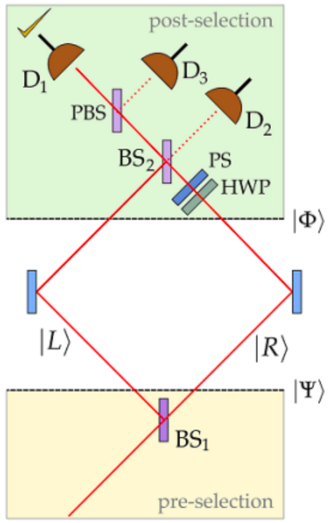
7.25: A Quantum Optical Cheshire Cat

The following is a summary of "Quantum Cheshire Cats" by Aharonov, Popescu, Rohrlich and Skrzypczyk which was published in the ***New Journal of Physics*** **15**, 113015 (2013) and can also be accessed at: arXiv:1202.0631v2.



In the absence of the half-wave plate (HWP) and the phase shifter (PS) a horizontally polarized photon entering the interferometer from the lower left (propagating to the upper right) arrives at D_2 with a 90 degree (π/2, i) phase shift. (By convention reflection at a beam splitter introduces a 90 degree phase shift.)

$$|R\rangle|H\rangle \xrightarrow{BS_1} \frac{1}{\sqrt{2}}[i|L\rangle + |R\rangle]|H\rangle \xrightarrow{BS_2} i|D_2\rangle|H\rangle$$

The state immediately after the first beam splitter is the pre-selected state.

$$|Psi\rangle = \frac{1}{\sqrt{2}}[i|L\rangle + |R\rangle]\,|H\rangle$$

The post-selected state is,

$$|\Phi\rangle = \frac{1}{\sqrt{2}}[|L\rangle|H\rangle + |R\rangle|V\rangle]$$

The HWP (converts |V> to |H> in the R-branch) and PS transform this state to,

$$|\Phi\rangle \xrightarrow[PS]{HWP} \frac{1}{\sqrt{2}}[|L\rangle + i|R\rangle]\,|H\rangle$$

which exits the second beam splitter through the left port to encounter a polarizing beam splitter which transmits horizontal polarization and reflects vertical polarization. Thus, the post-selected state is detected at D_1. The evolution of the post-selected state is summarized as follows:

$$|\Phi\rangle = \frac{1}{\sqrt{2}}[|L\rangle|H\rangle + |R\rangle|V\rangle] \xrightarrow[PS]{HWP} \frac{1}{\sqrt{2}}[|L\rangle + i|R\rangle]\,|H\rangle \xrightarrow{BS_2} i|L\rangle|H\rangle \xrightarrow{PBS} i|D_1\rangle|H\rangle$$

The last term on the right side below is the weak value of A multiplied by the probability of its occurrence for the preselected state Ψ and the post-selected state Φ.

$$\langle\Psi|\hat{A}|\Psi\rangle = \sum_j \langle\Psi|\Phi_j\rangle\langle\Phi_j|\hat{A}|\Psi\rangle = \sum_j \langle\Psi|\Phi_j\rangle\langle\Phi_j|\Psi\rangle\frac{\langle\Phi_j|\hat{A}|\Psi\rangle}{\langle\Phi_j|\Psi\rangle} = \sum_j p_j\frac{\langle\Phi_j|\hat{A}|\Psi\rangle}{\langle\Phi_j|\Psi\rangle}$$

The weak value calculations are carried out in a 4-dimensional Hilbert space created by the tensor product of the photon's direction of propagation and polarization vectors.

Direction of propagation vectors:

$$L = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$R = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Polarization state vectors:

$$H = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$V = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Pre-selected state:

$$\Psi = \frac{1}{\sqrt{2}}(iL + R)H = \frac{1}{\sqrt{2}}\left[i\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right] + \begin{pmatrix} 1 \\ 0 \end{pmatrix} = frac1\sqrt{2}\begin{pmatrix} i \\ 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\Psi = \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Post-selected state:

$$\Phi = \frac{1}{\sqrt{2}}(LH + RV) = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]$$

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Direction of propagation operators:

$$Left = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$Right = \begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Photon angular momentum operator:

$$Pang = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Identity operator:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The following weak value calculations show that for the pre- and post-selection ensemble of observations the photon is in the left arm of the interferometer while its angular momentum is in the right arm. Like the case of the Cheshire cat, a photon property has been separated from the photon.

