

### 4.3: Relation between Canonical and Microcanonical Ensembles

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We saw that the  $E(N, V, S)$  and  $A(N, V, T)$  could be related by a Legendre transformation. The partition functions  $\Omega(N, V, E)$  and  $Q(N, V, T)$  can be related by a Laplace transform. Recall that the Laplace transform  $\tilde{f}(\lambda)$  of a function  $f(x)$  is given by

$$\tilde{f}(\lambda) = \int_0^{\infty} dx e^{-\lambda x} f(x)$$

Let us compute the Laplace transform of  $\Omega(N, V, E)$  with respect to  $E$ :

$$\tilde{\Omega}(N, V, \lambda) = C_N \int_0^{\infty} dE e^{-\lambda E} \int dx \delta(H(x) - E)$$

Using the  $\delta$ -function to do the integral over  $E$ :

$$\tilde{\Omega}(N, V, \lambda) = C_N \int dx e^{-\lambda H(x)}$$

By identifying  $\lambda = \beta$ , we see that the Laplace transform of the microcanonical partition function gives the canonical partition function  $Q(N, V, T)$ .

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