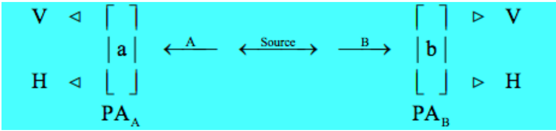
# 8.44: Yet Another Assault on Local Realism

The purpose of this tutorial is to review Nick Herbert's "simple proof of Bell's theorem" as presented in Chapter 12 of *Quantum Reality*.

A two-stage atomic cascade emits entangled photons (A and B) in opposite directions with the same circular polarization according to the observers in their path.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}[|L\rangle_A |L\rangle_B + |R\rangle_A |R\rangle_B]$$

The experiment involves the measurement of photon polarization states in the vertical/horizontal measurement basis, and allows that the polarization analyzers (PAs) can be oriented at different angles a and b. (The figure below is similar to the one on page 125 of Jim Baggott's *The Meaning of Quantum Theory*, which provides a thorough analysis of correlated two-photon experiments.)




The dramatize the quantum weirdness of this EPR experiment, Herbert places $P_A$ on Earth and $PA_B$ on Betelgeuse, 540 light years away. The source is a space ship midway between the PAs.

In the vertical/horizontal measurement basis the initial polarization state is (see the Appendix for a justification),

$$|\Psi\rangle = \frac{1}{\sqrt{2}}[|V\rangle_A |V\rangle_B - |H\rangle_A |H\rangle_B]$$

There are four measurement outcomes: both photons are vertically polarized, both are horizontally polarized, one is vertical and the other horizontal, and vice versa. In other words, the PAs behave the same or differently. The probabilities for these events are given below (see "Simulating Physics with Computers" by Richard Feynman, published in the *International Journal of Theoretical Physics*, volume 21, pages 481-485, or *The Meaning of Quantum Theory*, by Jim Baggott, pages 125-127).

$$P_{same} = P_{vv} + P_{hh} \quad P_{same}(a,\ b) = \cos(a-b)^2 \quad P_{diff} = P_{vh} + P_{hv} \quad P_{diff}(a,\ b) = \sin(a-b)^2$$

The following calculations show that if the PAs are oriented at the same angle they behave the same way 100% of the time. This is called perfect correlation.

$$P_{same}(0 \text{ deg}, 0 \text{ deg}) = 100\% \quad P_{same}(30 \text{ deg}, 30 \text{ deg}) = 100\% \quad P_{same}(90 \text{ deg}, 90 \text{ deg}) = 100\%$$
$$P_{diff}(0 \text{ deg}, 0 \text{ deg}) = 0\% \quad P_{diff}(30 \text{ deg}, 30 \text{ deg}) = 0\% \quad P_{diff}(90 \text{ deg}, 90 \text{ deg}) = 0\%$$

These results appear to support the notion that the linear polarization states of the photons are "elements of reality." In other words, they are photon properties that exist independent of observation. This position is not supported by further calculation and experimentation.

Perfect anti-correlation occurs when the relative angle between the PAs is 90 degrees.

$$P_{same}(0 \text{ deg}, 90 \text{ deg}) = 0\% \quad P_{diff}(0 \text{ deg}, 90 \text{ deg}) = 100\%$$

At 45 degrees there is not correlation between the detectors.

$$P_{same}(0 \text{ deg}, 45 \text{ deg}) = 50\% \quad P_{diff}(0 \text{ deg}, 45 \text{ deg}) = 50\%$$

Using 0 degrees for both PAs at the benchmark, Herbert's analysis proceeds by moving $PA_B$ to 30 degrees and noting that this leads to a 25% (1 in 4) discrepancy between the analyzers.

$$P_{same}(0 \text{ deg}, 30 \text{ deg}) = 75\% \quad P_{diff}(0 \text{ deg}, 30 \text{ deg}) = 25\%$$

If instead $PA_A$ had been moved to -30 degrees the result is the same, the PAs disagree 25% of the time.

$$P_{same}(\text{-30 deg}, 0 \text{ deg}) = 75\% \quad P_{diff}(\text{-30 deg}, 0 \text{ deg}) = 25\%$$

Now the locality principal is invoked. The PAs are spatially separated so that according to conventional intuition, the change in the orientation of $PA_B$ has no effect on the results at $PA_A$, and vice versa.

