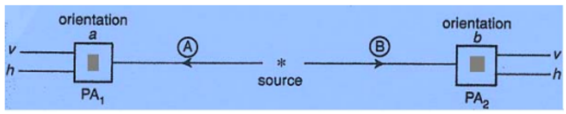
8.56: A Brief Description of Aspect's Experiment

The purpose of this tutorial is restricted to a brief computational summary of the EPR experiment reported by Aspect, Grangier and Roger, "Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedanken Experiment: A New Violation of Bell's Inequalities," in *Phys. Rev. Lett.* 49, 91 (1982). See Chapter 6 of *The Quantum Challenge* by Greenstein and Zajonc, Chapter 4 of Jim Baggott's *The Meaning of Quantum Theory*, and Chapter 12 of *Quantum Reality* by Nick Herbert for complete analyses of this historically important experiment.

A two-stage atomic cascade emits entangled photons (A and B) in opposite directions with the same circular polarization according to the observers in their path.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}[|L\rangle_A|L\rangle_B + |R\rangle_A|R\rangle_B]$$

The experiment involves the measurement of photon polarization states in the vertical/horizontal measurement basis, and allows that the polarization analyzers (PAs) can be oriented at different angles a and b. (The figure below is taken from Chapter 4 of Baggott's book.)



Naturally the bipartate photon wave function is identical in both the circular or linear polarization bases.

Left circular polarization:

$$L = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$$

Right circular polarization:

$$R = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$$

Vertical polarization:

$$V = \begin{pmatrix}1\\0\end{pmatrix}$$

Horizontal polarization:

$$H = \begin{pmatrix}0\\1\end{pmatrix}$$

$$|\Psi\rangle = \frac{1}{2\sqrt{2}}[|L\rangle_A|L\rangle_B + |R\rangle_A|R\rangle_B] = \frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\i\end{pmatrix}_A \otimes \begin{pmatrix}1\\i\end{pmatrix}_B + \begin{pmatrix}1\\-i\end{pmatrix}_A \otimes \begin{pmatrix}1\\-i\end{pmatrix}_B\right]$$

$$= \frac{1}{2\sqrt{2}}\left[\begin{pmatrix}1\\i\\i\\-1\end{pmatrix} + \begin{pmatrix}1\\-i\\-i\\-1\end{pmatrix}\right] = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}$$

$$|\Psi\rangle = \frac{1}{2\sqrt{2}}[|V\rangle_A|V\rangle_B + |H\rangle_A|H\rangle_B] = \frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}_A \otimes \begin{pmatrix}1\\0\end{pmatrix}_B - \begin{pmatrix}0\\1\end{pmatrix}_A \otimes \begin{pmatrix}0\\1\end{pmatrix}_B\right] = \frac{1}{2\sqrt{2}}\left[\begin{pmatrix}1\\0\\0\\0\end{pmatrix} - \begin{pmatrix}0\\0\\0\\1\end{pmatrix}\right]$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}$$

There are four measurement outcomes: both photons are vertically polarized, both are horizontally polarized, one is vertical and the other horizontal, and vice versa. The tensor representation of these measurement states are provided below.

$$|VV\rangle = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad |VH\rangle = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \quad |HV\rangle = \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \quad |HH\rangle = \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

We now write all states, Ψ and the measurement states, in Mathcad's vector format.

$$\Psi = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\-1\end{pmatrix} \quad \mathrm{VV} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad \mathrm{VH} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \quad \mathrm{HV} = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \quad \mathrm{HH} = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

Next, the operator representing the rotation of PA1 by angle a clockwise and PA_2 by angle b counter-clockwise (so that the PAs turn in the same direction) is constructed using matrix tensor multiplication. Kronecker is Mathcad's command for tensor matrix multiplication.

$$\mathrm{RotOp(a, b)} = \mathrm{kronecker}\left[\begin{pmatrix}\cos a & \sin a\\ -\sin a & \cos a\end{pmatrix}, \begin{pmatrix}\cos b & -\sin b\\ \sin b & \cos b\end{pmatrix}\right]$$

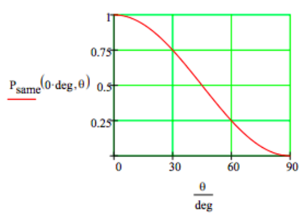
The probability that the detectors will behave the same or differently is calculated as follows.

