2.8: The Bohr Model for the Earth-Sun System

Data for the earth-sun system assuming a circular earth orbit:

$$\begin{array}{llll} \text{Mass of the earth:} & Me = 5.974(10)^{24}\text{kg} & \text{Mass of the sun:} & Ms = 1.989(10)^{30}\text{kg} \\ \text{Earth orbit radius:} & r = 1.496(10)^{11}m & \text{Gravitational constant:} & G = 6.674(10)^{-11}\frac{\text{N m}^2}{\text{kg}^2} \\ \text{Planck's constant:} & h = 6.62608(10)^{-34}\text{J s} & & \end{array}$$

Assuming the earth executes a circular orbit of radius r about the sun and has a deBroglie wavelength given by h/mv, yields a quantum mechanical kinetic energy for the earth which is the first term in the total energy expression below. The potential energy of the earth-sun interaction is well-known and is the second term in the total energy expression.

$$E = \frac{n^2h^2}{8\pi^2Mer^2} - \frac{GMeMs}{r} \quad \text{where n} = 1, 2, 3, \ldots$$

Setting the first derivative of the energy with respect to r equal to zero, yields the allowed values of r in terms of the quantum number, n.

$$\frac{d}{dr}\left(\frac{GMeMs}{r} - \frac{GMeMs}{r}\right) = 0 \quad \text{has solution(s)} \quad \frac{1}{4}n^2\frac{h^2}{G[Me^2(Ms\pi^2)]}$$

Substitution of this value of r in the total energy expression yields the energy of the earth-sun system as a function of the quantum number, n, Planck's constant, the gravitational constant, and the masses of the earth and the sun.

$$E = \frac{n^2h^2}{8\pi^2Mer^2} - \frac{GMeMs}{r} \quad \text{by substitution, yields} \quad E = \frac{-2}{n^2h^2}\pi^2Me^3G^2Ms^2$$

Given the radius of the earth's orbit listed above, calcuate the earth's quantum number.

$$r = \frac{1}{4}n^2\frac{h^2}{G[Me^2(Ms\pi^2)]} \quad \text{has solution(s)} \quad \begin{pmatrix} \frac{-2}{h}\sqrt{G}Me\sqrt{Ms}\pi\sqrt{r} \\ \frac{2}{h}\sqrt{G}Me\sqrt{Ms}\pi\sqrt{r} \end{pmatrix} = \begin{pmatrix} -2.524 \times 10^{74} \\ 2.524 \times 10^{74} \end{pmatrix}$$

The positive root n is used to calculate the energy of the earth-sun system.

$$n = 2.524(10)^{74} \quad E = \frac{-2}{n^2h^2}\pi^2Me^3G^2Ms^2 \quad E = -2.65 \times 10^{33}\text{J}$$

According to the virial theorem the classical expression for the energy of the earth-sun system with earth orbit radius r is half the potential energy. Note that this gives a value which is in agreement with the Bohr model for the earth-sun system. Is this a legitimate example of the correspondence principle?

$$E = -\frac{GMeMs}{2r} \quad E = -2.65 \times 10^{33}\text{J}$$

*Johnson and Pedersen, Problems and Solutions in Quantum Chemistry and Physics, pages 26-27.