# 9.6: The Physical State of a Quantum System

The physical state of a quantum system is represented by a vector denoted $|\Psi(t)\rangle$ which is a column vector, whose components are *probability amplitudes* for different states in which the system might be found if a measurement were made on it.

A probability amplitude $\alpha$ is a complex number, the square modulus of which gives the corresponding probability $P_\alpha$

$$P_\alpha = |\alpha|^2$$

The number of components of $|\Psi(t)\rangle$ is equal to the number of possible states in which the system might observed. The space that contains $|\Psi(t)\rangle$ is called a **Hilbert space** $\mathcal{H}$. The dimension of $\mathcal{H}$ is also equal $|\Psi(t)\rangle$ to the number of states in which the system might be observed. It could be finite or infinite (countable or not). $|\Psi(t)\rangle$ must be a unit vector. This means that the inner product:

$$\langle\Psi(t)|\Psi(t)\rangle = 1$$

In the above, if the vector $|\Psi(t)\rangle$, known as a Dirac "ket" vector, is given by the column

$$|\Psi(t)\rangle = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

then the vector $\langle\Psi(t)|$, known as a Dirac "bra" vector, is given by

$$\langle\Psi(t)| = (\psi_1^* \;\; \psi_2^* \;\; \cdots)$$

so that the inner product becomes

$$\langle\Psi(t)|\Psi(t)\rangle = \sum_i |\psi_i|^2 = 1$$

We can understand the meaning of this by noting that $\psi_i$, the components of the state vector, are probability amplitudes, and $|\psi_i|^2$ are the corresponding probabilities. The above condition then implies that the sum of all the probabilities of being in the various possible states is 1, which we know must be true for probabilities.