11.7: Examining Fourier Synthesis with Dirac Notation

The purpose of this tutorial is to use Dirac notation to examine Fourier synthesis. The first step is to write the function symbolically in Dirac notation.

$$f(x) = \langle x|f\rangle$$

Select an orthonormal basis set, |n>, for which the completeness relation holds

$$\sum_n |n\rangle\langle n| = 1$$

Expand |f> in terms of |n> by inserting equation (2) into the right side of equation (1). In other words write f(x) as a weighted () superposition using the basis set (the |n> basis set expressed in the coordinate representation).

$$f(x) = \sum_n \langle x|n\rangle\langle n|f\rangle$$

Evaluate the Fourier coefficient, , using the continuous completeness relation in coordinate space.

$$\int |x'\rangle\langle x'|dx' = 1$$

Equation (3) becomes,

$$f(x) = \sum_n \langle x|n\rangle \int \langle n|x'\rangle\langle x'|f\rangle dx'$$

Now select a function

$$\langle x'|f\rangle = x'^3(1-x')$$

over the interval (0,1). Choose the following orthonormal basis set over the same interval.

$$\langle x|n\rangle = \sqrt{2}sin(n\pi x)$$

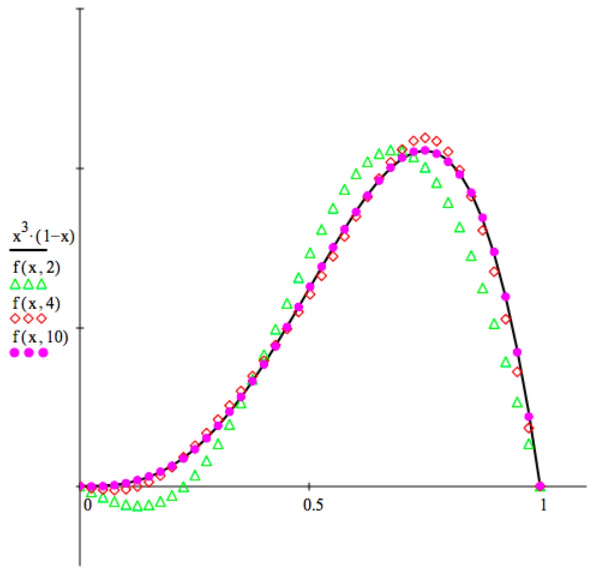
Substitution of equations (6) and (7) into (5) yields

$$f(x) = \sum_n \sqrt{2}sin(n\pi x)\int_0^1 \sqrt{2}(n\pi x')x'^3(1-x')dx'$$

The Fourier synthesis and the original function are shown for n = 2, 4, and 10 in the figure below.

$$x := 0, .025..1.0$$

$$f(x,n) := \sum_{i=1}^{n}[\sqrt{2}\cdot sin(i\cdot\pi\cdot x)\cdot\int_0^1 \sqrt{2}\cdot sin(i\cdot\pi\cdot x')\cdot x'^3\cdot(1-x')dx')]$$



$x^3 \cdot (1-x)$
f(x, 2)
f(x, 4)
f(x, 10)
0
0.5
1