1.63: The Wigner Function for the Single Slit Diffraction Problem

The quantum mechanical interpretation of the single-slit experiment is that position is measured at the slit screen and momentum is measured at the detection screen. Position and momentum are conjugate observables connected by a Fourier transform and governed by the uncertainty principle. Knowing the slit screen geometry makes it possible to calculate the momentum distribution at the detection screen.
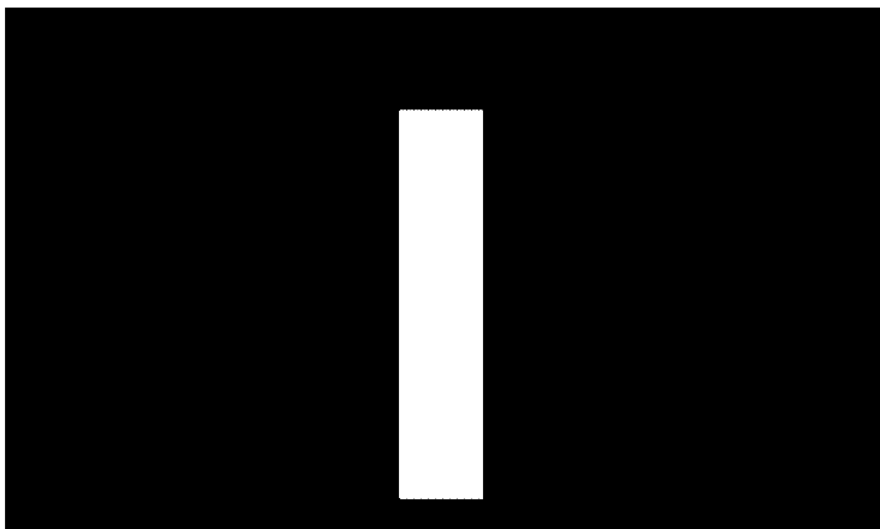
The slit-screen geometry and therefore the coordinate wavefunction is calculate as follows.

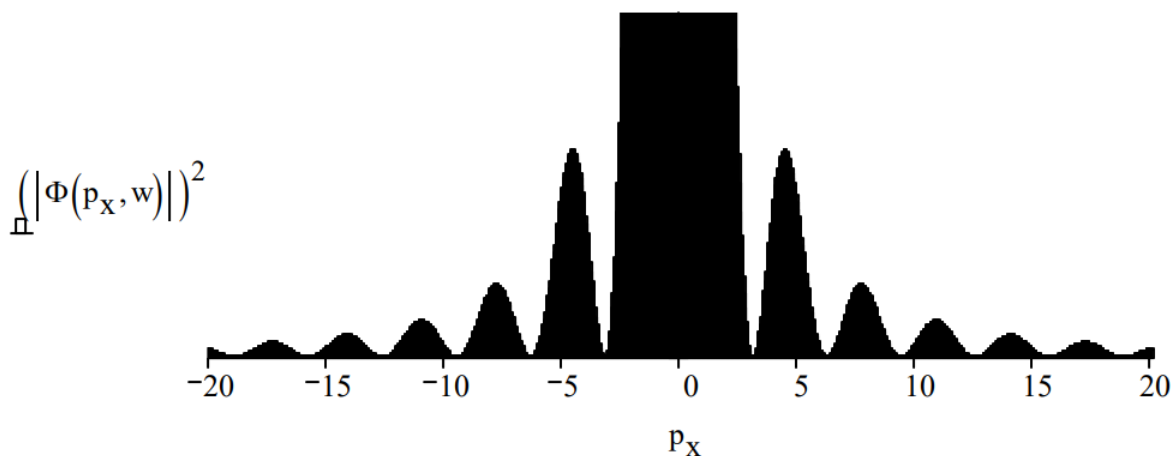Slit width: $w := 2$

Coordinate-space wave function:

$$\Psi(x, w) := \text{if}\left[\left(x \geq -\frac{w}{2}\right) \cdot \left(x \leq \frac{w}{2}\right), 1, 0\right]$$

$$x := \frac{-w}{2}, \frac{-w}{2} + .005 \ldots \frac{w}{2}$$

A Fourier transform of the coordinate-space wave function yields the momentum wave function and the momentum distribution function, which is the diffraction pattern.

$$\Phi(p_X, w) := \frac{1}{\sqrt{2 \cdot \pi \cdot w}} \cdot \int_{-\frac{w}{2}}^{\frac{w}{2}} \exp(-i \cdot p_X \cdot x) dx \text{ simplify} \to 2^{\frac{1}{2}} \cdot \frac{\sin\left(\frac{1}{2} \cdot w \cdot p_x\right)}{\pi^{\frac{1}{2}} \cdot w^{\frac{1}{2}} \cdot p_x}$$

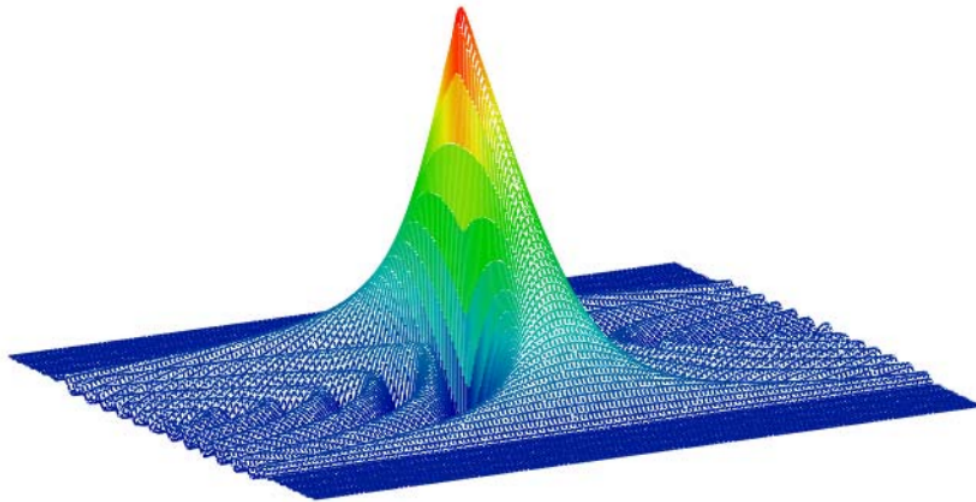$$\left(\left|\Phi(p_X, w)\right|\right)^2$$

$$p_X$$

The Wigner function for the single-slit screen geometry is generated using the momentum wave function. (Fifty is effectively infinity and is therefore as the limits of integration.)

$$W(x, p) := \frac{1}{2 \cdot \pi} \cdot \int_{-50}^{50} \overline{\Phi \left(p + \frac{s}{2}, w \right)} \cdot \exp(-i \cdot s \cdot x) \cdot \Phi \left(p - \frac{s}{2}, w \right) ds$$

The single-slit Wigner function is displayed graphically.

$$N := 150 \quad i := 0 \dots N \qquad x_i := -1.5 + \frac{3 \cdot i}{N}$$

$$j := 0 \dots N \qquad p_j := -20 + \frac{40 \cdot j}{N} \qquad \text{Wigner}_{i,j} := W(x_i, p_j)$$

Wigner

This page titled 1.63: The Wigner Function for the Single Slit Diffraction Problem is shared under a CC BY 4.0 license and was authored, remixed, and/or curated by Frank Rioux via source content that was edited to the style and standards of the LibreTexts platform.