10.10: Variation Method for a Particle in a Gravitational Field

The particle of unit mass in a gravitational field for which g = 1 has the energy operator shown below.

$$-\frac{1}{2}\frac{d^2}{dz^2}\blacksquare + z\blacksquare$$

$$\int_0^\infty \blacksquare dz$$

The following trial wave function for this problem is:

$$\Phi(\alpha, z) := 2\left(\frac{2\alpha}{\pi^{\frac{1}{3}}}\right)^{\frac{3}{4}} z\, exp(-\alpha z^2)$$

Determine whether or not the wave function is normalized.

$$\int_0^\infty \Psi(\alpha, z)^2 dz \Big|_{simplify}^{assume,\ \alpha>0} \rightarrow 1$$

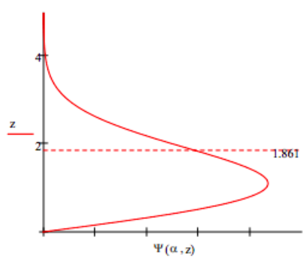
Evaluate the variational energy integral.

$$E(\alpha) := \int_0^\infty \Psi(\alpha, z) - \frac{1}{2}\frac{d^2}{dz^2}\Phi(\alpha, z) dz \ldots \Big|_{simplify}^{assume,\ \alpha>0} \rightarrow \frac{1}{2\pi^{\frac{1}{2}}}\frac{3\pi^{\frac{1}{2}}\alpha^2 + 2(2)^{\frac{1}{2}}\alpha^{\frac{1}{2}}}{\alpha} + \int_0^\infty z\Psi(\alpha, z)^2 dz$$

Minimize the energy with respect to the variational parameter α and report its optimum value and the ground-state energy.

$$\alpha := 1 \ \alpha := Minimize(E, \alpha) \ \alpha = 0.4136 \ E(\alpha) = 1.8611 \ E_{exact} := 1.8558$$

Plot the wave function with the distance of the particle from the surface on the vertical axis.



Tunneling threshold.

$$E(0.4136) = z \Big|_{float,4}^{solve,z} \rightarrow 1.861$$

Find that distance below which there is a 90% probability of finding the particle.

$$\alpha := 1$$

$$\text{Given } \int_0^a \Psi(\alpha, z)^2 dz = .90$$

Find (a) = 1.9440

Find the most probable value of the position of the particle from the surface.

$$\frac{d}{dz}\Psi(0.4136, z) = 0 \Big|_{float,\ 3}^{solve,\ z} \rightarrow \begin{pmatrix} -1.10 \\ 1.10 \end{pmatrix}$$

Calculate the probability that the particle will be found below the most probable distance from the surface.

$$\int_0^{1.10} \Psi(\alpha, z)^2 dz = 0.4279$$

Calculate the probability that tunneling is occurring: $\int_{1.861}^\infty \Psi(\alpha, z)^2 dz = 0.1256$

Kinetic energy: $\int_0^\infty \Psi(\alpha, z) - \frac{1}{2}\frac{d^2}{dz^2}\Psi(\alpha, z) dz = 0.6204$

Potential energy: $\int_0^\infty z\Psi(\alpha, z)^2 dz = 1.2407$

What is the apparent virial theorem for this system: $E = 3T = \frac{3}{2}V$