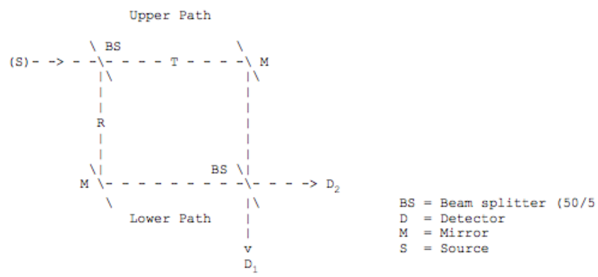
7.2: Single-Photon Interference - Second Version

Using Dirac Notation to Analyze Single Particle Interference

The schematic diagram below shows a Mach-Zehnder interferometer for photons. When the experiment is run so that there is only one photon in the apparatus at any time, the photon is always detected at D_2 and never at D_1. (1,2,3)

The quantum mechanical analysis of this striking phenomenon is outlined below. The photon leaves the source, S, and whether it takes the upper or lower path it interacts with a beam splitter, a mirror, and another beam splitter before reaching the detectors. At the beam splitters there is a 50% chance that the photon will be transmitted and a 50% chance that it will be reflected.



After the first beam splitter the photon is in an even linear superposition of being transmitted and reflected. Reflection involves a 90^o ($\pi/2$) phase change which is represented by $\exp(i\pi/2) = i$, where i = (-1)1/2. (See the appendix for a simple justification of the 90^o phase difference between transmission and reflection.) Thus the state after the first beam is given by equation 257.1.

$$|\psi = \left(\frac{[|T > +i|R]}{2}\right)^{\frac{1}{2}}$$

Now |T> and |R> will be written in terms of $|D_1>$ and $|D_2>$ the states they evolve to at detection. |T> reaches $|D_1>$ by transmission and $|D_2>$ by reflection.

$$|T >= \left(\frac{[|D1 > +i|D2 >]}{2}\right)^{\frac{1}{2}}$$

|R> reaches $|D_1>$ by reflection and $|D_2>$ by transmission.

$$|R >= \left(\frac{[i|D1 > +|D2 >]}{2}\right)^{\frac{1}{2}}$$

Equations 257.2 and 257.3 are substituted into equation 257.1.

$$|\psi >= \frac{[|D1 > +i|D2 > +i2|D1 > +i|D2 >]}{2}$$

It is clear ($i^2 = -1$) that the first and third terms cancel (the amplitudes are 180^o out of phase), so that we end up with a final state given by equation 257.5.

$$|\psi >= i|D_2 >$$

The probability of an event is the square of the absolute magnitude of the probability amplitude.

$$P(D_2) = |i|^2 = 1$$

Thus this analysis is in agreement with the experimental outcome that no photons are ever detected at D_1.

Appendix:

Suppose there is no phase difference between transmission and reflection. Then equations 257.1, 257.2, and 257.3 become

$$|\psi = \left(\frac{[|T > +|R >]}{2}\right)^{\frac{1}{2}}$$

$$|T>=\left(\frac{[|D1>+|D2>]}{2}\right)^{\frac{1}{2}}$$

$$|\psi=\left(\frac{[|D_1>+|D_2>|D_1>+D_2>]}{2}\right)^{\frac{1}{2}}$$

Substitution of equations 257.8 and 257.9 into equation 257.7 yields

$$|\psi>=|D_1>+|D_2>$$

Thus, the detection probabilities at the two detectors are:

$$P(D_1)=1andP(D_2)=1$$

This result violates the principle of conservation of energy because the original photon has a probability of 1 of being detected at D_1 and also a probability of 1 of being detected at D_2. In other words, the number of photons has doubled. Thus, there must be a phase difference between transmission and reflection, and a 90^o phase difference, as shown above, conserves energy.

References:

1. P. Grangier, G. Roger, and A. Aspect, "Experimental Evidence for Photon Anticorrelation Effects on a Beam Splitter: A New Light on Single Interferences," Europhys. Lett. 1, 173-179 (1986).

2. V. Scarani and A. Suarez, "Introducing Quantum Mechanics: One-particle Interferences," Am. J. Phys. 66, 718-721 (1998).

3. Kwiat, P, Weinfurter, H., and Zeilinger, A, "Quantum Seeing in the Dark, Sci. Amer. Nov. 1996, pp 72-78.