11.19: Age of the Elements

Analysis of cosmic microwave background radiation (CMBR) obtained by the Planck satellite telescope has added 80 million years to the age of the universe, making the best estimate of the current age 13.8 billion years. This tutorial deals with the more modest task of estimating the age of the elements found in the earth's crust.

As is well-known from examples such as carbon-14 dating, radioactivity can be used to tell time. Radioactive isotopes decay exponentially according to the following equation.

$$A_t = A_0 \left(\frac{1}{2}\right)^{\frac{1}{t_{1/2}}}$$

The amount of a radioactive isotope remaining at time t is equal to the original amount times 1/2 raised to the number of half-lives $(t/t_{1/2})$ that have occurred. Using this equation plus plausible assumptions, some information about uranium isotopes and rudimentary math, the time elapsed since the isotopes were produced is calculated.

Assume that ^{238}U and ^{235}U were produced in equal amounts originally. Today the $^{238}U/^{235}U$ ratio is 140/1. The half-lives of ^{238}U and ^{235}U are 4.5 billion years and 800 million years, respectively. How long ago where these isotopes synthesized?

$$\begin{bmatrix} U_{238} = \frac{1}{2}^{\frac{t}{4.5 \cdot 10^9\,year}} \\ U_{235} = \frac{1}{2}^{\frac{t}{0.8 \cdot 10^9\,year}} \\ U_{238} = 140 \cdot U_{235} \end{bmatrix} \begin{pmatrix} t \\ U_{238} \\ U_{235} \end{pmatrix} \Big|_{float,3} \to (0.694e10\ year\ 0.344\ 0.245e-2)$$

According to this rudimentary model, the uranium in the earth's crust is about half the age of the universe.