# 2.58: The Wigner Distribution for the 3s State of the 1D Hydrogen Atom

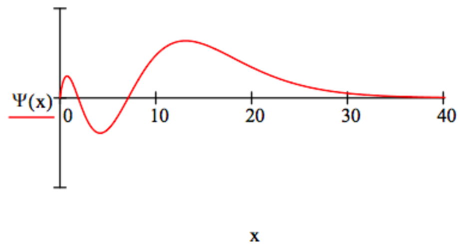
This tutorial presents three pictures of the 3s state of the one-dimensional hydrogen atom using its position, momentum and phase-space representations.

The energy operator for the one-dimensional hydrogen atom in atomic units is:

$$\frac{-1}{2}\frac{d^2}{dx^2}\blacksquare - \frac{1}{x}\blacksquare$$

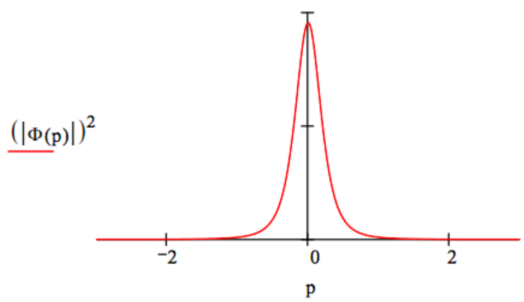
The position 3s wave function is:

$$\Psi(x) = \frac{2}{243}\sqrt{3}\mathrm{x}\left(27 - 18x + 2x^2\right)\exp\left(-\frac{x}{3}\right) \qquad \int_0^\infty \Psi(x)^2 dx = 1$$




The 3s energy is -0.065 $E_h$.

$$\Phi(p) = \frac{1}{\sqrt{2\pi}}\int_0^\infty \text{exp(-i p x)}\Psi(x)\text{dx} \rightarrow \left(-2^{\frac{1}{2}}\right)3^{\frac{1}{2}}\frac{9p^2 + 6ip - 1}{(3ip+1)^4\pi^{\frac{1}{2}}}$$

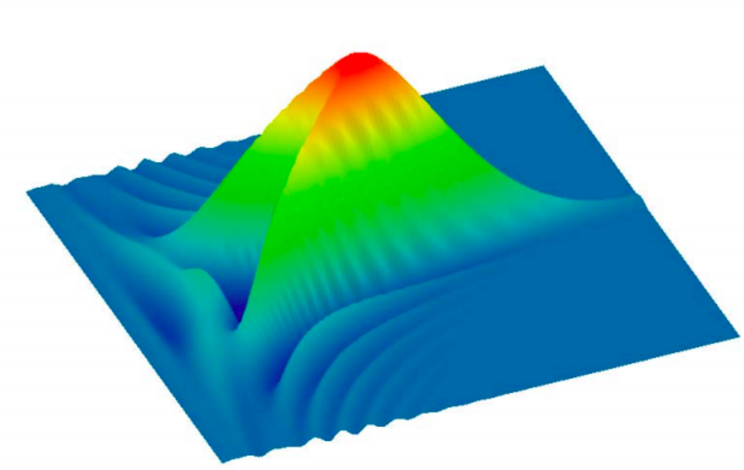



The Wigner function (phase-space representation) for the hydrogen atom 3s state is generated using the momentum wave function.

$$\mathrm{W(x,\,p)} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\overline{\Phi\left(\mathrm{p} + \frac{}{2}\right)}\,\text{exp(-i s x)}\Phi\left(\mathrm{p} - \frac{\mathrm{s}}{2}\right)\mathrm{ds}$$

The Wigner distribution is displayed graphically.

$$N = 100 \quad i = 0..N \quad x_i = \frac{30i}{N} \quad j = 0..N \quad p_j = -2 + \frac{4j}{N} \quad \mathrm{Wigner}_{i,\,j} = \mathrm{W}\left(x_i,\,p_j\right)$$



Wigner

Just as for the 2s state, the Wigner distribution for the 3s state takes on negative values.

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