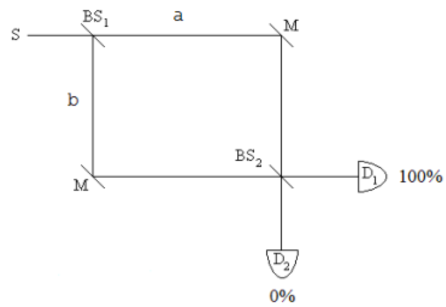
7.4: Single Photon Interference - Fourth Version

This analysis of the operation of a Mach-Zehnder Interferometer (MZI) will use tensor algebra and the creation and annihilation operators.



An interferometer arm can be occupied or unoccupied. These states are represented by the following vectors.

$$Unoccupied: |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$Occupied: |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

After the first beam splitter the photon is in an even superposition of being in both arms of the interferometer. By convention a 90 degree phase shift is assigned to arm b to preserve probability. In terms of the concept of occupation, the superposition takes the following form in tensor algebra.

$$|S\rangle \xrightarrow{i} \frac{1}{\sqrt{2}}[|1\rangle_a|0\rangle_b + i|0\rangle_a 1|\rangle_b] = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 \\ 1 \end{pmatrix}_a \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}_b + i\begin{pmatrix} 1 \\ 0 \end{pmatrix}_a \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}_b\right] = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ i \\ 1 \\ 0 \end{pmatrix}$$

$$\Psi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ i \\ 1 \\ 0 \end{pmatrix}$$

The matrix operators required for this analysis are as follows.

Creation:

$$C = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Annihilation:

$$A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Number:

$$N = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Identity:

$$I = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

The effect of the creation, annihilation and number operators on |0> and |1>:

$$C\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix}$$

$$A\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}$$

$$N\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

$$N\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix}$$

The creation operator is the Hermitian adjoint of the annihilation operator and the annihilation operator is the Hermitian adjoint of the creation operator.

$$\overline{(A^T)}=\begin{pmatrix}0&0\\1&0\end{pmatrix}$$

$$\overline{(C^T)}=\begin{pmatrix}0&1\\0&0\end{pmatrix}$$

The number operator is the product of the creation and annihilation operators.

$$CA=\begin{pmatrix}0&0\\0&1\end{pmatrix}$$

$$\overline{(A^T)}A=\begin{pmatrix}0&0\\0&1\end{pmatrix}$$

$$\overline{(C^T)}C=\begin{pmatrix}0&0\\0&1\end{pmatrix}$$

The eigenvectors of the number operator are |0> and |1> with eigenvalues 0 and 1, respectively:

$$eigenvecs(N)=\begin{pmatrix}1&0\\0&1\end{pmatrix}$$

$$eigenvals(N)=\begin{pmatrix}0\\1\end{pmatrix}$$

There are two paths to each detector. This provides the opportunity for constructive and destructive interference. To arrive at D_1 the a-arm photon state is reflected (90 degree phase shift) at BS_2 and the b-arm photon state is transmitted at BS_2. Therefore, photon detection requires the annihilation of the superposition of these paths to D_1. The annihilation is achieved with the following operator.

$$\frac{iA_a+A_b}{\sqrt{2}}$$

The product of this operator with its Hermitian conjugate (see above) creates the number operator for photon detection at D1.

$$N_{D1}=\frac{iC_a+C_b}{\sqrt{2}}+\frac{iA_a+A_b}{\sqrt{2}}$$

The D_1 number operator is formed using Mathcad's kronecker command as follows:

N_{D1} = $\frac{1}{2}$ (-i kronecker (C, I) + kronecker (I, C)) (i kronecker (A,I) + kronecker (I, A))

To arrive at D_2 the a-arm photon state is transmitted at BS_2 and the b-arm photon state is reflected (90 degree phase shift) at BS_2. Photon detection at D_2 requires the annihilation of the superposition of these paths to the detector. The annihilation is represented the following operator.

$$\frac{iA_a+A_b}{\sqrt{2}}$$

Therefore, the number operator for photon detection at D_2 is:

$$N_{D2} = \frac{C_a + iC_b}{\sqrt{2}} \frac{A_a + iA_b}{\sqrt{2}}$$

The D_2 number operator is formed using Mathcad's kronecker command as follows.

$N_{D2} = \frac{1}{2}$ (kronecker (C, I) - i kronecker (I, C)) (kronecker (A,I) + i kronecker (I, A))

We now show that the photon always arrives at D_1 and never at D_2 for an equal arm MZI.

Expectation value for photon detection at D_1:

$$\overline{\Psi^T} N_{D1} \Psi = 1$$

Expectation value for photon detection at D_2:

$$\overline{\Psi^T} N_{D2} \Psi = 1$$

Equivalent results can be obtained algebraically. Operating on Ψ with the D_1 number operator yields Ψ. In other words, Ψ is an eigenfunction of N_{D1} with eigenvalue 1.

$$\left[\frac{-C_a + C_b}{\sqrt{2}} \frac{iA_a + A_b}{\sqrt{2}}\right] \frac{1}{\sqrt{2}} \left[|1\rangle_a |0\rangle_b + i|0\rangle_a |1\rangle_b\right] = \frac{1}{\sqrt{2}} \left[|1\rangle_a |0\rangle_b + i|0_a|1\rangle_b\right]$$

Operating on Ψ with the D_2 number operator yields 0.

$$\left[\frac{C_a - iC_b}{\sqrt{2}} \frac{A_a + iA_b}{\sqrt{2}}\right] \frac{1}{\sqrt{2}} \left[|1\rangle_a |0\rangle_b + i|0\rangle_a |1\rangle_b\right] = 0$$