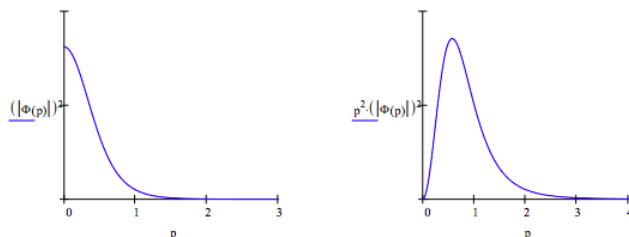


## 2.49: Momentum Wavefunctions and Distributions for the Hydrogen Atom

The Fourier transform for the 1s orbital

$$\Phi(p) = \frac{1}{\sqrt{8\pi^4}} \int_0^\infty \int_0^\pi \int_0^{2\pi} \exp(-r) \exp(-i p r \cos(\theta)) r^2 \sin(\theta) d\phi d\theta dr \rightarrow \frac{2^{\frac{1}{2}}}{\pi [(-1) + i p]^2 (1 + i p^2)}$$

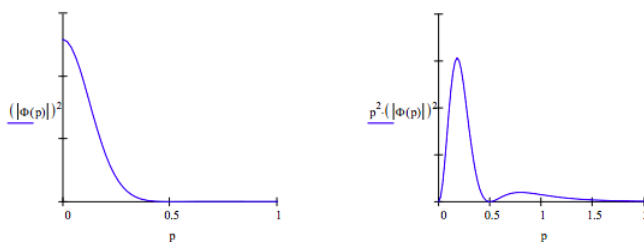
$p = 0, .02 \dots 5$



The Fourier transform for the 2s orbital

$$\Phi(p) = \frac{1}{16\pi^2} \int_0^\infty \int_0^\pi \int_0^{2\pi} (2-r) \exp\left(-\frac{r}{2}\right) \exp(-i p r \cos(\theta)) r^2 \sin(\theta) d\phi d\theta dr$$

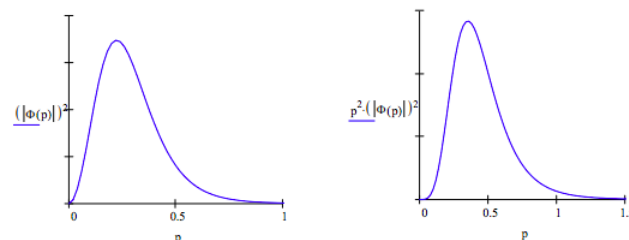
$$\text{yields } \Phi(p) = \frac{-16}{\pi} \frac{(-1) + 4p^2}{[(-1) + 2i p]^3 (1 + 2i p)^3} \quad p = 0, .02 \dots 2$$



The Fourier transform for the 2p<sub>z</sub> orbital

$$\Phi(p) = \frac{1}{16\pi^2} \int_0^\infty \int_0^\pi \int_0^{2\pi} r \exp\left(-\frac{r}{2}\right) \exp(-i p r \cos(\theta)) r^2 \sin(\theta) d\phi d\theta dr$$

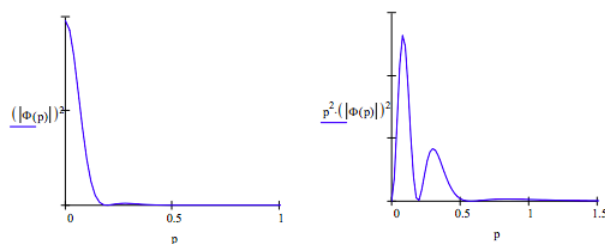
$$\text{yields } \Phi(p) = 64 \frac{i}{\pi} \frac{p}{[(-1) + 2i p]^3 (1 + 2i p)^3}$$



The Fourier transform for the 3s orbital

$$\Phi(p) = \frac{1}{162\sqrt{2}\pi^2} \int_0^\infty \int_0^\pi \int_0^{2\pi} (27 - 18r + 2r^2) \exp\left(-\frac{r}{3}\right) \exp(-i p r \cos(\theta)) r^2 \sin(\theta) d\phi d\theta dr$$

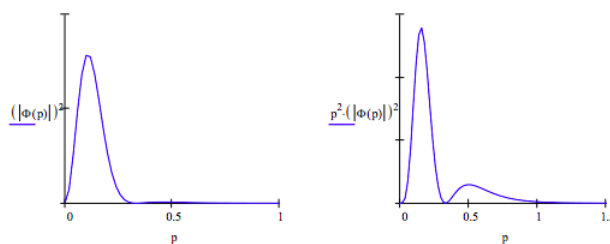
$$\text{yields } \Phi(p) = 18 \frac{6^{\frac{1}{2}}}{\pi} \frac{(-30)p^2 + 1 + 81p^4}{[(-1) + 3i p]^4 (1 + 3i p)^4}$$



The Fourier transform for the  $3p_z$  orbital

$$\Phi(p) = \frac{1}{162\pi^2} \int_0^\infty \int_0^\pi \int_0^{2\pi} (6r - r^2) \exp\left(-\frac{r}{3}\right) \exp(-i p r \cos(\theta)) \cos(\theta) r^2 \sin(\theta) d\phi d\theta dr$$

$$\text{yields } \Phi(p) = (-432) \frac{i}{\pi} p \frac{9p^2 - 1}{[(-1) + 3i p]^4 (1 + 3i p)^4}$$



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