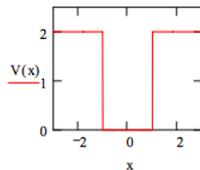


10.5: Variational Method for a Particle in a Finite Potential Well

Definite potential energy: $V(x) := if[(x \geq -1) \cdot (x \leq 1), 0, 2]$

Display potential energy:



Choose trial wavefunction: $\psi(x, \beta) := \left(\frac{2 \cdot \beta}{\pi}\right)^{\frac{1}{4}} \cdot (-\beta \cdot x^2)$

Demonstrate that the trial wavefunction is normalized.

$$\int_{-\infty}^{\infty} \psi(x, \beta)^2 dx \text{ assume, } \beta > 0 \rightarrow 1$$

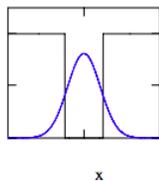
Evaluate the variational integral:

$$E(\beta) := \int_{-\infty}^{\infty} \psi(x, \beta) \cdot -\frac{1}{2} \cdot \frac{d^2}{dx^2} \psi(x, \beta) dx \dots \Big|_{\text{simplify}}^{\text{assume, } \beta > 0} \rightarrow \frac{1}{2} \cdot \beta + 2 - 2 \cdot \text{erf}\left(2^{\frac{1}{2}} \cdot \beta^{\frac{1}{2}}\right)$$

Minimize the energy integral with respect to the variational parameter, β .

$$\beta := 1 \quad \beta := \text{Minimize}(E, \beta) \quad \beta = 0.678 \quad E(\beta) = 0.538$$

Display wavefunction in the potential well and compare result with the exact energy, $0.530 E_h$.



$$\frac{E(\beta) - 0.530}{0.530} = 1.546\%$$

Calculate the fraction of time tunneling is occurring.

$$2 \cdot \int_1^{\infty} \psi(x, \beta)^2 dx = 0.1$$

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