

10.15: Variation Method for a Particle in an Ice Cream Cone

A Gaussian function is proposed as a trial wavefunction in a variational calculation for a particle experiencing a linear radial potential energy. Determine the optimum value of the parameter β and the optimum ground state energy. Use atomic units: $\hbar = 2\pi$, $m_e = 1$, $e = -1$.

$$\psi(r, \beta) := \left(\frac{2\beta}{\pi}\right)^{\frac{3}{4}} \exp(-\beta r^2)$$

$$T = \frac{1}{2r} \frac{d^2}{dr^2} (r \psi)$$

$$V = r$$

$$\int_0^\infty \psi^2 4\pi r^2 dr$$

a. Demonstrate the wave function is normalized.

$$\int_0^\infty \psi(r, \beta)^2 4\pi r^2 dr \Big|_{\text{simplify}}^{\text{assume, } \beta > 0} \rightarrow 1$$

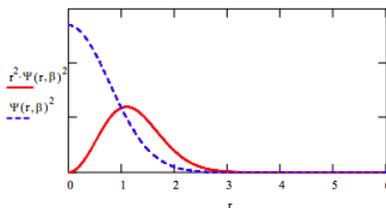
b. Evaluate the variational integral.

$$E(\beta) := \int_0^\infty \psi(r, \beta) \left[\left(-\frac{1}{2r}\right) \frac{d^2}{dr^2} (r\psi(r, \beta)) \right] 4\pi r^2 dr \dots \Big|_{\text{simplify}}^{\text{assume, } \beta > 0} \rightarrow \frac{1}{2} \frac{3\pi^{\frac{1}{2}} \beta^2 + (2)2^{\frac{1}{2}} \beta^{\frac{1}{2}}}{\pi^{\frac{1}{2}} \beta}$$

c. Minimize the energy with respect to the variational parameter β .

$$\beta := 1 \quad \beta := \text{Minimize}(E, \beta) \quad \beta = 0.414 \quad E(\beta) = 1.861$$

d. Plot the optimized trial wave function.



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