1.3: Dimensional Analysis

Sometimes you can catch an error in the form of an equation or expression, or in the dimensions of a quantity used for a calculation, by checking for dimensional consistency. Here are some rules that must be satisfied:

- In this e-book the *differential* of a function, such as $\mathrm{d}f$, refers to an *infinitesimal* quantity. If one side of an equation is an infinitesimal quantity, the other side must also be. Thus, the equation $\mathrm{d}f = a\,\mathrm{d}x + b\,\mathrm{d}y$ (where ax and by have the same dimensions as f) makes mathematical sense, but $\mathrm{d}f = ax + b\,\mathrm{d}y$ does not.

  Derivatives, partial derivatives, and integrals have dimensions that we must take into account when determining the overall dimensions of an expression that includes them. For instance:

  - Some examples of applying these principles are given here using symbols described in Sec. 1.2.

    Example 1. Since the gas constant R may be expressed in units of J K^{-1} mol^{-1}, it has dimensions of energy divided by thermodynamic temperature and amount. Thus, RT has dimensions of energy divided by amount, and nRT has dimensions of energy. The products RT and nRT appear frequently in thermodynamic expressions.

    Example 3. Find the dimensions of the constants a and b in the van der Waals equation

    $$p = \frac{nRT}{V - nb} - \frac{n^2 a}{V^2} \tag{1.3.1}$$

    Dimensional analysis tells us that, because nb is subtracted from V, nb has dimensions of volume and therefore b has dimensions of volume/amount. Furthermore, since the right side of the equation is a difference of two terms, these terms have the same dimensions as the left side, which is pressure. Therefore, the second term n^2a/V^2 has dimensions of pressure, and a has dimensions of pressure $\times$ volume^2 $\times$ amount^{-2}.

    Example 4. Consider an equation of the form

    $$\left(\frac{\partial \ln x}{\partial T}\right)_p = \frac{y}{R} \tag{1.3.2}$$

    What are the SI units of y? $\ln x$ is dimensionless, so the left side of the equation has the dimensions of $1/T$, and its SI units are K^{-1}. The SI units of the right side are therefore also K^{-1}. Since R has the units J K^{-1} mol^{-1}, the SI units of y are J K^{-2} mol^{-1}.