

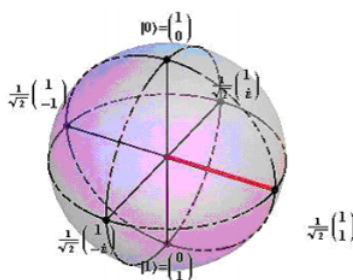
1.107: The Bloch Sphere

The eigenfunctions of the Pauli spin matrices.

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

are presented mathematically and shown on the Bloch sphere below. The Xu state is highlighted.

$$Z_u = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad Z_d = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad X_u = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad X_d = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad Y-u = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \quad Y_d = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$



This figure was taken from demonstrations.wolfram.com/QubitsOnThePoincareBlochSphere/ a contribution by Rudolf Muradian. The Bloch sphere is prepared in Cartesian coordinates using Mathcad graphics.

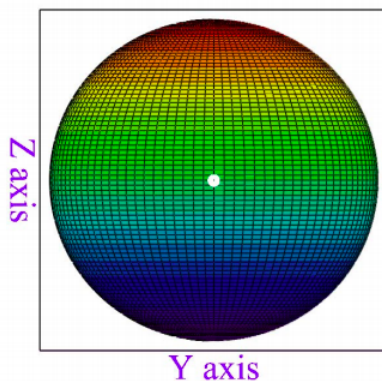
$$\text{numpts} = 100 \quad i = 0.. \text{numpts} \quad j = 0.. \text{numpts} \quad \theta_i = \frac{\pi i}{\text{numpts}} \quad \phi_j = \frac{2\pi j}{\text{numpts}}$$

$$X_{i,j} = \sin(\theta_i) \cos(\phi_j) \quad Y_{i,j} = \sin(\theta_i) \sin(\phi_j) \quad Z_{i,j} = \cos(\theta_i)$$

Next, the coordinates of a quantum qubit are calculated and displayed on the Bloch sphere as a white dot. As the polar and azimuthal angles are changed, you will need to rotate the figure to see where the white dot is on the surface of the Bloch sphere.

$$\theta_1 = \frac{\pi}{2} \quad \phi_1 = 0 \quad \Psi(\theta_1, \phi_1) = \cos\left(\frac{\theta_1}{2}\right) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \exp(i\phi_1) \sin\left(\frac{\theta_1}{2}\right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \Psi(\theta_1, \phi_1) = \begin{pmatrix} 0.707 \\ 0.707 \end{pmatrix}$$

$$XX_{i,j} = \sin(\theta_1) \cos(\phi_1) \quad YY_{i,j} = \sin(\theta_1) \sin(\phi_1) \quad ZZ_{i,j} = \cos(\theta_1)$$



(X,Y,Z),(XX,YY,ZZ)

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