10.1: Trial Wavefunctions for Various Potentials

This is list of functions and the potentials for which they would be suitable trial wave functions in a variation method calculation.

$$\psi(x,\alpha) = 2\cdot\alpha^{\frac{3}{2}}\cdot x\cdot exp(-\alpha\cdot x)$$

$$\psi(x,\alpha) = (\frac{128\cdot\alpha^3}{\pi})^{\frac{1}{4}}\cdot exp(-\alpha\cdot x^2)$$

- Particle in a gravitational field V(z) = mgz (z = 0 to ∞)
- Particle confined by a linear potential V(x) = ax (x = 0 to ∞)
- One-dimensional atoms and ions V(x) = -Z/x (x = 0 to ∞)
- Particle in semi-infinite potential well V(x) = if[x $\leq a, 0, b$] (x = 0 to ∞)
- Particle in semi-harmonic potential well V(x) = kx^2 (x = 0 to ∞)

$$\psi(x,\alpha) = (\frac{2\cdot\alpha}{\pi})^{\frac{1}{4}}\cdot exp(-\alpha\cdot x^2)$$

- Quartic oscillator V(x) = bx^4 (x = -∞ to ∞)
- Particle in the finite one-dimensional potential well V(x) = if[(x $\geq$ -1 · (x \leq 1), 0, 2] (x = -∞ to ∞)
- 1D Hydrogen atom ground state
- Harmonic oscillator ground state
- Particle in V(x) = | x | potential well

$$\psi(x,\alpha) = \sqrt{\alpha}\cdot exp(-\alpha\cdot|x|)$$

- This wavefunction is discontinuous at x = 0, so the following calculations must be made in momentum space
- Dirac hydrogen atom V(x) = - Δ (x)
- Harmonic oscillator ground state
- Particle in V(x) = | x | potential well
- Quartic oscillator V(x) = bx^4 (x = -∞ to ∞)

$$\psi(x) = \sqrt{30}\cdot x\cdot(1-x)$$

$$\Gamma(x) = \sqrt{105}\cdot x\cdot(1-x)^2$$

$$\Theta(x) = \sqrt{105}\cdot x^2\cdot(1-x)$$

- Particle in a one-dimensional, one-bohr box
- Particle in a slanted one-dimensional box
- Particle in a semi-infinite potential well (change 1 to variational parameter)
- Particle in a gravitational field (change 1 to variational parameter)

$$\Phi(r,a) = (a-r)$$

$$\Phi(r,a) = (a-r)^2$$

$$\Phi(r,a) = \frac{1}{\sqrt{2\cdot\pi\cdot a}}\cdot\frac{\sin\frac{\pi\cdot r}{a}}{r}$$

- Particle in a infinite spherical potential well of radius a
- Particle in a finite spherical potential well (treat a as a variational parameter)

$$\psi(r,\beta) = (\frac{2\cdot\beta}{\pi})^{\frac{3}{4}}\cdot exp(-\beta\cdot r^2)$$

- Particle in a finite spherical potential well
- Hydrogen atom ground state
- Helium atom ground state

$$\psi(r,\beta) = \sqrt{\frac{3\cdot\beta^3}{\pi^3}}\cdot sech(\beta\cdot r)$$

- Particle in a finite potential well
- Hydrogen atom ground state
- Helium atom ground state

$$\psi(x,\beta) = \sqrt{\frac{\beta}{2}} \cdot sech(\beta \cdot x)$$

- Harmonic oscillator
- Quartic oscillator
- Particle in a gravitational field
- Particle in a finite potential well

$$\psi(\alpha,\beta) = \sqrt{\frac{12\alpha^3}{\pi}} \cdot x \cdot sech(\alpha \cdot x)$$

- Particle in a semi-infinite potential well
- Particle in a gravitational field
- Particle in a linear potential well (same as above) V(x) = ax (x = 0 to ∞)
- 1D hydrogen atom or one-electron ion

Some finite potential energy wells.

V(x) = if[(x ≥ -1 · (x ≤ 1), 0, V_0]

V(x) = if[(x ≥ -1 · (x ≤ 1), 0, |x| - 1]

V(x) = if[(x ≥ -1 · (x ≤ 1), 0, $\sqrt{|x|-1}$]

Some semi-infinite potential energy well.

V(x) = if (x ≤ a, 0, b)

V(x) = if[(x ≤ 2), 0, $\frac{5}{x}$]

V(x) = if[(x ≥ 2), 0, (x - 2)]

V(x) = if[(x ≤ 2), 0, $\sqrt{x-2}$]
