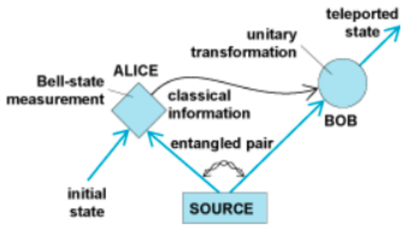
8.1: A Single Page Summary of Quantum Teleportation

As shown in the graphic below (Nature, December 11, 1997, page 576), quantum teleportation is a form of information transfer that requires pre-existing entanglement and a classical communication channel to send information from one location to another. Alice has the photon to be teleported and a photon of an entangled pair (β00) that she shares with Bob. She performs a measurement on her photons that projects them into one of the four Bell states and Bob's photon, via the entangled quantum channel, into a state that has a unique relationship to the state of the teleportee. Bob carries out one of four unitary operations on his photon depending on the results of Alice's measurement, which she sends him through a classical communication channel.



The teleportee and the Bell states indexed in binary notation:

Teleportee:

$$\begin{pmatrix} \sqrt{\frac{1}{3}} \\ \sqrt{\frac{2}{3}} \end{pmatrix}$$

Bell states:

$$\beta_{00} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad \beta_{01} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \quad \beta_{10} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} \quad \beta_{11} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$$

The three-cubit initial state is rewritten as a linear superposition of the four possible Bell states that Alice can find on measurement.

$$|\Psi\rangle = \begin{pmatrix} \sqrt{\frac{1}{3}} \\ \sqrt{\frac{2}{3}} \end{pmatrix} \otimes \beta_{00} = \frac{1}{2}\left[\beta_{00} \otimes \begin{pmatrix} \sqrt{\frac{1}{3}} \\ \sqrt{\frac{2}{3}} \end{pmatrix} + \beta_{01} \otimes \begin{pmatrix} \sqrt{\frac{2}{3}} \\ \sqrt{\frac{1}{3}} \end{pmatrix} + \beta_{10} \otimes \begin{pmatrix} \sqrt{\frac{1}{3}} \\ -\sqrt{\frac{2}{3}} \end{pmatrix} + \beta_{11} \otimes \begin{pmatrix} -\sqrt{\frac{2}{3}} \\ \sqrt{\frac{1}{3}} \end{pmatrix}\right]$$

Alice's Bell state measurement result (β_{00}, β_{01}, β_{10} or β_{11}) determines the operation (I, X, Z or ZX) that Bob performs on his photon. The matrices for these operations are as follows.

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad ZX = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Tabular summary of teleportation experiment:

$$\begin{pmatrix} \text{Alice Measurement Result} & \beta_{00} & \beta_{01} & \beta_{10} & \beta_{11} \\ \text{Bob's Action} & I & X & Z & XZ \end{pmatrix}$$

Summary of the quantum teleportation protocol: "Quantum teleportation provides a 'disembodied' way to transfer quantum states from one object to another at a distant location, assisted by previously shared entangled states and a classical communication channel." Nature **518**, 516 (2015)