8.81: A Brief Introduction to Quantum Dense Coding

> Quantum superdense coding reliably transmits two classical bits through an entangled pair of particles, even though only one member of the pair is handled by the sender. Charles Bennett, *Physics Today*, October 1995, p. 27

This tutorial is based on Brad Rubin's "Superdense Coding" at the Wolfram Demonstration Project: http:demonstrations.wolfram.com/SuperdenseCoding/. The quantum circuit shown below impliments quantum dense coding. Alice and Bob share the entangled pair of photons in the Bell basis shown at the left. Alice encodes two classical bits of information (four possible messages) on her photon, and Bob subsequently reads her message by performing a Bell state measurement on the modified entangled photon pair. In other words, although Alice encodes two bits on her photon Bob's readout requires a measurement involving both photons. In this example Alice sends |11> to Bob.

$$\begin{array}{cccccccc} \cdot & \boxed{X}^1 & \cdot & \boxed{Z}^1 & \cdot & \cdot & \boxed{H} & \cdot \\ \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} & & \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} & & \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} & \begin{array}{c} | \\ | \\ | \\ | \end{array} & & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ \cdot & \boxed{I} & \cdot & & & & & \\ fbox I & \cdot & \oplus & \boxed{I} & & & & \end{array}$$

The operation of the circuit is outlined in both matrix and algebraic format. The necessary truth tables and matrix operators are provided in the Appendix.

Matrix Method

$$\mathrm{H} \otimes \mathrm{I}\ \mathrm{CNOT}\ \mathrm{Z} \otimes \mathrm{I}\ \mathrm{X} \otimes \mathrm{I} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |11\rangle$$

Algebraic Method

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}} \xrightarrow{X^1 \otimes I} \frac{|10\rangle + |01\rangle}{\sqrt{2}} \xrightarrow{Z^1 \otimes 1} \frac{-|10\rangle + |01\rangle}{\sqrt{2}} \xrightarrow{CNOT} \frac{-|11\rangle + |01\rangle}{\sqrt{2}} \xrightarrow{H \otimes I} \frac{-(|0\rangle - |1\rangle)|1\rangle + (|0\rangle + |1\rangle)|1\rangle}{2}$$

$$= |11\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Appendix

Truth tables for the quantum circuit:

$$\mathrm{X} = \mathrm{NOT} \begin{pmatrix} 0 \text{ to } 0 \\ 1 \text{ to } 0 \end{pmatrix} \quad \mathrm{Z} \begin{pmatrix} 0 \text{ to } 0 \\ 1 \text{ to -}1 \end{pmatrix} \quad \mathrm{H} = \mathrm{Hadamard} \begin{bmatrix} 0 \text{ to } \frac{1}{\sqrt{2}}(0+1) \\ 1 \text{ to } \frac{1}{\sqrt{2}}(0-1) \end{bmatrix} \quad \mathrm{CNOT} \begin{pmatrix} 00 \text{ to } 00 \\ 01 \text{ to } 01 \\ 10 \text{ to } 11 \\ 11 \text{ to } 10 \end{pmatrix}$$

Circuit elements in matrix formats:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

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