1.85: The Difference Between Fermions and Bosons

$$n_1 := 1 \qquad n_2 := 2$$
$$\Psi(x) := \sqrt{2}\cdot\sin(n_1\cdot\pi\cdot x) \qquad \Phi(x) := \sqrt{2}\cdot\sin(n_2\cdot\pi\cdot x)$$

Calculate the average separation, $|x_1 - x_2|$, for two fermions and two bosons in a 1D box of unit length.

Fermions have antisymmetric wave functions:

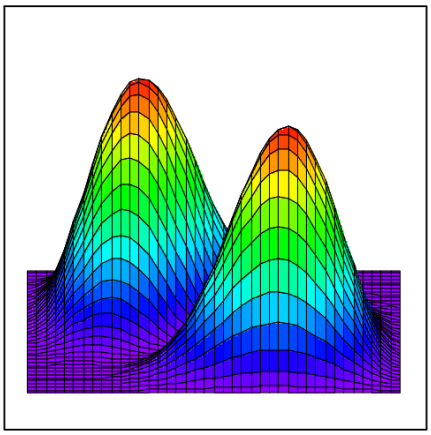
$$\Psi_f(x_1, x_2) := \frac{\Psi(x_1)\cdot\Phi(x_2) - \Psi(x_2)\cdot\Phi(x_1)}{\sqrt{2}}$$

The average particle separation for indistinquishable fermions:

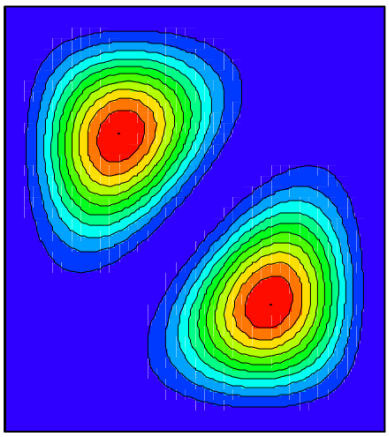
$$\int_0^1 \int_0^1 \Psi_f(x_1, x_2)\cdot|x_1 - x_2|\cdot\Psi_f(x_1, x_2)\,dx_1 dx_2 = 0.383$$

The particles are correlated so as to keep them apart.

$$N := 40 \qquad i := 0..N$$
$$x_{1_i} := \frac{1}{N} \quad j := 0..N \qquad x_2 := \frac{j}{N}$$
$$\Psi_{f_{i,j}} := \Psi_{f\left(x_{1_i}, x_{2_j}\right)^2}$$



Ψ_f



x_2

Ψ_f

Bosons have symmetric wave functions:

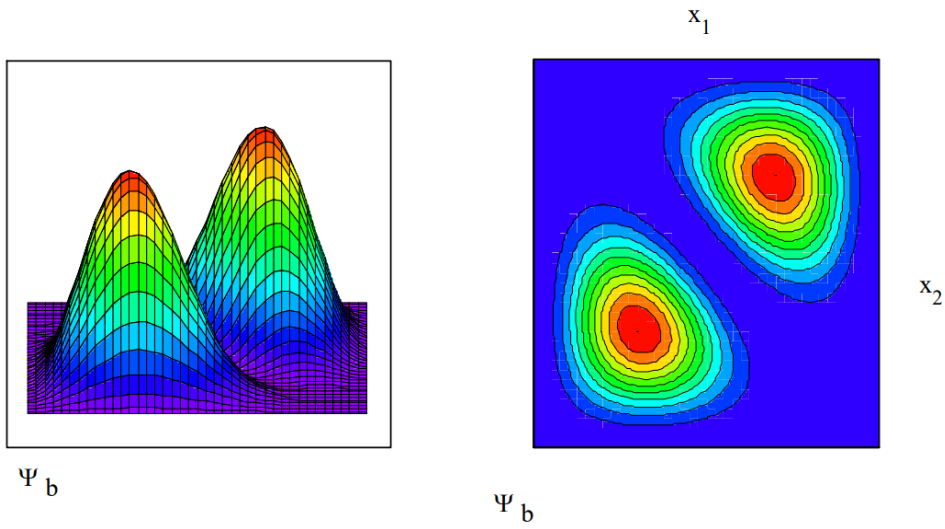
$$\Psi_b(x_1, x_2) = \frac{\Psi(x_1)\cdot\Phi(x_2) + \Psi(x_2)\cdot\Phi(x_1)}{\sqrt{2}}$$

The average particle separation for indistinquishable bosons:

$$\int_0^1 \int_0^1 \Psi_b(x_1, x_2)\cdot|x_1 - x_2|\cdot\Psi_b(x_1, x_2)\,dx_1 dx_2 = 0.157$$

The particles are correlated so as to bring them closer together.

$$N := 40 \qquad i := 0..N$$
$$x_{1_i} := \frac{1}{N} \quad j := 0..N \qquad x_2 := \frac{j}{N}$$
$$\Psi_{b_{i,j}} := \Psi_{b\left(x_{1_i}, x_{2_j}\right)^2}$$



All fundamental particles (electrons, neutrons, protons, photons, etc.) are either bosons or fermions. Composite entities such as the elements also fall into these two categories. The fundamental distinction is spin: bosons have integer spin (0, 1, 2, ...) while fermions have half-integer spin (1/2, 3/2, ...).

The dramatic difference in behavior between bosons and fermions has led to a sociology of fundamental particles. Bosons are social and gregarious, while fermions are antisocial and aloof.
