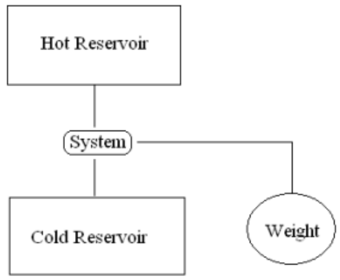
11.15: Global Thermodynamic Analyses of Heat Engines

The purpose of this tutorial is to demonstrate the clarity that the global method of doing thermodynamics brings to the analysis of heat engines, which under other names are power plants, heat pumps and refrigerators. The analyses provided are based on the schematic diagram shown below. The ″system″ is a working substance that undergoes a thermodynamic cycle ($\Delta E_{sys} = \Delta S_{sys} = 0$, under reversible conditions), so it does not appear in the mathematical analyses that follows.



Under the circumstances regarding the role of the system outlined above, the first and second laws of thermodynamics are:

$$\Delta E_{tot} = \Delta E_{hot} + \Delta E_{cold} + \Delta E_{wt} = 0$$

$$\Delta S_{tot} = \Delta S_{hot} + \Delta S_{cold} = \frac{\Delta E_{hot}}{T_{hot}} + \frac{\Delta E_{cold}}{T_{cold}} \geq 0$$

Since we will be interested in the best case scenario, we will use the equal sign for the entropy change, $\Delta S_{tot} = 0$, in our calculations.

Power Plant

A power plant harnesses the natural flow of thermal energy from a hot object to a cold object in order to generate useful energy, most likely in the form of electricity. Suppose a power plant operates with a high temperature thermal reservoir at 500K and the ambient temperature is taken to be 300K. How much electricity can, ideally ($\Delta S_{tot} = 0$), be generated per 100 J of thermal energy flowing from the high temperature reservoir to the low (ambient) temperature reservoir?

Input parameters: $\Delta E_{hot} := -100$ J $T_{hot} := 500$ K $T_{cold} := 300$ K

$$\begin{pmatrix} \Delta E_{hot} + \Delta E_{cold} + \Delta E_{wt} = 0 \\ \frac{\Delta E_{hot}}{T_{hot}} + \frac{\Delta E_{cold}}{T_{cold}} = 0 \end{pmatrix} \Big|_{float,3}^{solve, \begin{pmatrix} \Delta E_{wt} \\ \Delta E_{cold} \end{pmatrix}} \rightarrow (40.0J \; 60.0J)$$

It is clear from this analysis that the power plant is theoretically 40% efficient: $\left|\frac{40.0J}{-100J}\right| = 40\%$.

Heat Pump

Non-spontaneous processes occur when they are driven by some other spontaneous process, such as described in the previous example.

Heat pumps are in the news again today, as they were in the late 70s and early 80s. Here we ask the question of how much energy we can pump from the ambient thermal environment into a house by buying 100 J of energy from the local utility. We assume that the ambient thermal source temperature is 270K and the house is being maintained at 300K.

Clear ΔE_{hot} memory: $\Delta E_{hot} := \Delta E_{hot}$

Input parameters: $\Delta E_{wt} := -100$ J $T_{hot} := 300$ K $T_{cold} := 270$ K

$$\begin{pmatrix} \Delta E_{hot} + \Delta E_{cold} + \Delta E_{wt} = 0 \\ \frac{\Delta E_{hot}}{T_{hot}} + \frac{\Delta E_{cold}}{T_{cold}} = 0 \end{pmatrix} \Big|_{float,4}^{solve, \begin{pmatrix} \Delta E_{hot} \\ \Delta E_{cold} \end{pmatrix}} \rightarrow (1000.0J \; -900.0J)$$

This remarkable result shows that by buying 100 J from the local utility one can pump 1,000 J of thermal energy from the ambient source into the house. Again this is assuming the ideal, theoretical limit, $\Delta S_{tot} = 0$.

Refrigerator

A refrigerator is also a heat pump. However, with a refrigerator we are interested in how much energy we can pump out of the low temperature reservoir (the contents of the refrigerator) at a certain cost, rather than how much we can deliver to the high temperature reservoir. Here we ask how much energy we must purchase to pump 100 J out of the refrigerator.

Clear ΔE_{wt} memory: $\Delta E_{wt} := \Delta E_{wt}$

Input parameters: $\Delta E_{cold} := -100\ J$ $T_{hot} := 300\ K$ $T_{cold} := 280\ K$

$$\left(\begin{array}{c} \Delta E_{hot} + \Delta E_{cold} + \Delta E_{wt} = 0 \\ \frac{\Delta E_{hot}}{T_{hot}} + \frac{\Delta E_{cold}}{T_{cold}} = 0 \end{array}\right) \Big|_{float,4}^{solve, \left(\begin{array}{c} \Delta E_{hot} \\ \Delta E_{wt} \end{array}\right)} \rightarrow (107.1 J \quad -7.143 J)$$

This result tells us something we already know, but doesn′t obtrude on our senses: **operating a refrigerator is not particularly expensive**. One hundred joules can be pumped out of the refrigerator for a cost of just over 7 joules, under ideal circumstances.
