

8.14: Greenberger-Horne-Zeilinger (GHZ) Entanglement and Local Realism

This tutorial summarizes experimental results on GHZ entanglement reported by Anton Zeilinger and collaborators in the 3 February 2000 issue of Nature (pp. 515-519). The GHZ experiment employs three-photon entanglement to provide a stunning attack on local realism.

Definitions

First some definitions:

- **Realism** - experiments yield values for properties that exist independent of experimental observation
- **Locality** - the experimental results obtained at location A at time t, do not depend on the results at some other location B at time t.
- **H/V** = horizontal/vertical linear polarization. **R/L** = right/left circular polarization. **H'/V'** rotated by 45° with respect to H/V.

Next some relationships between the various photon polarization states: See the appendix for vector definitions of $|H\rangle$, $|V\rangle$, $|H'\rangle$, $|V'\rangle$, $|R\rangle$ and $|L\rangle$.

$$\begin{aligned} H' &= \frac{1}{\sqrt{2}}(H + V) & V' &= \frac{1}{\sqrt{2}}(H - V) & R &= \frac{1}{\sqrt{2}}(H + iV) & L &= \frac{1}{\sqrt{2}}(H - iV) \\ H &= \frac{1}{\sqrt{2}}(H' + V') & V &= \frac{1}{\sqrt{2}}(H' - V') & H &= \frac{1}{\sqrt{2}}(R + L) & V &= \frac{i}{\sqrt{2}}(L - R) \end{aligned}$$

The initial GHZ three-photon entangled state:

$$\Psi = \frac{1}{\sqrt{2}}(H_1 H_2 H_3 + V_1 V_2 V_3)$$

After preparation of the initial GHZ state (see figure 1 in the reference cited above), polarization measurements are performed on the three photons. Zeilinger and collaborators use y to stand for a circular polarization measurement and x for a linear polarization measurement. Initially they perform circular polarization measurements on two of the photons and a linear polarization measurement on the other photon. The quantum mechanically predicted results and actual experimental measurements are given below. The quantum predictions are obtained by substituting equations (2) into equation (3).

yyx - experiment

$$\Psi_{yyx} = \frac{1}{\sqrt{2}} \left[\frac{R_1 + L_1}{\sqrt{2}} \frac{R_2 + L_2}{\sqrt{2}} \frac{H'_3 + V'_3}{\sqrt{2}} + \frac{i(L_1 - R_1)}{\sqrt{2}} \frac{i(L_2 - R_2)}{\sqrt{2}} \frac{H'_3 - V'_3}{\sqrt{2}} \right]$$

on expansion yields

$$\Psi_{yyx} = \frac{1}{2} R_1 R_2 V'_3 + \frac{1}{2} R_1 L_2 H'_3 + \frac{1}{2} L_1 R_2 H'_3 + \frac{1}{2} L_1 R_2 H'_3 + \frac{1}{2} L_1 L_2 V'_3$$

Each measurement (R/L or H'/V') has two possible outcomes so, in principle, there could be 8 possible results. However, quantum mechanics predicts that only four equally probable, $(\frac{1}{2})^2 = 0.25$, outcomes are possible. The quantum mechanical prediction is in agreement with the experimental results shown in Figure 1 to within experimental error.

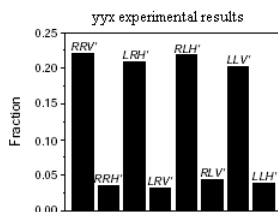


Figure 1.

For the two remaining experiments in this class (yxy and xyy), the agreement between theoretical prediction and experimental results is basically the same. While these agreements between quantum mechanics and experiment are impressive they do not directly challenge the local realist position. As will be shown later that will be accomplished by a fourth experiment involving the measurement of the linear polarization on all three photons - the xxx experiment.

yxy - experiment

$$\Psi_{yxy} = \frac{1}{\sqrt{2}} \left[\frac{R_1 + L_1}{\sqrt{2}} \frac{H'_2 + V'_2}{\sqrt{2}} \frac{R_3 + L_3}{\sqrt{2}} + \frac{i(L_1 - R_1)}{\sqrt{2}} \frac{i(H'_2 - V'_2)}{\sqrt{2}} \frac{L_3 - R_3}{\sqrt{2}} \right]$$

on expansion yields

$$\Psi_{yxy} = \frac{1}{2} R_1 H'_2 L_3 + \frac{1}{2} R_1 V'_2 R_3 + \frac{1}{2} L_1 H'_2 R_3 + \frac{1}{2} L_1 V'_2 L_3$$