Now Herbert moves $PA_B$ back to 30 degrees with the following result.

$$P_{same}(\text{-30 deg, 30 deg}) = 25\% \quad P_{diff}(\text{-30 deg, 30 deg}) = 75\%$$

The angular difference is now 60 degrees, and the PAs disagree 75% of the time. On the basis of local realism one would expect a discrepancy of no more than 50% . If the measurements at the PAs are independent of each other, we should simply be able to add 25% and 25%.

The experiment was performed by John Clauser and Stuart Freedman at Berkeley in 1972 and confirmed the quantum predictions. The agreement between quantum theory and experiment requires that some element of local realism must be abandoned. The consensus is that nature allows non-local interactions for entangled systems such as the photons in this example. The results at $PA_A$ and $PA_2$ (light years apart) are connected by a non-local interaction. This type of interaction is, in the words of Herbert, "unmediated, unmitigated and immediate."

Many other experiments besides those of Clauser and Freedman (most notably by Aspect and co-workers) have confirmed quantum mechanical predictions and refuted local realism. Anton Zeilinger described the current situation as follows:

> By now, a number of experiments have confirmed quantum predictions to such an extent that a local-realistic world view can no longer be maintained.
>
> It appears that, certainly at least for entangled quantum systems, it is wrong to assume that the features of the world which we observe, the measurement results, exist prior to and independently of our observation.

## Appendix

In vector notation the left- and right-circular polarization states are expressed as follows:

Left circular polarization:

$$L = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$$

Right circular polarization:

$$R = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

In tensor notation the initial photon state is,

$$|\Psi\rangle = \frac{1}{\sqrt{2}}[|L\rangle_A|L\rangle_B + |R\rangle_A|R\rangle_B] = \frac{1}{2\sqrt{2}}\left[\begin{pmatrix} 1 \\ i \end{pmatrix}_A \otimes \begin{pmatrix} 1 \\ i \end{pmatrix}_B + \begin{pmatrix} 1 \\ -i \end{pmatrix}_A \otimes \begin{pmatrix} 1 \\ -i \end{pmatrix}_B\right]$$
$$= \frac{1}{2\sqrt{2}}\left[\begin{pmatrix} 1 \\ i \\ i \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ -i \\ -i \\ -1 \end{pmatrix}\right] = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

Vertical polarization:

$$V = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Horizontal polarization:

$$H = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

It is easy to show that the equivalent vertical/horizontal polarization state is,

$$|\Psi\rangle = \frac{1}{\sqrt{2}}[|L\rangle_A|L\rangle_B + |R\rangle_A|R\rangle_B] = \frac{1}{2\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}_A \otimes \begin{pmatrix}1\\0\end{pmatrix}_B + \begin{pmatrix}0\\1\end{pmatrix}_A \otimes \begin{pmatrix}0\\1\end{pmatrix}_B\right] = \frac{1}{2\sqrt{2}}\left[\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + \begin{pmatrix}0\\0\\0\\1\end{pmatrix}\right]$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}$$

Naturally there are other ways to do this. The most direct would be to write |L> and |R> as superpositions of |V> and |H>, and substitute them into the initial state involving the circular polarization states.

$$\Psi = \frac{1}{\sqrt{2}}(L_A L_B + R_A R_B) \left| \begin{array}{l} \text{substitute, } L_A = \frac{1}{\sqrt{2}}(V_A + iH_A) \\ \text{substitute, } L_B = \frac{1}{\sqrt{2}}(V_B + iH_B) \\ \text{substitute, } R_A = \frac{1}{\sqrt{2}}(V_A - iH_A) \\ \text{substitute, } R_B = \frac{1}{\sqrt{2}}(V_B - iH_B) \\ \text{simplify} \end{array} \right. \rightarrow \Psi = \frac{\sqrt{2}V_A V_B}{2} - \frac{\sqrt{2}H_A H_B}{2}$$