7.4: General Correlation Functions

A general correlation function can be defined in terms of the probability distribution function $p^{(n)}(r_1, \cdots, r_n)$ according to

$$\begin{aligned} g^{(n)}(r_1, \cdots, r_n) &= \frac{1}{p^n} p^{(n)}(r_1, \cdots, r_n) \\ &= \frac{V^n N!}{Z_N N^n (N-n)!} \int dr_{n+1} \cdots dr_N e^{-\beta U(r_1, \cdots, r_N)} \end{aligned}$$

Another useful way to write the correlation function is

$$\begin{aligned} g^{(n)}(r_1, \cdots, r_n) &= \frac{V^n N!}{Z_N N^n (N-n)!} \int dr'_1 \cdots dr'_N e^{-\beta U(r_1, \cdots, r_N)} \delta(r_1 - r'_1) \cdots \delta(r_n - r'_n) \\ &= \frac{V^n N!}{Z_N N^n (N-n)!} \left\langle \Pi_{i=1}^n \delta(r_i - r'_i) \right\rangle_{r'_1, \cdots, r'_N} \end{aligned}$$

i.e., the general *n*-particle correlation function can be expressed as an ensemble average of the product of δ -functions, with the integration being taken over the variables $r'_1, \cdots, r'_N$.