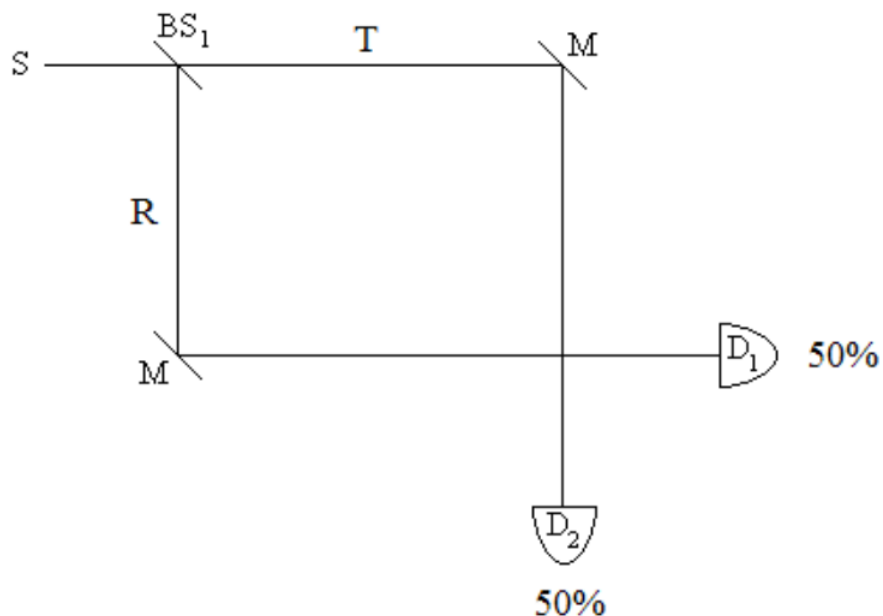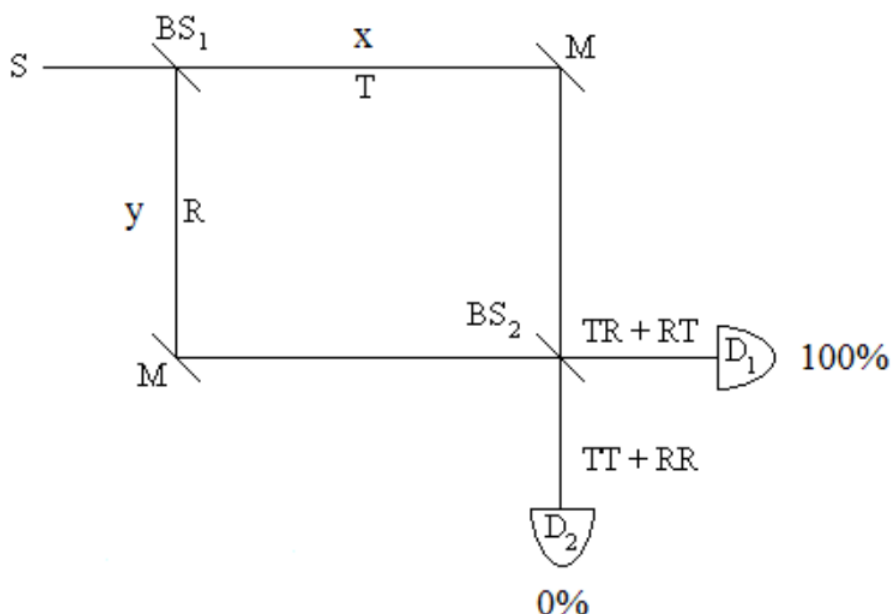# 1.56: The Paradox of Recombined Beams

French and Taylor illustrate the paradox of the recombined beams with a series of experiments using polarized photons in section 7-3 in *An Introduction to Quantum Physics*. It is my opinion that it is easier to demonstrate this so-called paradox using photons, beam splitters and mirrors. Of course, the paradox is only apparent, being created by thinking classically about a quantum phenomenon.

Single photons illuminate a 50-50 beam splitter and mirrors direct the photons to detectors D1 and D2. For a statistically meaningful number of observations, it is found that 50% of the photons are detected at D_1 and 50% at D_2. One might, therefore, conclude that each photon is either transmitted or reflected at the beam splitter.

