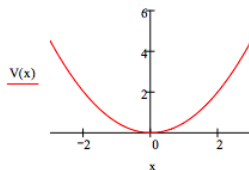


## 10.40: Variation Method Using the Wigner Function- The Harmonic Oscillator

Define potential energy:

$$V(x) = \frac{x^2}{2}$$

Display potential energy:



Choose trial wave function:

$$\psi(x, \beta) = \left(\frac{2\beta}{\pi}\right)^{\frac{1}{4}} \exp(-\beta x^2)$$

Calculate the Wigner distribution function:

$$W(x, p, \beta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi\left(x + \frac{s}{2}, \beta\right) \exp(isp) \psi\left(x - \frac{s}{2}, \beta\right) ds \Bigg|_{\text{simplify, assume } \beta > 0} \rightarrow \frac{1}{\pi} e^{-\frac{1}{2} \frac{4\beta^2 x^2 + p^2}{\beta}}$$

Evaluate the variational integral:

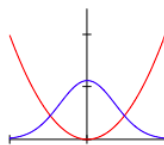
$$E(\beta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(x, p, \beta) \left(\frac{p^2}{2} + V(x)\right) dx dp$$

Minimize the energy integral with respect to the variational parameter,  $\beta$ .

$$\beta = 1 \quad \beta = \text{Minimize } (E, \beta) \quad \beta = 0.5 \quad E(\beta) = 0.5$$

Calculate and display the coordinate distribution function:

$$Px(x, \beta) = \int_{-\infty}^{\infty} W(x, p, \beta) dp$$



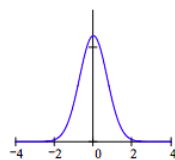
Classical turning point:  $x_{cl} = 0.5^{\frac{1}{2}} \quad x_{cl} = 0.707$

Probability that tunneling is occurring:

$$2 \int_{0.707}^{\infty} Px(x, \beta) dx = 0.317$$

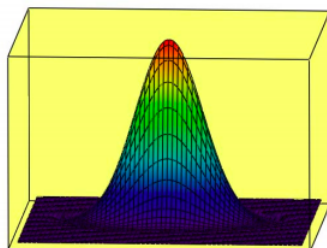
Calculate and display the momentum distribution function:

$$Pp(p, \beta) = \int_{-\infty}^{\infty} W(x, p, \beta) dx$$



Display the Wigner distribution function:

$$N = 60 \quad i = 0 \dots N \quad x_i = -3 + \frac{6i}{N} \quad j = 0 \dots N \quad p_j = -5 + \frac{10j}{N} \quad \text{Wigner}_{i,j} = W(x_i, p_j, \beta)$$



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