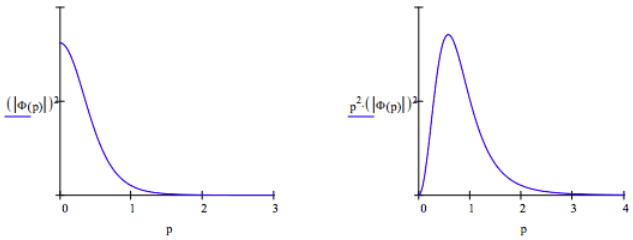
2.49: Momentum Wavefunctions and Distributions for the Hydrogen Atom

The Fourier transform for the 1s orbital

$$\Phi(p) = \frac{1}{\sqrt{8\pi^4}} \int_0^{\infty} \int_0^{\pi} \int_0^{2\pi} \exp(\text{-r}) \exp\left(-\text{i p r}\cos(\theta)\right) r^2 \sin(\theta) d\phi d\theta dr \rightarrow \frac{2^{\frac{1}{2}}}{\pi\left[(-1)+\text{i p}\right]^2 (1+\text{i p}^2}$$

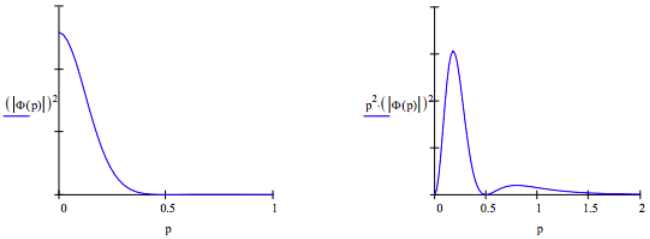
p = 0, .02 .. 5



The Fourier transform for the 2s orbital

$$\Phi(p) = \frac{1}{16\pi^2} \int_0^{\infty} \int_0^{\pi} \int_0^{2\pi} (2-r)\exp\left(-\frac{r}{2}\right) \exp\left(\text{-i p r}\cos(\theta)\right) r^2 \sin(\theta) d\phi d\theta dr$$

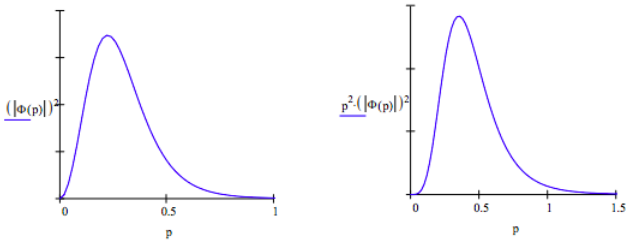
$$\text{yields} \quad \Phi(p) = \frac{-16}{\pi} \frac{(-1)+4p^2}{\left[(-1)+2\text{i p}\right]^3 (1+2\text{i p})^3} \qquad p = 0, .02..2$$



The Fourier transform for the $2p_z$ orbital

$$\Phi(p) = \frac{1}{16\pi^2} \int_0^{\infty} \int_0^{\pi} \int_0^{2\pi} \text{r} \exp\left(-\frac{r}{2}\right) \exp\left(\text{-i p r}\cos(\theta)\right) r^2 \sin(\theta) d\phi d\theta dr$$

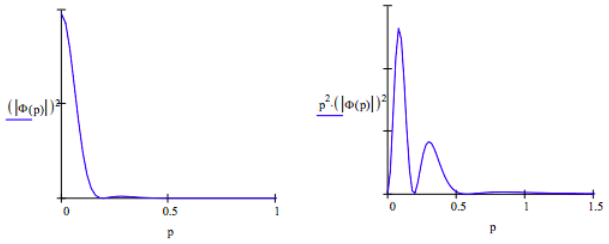
$$\text{yields} \quad \Phi(p) = 64 \frac{i}{\pi} \frac{p}{\left[(-1)+2\text{i p}^3 (1+2\text{i p}^3\right.}$$



The Fourier transform for the 3s orbital

$$\Phi(p) = \frac{1}{162\sqrt{2}\pi^2} \int_0^{\infty} \int_0^{\pi} \int_0^{2\pi} \left(27-18r+2r^2\right) \exp\left(-\frac{r}{3}\right) \exp\left(\text{-i p r}\cos(\theta)\right) r^2 \sin(\theta) d\phi d\theta dr$$

$$\text{yields} \quad \Phi(p) = 18 \frac{6^{\frac{1}{2}}}{\pi} \frac{(-30)p^2+1+81p^4}{\left[(-1)+3\text{i p}\right]^4 (1+3\text{i p})^4}$$



The Fourier transform for the $3p_z$ orbital

$$\Phi(p) = \frac{1}{162\pi^2} \int_0^{\infty} \int_0^{\pi} \int_0^{2\pi} \left(6r - r^2\right) \exp\left(-\frac{r}{3}\right) \exp\left(\text{-i p r cos}(\theta)\right) \cos(\theta) r^2 \sin(\theta) d\phi d\theta dr$$

$$\text{yields} \quad \Phi(p) = (-432)\frac{i}{\pi} p \frac{9p^2 - 1}{\left[(-1) + 3\text{i p}\right]^4 (1 + 3\text{i p})^4}$$