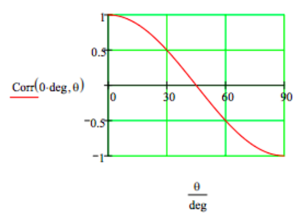
$$P_{\mathrm{same}}(a,\ b) = \left(VV^T \mathrm{RotOp(a, b)}\Psi\right)^2 + \left(HH^T \mathrm{RotOp(a, b)}\Psi\right)^2$$

$$P_{\mathrm{diff}}(a,\ b) = \left(VH^T \mathrm{RotOp(a, b)}\Psi\right)^2 + \left(HV^T \mathrm{RotOp(a, b)}\Psi\right)^2$$

The expectation value as a function of the relative orientation of the polarization detectors is the difference between these two expressions. This is generally called the correlation function. In other words, the composite eigenvalues are: ++ = -- = +1 (same) and +- = -+ = -1 (diff).

$$\mathrm{Corr(a, b)} = P_{same}\mathrm{(a, b)} - P_{diff}\mathrm{(a, b)} \qquad \Theta = 0\mathrm{deg},\ 2\mathrm{deg}\,..\,90\mathrm{deg}$$





These graphical representations of the Aspect experiment are in agreement with those presented in Aspect's paper and also in *The Quantum Challenge*, *The Meaning of Quantum Theory*, and *Quantum Reality*.

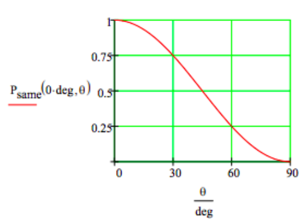
These calculations are now repeated for the three other Bell states.

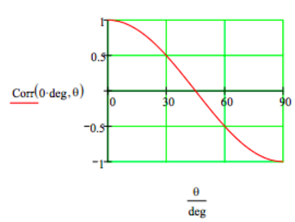
$$\Psi = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0 & 0 & 1\end{pmatrix}^T$$

$$P_{same}\mathrm{(a, b)} = \left(VV^T \mathrm{RotOp(a, b)}\Psi\right)^2 + \left(HH^T \mathrm{RotOp(a, b)}\Psi\right)^2$$

$$P_{diff}\mathrm{(a, b)} = \left(VH^T \mathrm{RotOp(a, b)}\Psi\right)^2 + \left(HV^T \mathrm{RotOp(a, b)}\Psi\right)^2$$

$$\mathrm{Corr(a, b)} = P_{same}\mathrm{(a, b)} - P_{diff}\mathrm{(a, b)}$$



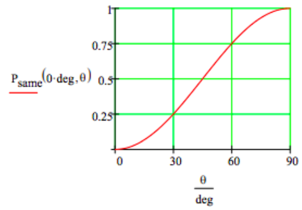


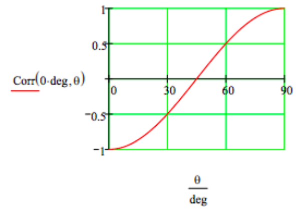
$$\Psi = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & 1 & 0\end{pmatrix}^T$$

$$P_{same}\mathrm{(a, b)} = \left(VV^T \mathrm{RotOp(a, b)}\Psi\right)^2 + \left(HH^T \mathrm{RotOp(a, b)}\Psi\right)^2$$

$$P_{diff}\mathrm{(a, b)} = \left(VH^T \mathrm{RotOp(a, b)}\Psi\right)^2 + \left(HV^T \mathrm{RotOp(a, b)}\Psi\right)^2$$

$$\mathrm{Corr(a, b)} = P_{same}\mathrm{(a, b)} - P_{diff}\mathrm{(a, b)}$$





$$\Psi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & -1 & 0 \end{pmatrix}^T$$

$$P_{same}(\mathrm{a},\mathrm{b}) = \left(VV^T \mathrm{RotOp}(\mathrm{a},\mathrm{b})\Psi\right)^2 + \left(HH^T \mathrm{RotOp}(\mathrm{a},\mathrm{b})\Psi\right)^2$$

$$P_{diff}(\mathrm{a},\mathrm{b}) = \left(VH^T \mathrm{RotOp}(\mathrm{a},\mathrm{b})\Psi\right)^2 + \left(HV^T \mathrm{RotOp}(\mathrm{a},\mathrm{b})\Psi\right)^2$$

$$\mathrm{Corr}(\mathrm{a},\mathrm{b}) = P_{same}(\mathrm{a},\mathrm{b}) - P_{diff}(\mathrm{a},\mathrm{b})$$