1.72: Superposition vs. Mixture

The Wigner function can be used to illustrate the difference between a superposition and a mixture. First consider the following linear superposition of Gaussian functions.

$$\Psi(x) := \exp\left[-(x-5)^2\right] + \exp\left[-(x+5)^2\right]$$

The Wigner distribution for this function is calculated and plotted below.

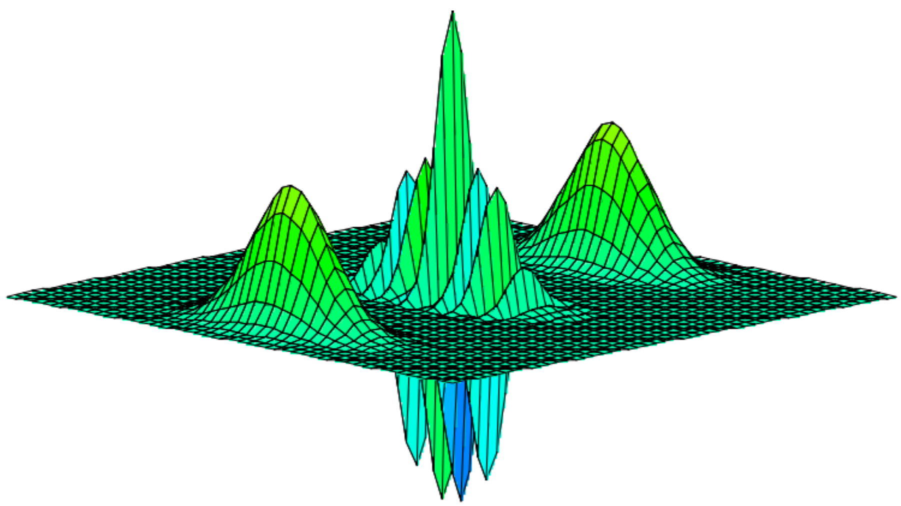
$$W(x,p) := \int_{-\infty}^{\infty} \left[\exp\left[-\left(x+\frac{s}{2}-5\right)^2\right] + \exp\left[-\left(x+\frac{s}{2}+5\right)^2\right]\right] \cdot \exp(i \cdot p \cdot s) \cdot \left[\exp\left[-\left(x-\frac{s}{2}-5\right)^2\right] + \exp\left[-\left(x-\frac{s}{2}+5\right)^2\right]\right] ds$$

Integration yields:

$$W(x,p) := \sqrt{2} \cdot \sqrt{\pi} \cdot \left(2 \cdot \exp\left(-2 \cdot x^2 - \frac{1}{2} \cdot p^2\right) \cdot \cos(10 \cdot p) + \exp\left(-2 \cdot x^2 + 20 \cdot x - 50 - \frac{1}{2} \cdot p^2\right) + \exp\left(-2 \cdot x^2 - 20 \cdot x - 50 - \frac{1}{2} \cdot p^2\right)\right)$$

$$N := 50 \qquad i := 0 \ldots N \qquad x_i := -7 + \frac{14 \cdot i}{N}$$

$$j := 0 \ldots N \qquad p_j := -6 + \frac{12 \cdot j}{N} \qquad \text{Wigner}_{i,j} := W\left(x_i, p_j\right)$$



Wigner

The signature of a superposition is the occurrence of interference fringes as seen in the center of the figure above.

The Wigner function for a classical mixture is the sum of Wigner functions for each member of the mixture. The interference region is clearly absent in the figure shown below.

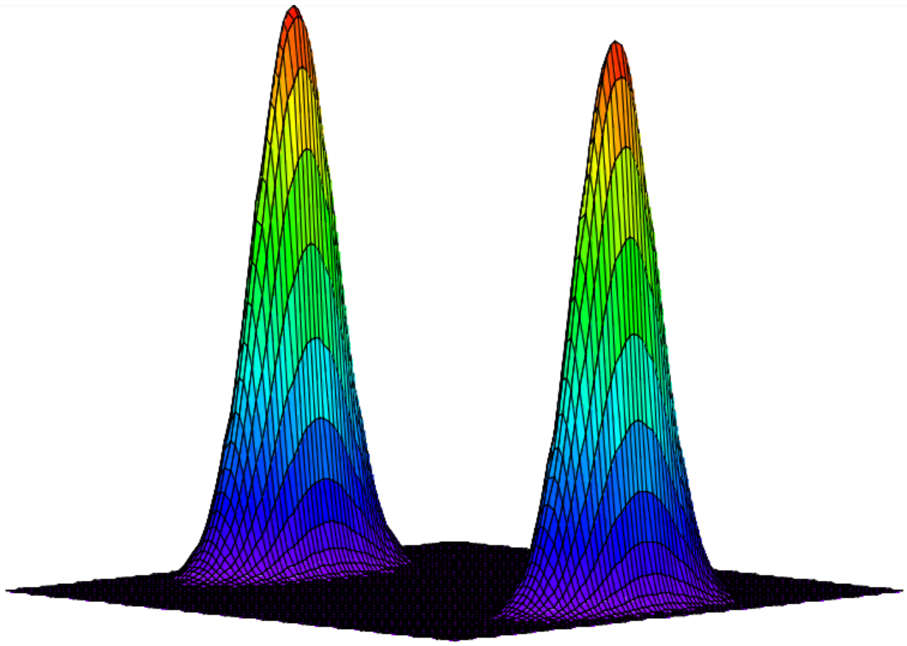
$$W(x,p) := \int_{-\infty}^{\infty} \exp\left[-\left(x+\frac{s}{2}-5\right)^2\right] \cdot \exp(i \cdot p \cdot s) \cdot \exp\left[-\left(x-\frac{s}{2}-5\right)^2\right] ds + \int_{-\infty}^{\infty} \exp\left[-\left(x+\frac{s}{2}+5\right)^2\right] \cdot \exp(i \cdot p \cdot s) \cdot \exp\left[-\left(x-\frac{s}{2}+5\right)^2\right] ds$$

Integration yields:

$$W(x,p) := \exp\left(-2\cdot x^2 + 20\cdot x - 50 - \frac{1}{2}\cdot p^2\right)\cdot\sqrt{2}\cdot\sqrt{\pi} + \exp\left(-2\cdot x^2 - 20\cdot x - 50 - \frac{1}{2}\cdot p^2\right)\cdot\sqrt{2}\cdot\sqrt{\pi}$$

$$N := 100 \qquad i := 0 \ldots N \qquad x_i := -7 + \frac{14\cdot i}{N}$$

$$j := 0 \ldots N \qquad p_j := -6 + \frac{12\cdot j}{N} \qquad \mathrm{Wigner}_{i,j} := W\left(x_i, p_j\right)$$



Wigner

Reference: Decoherence and the Transition form Quantum to Classical, Wojciech Jurek, Physics Today, October 1991, pages 36-44.