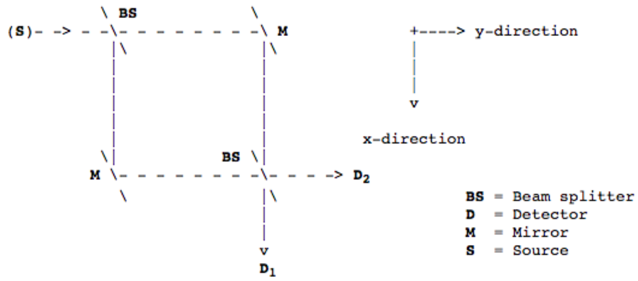
7.1: Single-Photon Interference - First Version

The schematic diagram below shows a Mach-Zehnder interferometer for photons. When the experiment is run so that there is only one photon in the apparatus at any time, the photon is always detected at D_2 and never at D_1.(1,2,3) The qualitative explanation is that there are two paths to each detector and, therefore, the probability amplitudes for these paths may interfere constructively or destructively. For detector D_2 the probability amplitudes for the two paths interfere *constructively*, while for detector D_1 they interfere *destructively*.



A quantitative quantum mechanical analysis of this striking phenomenon is outlined below. The photon leaves the source, S, traveling in the y-direction. Whether the photon takes the upper or lower path it interacts with a beam splitter, a mirror, and another beam splitter before reaching the detectors.

Orthonormal basis states: (1 x 2 vectors)

Photon moving in the x-direction: $|x >= \begin{vmatrix} 1 \\ 0 \end{vmatrix} \quad < x| = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad < x|x >= 1$

Photon moving in the y-direction: $|y >= \begin{vmatrix} 0 \\ 1 \end{vmatrix} \quad < y| = \begin{pmatrix} 0 & 1 \end{pmatrix} \quad < y|y >= 1$

< y | x > = < x | y > = 0

Operators: (2 x 2 matrices)

Operator for photon interaction with the mirror:

$$\hat{M} = \backslash(\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$$

Operator for photon interaction with the beam splitter:

$$\hat{BS} = \backslash(\begin{vmatrix} T & iR \\ iR & T \end{vmatrix}$$

T and R are the transmission and reflection amplitudes. For the half-silvered mirrors used in this example they are:

$$T = R = \left(\frac{1}{2}\right)^{\frac{1}{2}} = 0.707$$

Operations:

After interacting with a beam splitter, a photon is in a linear superposition of |x> and |y> in which the components are 90 degrees out of phase.

$$\hat{BS}|x\rangle = \frac{[|x + i|y]^{\frac{1}{2}}}{2}$$

$$\hat{BS}|y\rangle = \frac{[i|x + i|y]^{\frac{1}{2}}}{2}$$

BS M BS|y > = i|y>

Interaction with the mirror merely changes the direction of the photon.

$$\hat{M}|x\rangle = |y\rangle$$

$$\hat{M}|y\rangle = |x\rangle$$

Matrix elements:

< x | **M** | x > = 0 < y | **M** | x > = 1 < x | **M** | y > = 1 < y | **M** | y > = 0

< x | **BS** | x > = < y | **BS** | y > = $\frac{1}{2}^{\frac{1}{2}}$ < y | **BS** | x > = < x | **BS** | y > = $\frac{i}{2}^{\frac{1}{2}}$

Dirac brackets are read from right to left. In Dirac's notation < x | **M** | y > is the amplitude that a photon initially moving in the y-direction will be moving in the x-direction after interacting with the mirror. |< x | **M** | y >|^2 is the probability that a photon initially moving in the y-direction will be moving in the x-direction after interacting with the mirror. |< y | **BS** | y >|^2 is the probability that a photon initially moving in the y-direction will be found moving in the y-direction after interacting with the beam splitter.

(A) For the photon to be detected at D_1 it must be in the state |x> after interacting with two beam splitters and a mirror in the configuration shown above. The probability that a photon will be detected at D1:

< x | **BS M BS** | y > = 0 thus |< x | **BS M BS** |y>|2 = 0

(B) For the photon to be detected at D2 it must be in the state |y> after interacting with two beam splitters and a mirror in the configuration shown above. The probability that a photon will be detected at D_2:

< y |BS M BS| y > = i thus |< y |BS M BS| y >|2 = 1

It is also instructive to use Dirac's notation to examine upper and lower paths.

(A')

$$\langle D_1|y\rangle = \langle D_1|y\rangle_{upper} + \langle D_1|y\rangle_{lower} \tag{7.1.1}$$

$$= \langle x|\mathbf{BS}|x\rangle\langle x|\mathbf{M}|y\rangle\langle y|\mathbf{BS}|y\rangle + \langle x|\mathbf{BS}|y\rangle\langle y|\mathbf{M}|x\rangle\langle x|\mathbf{BS}|y > \tag{7.1.2}$$

$$= \frac{i}{2}^{\frac{1}{2}} \times 1 \times \frac{i}{2}^{\frac{1}{2}} + \frac{i}{2}^{\frac{1}{2}} \times 1 \times \frac{i}{2}^{\frac{1}{2}} \tag{7.1.3}$$

$$= \frac{i}{2} - \frac{i}{2} = i \tag{7.1.4}$$

This shows that upper and lower paths have the photon arriving 180 degrees out of phase. Thus the photon suffers destructive interference at D1.

(B') < D_2 | y > = < D_2 | y >$_{upper}$ + < D_2 | Y >$_{lower}$

= < y | **BS** | x >< x | **M** | y >< y | **BS** | y > + < y | **BS** | y >< y | **M** | x >< x | **BS** | y >

= $\frac{i}{2}^{\frac{1}{2}} \times 1 \times \frac{i}{2}^{\frac{1}{2}}$ + $\frac{i}{2}^{\frac{1}{2}} \times 1 \times \frac{i}{2}^{\frac{1}{2}}$

= $\frac{i}{2} - \frac{i}{2} = i$ i

Thus, |< D_2 | y >|2 = 1

This calculation shows that the upper and lower paths have the photon arriving in phase at D_2.

If either path (upper or lower) is blocked the interference no longer occurs and the photon reaches D1 25% of the time and D2 25%. Of course, 50% of the time it is absorbed by the blocker.

Lower path blocked:

Probability photon reaches D_1: |< x | **BS** | x >< x | **M** | y >< y | **BS** | y >|2 = $\frac{1}{4}$

Probability photon reaches D_2: |< y | **BS** | x >< x | **M** | y >< y | **BS** | y >|2 = $\frac{1}{4}$

Upper path blocked:

Probability photon reaches D_1: |< x | **BS** | y >< y | **M** | x >< x | **BS** | y >|2 = $\frac{1}{4}$

Probability photon reaches D_2: |< y | **BS** | y >< y | **M** | x >< x | **BS** | y >|2 = $\frac{1}{4}$

References:

1. P. Grangier, G. Roger, and A. Aspect, "Experimental Evidence for Photon Anticorrelation Effects on a Beam Splitter: A New Light on Single Photon Interferences," Europhys. Lett. 1, 173-179 (1986).
2. V. Scarani and A. Suarez, "Introducing Quantum Mechanics: One-particle Interferences," Am. J. Phys. 66, 718-721 (1998).
3. Kwiat, P, Weinfurter, H., and Zeilinger, A, "Quantum Seeing in the Dark," Sci. Amer. Nov. 1996, pp 72-78.
