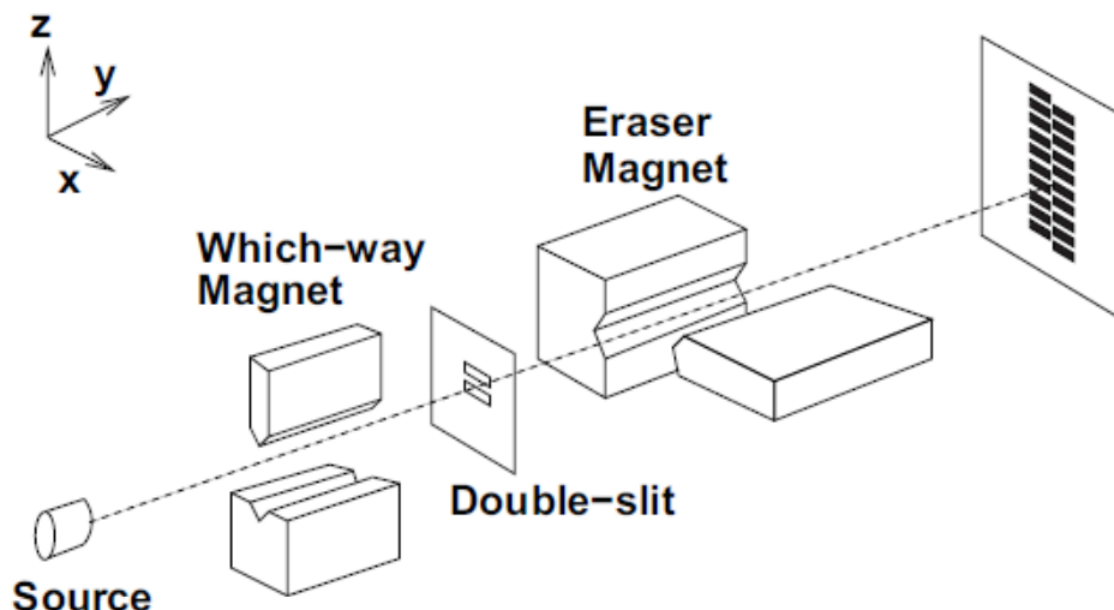


1.49: A Stern-Gerlach Quantum "Eraser"

This tutorial provides a brief mathematical analysis of a proposed quantum eraser experiment involving spin-1/2 particles which is available at [arXiv:quant-ph/0501010v2](https://arxiv.org/abs/quant-ph/0501010v2). Please see the two immediately preceding tutorials for another example of the quantum eraser and additional mathematical detail.



The first magnet attaches which-way information such that the spin-1/2 particles leaving the double-slit screen are described by the following entangled wave function,

$$|\Psi\rangle = \frac{1}{\sqrt{2}} [|\uparrow_z\rangle |z_1\rangle + |\downarrow_z\rangle |z_2\rangle]$$

where z_1 and z_2 represent the positions of the horizontal slits on the z -axis and the spin eigenstates in the z -direction are given below.

$$\Psi_{\text{zup}} := \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \Psi_{\text{zdown}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Recognizing that a diffraction pattern is actually a momentum distribution function, we project Ψ onto momentum space as follows (in atomic units, $\hbar = 2\pi$).

$$\langle p|\Psi\rangle = \frac{1}{\sqrt{2}} [|\uparrow_z\rangle \langle p|z_1\rangle + |\downarrow_z\rangle \langle p|z_2\rangle] = \frac{1}{\sqrt{2}} \left[|\uparrow_z\rangle \frac{\exp(-ipz_1)}{\sqrt{2\pi}} + |\downarrow_z\rangle \frac{\exp(-ipz_2)}{\sqrt{2\pi}} \right]$$

