1.73: Time-dependent Wigner Function for Harmonic Oscillator Transitions

Initial state:

$$m := 0 \qquad E_m := m + \frac{1}{2}$$

Final state:

$$n := 1 \qquad E_n := n + \frac{1}{2} \qquad t := \mathrm{FRAME}$$

Define Wigner distribution function for a linear superposition of the initial and final harmonic oscillator state.

$$W(x,p) := \frac{1}{\pi^{\frac{3}{2}}} \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2^n \cdot n! \sqrt{\pi}}} \cdot \mathrm{Her}\left(n, x + \frac{s}{2}\right) \cdot \exp\left[-\frac{\left(x + \frac{s}{2}\right)^2}{2} \right] \cdot \exp(i \cdot E_n \cdot t) + \frac{1}{\sqrt{2^m \cdot m! \cdot \sqrt{\pi}}} \cdot \mathrm{Her} \left(m, x + \frac{s}{2}\right) \cdot \exp\left[-\frac{\left(x + \frac{s}{2}\right)^2}{2} \right] \cdot \exp(i \cdot E_m \cdot t) \right]$$

$$\cdot \exp(i \cdot s \cdot p) \cdot \left[\frac{1}{\sqrt{2^n \cdot n! \cdot \sqrt{\pi}}} \cdot \mathrm{Her}\left(n, x - \frac{s}{2}\right) \cdot \exp\left[-\frac{\left(x - \frac{s}{2}\right)^2}{2} \right] \cdot \exp(-i \cdot E_n \cdot t) + \frac{1}{\sqrt{2^m \cdot m! \cdot \sqrt{\pi}}} \cdot \mathrm{Her}\left(m, x - \frac{s}{2}\right) \cdot \exp\left[-\frac{\left(x - \frac{s}{2}\right)^2}{2} \right] \cdot \exp(-i \cdot E_m \cdot t) \right] ds$$

Display Wigner distribution:

$$N = 60 \qquad i := 0 \ldots N \qquad x_i := -2.5 + \frac{5 \cdot i}{N}$$

$$j := 0 \ldots N \qquad P_j := -2.5 + \frac{5j}{N} \qquad \mathrm{Wigner}_{i,j} := W(x_i, p_j)$$