2.59: The Wigner Distribution for the 3p State of the 1D Hydrogen Atom

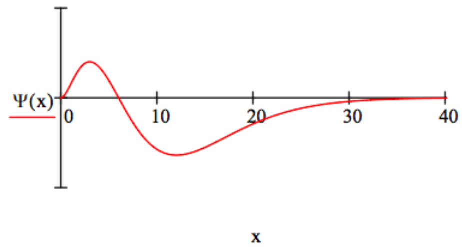
This tutorial presents three pictures of the 3p state of the one-dimensional hydrogen atom using its position, momentum and phase-space representations.

The energy operator for the one-dimensional hydrogen atom in atomic units is:

$$\frac{-1}{2}\frac{d^2}{dx^2}\blacksquare+\frac{L(L+1)}{2x^2}\blacksquare-\frac{1}{x}\blacksquare$$

The 3p wave function is:

$$\Psi(x)=\frac{8}{27\sqrt{6}}\left(1-\frac{x}{6}\right)x^2\exp\left(\frac{-x}{2}\right) \qquad \int_0^\infty \Psi(x)^2 dx=1$$

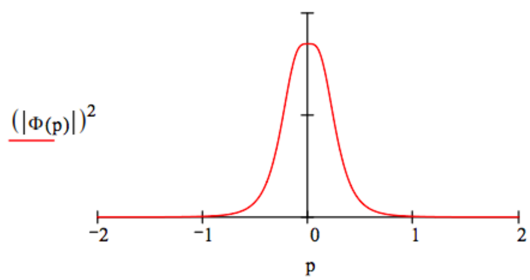


The 3p state energy is -0.0556 E_h.

$$\frac{-1}{2}\frac{d^2}{dx^2}\Psi(x)+\frac{1}{x^2}\Psi(x)-\frac{1}{x}\Psi(x)=E\Psi(x) \text{ solve, } E\rightarrow\frac{-1}{18}=-0.0556$$

The momentum wave function is generated by the following Fourier transform of the coordinate space wave function.

$$\Phi(p)=\frac{1}{\sqrt{2\pi}}\int_0^\infty \exp(\text{-i p x})\Psi(x)dx\rightarrow\frac{2}{3}2^{\frac{1}{2}}6^{\frac{1}{2}}\frac{(-1)+6\text{i p}}{(3\text{i p}+1)^4\pi^{\frac{1}{2}}}$$

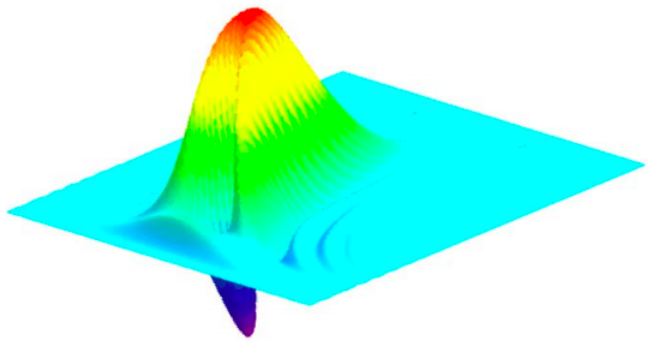


The Wigner function (phase-space representation) for the 3p state is generated using the momentum wave function.

$$W(x,p)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\overline{\Phi\left(p+\frac{s}{2}\right)}\exp(\text{-i s x})\Phi\left(p-\frac{s}{2}\right)ds$$

The Wigner distribution is displayed graphically.

$$N=150 \quad i=0..N \quad x_i=\frac{35i}{N} \quad j=0..N \quad p_j=-2+\frac{4j}{N} \quad \text{Wigner}_{i,j}=W(x_i,p_j)$$



Wigner