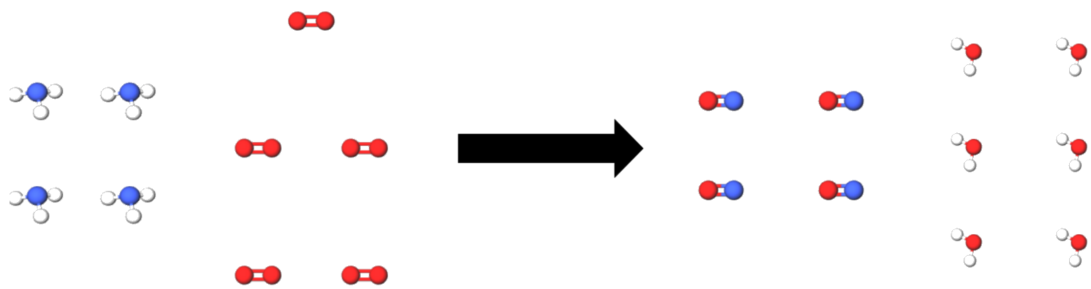
3.2: Equations and Mass Relationships

Consider the balanced chemical equation (i.e., catalytic oxidation of ammonia) such as

$$4\,NH_3(g) + 5O_2(g) \rightarrow 4\,NO(g) + 6\,H_2O(g) \tag{3.2.1}$$

not only tells how many molecules of each kind are involved in a reaction, it also indicates the *amount* of each substance that is involved. Equation 3.2.1 (represented molecularly by the image below it) says that 4 NH_3 *molecules* can react with 5 O_2 *molecules* to give 4 NO *molecules* and 6 H_2O *molecules*. It also says that 4 *mol* NH_3 would react with 5 *mol* O_2 yielding 4 *mol* NO and 6 *mol* H_2O.



A chemical equation is expressed in terms of its atomic view. 4 sets of blue spheres bonded to three white spheres reacts with 5 sets of double bonded red spheres to give 4 sets of double bonded red and blue spheres as well as 6 sets of red sphere bonded to two white spheres.

The balanced equation does more than this, though. It also tells us that $2 \cdot 4 = 8\text{mol NH}_3$ will react with $2 \cdot 5 = 10\text{mol O}_2$, and that $\frac{1}{2} \cdot 4 = 2\text{mol NH}_3$ requires only $\frac{1}{2} \cdot 5 = 2.5\text{mol O}_2$. In other words, the equation indicates that exactly 5 mol O_2 must react *for every* 4 mol NH_3 consumed. For the purpose of calculating how much O_2 is required to react with a certain amount of NH_3 therefore, the significant information contained in Equation 3.2.1 is the *ratio*

$$\frac{5\text{ mol O}_2}{4\text{ mol NH}_3} \tag{3.2.2}$$

We shall call such a ratio derived from a balanced chemical equation a **stoichiometric ratio** and give it the symbol *S*. Thus, for Equation 3.2.1,

$$\text{S}\left(\frac{\text{O}_2}{\text{NH}_3}\right) = \frac{5\text{ mol O}_2}{4\text{ mol NH}_3} \tag{3.2.3}$$

The word *stoichiometric* comes from the Greek words *stoicheion*, “element,“ and *metron*, “measure.“ Hence the stoichiometric ratio measures one element (or compound) against another.

Example 3.2.1: Stoichiometric Ratios

Derive all possible stoichiometric ratios from Equation 3.2.1.

Solution

Any ratio of amounts of substance given by coefficients in the equation may be used:

$$\text{S}\left(\frac{\text{NH}_3}{\text{O}_2}\right) = \frac{4\text{ mol NH}_3}{5\text{ mol O}_2} \qquad \text{S}\left(\frac{\text{O}_2}{\text{NO}}\right) = \frac{5\text{ mol O}_2}{4\text{ mol NO}}$$

$$\text{S}\left(\frac{\text{NH}_3}{\text{NO}}\right) = \frac{4\text{ mol NH}_3}{4\text{ mol NO}} \qquad \text{S}\left(\frac{\text{O}_2}{\text{H}_2\text{O}}\right) = \frac{5\text{ mol O}_2}{6\text{ mol H}_2\text{O}}$$

$$\text{S}\left(\frac{\text{NH}_3}{\text{H}_2\text{O}}\right) = \frac{4\text{ mol NH}_3}{6\text{ mol H}_2\text{O}} \qquad \text{S}\left(\frac{\text{NO}}{\text{H}_2\text{O}}\right) = \frac{4\text{ mol NO}}{6\text{ mol H}_2\text{O}}$$

There are six more stoichiometric ratios, each of which is the reciprocal of one of these. [Equation 3.2.3 gives one of them.]

