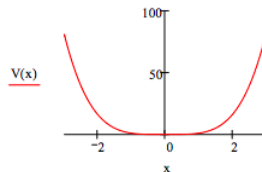# 10.41: Variation Method Using the Wigner Function - The Quartic Oscillator

Define potential energy:

$$V(x) = x^4$$

Display potential energy:

Choose trial wave function:

$$\psi(x, \beta = \left(\frac{2\beta}{\pi}\right)^{\frac{1}{4}} exp(-\beta x^2)$$

Calculate the Wigner distribution function:

$$W(x, p, \beta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi\left(x + \frac{s}{2}, \beta\right) exp(isp)\psi\left(\left(x - \frac{s}{2}, \beta\right) ds \Bigg|_{assume, \ \beta>0}^{simplify} \quad \rightarrow \frac{1}{\pi} e^{\frac{-1}{2} \frac{4\beta^2 x^2 + p^2}{\beta}}$$
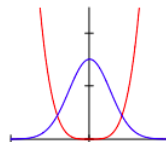
Evaluate the variational integral:

$$E(\beta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(x, p, \beta) \left(\frac{p^2}{2} + V(x)\right) dx \ dp$$

Minimize the energy integral with respect to the variational parameter, β.

$$\beta = 1 \quad \beta = \text{Minimize (E, } \beta) \quad \beta = 0.90856 \quad E(\beta) = 0.68142$$

Calculate and display the coordinate distribution function:

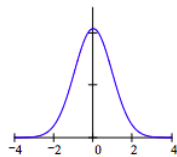$$Px(x, \beta) = \int_{-\infty}^{\infty} W(x, p, \beta) dp$$

Classical turning points: $x_{cl} = 0.681^{\frac{1}{4}} \quad x_{cl} = 0.90842$

Probability that tunneling is occurring:

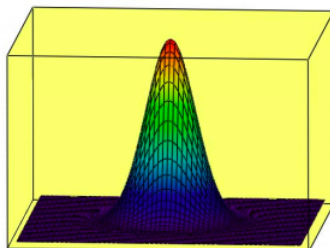$$2 \int_{0.908}^{\infty} Px(x, \beta) dx = 0.08345$$

Calculate and display the momentum distribution function:

$$Pp(p, \beta) = \int_{-\infty}^{\infty} W(x, p, \beta) dx$$

Display the Wigner distribution function:

$$N = 60 \quad i = 0 \ .. \ N \quad x_i = -3 + \frac{6i}{N} \quad j = 0 \ .. \ N \quad p_j = -5 + \frac{10j}{N} \quad \text{Wigner}_{i,\ j} = W(\ x_i,\ p_j,\ \beta)$$

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