

11.8: Finding Roots of Transcendental Equations

The attempt to find analytical solutions to Schrödinger's equation for some problems yields transcendental equations which must be solved by a combination of graphical and numerical techniques. Mathcad is particularly well-suited for such applications.

Solving Schrödinger's equation for the particle in the box with an internal barrier yields the transcendental equation $f(E)$ shown below. This equation is solved by plotting $f(E)$ vs E to find the approximate values of the bound energy states.

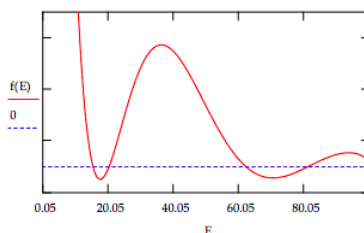
The box is 1 bohr wide and the barrier is 0.1 bohr thick and located in the center of the box.

- V_0 is the barrier height in hartrees. $V_0 := 100$
- The barrier thickness in bohrs. $BT := .1$
- Left barrier boundary in bohrs. $LB := .45$

$E := 0.05, .1 \dots 100$

$$f(E) := \tanh[BT \cdot \sqrt{2 \cdot (V_0 - E)}] \cdot \left(\frac{V_0 - E}{E} \cdot \sin(LB \cdot \sqrt{2 \cdot E})^2 + \cos(LB \cdot \sqrt{2 \cdot E})^2 \right) \cdots + 2 \cdot \sqrt{\frac{V_0 - E}{E}} \cdot \sin(LB \cdot \sqrt{2 \cdot E}) \cdot \cos(LB \cdot \sqrt{2 \cdot E})$$

For a derivation of this formula see: Johnson and Williams, Amer. J. Phys. 1982, 50, 239-244.



By inspection of the graph one can see that there are roots at approximately 15, 20, 62, and 80. The exact energy is found with Mathcad's root function using the approximate energy as a seed value as illustrated below.

$$E := 15 \text{ root}(f(E), E) = 15.43$$

$$E := 20 \text{ root}(f(E), E) = 20.29$$

$$E := 62 \text{ root}(f(E), E) = 62.24$$

$$E := 80 \text{ root}(f(E), E) = 81.07$$

This exercise can be extended by noting that this problem can also be solved by numerical integration of Schrödinger's equation. Comparisons of this sort help are helpful in strengthening the students understanding of the computational techniques available to the quantum chemist. Below the problem is solved by numerical integration of Schroedinger's equation.

Integration limit: $x_{\max} := 1$ Effective mass: $\mu := 1$ Barrier height: $V_0 := 100$

Barrier boundaries: $lb := .45$ $rb := .55$ Potential energy: $V(x) := if[(x \geq lb) \cdot (x \leq rb), V_0, 0]$

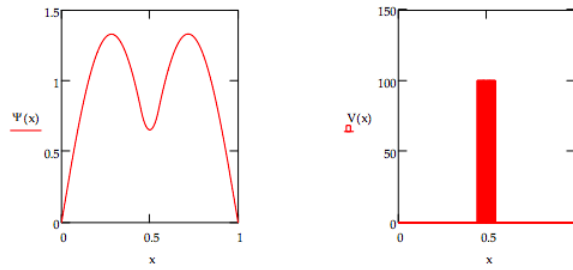
Numerical integration of Schrödinger's equation: $\frac{-1}{2 \cdot \mu} \cdot \frac{d^2}{dx^2} \psi(x) + V(x) \cdot \psi(x) = E \cdot \psi(x)$

$$\psi(0) = 0. \psi'(0) = 0.1$$

Enter energy guess: $E := 15.43$

$$\psi := \text{Odesolve}(x, x_{\max})$$

Normalize wave function: $\psi(x) := \frac{\psi(x)}{\sqrt{\int_0^{x_{\max}} \psi(x)^2 dx}}$



This page titled [11.8: Finding Roots of Transcendental Equations](#) is shared under a [CC BY 4.0](#) license and was authored, remixed, and/or curated by [Frank Rioux](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.