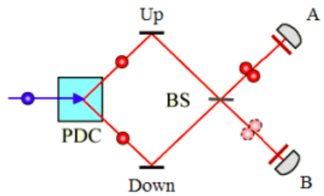
7.27: Two-particle Interference for Bosons and Fermions



A parametric down converter, PDC, transforms an incident photon into two lower energy photons. One photon takes the upper path and the other the lower path or vice versa. The principles of quantum mechanics require that the wave function for this event be written as the following entangled superposition.

Entangled superposition for **bosons**:

$$\frac{1}{\sqrt{2}}(U_1D_2+D_1U_2)$$

At the beam splitter, BS, the probability amplitude for transmission is $\frac{1}{\sqrt{2}}$ and the probability amplitude for reflection is $\frac{i}{\sqrt{2}}$. Therefore, for the four possible arrivals at the detectors we have,

$$U_1=\frac{1}{\sqrt{2}}(iA_1+B_1)$$

$$D_1=\frac{1}{\sqrt{2}}(A_1+iB_1)$$

$$U_2=\frac{1}{\sqrt{2}}(iA_2+B_2)$$

$$D_2=\frac{1}{\sqrt{2}}(A_2+iB_2)$$

Bosons are always observed at the same detector.

$$\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}(iA_1+B_1\sqrt{2}(A_2+iB_2+\sqrt{2}(A_1+iB_1\sqrt{2}(iA_2+B_2}\right]\ simplify\rightarrow\sqrt{2}\left(\frac{A_1A_2}{2}+\frac{B_1B_2}{2}\right)i$$

Entangled superposition for **fermions**:

$$\frac{1}{\sqrt{2}}U_1D_2-D_1U_2$$

Fermions are never observed at the same detector.

$$\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}(iA_1+B_1\sqrt{2}(A_2+iB_2-\sqrt{2}(A_1+iB_1\sqrt{2}(iA_2+B_2}\right]\ simplify\rightarrow\frac{\sqrt{2}A_2B_1}{2}+\frac{\sqrt{2}A_1B_2}{2}$$

In summary, the sociology of bosons and fermions can be briefly stated: bosons are gregarious and enjoy company; fermions are antisocial and prefer solitude.

Another way to do the calculation using Mathcad.

Bosons:

$$\frac{1}{\sqrt{2}}(U_1D_2 + D_1U_2) \left| \begin{array}{l} \text{substitute, } U_1 = \frac{1}{\sqrt{2}}(iA_1 + B_1) \\ \text{substitute, } D_2 = \frac{1}{\sqrt{2}}(A_2 + iB_2) \\ \text{substitute, } D_1 = \frac{1}{\sqrt{2}}(A_1 + iB_1) \\ \text{substitute, } U_2 = \frac{1}{\sqrt{2}}(iA_2 + B_2) \\ \text{simplify} \end{array} \right. \rightarrow \sqrt{2}\left(\frac{A_1A_2}{2} + \frac{B_1B_2}{2}\right) i$$

Fermions:

$$\frac{1}{\sqrt{2}}(U_1D_2 - D_1U_2) \left| \begin{array}{l} \text{substitute, } U_1 = \frac{1}{\sqrt{2}}(iA_1 + B_1) \\ \text{substitute, } D_2 = \frac{1}{\sqrt{2}}(A_2 + iB_2) \\ \text{substitute, } D_1 = \frac{1}{\sqrt{2}}(A_1 + iB_1) \\ \text{substitute, } U_2 = \frac{1}{\sqrt{2}}(iA_2 + B_2) \\ \text{simplify} \end{array} \right. \rightarrow \frac{\sqrt{2}A_2B_1}{2} - \frac{\sqrt{2}A_1B_2}{2}$$
