1.69: The Wigner Distribution Function for the Harmonic Oscillator

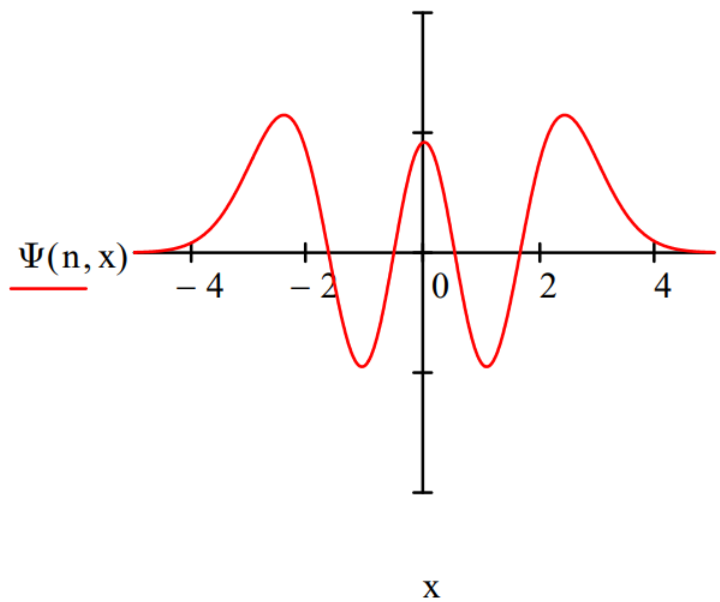
Given the quantum number this Mathcad file calculates the Wigner distribution function for the specified harmonic oscillator eigenstate using the coordinate wave function.

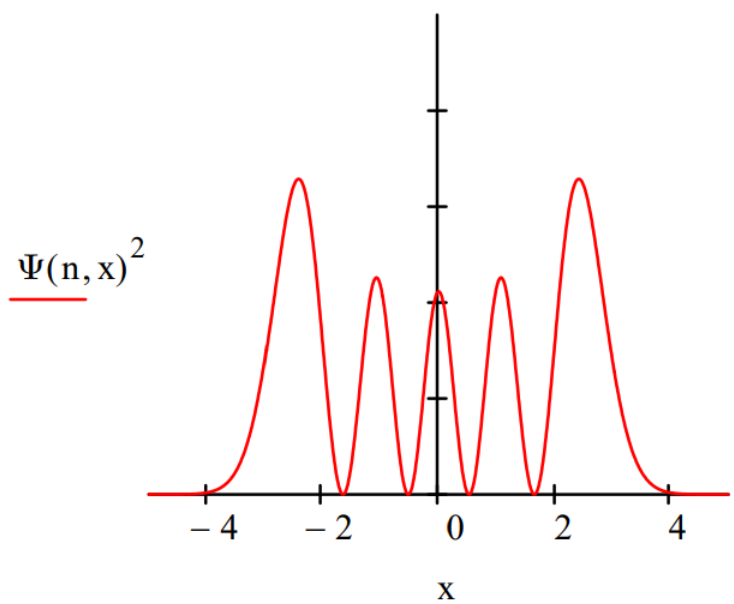
Quantum number: $n := 4$

Harmonic oscillator coordinate eigenstate:

$$\Psi(n, x) := \frac{1}{\sqrt{2^n \cdot n! \sqrt{\pi}}} \cdot \mathrm{Her}(n, x) \cdot \exp\left(-\frac{x^2}{2}\right)$$

Display coordinate wave function and distribution function:





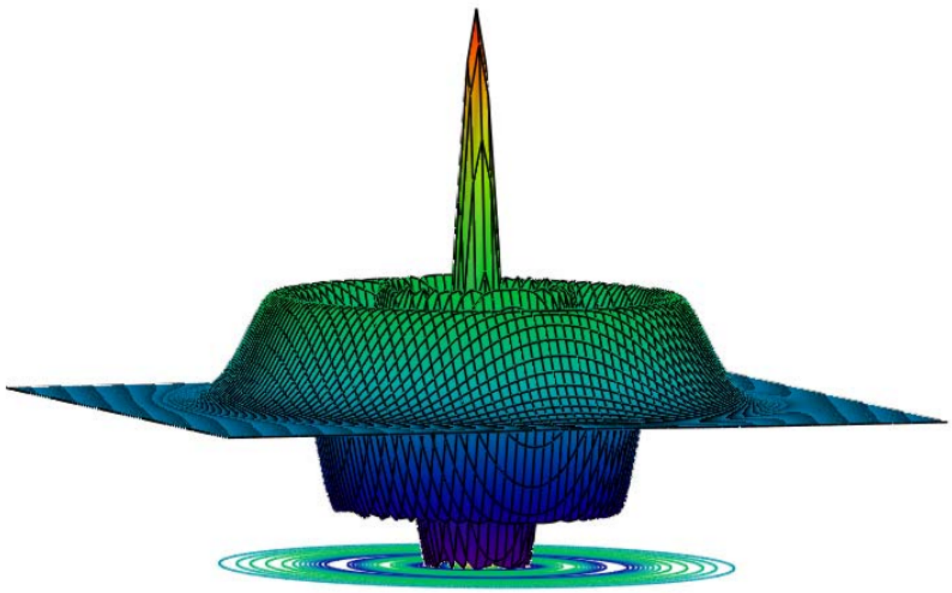
Calculate Wigner distribution:

$$W(n,x,p) := \frac{1}{\pi^{\frac{3}{2}}} \cdot \int_{-\infty}^{\infty} \Psi\left(n, x+\frac{s}{2}\right) \cdot \exp(i \cdot s \cdot p) \cdot \Psi\left(n, x-\frac{s}{2}\right) ds$$

Display Wigner distribution:

$$N := 80 \qquad i := 0 \ldots N \qquad x_i = -4 + \frac{8 \cdot i}{N}$$

$$j := 0 \ldots N \qquad p_j = -5 + \frac{10 \cdot j}{N} \qquad \text{Wigner}_{i,j} := W(n, x_i, p_j)$$



Calculate the momentum distribution function using the Wigner function:

$$\rho(p) := \int_{-\infty}^{\infty} W(n,x,p) dx \qquad p := -5, -4.95 \ldots 5$$