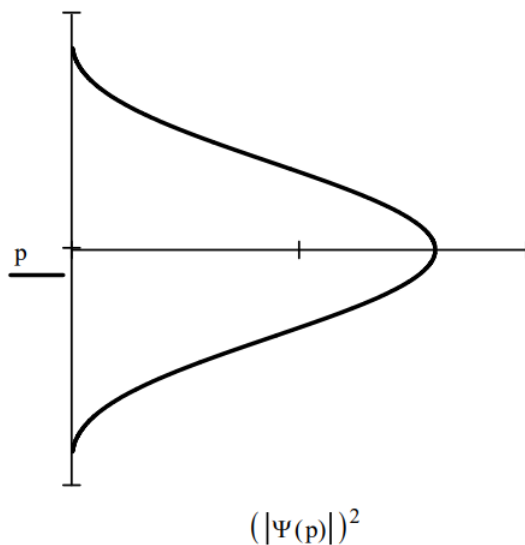
Here the exponential terms are the position eigenfunctions in momentum space for infinitesimally thin slits located at z_1 and z_2 . For slits of finite width $\langle p|\Psi\rangle$ is written as shown below. Again see the previous tutorials in this series for further mathematical detail. The slit positions and slit width chosen are arbitrary.

Slit positions: $z_1 := 1$ $z_2 := 2$	Slit width: $\delta := 0.2$
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$$\Psi(p) := \frac{1}{\sqrt{2}} \cdot \left(\Psi_{\text{zup}} \cdot \int_{z_1 - \frac{\delta}{2}}^{z_1 + \frac{\delta}{2}} \frac{1}{\sqrt{2 \cdot \pi}} \cdot \exp(-i \cdot p \cdot z) \cdot \frac{1}{\sqrt{\delta}} dz + \Psi_{\text{zdown}} \cdot \int_{z_2 - \frac{\delta}{2}}^{z_2 + \frac{\delta}{2}} \frac{1}{\sqrt{2 \cdot \pi}} \cdot \exp(-i \cdot p \cdot z) \cdot \frac{1}{\sqrt{\delta}} dz \right)$$

Because of the addition of path information there are no interference fringes associated with this two-slit wave function; the encoded orthogonal z -direction eigenstates destroy the interference cross terms as shown graphically below.

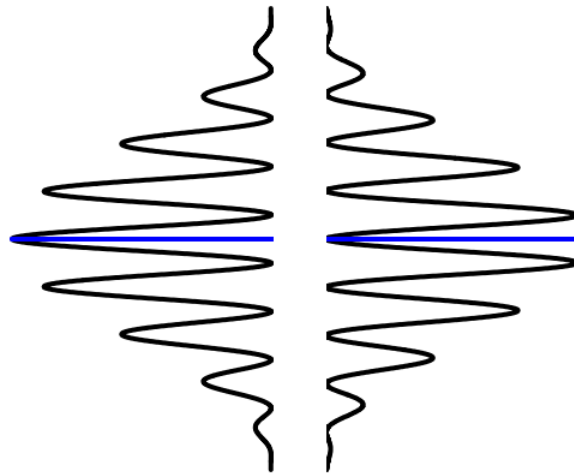
$$p := -30, -29.98 \dots 30$$



The second Stern-Gerlach magnet oriented in the x-direction, according to the conventional interpretation, "erases" the which-way information. This is shown by projecting the state after the first magnet and the slit screen, $\Psi(p)$, onto the x-direction spin eigenstates.

$$\Psi_{\text{xup}} = \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 & 1 \end{pmatrix} \quad \Psi_{\text{xdown}} = \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 & -1 \end{pmatrix}$$

$$\Psi_{\text{left}}(p) := \Psi_{\text{xup}} \cdot \Psi(p) \quad \Psi_{\text{right}}(p) := \Psi_{\text{xdown}} \cdot \Psi(p)$$



$$\left(|\Psi_{\text{left}}(p)| \right)^2$$

$$\left(|\Psi_{\text{right}}(p)| \right)^2$$

Figure 2b in the reference cited and shown below.

