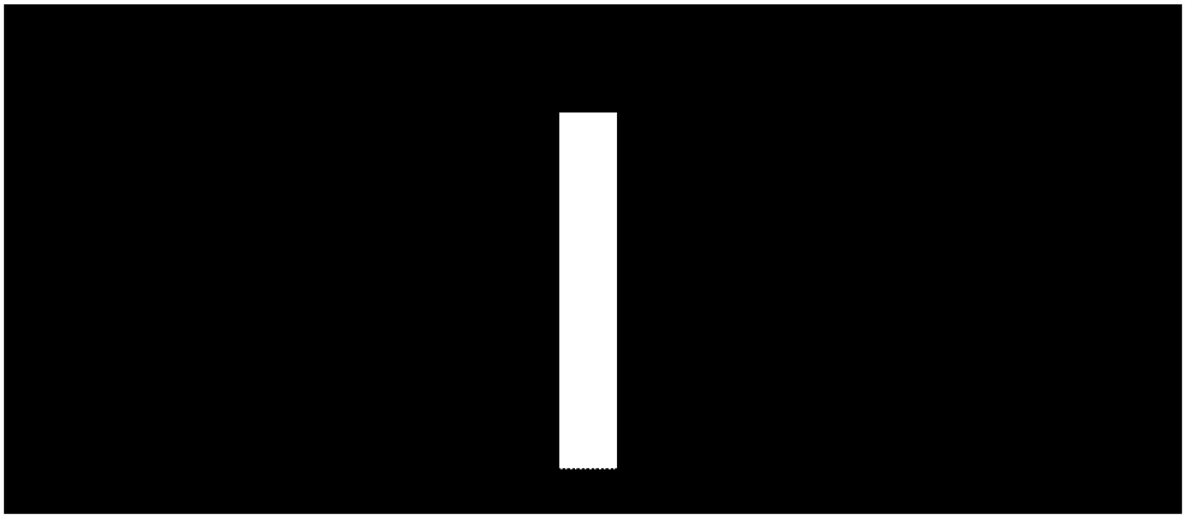
1.29: Single Slit Diffraction and the Fourier Transform

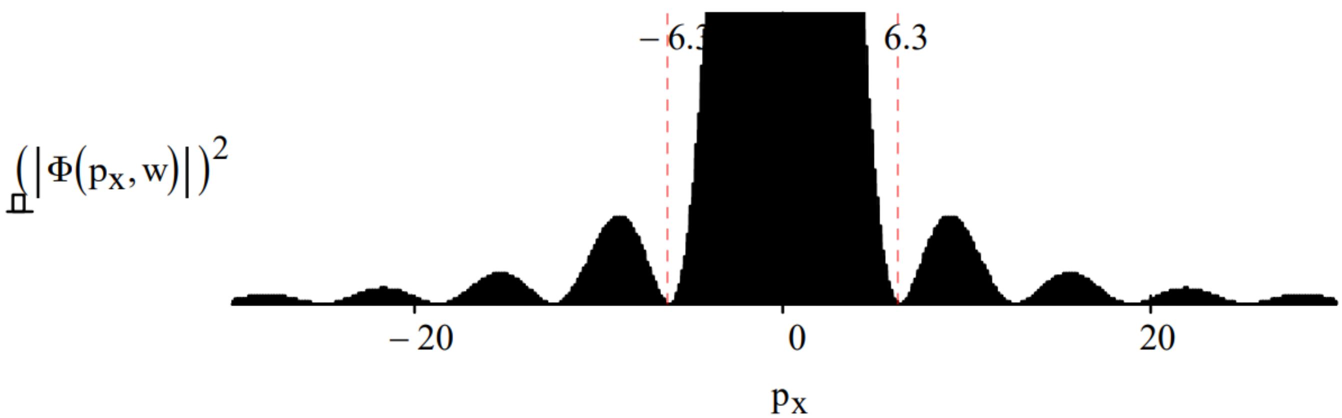
Slit width: $w := 1$	Coordinate-space wave function: $\Psi(x, w) := \text{ if } \left[\left(x \geq -\frac{w}{2}\right) \cdot \left(x \leq \frac{w}{2}\right), 1, 0\right]$

$$x := \frac{-w}{2}, \frac{-w}{2} + .005 \ldots \frac{w}{2}$$



A Fourier transform of the coordinate-space wave function yields the momentum wave function and the momentum distribution function, which is the diffraction pattern.

$$\Phi\left(\mathrm{p_x}, \mathrm{w}\right) := \frac{1}{\sqrt{2 \cdot \pi \cdot \mathrm{w}}} \cdot \int_{-\frac{\mathrm{w}}{2}}^{\frac{\mathrm{w}}{2}} \exp(-\mathrm{i} \cdot \mathrm{p_x} \cdot \mathrm{x}) d\mathrm{x} \text{ simplify } \rightarrow \frac{\sqrt{2} \cdot \sin\left(\frac{\mathrm{p_x} \cdot \mathrm{w}}{2}\right)}{\sqrt{\pi} \cdot \mathrm{p_x} \cdot \sqrt{\mathrm{w}}}$$



Now Fourier transform the momentum wave function back to coordinate space and display result. This is done numerically using large limits of integration for momentum.

$$\Psi(x, w) := \int_{-5000}^{5000} \frac{\frac{1}{2} \sin\left(\frac{1}{2} \cdot w \cdot p_x\right)}{\pi^{\frac{1}{2}} \cdot w^{\frac{1}{2}} \cdot p_x} \cdot \frac{\exp\left(i \cdot p_x \cdot x\right)}{\sqrt{2 \cdot \pi}} dp_x$$