



$$\text{Top qubit flipped: } \begin{pmatrix} 0 \\ \sqrt{\frac{1}{3}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} \sqrt{\frac{2}{3}} \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \sqrt{\frac{2}{3}} \\ \sqrt{\frac{1}{3}} \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{Decode} \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ \sqrt{\frac{2}{3}} \\ \sqrt{\frac{1}{3}} \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0.577 \\ 0 \\ 0 \\ 0 \\ 0.816 \end{pmatrix} \quad \begin{pmatrix} \sqrt{\frac{1}{3}} \\ \sqrt{\frac{2}{3}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \sqrt{\frac{1}{3}} \\ 0 \\ 0 \\ 0 \\ \sqrt{\frac{2}{3}} \end{pmatrix}$$

The circuit can also be expressed using Dirac notation. Truth tables for the gates are provided in the Appendix.

$$\begin{aligned} (\sqrt{\frac{1}{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle)|00\rangle &\xrightarrow{\text{encode}} \sqrt{\frac{1}{3}}|000\rangle + \sqrt{\frac{2}{3}}|111\rangle \xrightarrow{\text{flip top}} [\text{qubit}]\sqrt{\frac{1}{3}}|100\rangle + \sqrt{\frac{2}{3}}|011\rangle \\ &\xrightarrow{\text{CNOT, I}} \sqrt{\frac{1}{3}}|110\rangle + \sqrt{\frac{2}{3}}|011\rangle \xrightarrow{\text{CnNOT}} \sqrt{\frac{1}{3}}|111\rangle + \sqrt{\frac{2}{3}}|011\rangle \\ &\xrightarrow{\text{InTofoli}} \sqrt{\frac{1}{3}}|011\rangle + \sqrt{\frac{2}{3}}|111\rangle = (\sqrt{\frac{1}{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle)|11\rangle \end{aligned}$$

Naturally the ancillary qubits are also susceptible to errors. The following examples show that if a qubit flip occurs on the middle or bottom wire, the circuit still functions properly.

$$\text{Middle qubit flipped: } \begin{pmatrix} \sqrt{\frac{1}{3}} \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \sqrt{\frac{2}{3}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \sqrt{\frac{2}{3}} \\ 0 \\ 0 \\ 0 \\ \sqrt{\frac{1}{3}} \\ 0 \end{pmatrix} \quad \text{Decode} \quad \begin{pmatrix} 0 \\ 0 \\ \sqrt{\frac{1}{3}} \\ 0 \\ 0 \\ \sqrt{\frac{2}{3}} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0.577 \\ 0 \\ 0 \\ 0 \\ 0.816 \\ 0 \end{pmatrix} \quad \begin{pmatrix} \sqrt{\frac{1}{3}} \\ \sqrt{\frac{2}{3}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \sqrt{\frac{1}{3}} \\ 0 \\ 0 \\ 0 \\ \sqrt{\frac{2}{3}} \\ 0 \end{pmatrix}$$

$$\begin{aligned} (\sqrt{\frac{1}{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle)|00\rangle &\xrightarrow{\text{encode}} \sqrt{\frac{1}{3}}|000\rangle + \sqrt{\frac{2}{3}}|111\rangle \xrightarrow{\text{flip top}} [\text{qubit}]\sqrt{\frac{1}{3}}|010\rangle + \sqrt{\frac{2}{3}}|101\rangle \\ &\xrightarrow{\text{CNOT, I}} \sqrt{\frac{1}{3}}|010\rangle + \sqrt{\frac{2}{3}}|111\rangle \xrightarrow{\text{CnNOT}} \sqrt{\frac{1}{3}}|010\rangle + \sqrt{\frac{2}{3}}|110\rangle \\ &\xrightarrow{\text{InTofoli}} \sqrt{\frac{1}{3}}|010\rangle + \sqrt{\frac{2}{3}}|110\rangle = (\sqrt{\frac{1}{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle)|10\rangle \end{aligned}$$

$$\text{Bottom qubit flipped: } \begin{pmatrix} \sqrt{\frac{1}{3}} \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ \sqrt{\frac{2}{3}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \sqrt{\frac{1}{3}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \sqrt{\frac{2}{3}} \end{pmatrix} \quad \text{Decode} \quad \begin{pmatrix} 0 \\ \sqrt{\frac{1}{3}} \\ 0 \\ 0 \\ 0 \\ 0 \\ \sqrt{\frac{2}{3}} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0.577 \\ 0 \\ 0 \\ 0 \\ 0.816 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} \sqrt{\frac{1}{3}} \\ \sqrt{\frac{2}{3}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \sqrt{\frac{1}{3}} \\ 0 \\ 0 \\ 0 \\ 0 \\ \sqrt{\frac{2}{3}} \\ 0 \end{pmatrix}$$

$$\begin{aligned} (\sqrt{\frac{1}{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle)|00\rangle &\xrightarrow{\text{encode}} \sqrt{\frac{1}{3}}|000\rangle + \sqrt{\frac{2}{3}}|111\rangle \xrightarrow{\text{flip top}} [\text{qubit}]\sqrt{\frac{1}{3}}|001\rangle + \sqrt{\frac{2}{3}}|110\rangle \\ &\xrightarrow{\text{CNOT, I}} \sqrt{\frac{1}{3}}|001\rangle + \sqrt{\frac{2}{3}}|100\rangle \xrightarrow{\text{CnNOT}} \sqrt{\frac{1}{3}}|001\rangle + \sqrt{\frac{2}{3}}|101\rangle \\ &\xrightarrow{\text{InTofoli}} \sqrt{\frac{1}{3}}|001\rangle + \sqrt{\frac{2}{3}}|101\rangle = (\sqrt{\frac{1}{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle)|01\rangle \end{aligned}$$

## Appendix

CNOT

Decimal	Binary	to	Binary	Decimal
0	00	to	00	0
1	01	to	01	1
2	10	to	11	3
3	11	to	10	2

CnNOT

Decimal	Binary	to	Binary	Decimal
0	000	to	000	0
1	001	to	001	1
2	010	to	010	2
3	011	to	011	3
4	100	to	101	5
5	101	to	100	4
6	110	to	111	7
7	111	to	110	6

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Decimal	Binary	to	Binary	Decimal
0	000	to	000	0
1	001	to	001	1
2	010	to	010	2
3	011	to	111	7
4	100	to	100	4
5	101	to	101	5
6	110	to	110	6
7	111	to	011	3

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