1.87: Planck's Radiation Equation Fit to Experimental Data

$$n := 42 \qquad i := 1..n$$

$\rho_i :=$	$\lambda_i :=$
0.07	0.667
0.096	00.720
0.10	0.737
0.190	0.811
0.210	0.383
0.398	0.917
0.420	0.917
0.680	1.027
0.708	1.021
1.036	1.167
1.062	1.172
1.258	1.247
1.669	1.484
1.770	1.697
1.776	1.831
1.730	2.039
1.685	2.170
1.640	2.275
1.551	2.406
1.392	2.563
1.145	2.27
1.115	2.824
1.071	2.916
1.042	2.921
0.974	3.050
0.918	2.151
0.797	3.344
0.760	3.450
0.742	3.556
0.698	3.661
0.667	3.754
0.570	4.027

0.426	4.427
0.378	4.613
0.345	4.805
0.310	4.968
0.280	5.128
0.250	5.296
0.220	5.469
0.205	5.632
0.175	5.783
0.155	6.168

The data for this exercise is taken from page 19 of Eisberg and Resnick, *Quantum Physics.*

The values of rho are given in units of 10^3 joules/m^3 and the values of lambda are given in 10^{-6} m. The temperature is 1595 K.

Two pairs of data points are used to get approximate values for the parameters a and b in the Planck equation.

$$a := 1 \qquad \mathrm{b} := 1$$

Given

$$\rho_{16} = \frac{a \cdot (\lambda_{16})^{-5}}{e^{\frac{b}{\lambda_{16}}}} \qquad \rho_{22} = \frac{a \cdot (\lambda_{22})^{-5}}{e^{\frac{b}{\lambda_{22}}} - 1}$$

$$\begin{pmatrix} \mathrm{a} \\ \mathrm{b} \end{pmatrix} := \text{Find}(\mathrm{a}, \mathrm{b}) \qquad \mathrm{a} = 3.84 \times 10^3 \qquad \mathrm{b} = 8.479$$

Planck's Equation is fit to data:

$$\mathrm{F}(\lambda, \mathrm{a}, \mathrm{b}) := \frac{\mathrm{a} \cdot \lambda^{-5}}{e^{\frac{\mathrm{b}}{\lambda}} - 1}$$

$$\mathrm{SSD}(\mathrm{a}, \mathrm{b}) := \sum_{\mathrm{i}} (\rho_{\mathrm{i}} - \mathrm{F}(\lambda_{\mathrm{i}}, \mathrm{a}, \mathrm{b}))^2$$

Given

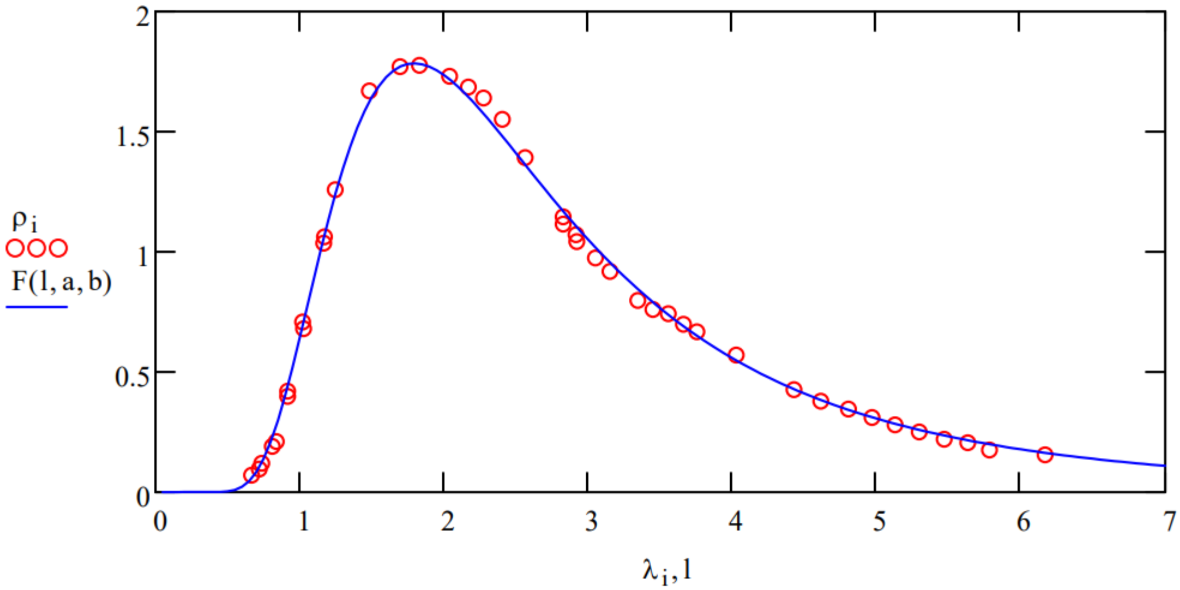
$$\mathrm{SSD}(\mathrm{a}, \mathrm{b}) = 0 \qquad \mathrm{a} > 0 \qquad \mathrm{b} > 0 \qquad \begin{pmatrix} \mathrm{a} \\ \mathrm{b} \end{pmatrix} := \mathrm{Minerr}(\mathrm{a}, \mathrm{b})$$

Display optimum values of a and b:

$$\mathrm{a} = 4.715 \times 10^3 \qquad \mathrm{b} = 8.906$$

Plot of fit:

$$1 := 0.05, 0.1 \ldots 7$$



Calculate Planck's constant using the value of b, which is equal to (hc)/(kT).

$$h := \frac{b \cdot 10^{-6} \cdot 1.381 \cdot 10^{-23} \cdot 1595}{2.9979 \cdot 10^{8}} \qquad h = 6.544 \times 10^{-34}$$