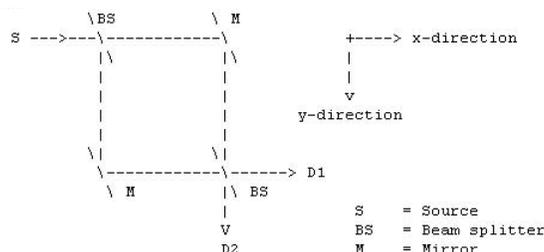


## 7.5: Single Photon Interference - Mathcad version

The schematic diagram below shows a Mach-Zehnder interferometer for photons. When the experiment is run so that there is only one photon in the apparatus at any time, the photon is always detected at  $D_1$  and never at  $D_2$ .

This surprising phenomenon will be analyzed using matrix mechanics. State vectors for photon motion in the x- and y-direction, plus matrix operators for beam splitters and mirrors are defined below. For background and references to the primary literature see: V. Scarani and A. Suarez, "Introducing Quantum Mechanics: One-particle Interferences," Am. J. Phys. 66, 718-721 (1998).



### Orthonormal basis states:

Photon moving in x-direction:

$$x = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad x^T x = 1$$

Photon moving in y-direction:

$$y = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad y^T y = 1 \quad x^T y = 0 \quad y^T x = 0$$

### Operators:

Operator for interaction with the mirror:

$$M = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Operator for interaction with a 50/50 beam splitter:

$$BS = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$$

### Operations:

$$M(x) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad M(y) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad BS(x) = \begin{bmatrix} 0.707 \\ 0.707i \end{bmatrix} \quad BS(y) = \begin{bmatrix} 0.707i \\ 0.707 \end{bmatrix} \quad BS(M)BS(x) = \begin{bmatrix} i \\ 0 \end{bmatrix}$$

Quantum Mechanical Calculation of Experimental Results:

To be detected at  $D_1$  the photon must be moving in the x-direction (photon state =  $|x\rangle$ ). To be detected at  $D_2$  the photon must be moving in the y-direction (photon state =  $|y\rangle$ ). It is shown below that the probability the photon is moving in the x-direction is 1 and the probability it is moving in the y-direction is 0. In its course from source to detector the photon encounters a beam splitter, a mirror, and another beam splitter. Thus,

Probability photon will arrive at detector  $D_1$ :

$$(|x^T BS(M)BS(x)|)^2 = 1$$

Probability photon will arrive at detector  $D_2$ :

$$(|x^T BS(M)BS(x)|)^2 = 1$$

### Further analysis:

The photon leaves the source traveling in the x-direction. Interaction with the beam splitter puts the photon in an even linear superposition of traveling in the x- and y-directions with a  $90^\circ$  phase shift ( $\frac{\pi}{2}$  or  $i$ ) assigned by convention to motion in the y-direction (see note below).

$$BS(x) = \begin{bmatrix} 0.707 \\ 0.707i \end{bmatrix} \frac{x + iy}{\sqrt{2}} = \begin{bmatrix} 0.707 \\ 0.707i \end{bmatrix}$$

The interaction of this state with the mirrors transfers the  $90^\circ$  phase shift to motion in the x-direction.

$$(M)BS(x) = \begin{bmatrix} 0.707i \\ 0.707 \end{bmatrix} = \frac{ix + y}{\sqrt{2}} = \begin{bmatrix} 0.707i \\ 0.707 \end{bmatrix}$$

Finally the interaction of this state with the second beam splitter yields a  $90^\circ$  phase-shifted photon travelling in the x-direction.

$$BS(M)BS(x) = \begin{bmatrix} i \\ 0 \end{bmatrix} \quad ix = \begin{bmatrix} i \\ 0 \end{bmatrix}$$

Thus, the probability amplitude that it will be detected at  $D_1$  is  $i$  and the probability amplitude that it will be detected at  $D_2$  is  $0$ .

$$x^T BS(M)BS(x) = (i) \quad y^T BS(M)BS(x) = (0)$$

The probability for an event is the square of the absolute magnitude of the probability amplitude, so the probability that the photon will be detected at  $D_1$  is 1.

**Note:** The justification for the  $90^\circ$  ( $\frac{\pi}{2}$  or  $i$ ) phase shift between transmission and reflection at the beam splitter is conservation of energy. Assuming there is no phase shift requires that the BS operator be defined as:

$$BS = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

This leads to the following calculation for the probability of the detection events:

$$\text{Probability photon will arrive at detector } D_1: (|x^T BS(M)BS(x)|)^2 = 1$$

$$\text{Probability photon will arrive at detector } D_2: (|y^T BS(M)BS(x)|)^2 = 1$$

According to this analysis, the single photon leaving the source has arrived at both detectors. One photon has become two, an obvious violation of the energy conservation principle.

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