

2.58: The Wigner Distribution for the 3s State of the 1D Hydrogen Atom

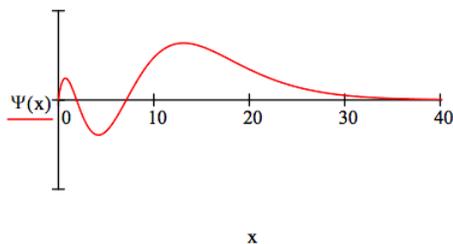
This tutorial presents three pictures of the 3s state of the one-dimensional hydrogen atom using its position, momentum and phase-space representations.

The energy operator for the one-dimensional hydrogen atom in atomic units is:

$$\frac{-1}{2} \frac{d^2}{dx^2} - \frac{1}{x}$$

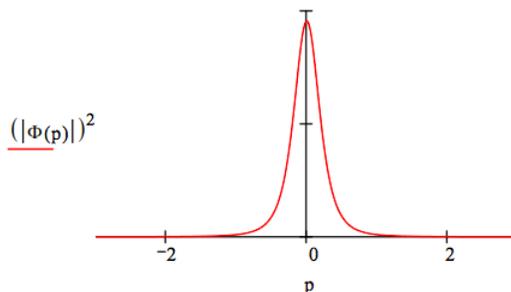
The position 3s wave function is:

$$\Psi(x) = \frac{2}{243} \sqrt{3x} (27 - 18x + 2x^2) \exp\left(-\frac{x}{3}\right) \quad \int_0^\infty \Psi(x)^2 dx = 1$$



The 3s energy is $-0.065 E_h$.

$$\Phi(p) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \exp(-i p x) \Psi(x) dx \rightarrow \left(-2\frac{1}{2}\right) 3^{\frac{1}{2}} \frac{9p^2 + 6ip - 1}{(3ip + 1)^4 \pi^{\frac{1}{2}}}$$

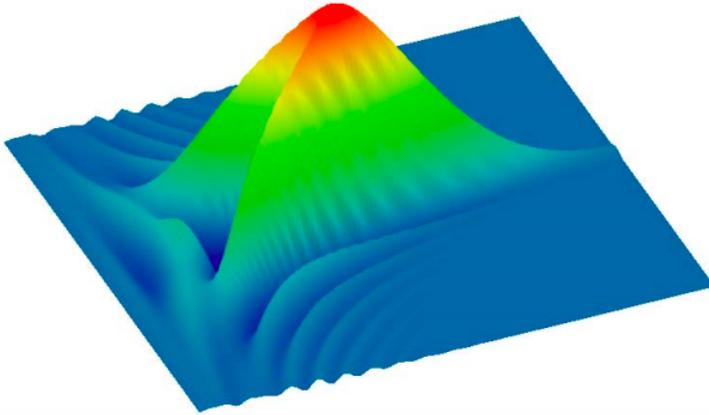


The Wigner function (phase-space representation) for the hydrogen atom 3s state is generated using the momentum wave function.

$$W(x, p) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{\Phi\left(p + \frac{s}{2}\right)} \exp(-i s x) \Phi\left(p - \frac{s}{2}\right) ds$$

The Wigner distribution is displayed graphically.

$$N = 100 \quad i = 0..N \quad x_i = \frac{30i}{N} \quad j = 0..N \quad p_j = -2 + \frac{4j}{N} \quad \text{Wigner}_{i,j} = W(x_i, p_j)$$



Wigner

Just as for the 2s state, the Wigner distribution for the 3s state takes on negative values.

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