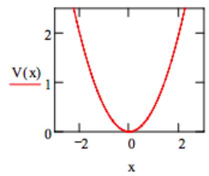
# 10.18: Trigonometric Trial Wave Function for the Harmonic Potential Well

Definte potential energy: V(x) := $\frac{s^2}{2}$

Display potential energy:




Choose trial wave function: $\Psi(x,\beta) := \sqrt{\frac{\beta}{2}}sech(\beta x)$

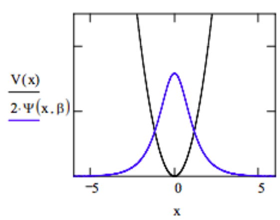
Set up variational energy integral:

$$E(\beta) := \frac{\int_{-\infty}^{\infty}\Psi(x,\beta)\frac{-1}{2}\frac{d^2}{dx^2}\Psi(x,\beta)dx + \int_{-\infty}^{\infty}\Psi(x,\beta)\frac{x^2}{2}\Psi(x,\beta)dx}{\int_0^{\infty}\Psi(x,\beta)^2dx}\Big|_{simplify}^{assume,\ \beta>0} \rightarrow \frac{1}{24}\frac{4\beta^2+\pi^2}{\beta^2}$$

Minimize the energy integral with respect to the variational parameter, $\beta$.

$\beta$ := 0.2 $\beta$ := Minimize (E, $\beta$) $\beta$ = 1.253 E( $\beta$) = 0.524

Display wave function in the potential well.




Calculate the probability that the particle is in the potential barrier.

$$2\int_0^{\infty}\Psi(x,\beta)^2dx = 1$$

Define quantum mechanical tunneling.

> Tunneling occurs when a quon (a quantum mechanical particle) has probability of being in a nonclassical region. In other words, a region in which the total energy is less than the potential energy.

Calculate the probability that tunneling is occurring.

> Calculate the classical turning point.

$$\frac{x^2}{2} = 0.524\Big|_{float,\ 4}^{solve,\ x} \rightarrow \begin{pmatrix} -1.024 \\ 1.024 \end{pmatrix}$$

$$2\int_{1.024}^{\infty}\Psi(x,\beta)^2dx = 0.143$$

Calculate the kinetic and potential energy contributions to the total energy.

> Kinetic energy:

$$\int_{-infty}^{\infty}\Psi(x,\beta)\frac{-1}{2}\frac{d^2}{dx^2}\Psi(x,\beta)dx = 0.262$$

> Potential energy:

$$\int_{-\infty}^{\infty}V(x)\Psi(x,\beta)^2dx = 0.262$$

Is the virial theorem satisfied?

Yes, for the harmonic potential the virial theorem is T = V = E/2.