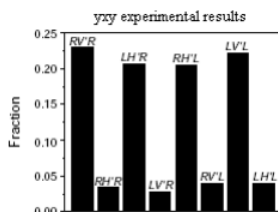


Figure 2.

xyy - experiment

$$\Psi_{xyy} = \frac{1}{\sqrt{2}} \left[\frac{H'_1 + V'_1}{\sqrt{2}} \frac{R_2 + L_2}{\sqrt{2}} \frac{R_3 + L_3}{\sqrt{2}} + \frac{H'_1 - V'_1}{\sqrt{2}} \frac{i(L_2 - R_2)}{\sqrt{2}} \frac{i(L_3 - R_3)}{\sqrt{2}} \right]$$

on expansion yields

$$\Psi_{xyy} = \frac{1}{2} H'_1 R_2 L_3 + \frac{1}{2} H'_1 L_2 R_3 + \frac{1}{2} V'_1 R_2 R_3 + \frac{1}{2} V'_1 L_2 L_3$$

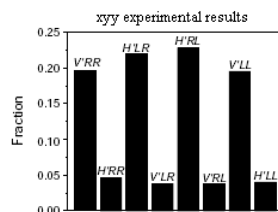


Figure 3.

Now for the critical experiment.

xxx - experiment

$$\Psi_{xxx} = \frac{1}{\sqrt{2}} \left[\frac{H'_1 + V'_1}{\sqrt{2}} \frac{H'_2 + V'_2}{\sqrt{2}} \frac{H'_3 + V'_3}{\sqrt{2}} + \frac{H'_1 - V'_1}{\sqrt{2}} \frac{i(H'_2 - V'_2)}{\sqrt{2}} \frac{H'_3 - V'_3}{\sqrt{2}} \right]$$

on expansion yields

$$\Psi_{xxx} = \frac{1}{2} H'_1 H'_2 H'_3 + \frac{1}{2} H'_1 V'_2 V'_3 + \frac{1}{2} V'_1 H'_2 V'_3 + \frac{1}{2} V'_1 V'_2 H'_3$$

This quantum mechanical prediction is displayed graphically in Figure 4.

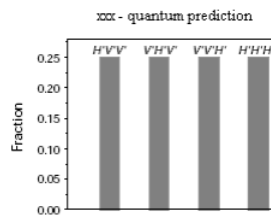


Figure 4.

According to local realism the experimental outcome should be as shown in Figure 5. The origin of this prediction will be outlined shortly.

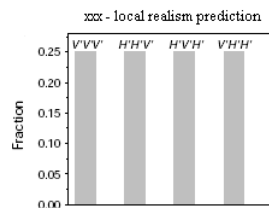


Figure 5.

The quantum mechanical prediction for the xxx experiment agrees with experiment, the local realist prediction doesn't.

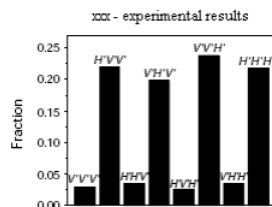


Figure 6.

To explain how the local realist prediction shown in Figure 5 is derived, we recall that this position assumes that physical properties exist independent of measurement. We associate with polarization measurements the following eigenvalues: $H' = +1$, $V' = -1$, $R = +1$ and $L = -1$. Substituting these measurement eigenvalues into the first three experimental results (yyx, xyx, xyy) yields,

$$Y_1 Y_2 Y_3 = Y_1 X_2 Y_3 = X_1 Y_2 Y_3 = -1$$

Therefore,

$$(Y_1 Y_2 Y_3)(Y_1 X_2 Y_3)(X_1 Y_2 Y_3) = -1$$

However, this means that $X_1 X_2 X_3 = -1$ because $Y_1 Y_1 = Y_2 Y_2 = Y_3 Y_3 = 1$. According to the local realist view there are four ways to achieve this result V1V2V3, H1H2V3, H1V2H3 and V1H2H3, none of which are observed at a statistically meaningful level in the GHZ experiment.

For the xxx experiment, the mathematical predictions of quantum mechanics (Figure 4) and the local realist view (Figure 5) when compared with the actual experimental results (Figure 6) present a convincing refutation of local realism.

Appendix

$$H = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad V = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad H' = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad V' = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad L = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} \quad R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$$

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