13.1.1: The Hamiltonian

Consider a quantum system with a Hamiltonian H_0. Suppose this system is subject to an external driving force $F_e(t)$ such that the full Hamiltonian takes the form

$$H = H_0 - BF_e(t) = H_0 + H'$$

where B is an operator through which this coupling occurs. This is the situation, for example, when the infrared spectrum is measured experimentally - the external force $F_e(t)$ is identified with an electric field $E(t)$ and B is identified with the electric dipole moment operator. If the field $F_e(t)$ is inhomogeneous, then H takes the more general form

$$\begin{aligned} H &= H_0 - \int d^3x\ B(\mathbf{x})F_e(\mathbf{x},t) \\ &= H_0 - \sum_{\mathbf{k}} B_{\mathbf{k}} F_{e,\mathbf{k}}(t) \end{aligned}$$

where the sum is taken over Fourier modes. Often, B is an operator such that, if $F_e(t) = 0$, then

$$\langle B\rangle = \frac{\mathrm{Tr}\left(Be^{-\beta H}\right)}{\mathrm{Tr}\left(e^{-\beta H}\right)}$$

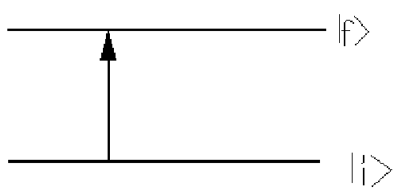
Suppose we take $F_e(t)$ to be a *monochromatic* field of the form

$$F_e(t) = F_\omega e^{i\omega t}$$

Generally, the external field can induce transitions between eigenstates of H_0 in the system. Consider such a transition between an initial state $|i\rangle$ and a final state $|f\rangle$, with energies E_i and E_f, respectively:

$$\begin{aligned} H_0|i\rangle &= E_i|i\rangle \\ H_0|f\rangle &= E_f|f\rangle \end{aligned}$$

(see figure below).



This transition can only occur if

$$E_f = E_i + \hbar\omega$$