7.5: Single Photon Interference - Mathcad version

The schematic diagram below shows a Mach-Zehnder interferometer for photons. When the experiment is run so that there is only one photon in the apparatus at any time, the photon is always detected at D_1 and never at D_2.

This surprising phenomenon will be analyzed using matrix mechanics. State vectors for photon motion in the x- and y-direction, plus matrix operators for beam splitters and mirrors are defined below. For background and references to the primary literature see: V. Scarani and A. Suarez, "Introducing Quantum Mechanics: One-particle Interferences," Am. J. Phys. 66, 718-721 (1998).

```
         \BS          \ M
S --->---\------------\            +----> x-direction
         |\           |\           |
         |            |            |
         |            |            v
         |            |       y-direction
         |            |
        \|           \|
         \------------\------> D1
          \ M         |\ BS
                      |            S    = Source
                      v            BS   = Beam splitter
                      D2           M    = Mirror
```

Orthonormal basis states:

Photon moving in x-direction:

$$x = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad x^T x = 1$$

Photon moving in y-direction:

$$y = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad y^T y = 1 \quad x^T y = 0 \quad y^T x = 0$$

Operators:

Operator for interaction with the mirror:

$$M = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Operator for interaction with a 50/50 beam splitter:

$$BS = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$$

Operations:

$$M(x) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad M(y) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad BS(x) = \begin{bmatrix} 0.707 \\ 0.707i \end{bmatrix} \quad BS(y) = \begin{bmatrix} 0.707i \\ 0.707 \end{bmatrix} \quad BS(M)BS(x) = \begin{bmatrix} i \\ 0 \end{bmatrix}$$

Quantum Mechanical Calculation of Experimental Results:

To be detected at D_1 the photon must be moving in the x-direction (photon state = |x>). To be detected at D_2 the photon must be moving in the y-direction (photon state = |y>). It is shown below that the probability the photon is moving in the x-direction is 1 and the probability it is moving in the y-direction is 0. In its course from source to detector the photon encounters a beam splitter, a mirror, and another beam splitter. Thus,

Probability photon will arrive at detector D_1:

$$(|x^T BS(M)BS(x)|)^2 = 1$$

Probability photon will arrive at detector D_2:

$$(|x^T BS(M)BS(x)|)^2 = 1$$

Further analysis:

The photon leaves the source traveling in the x-direction. Interaction with the beam splitter puts the photon in an even linear superposition of traveling in the x- and y-directions with a 90° phase shift ($\frac{\pi}{2}$ or i) assigned by convention to motion in the y-direction (see note below).

$$BS(x) = \begin{bmatrix} 0.707 \\ 0.707i \end{bmatrix} \quad \frac{x+iy}{\sqrt{2}} = \begin{bmatrix} 0.707 \\ 0.707i \end{bmatrix}$$

The interaction of this state with the mirrors transfers the 90° phase shift to motion in the x-direction.

$$(M)BS(x) = \begin{bmatrix} 0.707i \\ 0.707 \end{bmatrix} = \frac{ix+y}{\sqrt{2}} = \begin{bmatrix} 0.707i \\ 0.707 \end{bmatrix}$$

Finally the interaction of this state with the second beam splitter yields a 90° phase-shifted photon travelling in the x-direction.

$$BS(M)BS(x) = \begin{bmatrix} i \\ 0 \end{bmatrix} \quad ix = \begin{bmatrix} i \\ 0 \end{bmatrix}$$

Thus, the probability amplitude that it will be detected at D_1 is i and the probability amplitude that it will be detected at D_2 is 0.

$$x^T BS(M)BS(x) = (i) \quad y^T BS(M)BS(x) = (0)$$

The probability for an event is the square of the absolute magnitude of the probability amplitude, so the probability that the photon will be detected at D_1 is 1.

Note: The justification for the 90 ($\frac{\pi}{2}$ or i) phase shift between transmission and reflection at the beam splitter is conservation of energy. Assuming there is no phase shift requires that the BS operator be defined as:

$$BS = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

This leads to the following calculation for the probability of the detection events:

Probability photon will arrive at detector D_1: $(|x^T BS(M)BS(x)|)^2 = 1$

Probability photon will arrive at detector D_2: $(|y^T BS(M)BS(x)|)^2 = 1$

According to this analysis, the single photon leaving the source has arrived at both detectors. One photon has become two, an obvious violation of the energy conservation principle.