1.4: Phase Space

We construct a Cartesian space in which each of the $6N$ coordinates and momenta is assigned to one of $6N$ mutually orthogonal axes. Phase space is, therefore, a $6N$ dimensional space. A point in this space is specified by giving a particular set of values for the $6N$ coordinates and momenta. Denote such a point by

$$x = (p_1, \cdots, p_N, r_1, \cdots, r_N)$$

x is a $6N$ dimensional vector. Thus, the time evolution or trajectory of a system as specified by Hamilton's equations of motion, can be expressed by giving the phase space vector, x as a function of time.

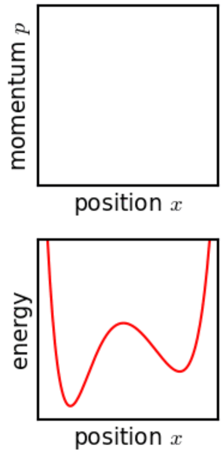

Figure 1.4.1: Evolution of an ensemble of classical systems in phase space (top). The systems are a massive particle in a one-dimensional potential well (red curve, lower figure). The initially compact ensemble becomes swirled up over time.

The law of conservation of energy, expressed as a condition on the phase space vector:

$$H(x(t)) = \text{const} = E$$

defines a $6N-1$ dimensional hypersurface in phase space on which the trajectory must remain.