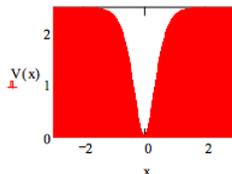


10.33: Variational Method for the Feshbach Potential

Define potential energy: $V_0 = 2.5$ $d = 0.5$ $V(x) = V_0 \tanh\left(\frac{x}{d}\right)^2$

Display potential energy:



Choose Gaussian trial wavefunction:

$$\psi(x, \beta) = \left(\frac{2\beta}{\pi}\right)^{\frac{1}{4}} \exp(-\beta x^2)$$

Demonstrate that the trial wavefunction is normalized.

$$\int_{-\infty}^{\infty} \psi(x, \beta)^2 dx \quad \text{assume, } \beta > 0 \rightarrow 1$$

Evaluate the variational integral.

$$E(\beta) = \int_{-\infty}^{\infty} \psi(x, \beta) \frac{-1}{2} \frac{d^2}{dx^2} \psi(x, \beta) dx + \int_{-\infty}^{\infty} V(x) \psi(x, \beta)^2 dx$$

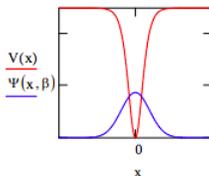
Minimize the energy integral with respect to the variational parameter, β .

$$\beta = 1 \quad \beta = \text{Minimize } (E, \beta) \quad \beta = 0.913 \quad E(\beta) = 1.484$$

Calculate the % error given that numerical integration of Schrödinger's equation (see next tutorial) yields $E = 1.44949 E_h$.

$$\frac{E(\beta) - 1.44949}{1.44949} \times 100 = 2.36$$

Display wavefunction in the potential well.



Calculate the probability that tunneling is occurring.

$$V(x) = 1.484 \Big|_{\text{float, 3}}^{\text{solve, x}} \rightarrow \begin{pmatrix} -1.511 \\ 0.511 \end{pmatrix}$$

$$2 \int_{0.511}^{\infty} \psi(x, \beta)^2 dx = 0.329$$

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