10.5: The quantum equilibrium ensembles

At equilibrium, the density operator does not evolve in time; thus, $\frac{\partial \rho}{\partial t} = 0$. Thus, from the equation of motion, if this holds, then $[H, \rho] = 0$, and $\rho(t)$ is a constant of the motion. This means that it can be simultaneously diagonalized with the Hamiltonian and can be expressed as a pure function of the Hamiltonian

$$\rho = f(H)$$

Therefore, the eigenstates of ρ, the vectors, we called $|w_k\rangle$ are the eigenvectors $|E_i\rangle$ of the Hamiltonian, and we can write H and ρ as

$$H = \sum_i E_i |E_i\rangle\langle E_i|$$

$$\rho = \sum_i f(E_i) |E_i\rangle\langle E_i|$$

The choice of the function f determines the ensemble.