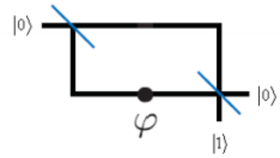
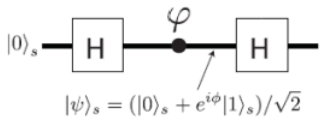
7.22: A Quantum Circuit for a Michelson Interferometer

Schematic diagram of a Mach-Zehnder interferometer (MZI).



The following quantum circuit simulates the MZI.



The arms of the MZI are represented by the following orthonormal basis.

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The matrices representing the Hadamard and phase shift gates are:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$A(\theta) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

Step-by-step through the circuit. The first Hadamard gate creates a superposition of the |0 > and |1 > states. The phase shifter operates on the lower arm of the MZI. The final Hadamard gate allows interference between the two arms of the MZI.

$$H \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix}$$

$$A(\phi) H \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2} e^{\phi i}}{2} \end{pmatrix}$$

$$H A(\phi) H \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{e^{\phi i}}{2} + \frac{1}{2} \\ \frac{1}{2} - \frac{e^{\phi i}}{2} \end{pmatrix}$$

Probability of detection at the |0 > port:

$$P_0(\phi) = \left[\left| \begin{pmatrix} 1 & 0 \end{pmatrix} H A(\phi) H \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right| \right]^2 \Big|_{simplify}^{assume,\ \phi = real} \rightarrow \frac{\cos\phi}{2} + \frac{1}{2}$$

Probability of detection at the |1 > port:

$$P_1(\phi) = \left[\left| \begin{pmatrix} 0 & 1 \end{pmatrix} H A(\phi) H \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right| \right]^2 \Big|_{simplify}^{assume,\ \phi = real} \rightarrow \frac{1}{2} - \frac{\cos\phi}{2}$$

A graphical representation of the above calculations shows the interference effects as a function of ϕ.

$$\phi := 0, .01\cdot\pi .. 4\cdot\pi$$