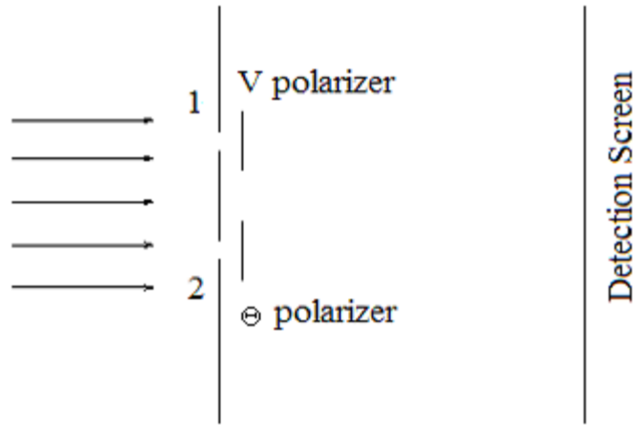
1.41: The Double-Slit Experiment with Polarized Light

Fresnel and Arago ″using an apparatus based on Young′s [double-slit] experiment″ observed that ″two beams polarized in mutually perpendicular planes never yield fringes.″ The purpose of this tutorial is to examine this phenomenon from a quantum mechanical perspective.

A schematic diagram of the double-slit experiment with polarizers behind the slits is shown below.V and Θ stand for vertical and Θ polarizers, respectively.



Assuming infinitessimally thin slits, the photon wave function is a superposition of being at slit 1 with vertical polarization and slit 2 with polarization at an angle θ relative to the vertical.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}[|x_1\rangle\,|\mathrm{V}\rangle + |x_2\rangle|\Theta\rangle]$$

The vertical and θ polarization states are represented by the following vectors.

$$|\mathrm{V}\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } |\Theta\rangle = \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix}$$

The diffraction pattern is the Fourier transform of the state above into the momentum representation.

$$\langle p|\Psi\rangle = \frac{1}{\sqrt{2}}[\langle p|x_1\rangle\,|\mathrm{V}\rangle + \langle p|x_2\rangle\,|\Theta\rangle]$$

This calculation is implemented in Mathcad for slits of finite width as follows.

Slit positions:

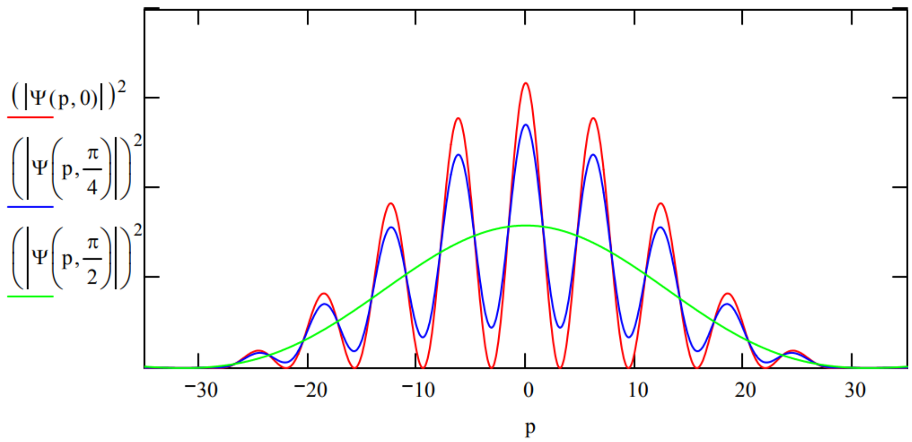
$$\mathrm{x}_1 := 1 \qquad \mathrm{x}_2 := 2$$

Slit width:

$$\delta := 0.2$$

$$\Psi(p,\theta) := \frac{\int_{\mathrm{x}_1-\frac{\delta}{2}}^{\mathrm{x}_1+\frac{\delta}{2}} \frac{1}{\sqrt{2\cdot\pi}}\cdot\exp(-\mathrm{i}\cdot\mathrm{p}\cdot\mathrm{x})\cdot\frac{1}{\sqrt{\delta}}\,\mathrm{dx}\cdot\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \int_{x_2-\frac{\delta}{2}}^{x_2+\frac{\delta}{2}} \frac{1}{\sqrt{2\cdot\pi}}\cdot\exp(-\mathrm{i}\cdot\mathrm{p}\cdot\mathrm{x})\cdot\frac{1}{\sqrt{\delta}}\,\mathrm{dx}\cdot\begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix}}{\sqrt{2}}$$

To confirm the assertion of Fresnel and Arago the momentum distributions for three angles of the polarizer at the right slit are calculated.



We see that when the polarizers are oriented at the same angle, the diffraction pattern is the usual one for the Young double-slit experiment. When the polarizers are crossed the fringes, as Fresnel and Arago assert, disappear. Finally, when the relative angle of the two polarizers is 45 degrees, we see a reduced interference pattern.