14.2: The Random Force Term

Within the context of a harmonic bath, the term "random force" is something of a misnomer, since $R(t)$ is completely deterministic and not random at all!!! We will return to this point momentarily, however, let us examine particular features of $R(t)$ from its explicit expression from the harmonic bath dynamics. Note, first of all, that it does not depend on the dynamics of the system coordinate q (except for the appearance of $q(0)$). In this sense, it is independent or "orthogonal" to q within a phase space picture. From the explicit form of $R(t)$, it is straightforward to see that the correlation function

$$\langle \dot{q}(0)R(t)\rangle = 0$$

i.e., the correlation function of the system velocity $\dot{q}$ with the random force is 0. This can be seen by substituting in the expression for $R(t)$ and integrating over initial conditions with a canonical distribution weighting. For certain potentials $\phi(q)$ that are even in q (such as a harmonic oscillator), one can also show that

$$\langle q(0)R(t)\rangle = 0$$

Thus, $R(t)$ is completely uncorrelated from both q and $\dot{q}$, which is a property we might expect from a truly random process. In fact, $R(t)$ is determined by the detailed dynamics of the bath. However, we are not particularly interested or able to follow these detailed dynamics for a large number of bath degrees of freedom. Thus, we could just as well model $R(t)$ by a completely random process (satisfying certain desirable features that are characteristic of a more general bath), and, in fact, this is often done. One could, for example, postulate that $R(t)$ act over a maximum time t_{max} at discrete points in time $k\Delta t$, giving $N = t_{\max}/\Delta t$ values of $R_k = R(k\Delta t)$, and assume that R_k takes the form of a *gaussian random process*:

$$R_k = \sum_{j=1}^{N}\left[a_j e^{2\pi ijk/N} + b_j e^{-2\pi ijk/N}\right]$$

where the coefficients $\{a_j\}$ and $\{b_j\}$ are chosen at random from a gaussian distribution function. This might be expected to be suitable for a bath of high density, where strong collisions between the system and a bath particle are essentially nonexistent, but where the system only sees feels the relatively "soft" fluctuations of the less mobile bath. For a low density bath, one might try modeling $R(t)$ as a Poisson process of very strong collisions.

Whatever model is chosen for $R(t)$, if it is a truly random process that can only act at discrete points in time, then the GLE takes the form of a stochastic (based on random numbers) integro-differential equation. There is a whole body of mathematics devoted to the properties of such equations, where heavy use of an c*alculus* is made.