4.3: Relation between Canonical and Microcanonical Ensembles

We saw that the $E(N,V,S)$ and $A(N,V,T)$ could be related by a Legendre transformation. The partition functions $\Omega(N,V,E)$ and $Q(N,V,T)$ can be related by a Laplace transform. Recall that the Laplace transform $\tilde{f}(\lambda)$ of a function $f(x)$ is given by

$$\tilde{f}(\lambda) = \int_0^\infty dx e^{-\lambda x} f(x)$$

Let us compute the Laplace transform of $\Omega(N,V,E)$ with respect to E:

$$\tilde{\Omega}(N,V,\lambda) = C_N \int_0^\infty dE e^{-\lambda E} \int dx \delta(H(x) - E)$$

Using the δ-function to do the integral over E:

$$\tilde{\Omega}(N,V,\lambda) = C_N \int dx e^{-\lambda H(x)}$$

By identifying $\lambda = \beta$, we see that the Laplace transform of the microcanonical partition function gives the canonical partition function $Q(N,V,T)$.