When any chemical reaction occurs, the amounts of substances consumed or produced are related by the appropriate stoichiometric ratios. Using Equation 3.2.2 as an example, this means that the ratio of the amount of O_2 consumed to the amount of NH_3 consumed must be the stoichiometric ratio S(O_2/NH_3):

$$\frac{n_{\text{O}_2\text{ consumed}}}{n_{\text{NH}_3\text{ consumed}}} = \text{S}\left(\frac{\text{O}_2}{\text{NH}_3}\right) = \frac{5\text{ mol O}_2}{4\text{ mol NH}_3} \tag{3.2.4}$$

Similarly, the ratio of the amount of H_2O produced to the amount of NH_3 consumed must be S(H_2O/NH_3):

$$\frac{n_{\text{H}_2\text{O produced}}}{n_{\text{NH}_3\text{ consumed}}} = \text{S}\left(\frac{\text{H}_2\text{O}}{\text{NH}_3}\right) = \frac{6\text{ mol H}_2\text{O}}{4\text{ mol NH}_3} \tag{3.2.5}$$

In general we can say that

$$\text{Stoichiometric ratio}\left(\frac{\text{X}}{\text{Y}}\right) = \frac{\text{amount of X consumed or produced}}{\text{amount of Y consumed or produced}} \tag{3.2.6}$$

or, in symbols,

$$\text{S}\left(\frac{\text{X}}{\text{Y}}\right) = \frac{n_{\text{X consumed or produced}}}{n_{\text{Y consumed or produced}}} \tag{3.2.7}$$

Note that in the word Equation 3.2.6 and the symbolic Equation 3.2.7, X and Y may represent *any* reactant or *any* product in the balanced chemical equation from which the stoichiometric ratio was derived. No matter how much of each reactant we have, the amounts of reactants *consumed* and the amounts of products *produced* will be in appropriate stoichiometric ratios.

Example 3.2.2: Ratio of Water

Find the amount of water produced when 3.68 mol NH_3 is consumed according to Equation 3.2.5.

Solution

The amount of water produced must be in the stoichiometric ratio S(H_2O/NH_3) to the amount of ammonia consumed:

$$S\left(\frac{H_2O}{NH_3}\right) = \frac{n_{H_2O\text{ produced}}}{n_{NH_3\text{ consumed}}}$$

Multiplying both sides n_{NH3} consumed, by we have

$$n_{H_2O\text{ produced}} = n_{NH_3\text{ consumed}} \cdot S\left(\frac{H_2O}{NH_3}\right) \tag{3.2.8}$$

$$\tag{3.2.9}$$

$$= 3.68\text{ mol }NH_3 \cdot \frac{6\text{ mol }H_2O}{4\text{ mol }NH_3} \tag{3.2.10}$$

$$= 5.52\text{ mol }H_2O \tag{3.2.11}$$

This is a typical illustration of the use of a stoichiometric ratio as a conversion factor. Example 3.2.2 is analogous to Examples 1 and 2 from Conversion Factors and Functions, where density was employed as a conversion factor between mass and volume. Example 3.2.2 is also analogous to Examples 2.4 and 2.6, in which the Avogadro constant and molar mass were used as conversion factors. As in these previous cases, there is no need to memorize or do algebraic manipulations with Equation 3.2.4 when using the stoichiometric ratio. Simply remember that the coefficients in a balanced chemical equation give stoichiometric ratios, and that the proper choice results in cancellation of units. In road-map form

$$\text{amount of X consumed or produced} \overset{\substack{\text{stoichiometric} \\ \text{ratio X/Y}}}{\longleftrightarrow} \text{amount of Y consumed or produced}$$

or symbolically.

$$n_{\text{X consumed or produced}} \overset{S(X/Y)}{\longleftrightarrow} n_{\text{Y consumed or produced}}$$

When using stoichiometric ratios, be sure you *always* indicate moles *of what*. You can only cancel moles of the same substance. In other words, 1 mol NH_3 cancels 1 mol NH_3 but does not cancel 1 mol H_2O.

The next example shows that stoichiometric ratios are also useful in problems involving the mass of a reactant or product.

