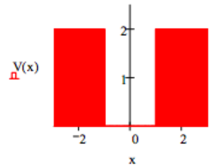
10.38: Variation Method Using the Wigner Function- Finite Potential Well

Define potential energy:

$$V(x) = if[(x \geq -1)(x \leq 1), 0, 20$$

Display potential energy:



Choose trial wave function:

$$\psi(x,\beta) = \left(\frac{2\beta}{\pi}\right)^2 exp(-\beta x^2)$$

Calculate the Wigner distribution function:

$$W(x,p,\beta) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \psi\left(x + \frac{s}{2}, \beta\right) ds\Big|_{assume,\ \beta>0}^{simplify} \rightarrow \frac{1}{\pi} e^{\frac{-1}{2}\ \frac{4\beta^2 x^2 + p^2}{\beta}}$$

Evaluate the variational integral:

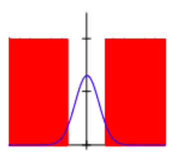
$$E(\beta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (W, x, p, \beta)\left(\frac{p^2}{2} V(x)\right) dx\ dp$$

Minimize the energy integral with respect to the variational parameter, β.

β = 1 β = Minimize (E, β) β = 0.678 E(β) = 0.538

Calculate and display the coordinate distribution function:

$$P(x,\beta) = \int_{-\infty}^{\infty} W(x,p,\beta) dp$$

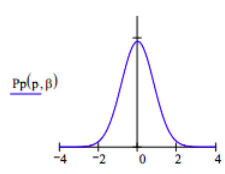


Probability that tunneling is occuring:

$$2\int_{1}^{\infty} P(x,\beta) dx = 0.1$$

Calculate and display the momentum distribution function:

$$Pp(p,\beta) = \int_{-\infty}^{\infty} W(x,p,\beta) dx$$



Display the Wigner distribution function:

N = 60 i = 0 .. N $x_i = -3 + \frac{6j}{N}$ j =) .. N $p_j = -5 + \frac{10j}{N}$ $Wigner_{i,j} = W(x_i, p_j, \beta$