1.2.4: Scientific Dimensional Analysis

Conversion Factors

Many quantities can be expressed in several different ways. The English system measurement of 4 cups is also equal to 2 pints, 1 quart, and $\frac{1}{4}$ of a gallon.

$$4 \text{ cups} = 2 \text{ pints} = 1 \text{ quart} = 0.25 \text{ gallon}$$

Notice that the numerical component of each quantity is different, while the actual amount of material that it represents is the same. This is because the units are different. We can establish the same set of equalities for the metric system:

$$1 \text{ meter} = 10 \text{ decimeters} = 100 \text{ centimeters} = 1000 \text{ millimeters}$$

The metric system's use of powers of 10 for all conversions makes this quite simple.

Whenever two quantities are equal, a ratio can be written that is numerically equal to 1. Using the metric examples above:

$$\frac{1 \text{ m}}{100 \text{ cm}} = \frac{100 \text{ cm}}{100 \text{ cm}} = \frac{1 \text{ m}}{1 \text{ m}} = 1$$

The $\frac{1 \text{ m}}{100 \text{ cm}}$ is called a **conversion factor**. A conversion factor is a ratio of equivalent measurements. Because both 1 m and 100 cm represent the exact same length, the value of the conversion factor is 1. The conversion factor is read as "1 meter per 100 centimeters". Other conversion factors from the cup measurement example can be:

$$\frac{4 \text{ cups}}{2 \text{ pints}} = \frac{2 \text{ pints}}{1 \text{ quart}} = \frac{1 \text{ quart}}{0.25 \text{ gallon}} = 1$$

Since the numerator and denominator represent equal quantities in each case, all are valid conversion factors.

Scientific Dimensional Analysis

Conversion factors are used in solving problems in which a certain measurement must be expressed with different units. When a given measurement is multiplied by an appropriate conversion factor, the numerical value changes, but the actual size of the quantity measured remains the same. **Dimensional analysis** is a technique that uses the units (dimensions) of the measurement in order to correctly solve problems. Dimensional analysis is best illustrated with an example.

Example 1.2.4.1

How many seconds are in a day?

Solution

Step 1: List the known quantities and plan the problem.

Known

- 1 day = 24 hours
- 1 hour = 60 minutes
- 1 minute = 60 seconds

Unknown

The known quantities above represent the conversion factors that we will use. The first conversion factor will have day in the denominator so that the "day" unit will cancel. The second conversion factor will then have hours in the denominator, while the third conversion factor will have minutes in the denominator. As a result, the unit of the last numerator will be seconds and that will be the units for the answer.

Step 2: Calculate.

$$1 \text{ d} \times \frac{24 \text{ hr}}{1 \text{ d}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{60 \text{ s}}{1 \text{ min}} = 86,400 \text{ s}$$

Applying the first conversion factor, the "d" unit cancels and $1 \times 24 = 24$. Applying the second conversion factor, the "hr" unit cancels and $24 \times 60 = 1440$. Applying the third conversion factor, the "min" unit cancels and $1440 \times 60 = 86,400$. The

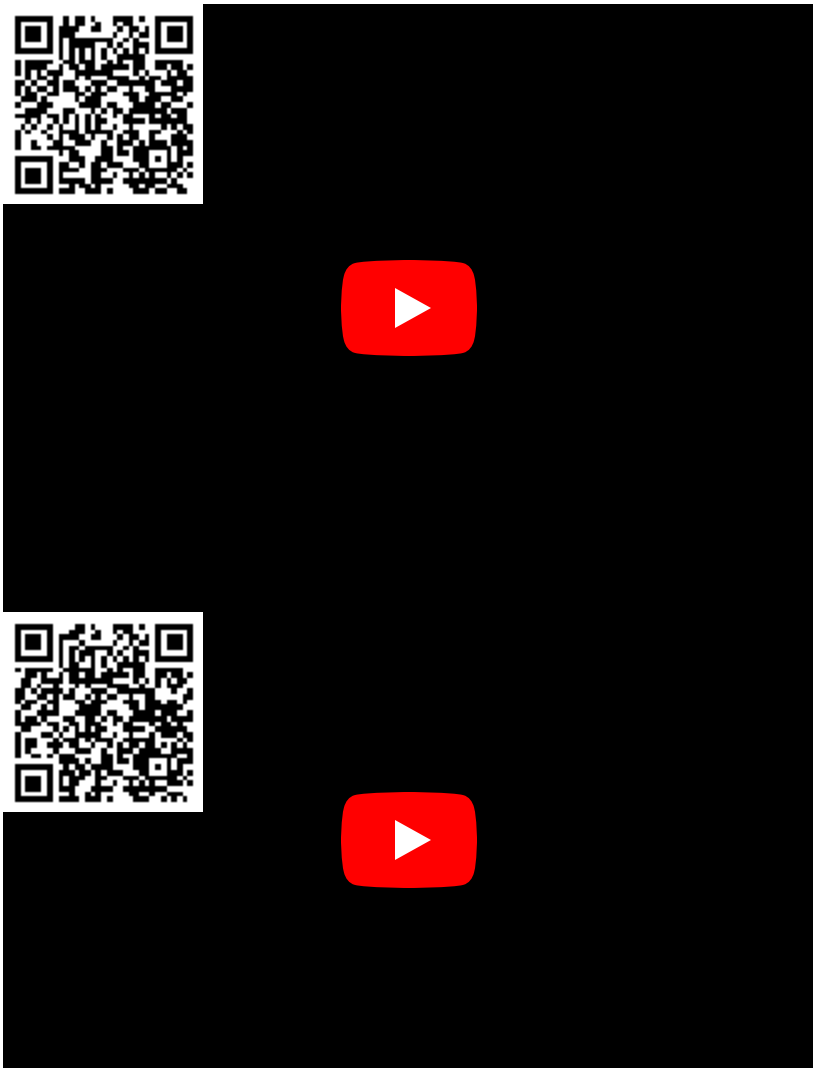
unit that remains is "s" for seconds.

Step 3: Think about your result.

Seconds is a much smaller unit of time than days, so it makes sense that there are a very large number of seconds in one day.

Summary

- A conversion factor is a ratio of equivalent measurements.
- Dimensional analysis is a technique that uses the units (dimensions) of the measurement to solve problems.



Review

1. What is a conversion factor?
2. What is dimensional analysis?
3. How many meters are in 3.7 km?
4. How many kg in 12980 g?

Explore More

Use the link below to answer the following questions**:** http://www.felderbooks.com/papers/units.html

1. What do we always need to express measurements correctly?
2. What does dimensional analysis tell you?
3. How do you know that you have set the problem up incorrectly?

4. How do you know that you have set the problem up correctly?
