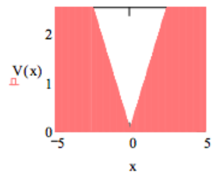
10.14: Variation Method for a Particle in a 1D Ice Cream Cone

Define potential energy: V(x) := |x|

Display potential energy:



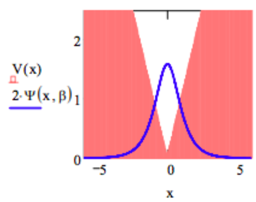
Choose trial wave function: $\Psi(x,\beta) := \sqrt{\frac{\beta}{2}}sech(\beta x)$

$$E(\beta) := \int_{-\infty}^{\infty}\Psi(x,\beta)\frac{1}{2}\frac{d^2}{dx^2}\Psi(x,\beta)dx + \int_{-\infty}^{\infty}V(x)\Psi(x,\beta)^2dx$$

Minimize the energy integral with respect to the variational parameter, β.

β := 2 β := Minimize(E, β) β = 1.276 E(β) = 0.815

Display wave function in the potential well.



Calculate the probability that the particle is in the potential barrier.

$$2\int_{0}^{\infty}\Psi(x,\beta)^2dx = 1$$

Define quantum mechanical tunneling.

> Tunneling occurs when a quon (a quantum mechanical particle) has probability of being in a nonclassical region. In other words, a region in which the total energy is less than the potential energy.

Calculate the probability that tunneling is occurring.

$$|x| = 0.815|^{solve,\,x}_{float,\,4} \rightarrow \begin{pmatrix} 0.8150 \\ -0.8150 \end{pmatrix}$$

$$2\int_{0.815}^{\infty}\Psi(x,\beta)^2dx = 0.222$$

Calculate the kinetic and potential energy contributions to the total energy.

Kinetic energy:

$$\int_{-\infty}^{\infty}\Psi(x,\beta) - (\frac{1}{2})\frac{d^2}{dx^2}\Psi(x,\beta)dx = 0.272$$

Potential energy:

$$\int_{-\infty}^{\infty}V(x)\Psi(x,\beta)^2dx = 0.543$$