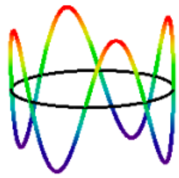
2.6: The de Broglie-Bohr Model for a Hydrogen Atom Held Together by a Gravitational Interaction



$$\lambda = \frac{h}{mv}$$

de Broglie's hypothesis that matter has wave-like properties.

$$n\lambda = 2\pi r$$

The consequence of de Broglie's hypothesis; an integral number of wavelengths must fit within the circumference of the orbit. This introduces the quantum number, n, which can have values 1,2,3,...

$$mv = \frac{nh}{2\pi r}$$

Substitution of the first equation into the second equation reveals that linear momentum is quantized.

$$T = \frac{1}{2}mv^2 = \frac{n^2h^2}{8\pi^2 m_e r^2}$$

If momentum is quantized, so is kinetic energy.

$$E = T + V = \frac{n^2h^2}{8\pi^2 m_e r^2} - \frac{Gm_pm_e}{r}$$

Which means that total energy is quantized, where $-\frac{Gm_pm_e}{r}$ is the gravitational potential energy interaction between a proton and an electron.

$$\frac{d}{dr}\left(\frac{n^2h^2}{8\pi^2 m_e r^2} - \frac{Gm_pm_e}{r}\right) = 0 \text{ solve, r} \rightarrow \frac{h^2n^2}{4\pi^2 G m_e^2 m_p}$$

Minimization of the energy with respect to orbit radius yields the optimum values of r. This expression is subtituted back in the energy expression below to find the allowed energies.

$$E = \frac{n^2h^2}{8\pi^2 m_e r^2} - \frac{Gm_pm_e}{r} \text{substitute, r} = \frac{h^2n^2}{4\pi^2 G m_e^2 m_p} \rightarrow E = \frac{2\pi^2 G^2 m_e^3 m_p^2}{h^2n^2}$$

$$\text{Fundamental constants:} \qquad m_p = 1.67262(10)^{-27}\text{kg} \qquad m_e = 9.10939(10)^{-31}\text{kg}$$
$$h = 6.62608(10)^{-34}\text{joule sec} \quad G = 6.67259(10)^{-11}\frac{m^3}{\text{kg s}^2}$$

Energy:

$$E(n) = -\frac{2\pi^2 G^2 m_e^3 m_p^2}{h^2n^2}$$

Orbit radius:

$$r(n) = \frac{h^2n^2}{4\pi^2 G m_e^2 m_p^2}$$

Calculate the first four energy levels and orbit radii.

$$n = 1..4 \quad \frac{E(n)}{J} = \begin{pmatrix} -4.233 \times 10^{-97} \\ -1.058 \times 10^{-97} \\ -4.704 \times 10^{-98} \\ -2.646 \times 10^{-98} \end{pmatrix} \quad \frac{r(n)}{m} = \begin{pmatrix} 1.201 \times 10^{29} \\ 4.803 \times 10^{29} \\ 1.081 \times 10^{30} \\ 1.921 \times 10^{30} \end{pmatrix}$$

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