

## 10.32: Momentum-Space Variation Method for the Abs(x) Potential

The energy operator in atomic units in coordinate space for a unit mass particle with potential energy  $V = |x|$  is given below.

$$H = -\frac{1}{2} \frac{d^2}{dx^2} + |x|$$

Suggested trial wave function:

$$\Psi(x, \beta) := \left(\frac{2\beta}{\pi}\right)^{\frac{1}{4}} \exp(-\beta x^2)$$

Demonstrate that the wave function is normalized.

$$\int_{-\infty}^{\infty} \Psi(x, \beta)^2 dx \text{ assume, } \beta > 0 \rightarrow 1$$

Carry out Fourier transform to get momentum wave function:

$$\Phi(p, \beta) := \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp(-ipx) \Psi(x, \beta) dx \stackrel{\text{assume, } \beta > 1}{\text{simplify}} \rightarrow \frac{1}{2} \frac{2^{\frac{3}{4}}}{\pi^{\frac{1}{4}}} \frac{e^{-\frac{1}{4} \frac{p^2}{\beta}}}{\beta^{\frac{1}{4}}}$$

Demonstrate that the momentum wave function is normalized.

$$\int_{-\infty}^{\infty} \overline{\Phi(p, \beta)} \Phi(p, \beta) dp \text{ assume, } \beta > 0 \rightarrow 1$$

The energy operator in momentum space is:

$$H = \frac{p^2}{2} + |i + \frac{d}{dp}|$$

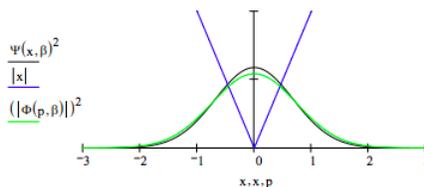
Evaluate the variational energy integral:

$$E(\beta) := \int_{-\infty}^{\infty} \overline{\Phi(p, \beta)} \frac{p^2}{2} \Phi(p, \beta) dp + \int_{-\infty}^{\infty} \overline{\Phi(p, \beta)} |i + \frac{d}{dp} \Phi(p, \beta)| dp \stackrel{\text{assume, } \beta > 0}{\text{simplify}} \frac{1}{2} \frac{\pi^{\frac{1}{2}} \beta^{\frac{3}{2}} + 2^{\frac{1}{2}}}{\beta^{\frac{1}{2}} \pi^{\frac{1}{2}}}$$

Minimize the energy with respect to the variational parameter  $\beta$  and report its optimum value and the ground-state energy.

$$\beta := 1 \quad \beta := \text{Minimize}(E, \beta) \quad \beta = 0.542 \quad E(\beta) = 0.813$$

Plot the coordinate and momentum wave functions and the potential energy on the same graph.



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