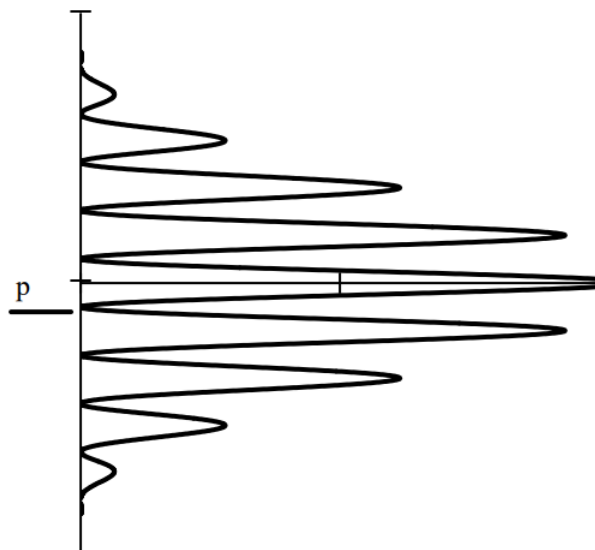


The horizontal blue line marks $p = 0$ on the z -axis. On the left is the interference pattern of the part of the beam emerging from the x-up magnet direction with spin state Ψ_{xup} , and on the right is the interference pattern of the part of the beam emerging from the x-down magnet direction with spin state Ψ_{xdown} . As shown in the Summary, $\Psi(p)$ can be rewritten in terms of the x-direction spin states clearly show in the superpositions responsible for the interference fringes on the left and right.

$$\langle p | \Psi \rangle = \frac{1}{2} [|\uparrow_x\rangle (\langle p | z_1 \rangle + \langle p | z_2 \rangle) + |\downarrow_x\rangle (\langle p | z_1 \rangle - \langle p | z_2 \rangle)]$$

In the absence of both Stern-Gerlach magnets the usual double-slit interference pattern is observed.

$$\Psi(p) = \frac{1}{\sqrt{2}} \cdot \left(\int_{z_1 - \frac{\delta}{2}}^{z_1 + \frac{\delta}{2}} \frac{1}{\sqrt{2 \cdot \pi}} \cdot \exp(-i \cdot p \cdot z) \cdot \frac{1}{\sqrt{\delta}} dz + \int_{z_2 - \frac{\delta}{2}}^{z_2 + \frac{\delta}{2}} \frac{1}{\sqrt{2 \cdot \pi}} \cdot \exp(-i \cdot p \cdot z) \cdot \frac{1}{\sqrt{\delta}} dz \right)$$



$$(|\Psi(p)|)^2$$

Alternative Analysis

It is possible to express the mathematics in an alternative but equivalent form. The first wave function,

$$\frac{1}{\sqrt{2}} [|\uparrow_z\rangle |z_1\rangle + |\downarrow_z\rangle |z_2\rangle]$$

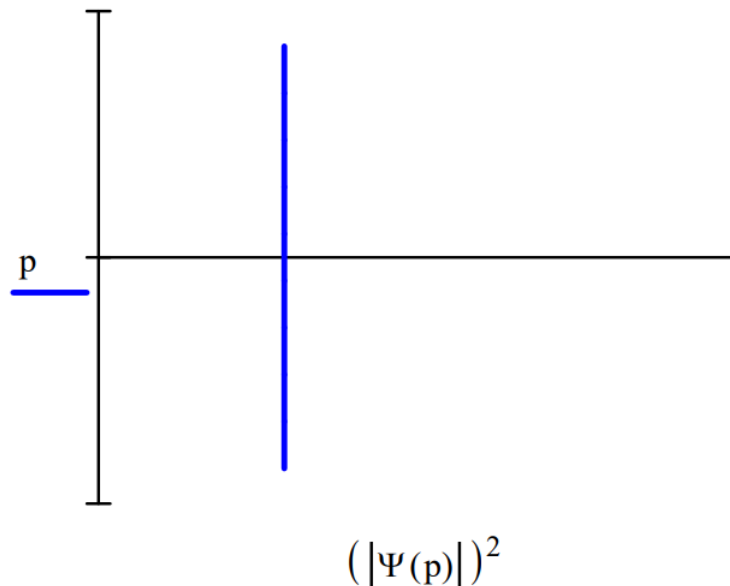
can be expressed explicitly in vector format in the momentum representation. This analysis will be based on infinitesimally thin slits as introduced earlier.