Recombining the paths with a second beam splitter creates a Mach-Zehnder interferometer (MZI). On the basis of the previous reasoning one might expect again that each detector would fire 50% of the time. Half of the photons are in the T branch of the interferometer and they have a 50% chance of being transmitted to D_2 and a 50% chance of being reflected to D_1 at the second beam splitter. The same reasoning applies to the photons in the R branch. However what is observed in an equal arm MZI is that all the photons arrive at D_1.

The reasoning used to explain the first result is plausible, but we see that the attempt to extend it to the MZI shown below leads to a contradiction with actual experimental results. It is clear that some new concepts are required. As will be shown *probability amplitude* and the quantum *superposition* are the required concepts. They will yield predictions that are consistent with all experimental results, but they will require a non-classical, quantum way of thinking that most people find bizarre and a bit unsettling.

A Beam Splitter Creates a Quantum Superposition

The probability amplitudes for transmission and reflection at a beam splitter are given below. By convention a 90 degree phase shift (i) is assigned to reflection to conserve probability.

Probability amplitude for transmission at a 50-50 beam splitter:

$$\langle T|S\rangle = \frac{1}{\sqrt{2}}$$

Probability amplitude for reflection at a 50-50 beam splitter:

$$\langle R|S\rangle = \frac{i}{\sqrt{2}}$$

After the beam splitter the photon is in a superposition of being transmitted and reflected. In other words according to quantum theory prior to observations the photon state is not |T> or |R>, but |T> and |R>. Observation causes the superposition to collapse with equal probability to either |T> ($|D_2\rangle$) or |R> ($|D_1\rangle$) according to the Born rule and the Copenhagen interpretation.

$$S = \frac{1}{\sqrt{2}} \cdot (T + iR)$$

As shown in the first diagram, mirrors direct the transmitted photon to D_2 and the reflected photon to D_1. This can be expressed quantum mechanically as shown below.

$$T = D_2 \qquad R = D_1$$

Expressing the source photon in the basis of the detectors we have,

$$S = \frac{1}{\sqrt{2}} \cdot (T + iR) \Big|_{\substack{\text{substitute, R=D}_1 \\ \text{substitute, T=D}_2}} \rightarrow S = \sqrt{2} \cdot \left(\frac{D_2}{2} + \frac{D_1 \cdot i}{2} \right)$$

The magnitude squared of the coefficients of D_1 and D_2 gives the probabilities that the photon will be detected at D_1 and D_2. As shown below, each detector registers photons 50% of the time in agreement with experiment.

$$\text{Probability } D_1 = \left(\left| \frac{i \cdot \sqrt{2}}{2} \right| \right)^2 \rightarrow \text{Probability } D_1 = \frac{1}{2}$$

$$\text{Probability } D_2 = \left(\left| \frac{\sqrt{2}}{2} \right| \right)^2 \rightarrow \text{Probability } D_2 = \frac{1}{2}$$

A Second Beam Splitter Allows Probability Amplitudes to Interfere

The presence of the second beam splitter provides two paths to each detector and the opportunity for the interference of the probability amplitudes. The probability amplitudes for the paths to D_1 interfere constructively, while the probability amplitudes for the paths to D_2 interfere destructively. Thus, the evolution of $|T>$ and $|R>$ at the second beam splitter result in the photon always arriving at D_1.

$$T = \frac{1}{\sqrt{2}} \cdot (i \cdot D_1 + D_2) \qquad R = \frac{1}{\sqrt{2}} \cdot (D_1 + i \cdot D_2)$$

$$S = \frac{1}{\sqrt{2}} \cdot (T + iR) \left| \begin{array}{c} \text{substitute}, T = \frac{1}{\sqrt{2}} \cdot (i \cdot D_1 + D_2) \\ \text{substitute}, R = \frac{1}{\sqrt{2}} \cdot (D_1 + i \cdot D_2) \\ \text{simplify} \end{array} \right. ^{\to S = D_1 \cdot i}$$

$$\text{Probability } D_1 = (|i|)^2 \to \text{Probability } D_1 = 1$$

"The things that interfere in quantum mechanics are are the probability amplitudes for certain events. It is the fact that probability amplitudes add up like complex numbers that accounts for all quantum mechanical interferences." [Roy Glauber, *American Journal of Physics* **63**, 12 (1995)]