

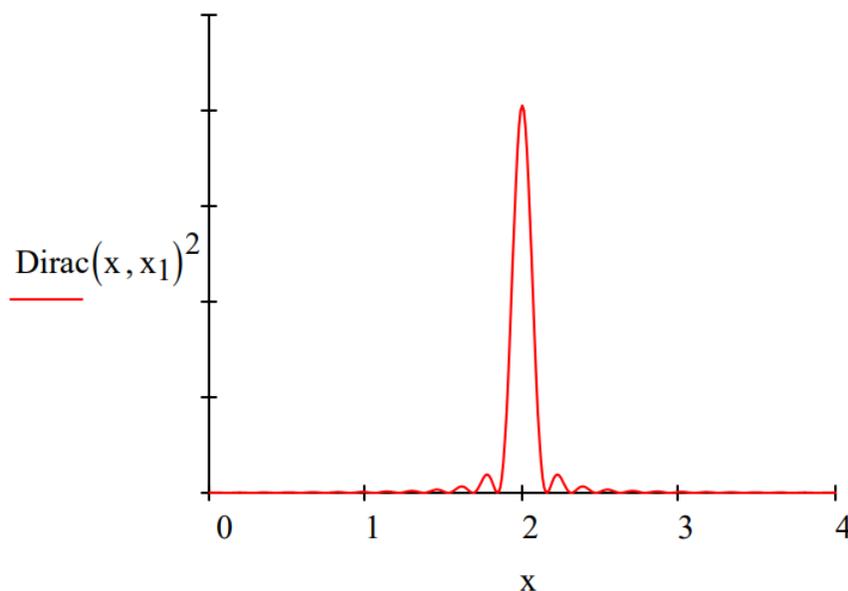
1.25: The Dirac Delta Function

The Dirac delta function expressed in Dirac notation is: $\Delta(x - x_1) = \langle x | x_1 \rangle$. The $\langle x | x_1 \rangle$ bracket is evaluated using the momentum completeness condition. See the Mathematical Appendix for definitions of the required Dirac brackets and other mathematical tools used in the analysis that follows.

$$\langle x | x_1 \rangle = \int_{-\infty}^{\infty} \langle x | p \rangle \langle p | x_1 \rangle dp = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(ipx) \exp(-ipx_1) dp = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp[ip(x - x_1)] dp$$

Evaluation of this integral over a finite range of momentum values shows that the delta function is small except in the immediate neighborhood of x_1 . Integrating from -20 to 20 to reduce computational time shows that $\langle x | x_1 = 2 \rangle$ is small except in the area $x = 2$.

$$x_1 = 2 \quad x = 0, 0.01 \dots 4 \quad \text{Dirac}(x, x_1) = \frac{1}{2 \cdot \pi} \cdot \int_{-20}^{20} \exp[i \cdot p \cdot (x - x_1)] dp$$



The Fourier transform of the Dirac delta function into the momentum representation yields the following result.

$$\int_{-\infty}^{\infty} \langle p | x \rangle \langle x | x_1 \rangle dx = \frac{1}{\sqrt{2\pi}} \exp(-ipx_1) = \langle p | x_1 \rangle$$

The normalization constant is omitted for clarity of expression and the previous value of x_1 is cleared to allow symbolic calculation.

$$x_1 = x_1 \quad \int_{-\infty}^{\infty} \exp(-i \cdot p \cdot x) \cdot \Delta(x - x_1) dx \text{ simplify } \rightarrow e^{-p \cdot x_1 \cdot i}$$

Mathematical Appendix

The position and momentum completeness conditions:

$$\int |x\rangle \langle x| dx = 1 \quad \int |p\rangle \langle p| dp = 1$$

The momentum eigenstate in the coordinate representation:

$$\langle x | p \rangle = \frac{1}{\sqrt{2\pi}} \exp(ipx)$$

The position eigenstate in the momentum representation:

$$\langle p|x\rangle = \frac{1}{\sqrt{2\pi}} \exp(-ipx)$$

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