$$\frac{1}{\sqrt{2}} \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} |z_1\rangle + \begin{pmatrix} 0 \\ 1 \end{pmatrix} |z_2\rangle \right] = \frac{1}{\sqrt{2}} \begin{pmatrix} |z_1\rangle \\ |z_2\rangle \end{pmatrix} \xrightarrow{\langle p|} \frac{1}{\sqrt{2}} \begin{pmatrix} \langle p|z_1\rangle \\ \langle p|z_2\rangle \end{pmatrix}$$

It is easily shown that this wave function does not lead to interference fringes at the detection screen by calculating the square of its absolute magnitude.

$$\frac{1}{2} (\langle z_1|p\rangle \quad \langle z_2|p\rangle) \begin{pmatrix} \langle p|z_1\rangle \\ \langle p|z_2\rangle \end{pmatrix} = \frac{1}{2} [|\langle p|z_1\rangle|^2 + |\langle p|z_2\rangle|^2]$$

$$\Psi(p) := \frac{1}{2 \cdot \sqrt{\pi}} \cdot \begin{pmatrix} \exp(-i \cdot p \cdot z_1) \\ \exp(-i \cdot p \cdot z_2) \end{pmatrix}$$

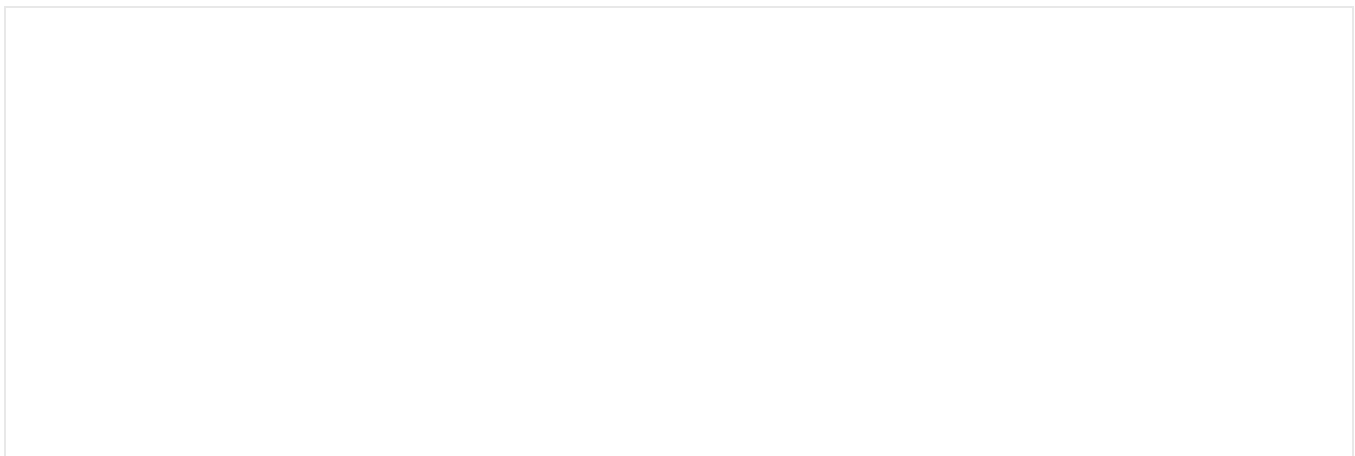


However, in the presence of the second Stern-Gerlach magnet vector B is projected onto the two output channels of the magnet.

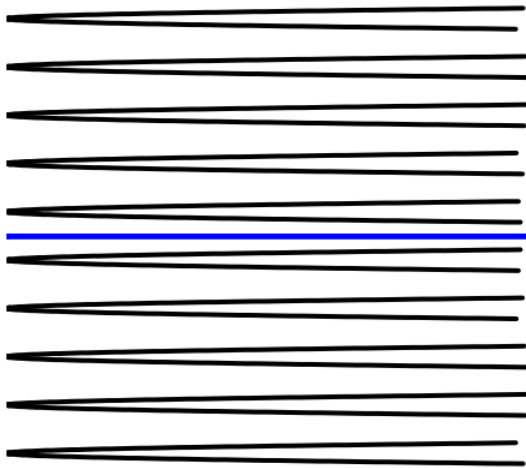
$$\frac{1}{2} \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \langle p|z_1\rangle \\ \langle p|z_2\rangle \end{pmatrix} = \frac{1}{2} [\langle p|z_1\rangle + \langle p|z_2\rangle]$$

$$\frac{1}{2} \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} \langle p|z_1\rangle \\ \langle p|z_2\rangle \end{pmatrix} = \frac{1}{2} [\langle p|z_1\rangle - \langle p|z_2\rangle]$$

