

2.56: The Wigner Distribution for the 2s State of the 1D Hydrogen Atom

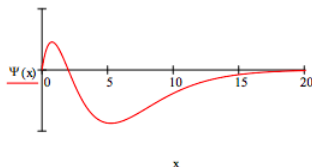
This tutorial presents three pictures of the 2s state of the one-dimensional hydrogen atom using its position, momentum and phase-space representations.

The energy operator for the one-dimensional hydrogen atom in atomic units is:

$$\frac{-1}{2} \frac{d^2}{dx^2} - \frac{1}{x}$$

The 2s wave function is:

$$\Psi(x) = \frac{1}{\sqrt{8}} x(2-x) \exp\left(-\frac{x}{2}\right) \quad \int_0^\infty \Psi(x)^2 dx = 1$$

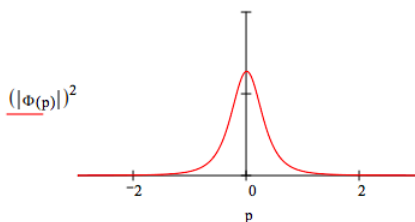


The 2s state energy is $-0.125 E_h$.

$$\frac{\frac{-1}{2} \frac{d^2}{dx^2} \Psi(x) - \frac{1}{x} \Psi(x)}{\Psi(x)} \text{ simplify } \rightarrow \frac{-1}{8}$$

The momentum wave function is generated by the following Fourier transform of the coordinate space wave function.

$$\Phi(p) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \exp(-i p x) \Psi(x) dx \rightarrow \frac{2}{\pi^{\frac{1}{2}}} \frac{2i p - 1}{(2i p + 1)^3}$$

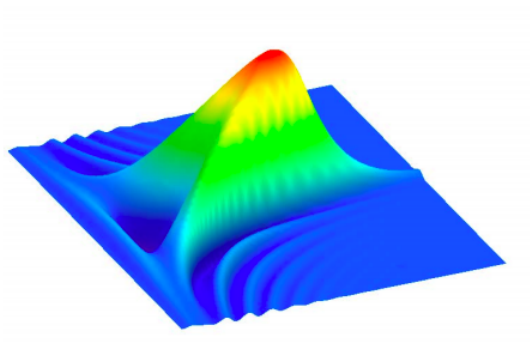


The Wigner function (phase-space representation) for the 2s state is generated using the momentum wave function.

$$W(x, p) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{\Phi\left(p + \frac{s}{2}\right)} \exp(-i s x) \Phi\left(p - \frac{s}{2}\right) ds$$

The Wigner distribution is displayed graphically.

$$N = 100 \quad i = 1 \dots N \quad x_i = \frac{15i}{N} \quad j = 0 \dots N \quad p_j = -3 \frac{6j}{N} \quad \text{Wigner}_{i,j} = W(x_i, p_j)$$



Wigner

If we rotate this figure to look below the plane, we see that for the 2s state of the 1D hydrogen atom the Wigner distribution takes on negative values.

This page titled [2.56: The Wigner Distribution for the 2s State of the 1D Hydrogen Atom](#) is shared under a [CC BY 4.0](#) license and was authored, remixed, and/or curated by [Frank Rioux](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.