$$\begin{pmatrix} "" & "\ Left\ Arm\ " & "\ Right\ Arm\ " \\ "\ Arm\ " & \frac{\Phi^T kronecker(Left, I)\Psi}{\Phi^T\Psi} & \frac{\Phi^T kronecker(Right, I)\Psi}{\Phi^T\Psi} \\ "\ Pang\ " & \frac{\Phi^T kronecker(Left, Pang)\Psi}{\Phi^T\Psi} & \frac{\Phi^T kronecker(Right, Pang)\Psi}{\Phi^T\Psi} \end{pmatrix} = \begin{pmatrix} "" & "\ Left\ Arm\ " & "\ Right\ Arm \\ "\ Arm\ " & 1 & 0 \\ "\ Pang\ " & 0 & 1 \end{pmatrix}$$

The following shows the evolution of the pre-selected state to the final state at the detectors. The intermediate is the state illuminating BS_2. The polarization state at the detectors is ignored.

$$|\Psi\rangle \rightarrow \frac{i}{\sqrt{2}}[|L\rangle|H\rangle + |R\rangle|V\rangle] \rightarrow -\frac{1}{2}|D_1\rangle + \frac{i}{2}|D_3\rangle + \frac{(i-1)}{2}|D_2\rangle$$

Squaring the magnitude of the probability amplitudes shows that the probabilities that D_1, D_3 and D_2 will fire are 1/4, 1/4 and 1/2, respectively. The probability at D_1 is consistent with the probability that the post-selected state is contained in the pre-selected state. A photon in the post-selected state has a probability of 1 of reaching D_1 and it represents a 25% contribution to the pre-selected state.

$$(|\Phi^T\Psi|)^2 \rightarrow \frac{1}{4}$$

Note that the expectation values for the pre-selected state show no path-polarization separation.

$$\left(\begin{array}{ccc} "" & "\ Left\ Arm\ " & "\ Right\ Arm\ " \\ "\ Arm\ " & (\overline{\Psi})^T kronecker(Left,\ I)\Psi & (\overline{\Psi})^T kronecker(Right,\ I)\Psi \\ "\ Pang\ " & (\overline{\Psi})^T kronecker(Left,\ Pang)\Psi & (\overline{\Psi})^T kronecker(Right,\ Pang)\Psi \\ "\ Hop\ " & (\overline{\Psi})^T kronecker(Left,\ HH^T)\Psi & (\overline{\Psi})^T kronecker(Right,\ HH^T)\Psi \\ "\ Vop\ " & (\overline{\Psi})^T kronecker(Left,\ VV^T)\Psi & (\overline{\Psi})^T kronecker(Right,\ VV^T)\Psi \end{array}\right) = \left(\begin{array}{ccc} "" & "\ Left\ Arm\ " & "\ Right\ Arm\ " \\ "\ Arm\ " & 0.5 & 0.5 \\ "\ Pang\ " & 0 & 0 \\ "\ Hop\ " & 0.5 & 0.5 \\ "\ Vop\ " & 0 & 0 \end{array}\right)$$

In addition the following table shows that linear polarization (HV) is not separated from the photon's path.

$$HV = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\left(\begin{array}{ccc} "" & "\ Left\ Arm\ " & "\ Right\ Arm\ " \\ "\ Arm\ " & \frac{\Phi^T kronecker(Left,\ I)\Psi}{\Phi^T\Psi} & \frac{\Phi^T kronecker(Right,\ I)\Psi}{\Phi^T\Psi} \\ "\ HV\ " & \frac{\Phi^T kronecker(Left,\ HV)\Psi}{\Phi^T\Psi} & \frac{\Phi^T kronecker(Right,\ HV)\Psi}{\Phi^T\Psi} \end{array}\right) = \left(\begin{array}{ccc} "" & "\ Left\ Arm\ " & "\ Right\ Arm \\ "\ Arm\ " & 1 & 0 \\ "\ HV\ " & 1 & 0 \end{array}\right)$$

The "Complete Quantum Cheshire Cat" by Guryanova, Brunner and Popescu (arXiv 1203.4215) provides an optical set-up which achieves complete path-polarization separation for the photon.