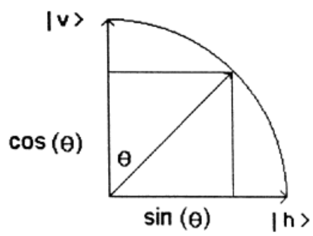
7.17: Polarized Light and Quantum Mechanics

The Linear Superposition

Unpolarized light consists of photons of all possible polarization angles. A photon polarized at an angle θ with respect to the vertical can be written as a linear superposition of a vertically polarized photon, $|v\rangle$, and a horizontally polarized photon, $|h\rangle$. $|v\rangle$ and $|h\rangle$ are the polarization basis states.

$$|\theta\rangle = |v\rangle\langle v|\theta\rangle + |h\rangle\langle h|\theta\rangle$$



From the figure above it can be seen that the projection of $|\theta\rangle$ onto $|v\rangle$ and $|h\rangle$ are $\cos\theta$ and $\sin\theta$ respectively.

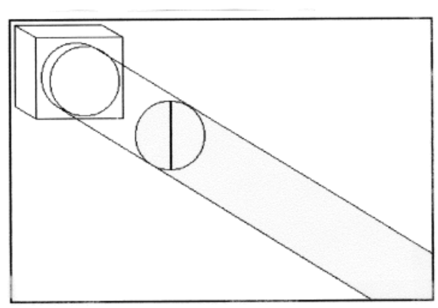
$$|\theta\rangle = |v\rangle\cos\theta + |h\rangle\sin\theta$$

The probability that a photon polarized at an angle θ will pass a vertical polarizer is

$$|\langle|\theta\rangle|^2 = \cos^2\theta$$

By integrating this function over all possible angles we find that half of the incident light passes through a vertical polarizer. See the figure below.

$$\frac{\int_0^{2\pi}\cos^2\theta d\theta}{2\pi} = 0.5$$



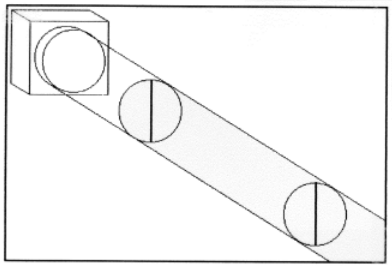
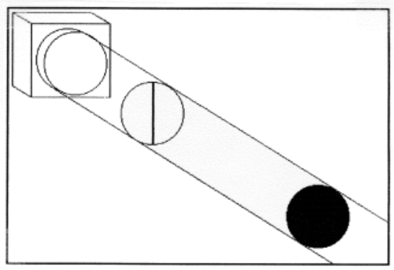
The photons that pass the vertical polarizer are now vertically polarized. That is they are eigenfunctions of that measurement operator.

The probability that a vertically polarized photon will pass a second filter that is vertically polarized is one.

$$|\langle v|v\rangle|^2 = \cos^2 0^o = 1$$

The probability that a vertically polarized photon will pass a second filter that is horizontally polarized is zero.

$$|\langle h|v\rangle|^2 = \cos^2 90^o = 0$$

The vertically polarized photon can be written as linear superposition of any other set of orthogonal basis states, for example $|45^o\rangle$ and $|-45^o\rangle$.

$$|v\rangle = |45^o\rangle\langle 45^o|v\rangle + |-45^o\rangle\langle -45^o|v\rangle$$

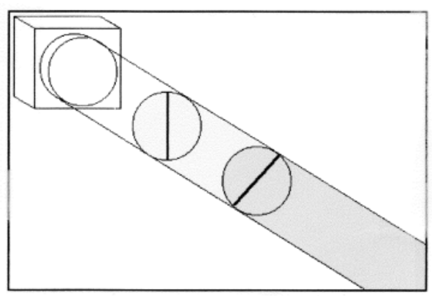
$$|v\rangle = |45^o\rangle \cos 45^o + |-45^o\rangle \cos -45^o$$

$$|v\rangle = |45^o\rangle 0.707 + |-45^o\rangle 0.707$$

Now if a 45^o polarizer is inserted in between the vertical and horizontal polarizers photons get through the horizontal polarizer that stopped them previously (see the last figure).

Here's the explanation. The probability that a vertically polarized photon will get through a polarizer oriented at an angle of 45^o is 0.5. See the figure below.

$$|\langle 45^o|v\rangle|^2 = \cos^2 45^o = 0.5$$



Now the photon is in the state of $|45^o\rangle$ which can be written as a linear superposition of $|v\rangle$ and $\backslash\mathrm{h}\rangle$.

$$|45^o\rangle = |v\rangle\langle v|45^o\rangle + |h\rangle\langle h|45^o\rangle$$

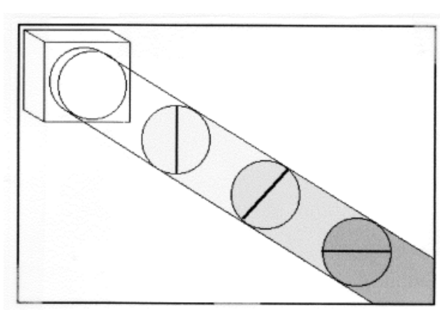
$$|45^o\rangle = |v\rangle \cos 45^o + |h\rangle \sin 45^o$$

$$|45^o\rangle = |v\rangle 0.707 + |h\rangle 0.707$$

Thus, the probability that this photon will pass the final horizontally oriented polarizer is

$$|\langle H|45^o\rangle|^2 = \sin^2 45^o = 0.5$$

See the figure below.



In this last figure the intensity of the light emerging from the final horizontal polarizing filter can be calculated compactly as

$$\frac{\int_0^{2\pi} \left| \langle h | 45^o \rangle \langle 45^o | v \rangle \langle v | \theta \rangle \right|^2 d\theta}{2\pi} = \frac{1}{8}$$

The term inside the integral is the probability that a photon with polarization θ will pass through the three filters. This expression is integrated over all values of θ.