4.13: The Quantum Jump in Momentum Space

This tutorial is a companion to "The Quantum Jump" which deals with the quantum jump from the perspective of the coordinate-space wave function. This tutorial accomplishes the same thing in momentum space.

The time-dependent momentum wave function for a particle in a one-dimensional box of width $1a_0$ is shown below.

$$\psi(n,\ p,\ t) = n\sqrt{\pi}\left[\frac{1-(-1)^n \exp(-ip)}{n^2-\pi^2-p^2}\right]\exp(-iE_i t)$$

The n = 1 to n = 2 Transition for the Particle in a Box is Allowed

This transition is allowed because it yields a momentum distribution that is asymmetric in time as is shown in the figure below. Consequently it allows for coupling with the perturbing electromagnetic field.

$$\text{Momentum increment} \quad P=100 \quad \text{Time Increment} \quad T=100 \quad \text{Initial} \quad n_i=1 \quad \text{Final state} \quad n_f=2$$

Initial and final energy states for the transition under study:

$$E_i = \frac{n_i^2\pi^2}{2} \qquad E_f = \frac{n_f^2\pi^2}{2}$$

Plot the wavefunction:

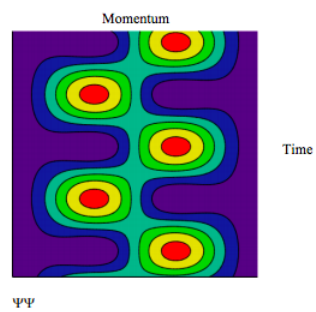
$$j=0..P \quad p_j = -10+\frac{20j}{P} \quad k=0..T \quad t_k=\frac{k}{T}$$

In the presence of electromagnetic radiation the particle in the box goes into a linear superposition of the stationary states. The linear superposition for the n = 1 and n = 2 states is given below.

$$\Psi(p,\ t) = n_i\sqrt{\pi}\left[\frac{1-(-1)^{n_i}\exp(-ip)}{n_i^2\pi^2-p^2}\right]\exp(-iE_i t)+n_f\sqrt{\pi}\left[\frac{1-(-1)^{n_f}\exp(-ip)}{n_f^2\pi^2-p^2}\right]\exp(-iE_f t)$$

Calculate and plot the momentum distribution: Ψ*Ψ:

$$\Psi\Psi_{(j,\ k)} = \left(\left|\Psi(p_j,\ t_k)\right|\right)^2$$



The n = 1 to n = 3 Transition for the Particle in a Box is Not Allowed

This transition is allowed because it yields a momentum distribution that is symmetric in time as is shown in the figure below. Consequently it does not allow for coupling with the perturbing electromagnetic field.

$$\text{Momentum increment} \quad P=100 \quad \text{Time Increment} \quad T=100 \quad \text{Initial} \quad n_i=1 \quad \text{Final state} \quad n_f=3$$

Initial and final energy states for the transition under study:

$$E_i = \frac{n_i^2\pi^2}{2} \qquad E_f = \frac{n_f^2\pi^2}{2}$$

Plot the wavefunction:

$$j=0..P \quad p_j = -10+\frac{20j}{P} \quad k=0..T \quad t_k=\frac{k}{T}$$

In the presence of electromagnetic radiation the particle in the box goes into a linear superposition of the stationary states. The linear superposition for the n = 1 and n = 3 states is given below.

$$\Psi(p,\ t) = n_i\sqrt{\pi}\left[\frac{1-(-1)^{n_i}\exp(-ip)}{n_i^2\pi^2 - p^2}\right]\exp(-iE_it) + n_f\sqrt{\pi}\left[\frac{1-(-1)^{n_f}\exp(-ip)}{n_f^2\pi^2 - p^2}\right]\exp(-iE_ft)$$

Calculate and plot the momentum distribution: Ψ*Ψ:

$$\Psi\Psi_{(j,\ k)} = (|\Psi(p_j,\ t_k)|)^2$$