Example 3.2.3: Mass Produced

Calculate the mass of sulfur dioxide (SO_2) produced when 3.84 mol O_2 is reacted with FeS_2 according to the equation

$$4\,FeS_2 + 11\,O_2 \longrightarrow 2\,Fe_2O_3 + 8\,SO_2$$

Solution

The problem asks that we calculate the mass of SO_2 produced. As we learned in Example 2 of The Molar Mass, the molar mass can be used to convert from the amount of SO_2 to the mass of SO_2. Therefore this problem in effect is asking that we calculate the amount of SO_2 produced from the amount of O_2 consumed. This is the same problem as in Example 2. It requires the stoichiometric ratio:

$$S\left(\frac{SO_2}{O_2}\right) = \frac{8\text{ mol }SO_2}{11\text{ mol }O_2}$$

The *amount* of SO_2 produced is then:

$$\begin{aligned} n_{SO_2\text{ produced}} &= n_{O_2\text{ consumed}} \cdot \text{conversion factor} \\ &= 3.84\text{ mol }O_2 \cdot \frac{8\text{ mol }SO_2}{11\text{ mol }O_2} \\ &= 2.79\text{ mol }SO_2 \end{aligned}$$

The *mass* of SO_2 is:

$$\begin{aligned} m_{SO_2} &= 2.79\text{ mol }SO_2 \cdot \frac{64.06\text{ g }SO_2}{1\text{ mol }SO_2} \\ &= 179\text{ g }SO_2 \end{aligned}$$

With practice this kind of problem can be solved in one step by concentrating on the units. The appropriate stoichiometric ratio will convert moles of O_2 to moles of SO_2 and the molar mass will convert moles of SO_2 to grams of SO_2. A schematic road map for the one-step calculation can be written as:

$$n_{O_2} \xrightarrow{S(SO_2/O_2)} n_{SO_2} \xrightarrow{M_{SO_2}} m_{SO_2}$$

Thus:

$$m_{SO_2} = 3.84\text{ mol }O_2 \cdot \frac{8\text{ mol }SO_2}{11\text{ mol }O_2} \cdot \frac{64.06\text{ g}}{1\text{ mol }SO_2} = 179\text{ g}$$

These calculations can be organized as a table, with entries below the respective reactants and products in the chemical equation. You may verify the additional calculations.

Calculations

	$4\,FeS_2$	$+11\,O_2$	$\rightarrow 2Fe_2O_3$	$+8SO_2$
m (g)	168	123	111	179
M (g/mol)	120.0	32.0	159.7	64.06
n (mol)	1.40	3.84	0.698	2.79

The chemical reaction in this example is of environmental interest. Iron pyrite (FeS_2) is often an impurity in coal, and so burning this fuel in a power plant produces sulfur dioxide (SO_2), a major air pollutant. Our next example also involves burning a fuel and its effect on the atmosphere.

Example 3.2.4: Mass of Oxygen

What mass of oxygen would be consumed when 3.3×10^{15} g, 3.3 Pg (petagrams), of octane (C_8H_{18}) is burned to produce CO_2 and H_2O?

Solution

First, write a balanced equation

$$2\,C_8H_{18} + 25\,O_2 \longrightarrow 16\,CO_2 + 18\,H_2O$$

The problem gives the mass of C_8H_{18} burned and asks for the mass of O_2 required to combine with it. Thinking the problem through before trying to solve it, we realize that the molar mass of octane could be used to calculate the amount of octane consumed. Then we need a stoichiometric ratio to get the amount of O_2 consumed. Finally, the molar mass of O_2 permits calculation of the mass of O_2. Symbolically

$$m_{\mathrm{C_8H_{18}}} \xrightarrow{M_{\mathrm{C_8H_{18}}}} n_{\mathrm{C_8H_{18}}} \xrightarrow{S(\mathrm{SO_2/C_8H_{18}})} n_{\mathrm{O_2}} \xrightarrow{M_{\mathrm{O_2}}} m_{\mathrm{O_2}}$$

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\begin{align} m_{\text{O}_{\text{2}}} & =\text{3}\text{.3 }\cdot\text{ 10}^{\text{15}}\text{ g }\cdot\text{ }\frac{\text{1 mol C}_{\text{8}}\text{H}_{\text{18}}}{\text{114 g}}\text{ }\cdot\text{ }\frac{\text{25 mol
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Thus 12 Pg (petagrams) of O_2 would be needed.

The large mass of oxygen obtained in this example is an estimate of how much O_2 is removed from the earth's atmosphere each year by human activities. Octane, a component of gasoline, was chosen to represent coal, gas, and other fossil fuels. Fortunately, the total mass of oxygen in the air (1.2×10^{21} g) is much larger than the yearly consumption. If we were to go on burning fuel at the present rate, it would take about 100 000 years to use up all the O_2. Actually we will consume the fossil fuels long before that! One of the least of our environmental worries is running out of atmospheric oxygen.