality *R*,

$$V = R\,\frac{nT}{P} \tag{9.10.2}$$

Equation 9.10.2 applies to all gases at low pressures and high temperatures and is a very good approximation under nearly all conditions. The value of *R*, the **gas constant**, is independent of the kind of gas, the temperature, or the pressure. To calculate *R*, we rearrange Equation 9.10.2:

$$R = \frac{PV}{nT} \tag{9.10.3}$$

and substitute appropriate values of *P*, *V* , *n*, and *T*. From Table 9.1 we saw that the molar volumes of several gases at 0°C (273.15 K) and 1 atm (101.3 kPa) were close to 22.4 liters (22.4 dm^3). Substituting into Equation 9.10.3,

$$R = \frac{1\text{ atm } \times \ 22.4\text{ liters}}{1\text{ mol } \times \ 273.15\text{ K}} = 0.0820\,\frac{\text{liter atm}}{\text{mol K}}$$

If we use SI units for pressure and volume, as well as for amount of substance and temperature,

$$R = \frac{101.3\text{ kPa } \times \ 22.4\text{ dm}^3}{1\text{ mol } \times \ 273.15\text{ K}} = 8.31\,\frac{\text{kPa dm}^3}{\text{mol K}} = 8.31\text{ J mol}^{-1}\text{ K}^{-1}$$

Thus the gas constant has units of energy divided by amount of substance and thermodynamic temperature.

Equation 9.10.2 is usually rearranged by multiplying both sides by P, so that it reads:

$$PV = nRT \tag{9.10.4}$$

This is called the ideal gas equation or the ideal gas law. With the ideal gas equation we can convert from volume of a gas to amount of substance (provided that P and T are known). This is very useful since the volume, pressure, and temperature of a gas are easier to measure than mass, and because knowledge of the molar mass is unnecessary.

Example 9.10.1 : Amount of Gas

Calculate the amount of gas in a 100-cm³ sample at a temperature of 300 K and a pressure of 750 mmHg.

Solution

Either of the two values of the gas constant may be used, so long as the units P, V, and T are adjusted properly. Using R = 0.0820 liter atm mol^{-1} K^{-1},

$$V = 100 \text{ cm}^3 \times \frac{1 \text{ liter}}{1000 \text{ cm}^3} = 0.100 \text{ liter}$$

$$P = 750 \text{ mmHg} \times \frac{1 \text{ atm}}{760 \text{ mmHg}} = 0.987 \text{ atm}$$

$$T = 300 \text{ K}$$

Rearranging Equation 9.10.4) and substituting,

$$n = \frac{PV}{RT} = \frac{0.987 \text{ atm} \times 0.100 \text{ liter}}{0.0820 \text{ liter atm mol}^{-1} \text{ K}^{-1} \times 300 \text{ K}} = 4.01 \times 10^{-3} \text{ mol}$$

Essentially the same calculations are required when SI units are used:

$$V = 100 \text{ cm}^3 \times \left(\frac{1 \text{ dm}}{10 \text{ cm}}\right)^3 = 0.100 \text{ dm}^3$$

$$P = 750 \text{ mmHg} \times \frac{101.3 \text{ kPa}}{760 \text{ mmHg}} = 100.0 \text{ kPa}$$

$$T = 300 \text{ K}$$

$$n = \frac{PV}{RT} = \frac{100.0 \text{ kPa} \times 0.100 \text{ dm}^3}{8.31 \text{ J mol}^{-1} \text{ K}^{-1} \times 300\text{K}} = 4.01 \times 10^{-3} \text{ mol}$$

Note

Note: Since 1 kPa dm^3 = 1 J, the units cancel as shown. For this reason it is convenient to use cubic decimeters as the unit of volume when kilopascals are used as the unit of pressure in the ideal gas equation.

The ideal gas law enables us to find the amount of substance, provided we can measure V, P, and T. If we can also determine the mass of a gas, it is possible to calculate molar mass (and molecular weight). One way to do this is vaporize a volatile liquid (one which has a low boiling temperature) so that it fills a flask of known volume. When the flask is cooled, the vapor condenses to a liquid and can easily be weighed.

Example 9.10.1 : Molar Mass

The empirical formula of benzene is CH. When heated to 100°C in a flask whose volume was 247.2 ml, a sample of benzene vaporized and drove all air from the flask. When the benzene was condensed to a liquid, its mass was found to be 0.616 g. The barometric pressure was 742 mmHg. Calculate (a) the molar mass and (b) the molecular formula of benzene.

Solution

a) Molar mass is mass divided by amount of substance. The latter quantity can be obtained from the volume, temperature, and pressure of benzene vapor:

$$V = 247.2\ \text{cm}^3 \times \frac{1\ \text{liter}}{10^3\ \text{cm}^3} = 0.2472\ \text{liter}$$

$$T = (273.15 + 100)\ \text{K} = 373\ \text{K}$$

$$P = 742\ \text{mmHg} \times \frac{1.00\ \text{atm}}{760\ \text{mmHg}} = 0.976\ \text{atm}$$

$$n = \frac{PV}{RT} = \frac{0.976\ \text{atm} \times 0.2472\ \text{liter}}{0.0820\ \text{liter atm mol}^{-1}\ \text{K}^{-1} \times 373\ \text{K}} = 7.89 \times 10^{-3}\ \text{mol}$$

The molar mass is:

$$M = \frac{m}{n} = \frac{0.616\ \text{g}}{7.89 \times 10^{-3}\ \text{mol}} = 78.1\ \text{g mol}^{-1}$$

b) The empirical formula CH would imply a molar mass of (12.01 + 1.008) g mol^{-1}, or 13.02 g mol^{-1}. The experimentally determined molar mass is 6 times larger:

$$\frac{78.1\ \text{g mol}^{-1}}{13.02\ \text{g mol}^{-1}} = 6.00$$

and so the molecular formula must be C_6H_6.