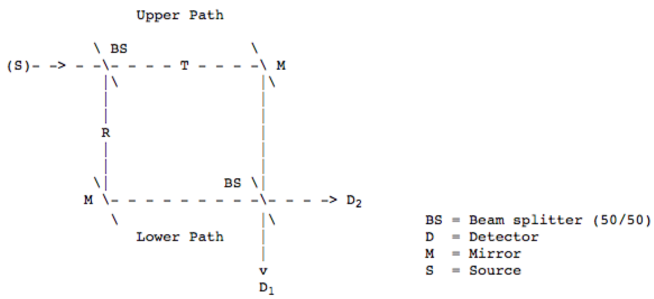
7.3: Single-photon Interference - Third Version

The schematic diagram below shows a Mach-Zehnder interferometer for photons. When the experiment is run so that there is only one photon in the apparatus at any time, the photon is always detected at D_2 and never at D_1.(1,2,3) The quantum mechanical analysis of this striking phenomenon is outlined below. There are two paths (upper and lower) to each detector, and they both contain a beam splitter, a mirror, and another beam splitter before the detectors are reached. At the beam splitters the the probability amplitude for transmission is $\left(\frac{1}{2}\right)^{\frac{1}{2}}$, while for reflection it is $\left(\frac{i}{2}\right)^{\frac{1}{2}}$. The origin of the 90° phase difference between transmission and reflection is found in the principle of energy conservation.



Because there are two paths to each detector the probability amplitudes for these paths may interfere constructively or destructively when added. For detector D_2 the probability amplitudes for the two paths interfere constructively, while for detector D_1 they interfere destructively.

For example, the probability for the photon being detected at D_2 is calculated as follows:

$$P(D_2) = |<D_2|S>|^2 = |<D_2|T><T|S> + <D_2|R><R|S>|^2 = \left|\left(\frac{i}{2}\right)^{\frac{1}{2}}\left(\frac{1}{2}\right)^{\frac{1}{2}} + \left(\frac{1}{2}\right)^{\frac{1}{2}}\left(\frac{i}{2}\right)^{\frac{1}{2}}\right|^2 = 1 \tag{7.3.1}$$

The probability that the photon will be detected at D_1 is:

$$P(D_1) = |<D_1|S>|^2 = |<D_1|T><T|S> + <D_1|R><R|S>|^2 = \left|\left(\frac{1}{2}\right)^{\frac{1}{2}}\left(\frac{1}{2}\right)^{\frac{1}{2}} + \left(\frac{i}{2}\right)^{\frac{1}{2}}\left(\frac{i}{2}\right)^{\frac{1}{2}}\right|^2 = 0 \tag{7.3.2}$$

energy conservation

Suppose there is no phase difference between transmission and reflection. Then the probability amplitudes for transmission and reflection are both $\left(\frac{1}{2}\right)^{\frac{1}{2}}$. Under these circumstances Equations 7.3.1 and 7.3.2 become

$$P(D_2) = \left|\left(\frac{1}{2}\right)^{\frac{1}{2}}\left(\frac{1}{2}\right)^{\frac{1}{2}} + \left(\frac{1}{2}\right)^{\frac{1}{2}}\left(\frac{1}{2}\right)^{\frac{1}{2}}\right|^2 = 1$$

$$P(D_1) = \left|\left(\frac{1}{2}\right)^{\frac{1}{2}}\left(\frac{1}{2}\right)^{\frac{1}{2}} + \left(\frac{1}{2}\right)^{\frac{1}{2}}\left(\frac{1}{2}\right)^{\frac{1}{2}}\right|^2 = 1$$

This result violates the principle of conservation of energy because the original photon has a probability of 1 of being detected at D_1 and also a probability of 1 of being detected at D_2. In other words, the number of photons has **doubled**. Thus, there must be a phase difference between transmission and reflection, and a 90° phase difference, as shown above, conserves energy.

References

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2. V. Scarani and A. Suarez, "Introducing Quantum Mechanics: One-particle Interferences," Am. J. Phys. 66, 718-721 (1998).
3. Kwiat, P, Weinfurter, H., and Zeilinger, A, "Quantum Seeing in the Dark," Sci. Amer. Nov. 1996, pp 72-78.