The probability distributions of these states show interference fringes.

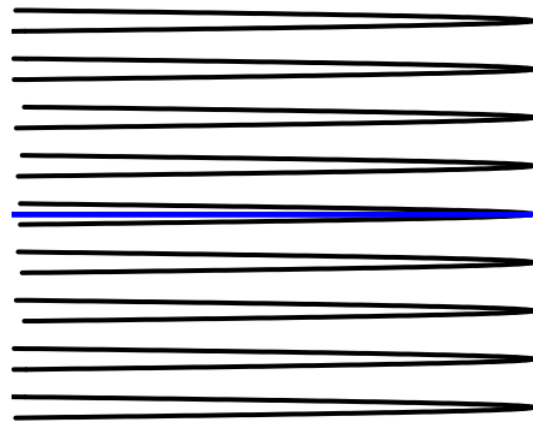


$$\Psi_{\text{left}}(p) := \frac{1}{\sqrt{2}} \cdot (1 \quad 1) \cdot \frac{1}{2 \cdot \sqrt{\pi}} \cdot \begin{pmatrix} \exp(-i \cdot p \cdot z_1) \\ \exp(-i \cdot p \cdot z_2) \end{pmatrix}$$



$$\left(|\Psi_{\text{left}}(p)| \right)^2$$

$$\Psi_{\text{right}}(p) := \frac{1}{\sqrt{2}} \cdot (1 \quad -1) \cdot \frac{1}{2 \cdot \sqrt{\pi}} \cdot \begin{pmatrix} \exp(-i \cdot p \cdot z_1) \\ \exp(-i \cdot p \cdot z_2) \end{pmatrix}$$



$$\left(|\Psi_{\text{right}}(p)| \right)^2$$

Summary

The z-direction Stern-Gerlach magnet and the slit screen create the following entangled superposition which does not produce interference fringes due to the orthogonality of the spin states marking the slits.

$$\langle p | \Psi \rangle = \frac{1}{\sqrt{2}} [|\uparrow_z\rangle \langle p | z_1 \rangle + |\downarrow_z\rangle \langle p | z_2 \rangle]$$

To understand what happens at the x-direction magnet this state is rewritten in the x-direction spin basis.

$$\langle p | \Psi \rangle = \frac{1}{\sqrt{2}} \left[\frac{1}{\sqrt{2}} (|\uparrow_x\rangle + |\downarrow_x\rangle) \langle p | z_1 \rangle + \frac{1}{\sqrt{2}} (|\uparrow_x\rangle - |\downarrow_x\rangle) \langle p | z_2 \rangle \right]$$

Collecting terms on the x-direction spin eigenstates yields,

$$\langle p | \Psi \rangle = \frac{1}{2} [|\uparrow_x\rangle (\langle p | z_1 \rangle + \langle p | z_2 \rangle) + |\downarrow_x\rangle (\langle p | z_1 \rangle - \langle p | z_2 \rangle)]$$

The in-phase and out-of-phase superpositions, highlighted in blue and red, exit the magnet in opposite directions. Because of this the superpositions become spatially separated which leads to two sets of interference fringes with a one-fringe relative phase shift at the detection screen.

It's clear to me that erasure is not a satisfactory explanation for this process. Because fringes appear after the x-direction magnet it might seem plausible, at first glance, to assume that the which-way markers have been erased. But actually the x-direction magnet sorts $\langle p | \Psi \rangle$ into two components in terms of the x-direction spin eigenstates. Nothing has been erased.

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