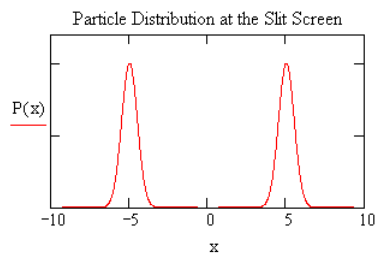
5.18: Another Look at the Double-Slit Experiment

Illumination of a double-slit screen with a coherent particle beam leads to a Schrödinger "cat state" that can be represented by a linear superposition (unnormalized) of two Gaussian wavepackets. The probability distribution function in coordinate space, |Y(x)|^2, at the slit-screen for this "cat state" is shown below.

$$x = -10, \ -9.99..10 \quad \Psi(x) = exp\left[-(x-5)^2\right] + exp\left[-(x+5)^2\right] \quad P(x) = (|\Psi(x)|)^2$$



The slits localize the particle in the x-direction which leads to a spread in the x-component of the momentum required by the uncertainty principle, DxDp_x > h/4p. The momentum wave function is obtained by a Fourier transform of the coordinate-space wave function.

$$\Phi(p_x) = \frac{1}{\sqrt{2\pi}}\left[\int_{-\infty}^{\infty} exp(-ip_x x)\left[exp\left[-(x-5)^2\right] + exp\left[-(x+5)^2\right]\right] dx\right]$$

Evaluation of the integral yields,

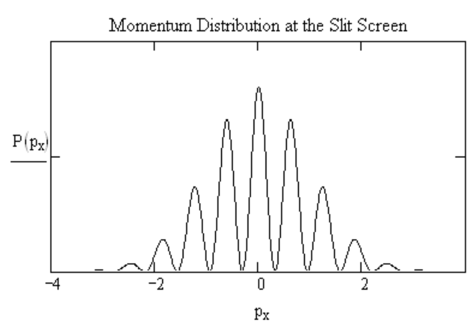
$$\Phi(p_x) = \frac{1}{\sqrt{2}}\left[exp\left[\frac{-1}{4}p_x(p_x+20i)\right] + exp\left[\frac{-1}{4}p_x(p_x-20i\right]\right]$$

The momentum probability function in the x-direction is |F(p_x)|^2 and simplifies to the expression given below when evaluated.

$$P(p_x) = 2exp\left(\frac{-1}{2}p_x^2\right)\cos(5p_x)^2$$

This momentum probability function is displayed below.

$$p_x = -4, -3.99..4$$



Because the arrival at position *x* on the detection screen is proportional to p_x it is also proportional to |F(p_x)|^2. In other words, the particle distribution at the detection screen is determined by the momentum distribution at the slit screen. This means the position measurement at the detection screen is effectively a measurement of the p_x. Therefore, the particle distribution at the detector screen will have the same shape as shown in the figure above.

In summary, the double-slit experiment clearly reveals the three essential steps in a quantum mechanical experiment:

1. State preparation (interaction of the incident beam with the slit-screen)
2. Measurement of an observable (arrival of scattered beam at the detection screen)
3. Calculation of expected results of the measurement step

*The preparation of this tutorial was stimulated by reading "Quantum interference with slits" by Thomas Marcella which appeared in *European Journal of Physics* **23**, 615-621 (2002). This paper offers a lucid and novel quantum mechanical analysis of a very important experiment.

Additional references:

R. P. Feynman, R. B. Leighton, and M.Sands, *The Feynman Lectures on Physics*, Volume 3; Addison-Wesley; Reading, 1965, Chapters 1 and 3.

R. P. Feynman, *The Character of Physical Law*; MIT Press: Cambridge, 1967, Chapter 6.

A. Tonomura, J. Endo, T. Matsuda, T. Kawasaki, and H. Exawa, "Demonstration of single-electron buildup of an interference pattern" *Am. J. Phys*. **57**, 117-120 (1989).

D. Leibfried, T. Pfau, and C. Monroe, "Shadows and Mirrors: Reconstructing Quantum States of Atom Motion" *Phys. Today* **51(4)**, 22-28 (1998).

The double-slit experiment with single electrons was recently selected (informally) as physics most beautiful experiment. The following web reference traces the history of double-slit interference experiments from the time of Thomas Young to the present, presenting numerous literature references in the process: http://physicsweb.org/article/world/15/9/1.

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