5.8: Is a Two-dimensional Fibonacci Array a Quasilattice?

A two-dimensional Fibonacci lattice lacks translational periodicity but has a discrete diffraction pattern, just like a quasicrystal. However, it does not fit the definition of a quasilattice because it does not posses one of the 'forbidden' n-fold rotational symmetries (n = 5 or greater than 6) that are characteristic of quasicrystals and incompatible with translational periodicity. R. Lifshitz1, therefore, recommends that the symmetry requirement be relaxed so that two- and three-dimensional Fibonacci lattices can have quasilattice stature.

A one-dimensional Fibonacci grid consists of a sequence of long (L) and short (S) segments such as LSLLSLSLLS.... with L/S = 1.618, the golden ratio. A two-dimensional array is created by superimposing two such grids at a 90^{o} angle and placing atomic scatterers at the vertices.

$$\text{Dimension of grid:} \quad A = 10 \quad m = 1..A \quad n = 1..A \quad \tau = \frac{1+\sqrt{5}}{2}$$

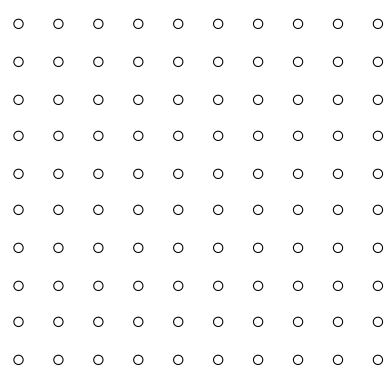
Calculate the coordinates of the Fibonacci vertices in a two-dimensional lattice (see Lifshitz).

$$x_m = \text{floor}\left(\frac{m}{\tau}\right)\tau + \left(m - \text{floor}\left(\frac{m}{\tau}\right)\right) - 1 \qquad y_n = \text{floor}\left(\frac{n}{\tau}\right)\tau + \left(n - \text{floor}\left(\frac{n}{\tau}\right)\right) - 1$$

$$x^T = (0 \quad 1.618 \quad 2.618 \quad 4.236 \quad 5.854 \quad 6.854 \quad 8.472 \quad 9.472 \quad 11.09 \quad 12.708)$$
$$y^T = (0 \quad 1.618 \quad 2.618 \quad 4.236 \quad 5.854 \quad 6.854 \quad 8.472 \quad 9.472 \quad 11.09 \quad 12.708)$$

Display the two-dimensional Fibonacci array:

2D Fibonacci Lattice



The diffraction pattern is the Fourier transform of the spatial Fibonacci array into the momentum representation.

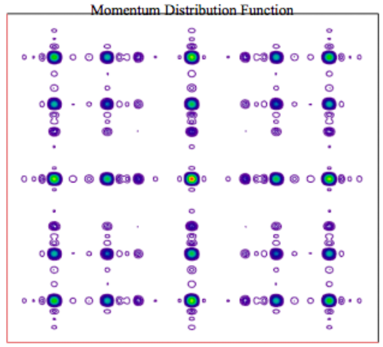
Calculate momentum-space wave function:

$$\Phi(p_x, p_y) = \frac{1}{2\pi}\sum_{m=1}^{A} exp(-ip_x x_m)\sum_{n=1}^{A} exp(-ip_y y_n)$$

Display momentum-space distribution function (diffraction pattern).

$$\Delta = 10 \quad N = 200 \quad j = 1..N \quad px_j = -\Delta + \frac{2\Delta j}{N} \quad k = 1..N \quad py_k = -\Delta + \frac{2\Delta k}{N}$$

$$\text{Diffraction Pattern}_{j,k} = \left(\left|\Phi(px_j, py_k)\right|\right)^2$$



DiffractionPattern

1. R. Lifshitz, "The square Fibonacci tiling," Journal of Alloys and Compounds 342, 186-190 (2002).