

## 8.50: EPR Analysis for a Composite Singlet Spin System - Short Version

A spin-1/2 pair is prepared in an entangled singlet state and the individual particles travel in opposite directions on the y-axis to a pair of Stern-Gerlach detectors which are set up to measure spin in the x-z plane. Particle 1's spin is measured along the z-axis, and particle 2's spin is measured at an angle  $\theta$  with respect to the z-axis.



For the singlet state the arrows below indicate the spin orientation for any direction in the x-z plane.

$$\begin{aligned}
 |\Psi\rangle &= \frac{1}{\sqrt{2}} [|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2] = \frac{1}{\sqrt{2}} \left[ \begin{pmatrix} \cos(\frac{\theta}{2}) \\ \sin(\frac{\theta}{2}) \end{pmatrix}_1 \otimes \begin{pmatrix} -\sin(\frac{\theta}{2}) \\ \cos(\frac{\theta}{2}) \end{pmatrix}_2 - \begin{pmatrix} -\sin(\frac{\theta}{2}) \\ \cos(\frac{\theta}{2}) \end{pmatrix}_1 \otimes \begin{pmatrix} \cos(\frac{\theta}{2}) \\ \sin(\frac{\theta}{2}) \end{pmatrix}_2 \right] \\
 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}
 \end{aligned}$$

The single particle spin operator in the x-z plane is constructed from the Pauli spin operators in the x- and z-directions.  $\phi$  is the angle of orientation of the measurement magnet with the z-axis.

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad S(\theta) = \cos(\theta)\sigma_z + \sin(\theta)\sigma_x \rightarrow \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{pmatrix} \quad S(0) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

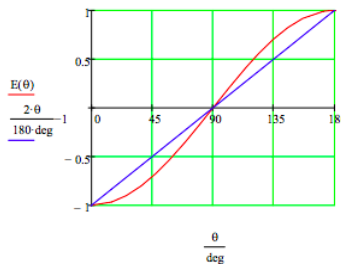
Tensor multiplication of  $S(0)$  and  $S(\phi)$  creates a joint spin measurement operator.

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta & 0 & 0 \\ \sin \theta & -\cos \theta & 0 & 0 \\ 0 & 0 & -\cos \theta & -\sin \theta \\ 0 & 0 & \sin \theta & \cos \theta \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$$

The expectation value as a function of the measurement angle of particle 2 is calculated and the result displayed graphically.

$$E(\theta) = \frac{1}{\sqrt{2}} (0 \quad 1 \quad -1 \quad 0) \begin{pmatrix} \cos \theta & \sin \theta & 0 & 0 \\ \sin \theta & -\cos \theta & 0 & 0 \\ 0 & 0 & -\cos \theta & -\sin \theta \\ 0 & 0 & \sin \theta & \cos \theta \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \rightarrow -\cos \theta$$

For  $\theta = 0^\circ$  there is perfect anti-correlation; for  $\theta = 180^\circ$  perfect correlation; for  $\theta = 90^\circ$  no correlation. A correlation function based on a local-realistic, hidden-variable model (see Fig. 11.2 and related text in A. I. M. Rae's Quantum Mechanics, 2nd Ed.) and the quantum mechanical correlation function,  $E(\theta)$ , are compared on the graph below. Quantum theory and local realism disagree at all angles except 0, 90 and 180 degrees.



This example illustrates Bell's theorem: no local hidden-variable theory can reproduce all the predictions of quantum mechanics for entangled composite systems. As the quantum predictions are confirmed experimentally, the local hidden-variable approach to reality must be abandoned.

---

This page titled [8.50: EPR Analysis for a Composite Singlet Spin System - Short Version](#) is shared under a [CC BY 4.0](#) license and was authored, remixed, and/or curated by [Frank Rioux](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.