

1.72: Superposition vs. Mixture

The Wigner function can be used to illustrate the difference between a superposition and a mixture. First consider the following linear superposition of Gaussian functions.

$$\Psi(x) := \exp[-(x-5)^2] + \exp[-(x+5)^2]$$

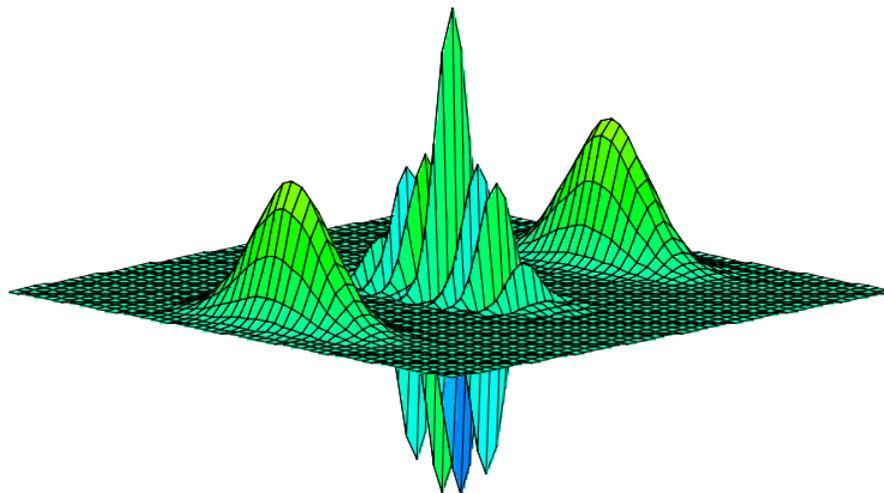
The Wigner distribution for this function is calculated and plotted below.

$$W(x, p) := \int_{-\infty}^{\infty} \left[\exp\left[-\left(x + \frac{s}{2} - 5\right)^2\right] + \exp\left[-\left(x + \frac{s}{2} + 5\right)^2\right] \right] \cdot \exp(i \cdot p \cdot s) \\ \cdot \left[\exp\left[-\left(x - \frac{s}{2} - 5\right)^2\right] + \exp\left[-\left(x - \frac{s}{2} + 5\right)^2\right] \right] ds$$

Integration yields:

$$W(x, p) := \sqrt{2} \cdot \sqrt{\pi} \\ \cdot \left(2 \cdot \exp\left(-2 \cdot x^2 - \frac{1}{2} \cdot p^2\right) \cdot \cos(10 \cdot p) + \exp\left(-2 \cdot x^2 + 20 \cdot x - 50 - \frac{1}{2} \cdot p^2\right) + \exp\left(-2 \cdot x^2 - 20 \cdot x - 50 - \frac{1}{2} \cdot p^2\right) \right)$$

$$N := 50 \quad i := 0 \dots N \quad x_i := -7 + \frac{14 \cdot i}{N} \\ j := 0 \dots N \quad p_j := -6 + \frac{12 \cdot j}{N} \quad \text{Wigner}_{i,j} := W(x_i, p_j)$$



Wigner

The signature of a superposition is the occurrence of interference fringes as seen in the center of the figure above.

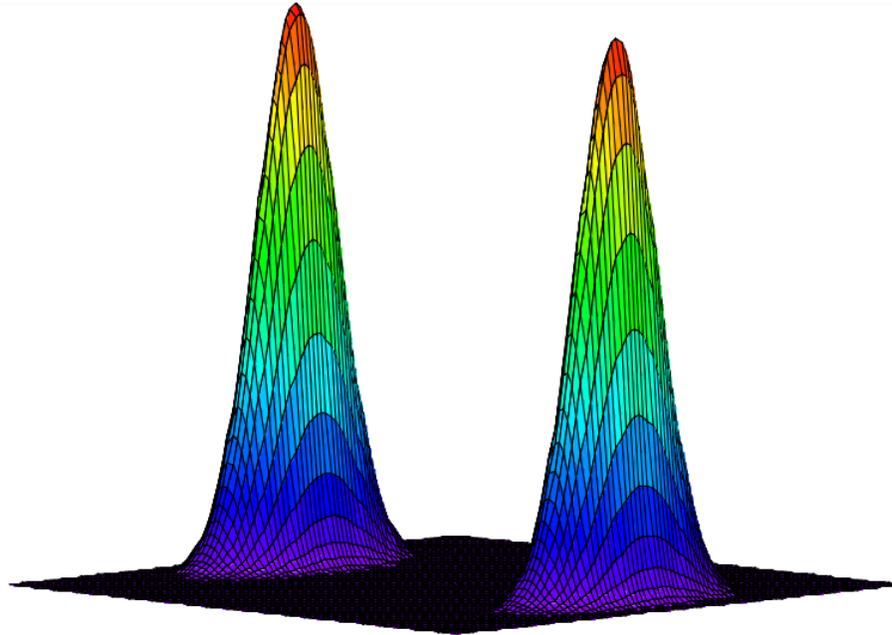
The Wigner function for a classical mixture is the sum of Wigner functions for each member of the mixture. The interference region is clearly absent in the figure shown below.

$$W(x, p) := \int_{-\infty}^{\infty} \exp\left[-\left(x + \frac{s}{2} - 5\right)^2\right] \cdot \exp(i \cdot p \cdot s) \cdot \exp\left[-\left(x - \frac{s}{2} - 5\right)^2\right] ds + \int_{-\infty}^{\infty} \exp\left[-\left(x + \frac{s}{2} + 5\right)^2\right] \cdot \exp(i \cdot p \cdot s) \cdot \exp\left[-\left(x - \frac{s}{2} + 5\right)^2\right] ds$$

Integration yields:

$$W(x, p) := \exp\left(-2 \cdot x^2 + 20 \cdot x - 50 - \frac{1}{2} \cdot p^2\right) \cdot \sqrt{2} \cdot \sqrt{\pi} + \exp\left(-2 \cdot x^2 - 20 \cdot x - 50 - \frac{1}{2} \cdot p^2\right) \cdot \sqrt{2} \cdot \sqrt{\pi}$$

$$\begin{aligned} N &:= 100 & i &:= 0 \dots N & x_i &:= -7 + \frac{14 \cdot i}{N} \\ j &:= 0 \dots N & p_j &:= -6 + \frac{12 \cdot j}{N} & \text{Wigner}_{i,j} &:= W(x_i, p_j) \end{aligned}$$



Wigner

Reference: Decoherence and the Transition from Quantum to Classical, Wojciech Jurek, Physics Today, October 1991, pages 36-44.

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