

1.73: Time-dependent Wigner Function for Harmonic Oscillator Transitions

Initial state:

$$m := 0 \quad E_m := m + \frac{1}{2}$$

Final state:

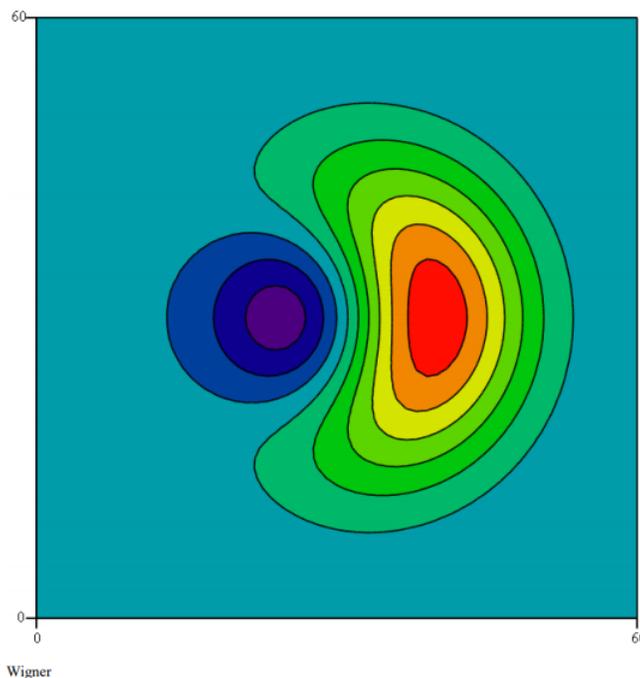
$$n := 1 \quad E_n := n + \frac{1}{2} \quad t := \text{FRAME}$$

Define Wigner distribution function for a linear superposition of the initial and final harmonic oscillator state.

$$\begin{aligned}
 W(x, p) := & \frac{1}{\pi^{\frac{3}{2}}} \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2^n \cdot n! \cdot \sqrt{\pi}}} \cdot \text{Her}\left(n, x + \frac{s}{2}\right) \cdot \exp\left[-\frac{\left(x + \frac{s}{2}\right)^2}{2}\right] \cdot \exp(i \cdot E_n \cdot t) + \frac{1}{\sqrt{2^m \cdot m! \cdot \sqrt{\pi}}} \cdot \text{Her}\right. \\
 & \left. \left(m, x + \frac{s}{2}\right) \cdot \exp\left[-\frac{\left(x + \frac{s}{2}\right)^2}{2}\right] \cdot \exp(i \cdot E_m \cdot t) \right] \\
 & \cdot \exp(i \cdot s \cdot p) \cdot \left[\frac{1}{\sqrt{2^n \cdot n! \cdot \sqrt{\pi}}} \cdot \text{Her}\left(n, x - \frac{s}{2}\right) \cdot \exp\left[-\frac{\left(x - \frac{s}{2}\right)^2}{2}\right] \cdot \exp(-i \cdot E_n \cdot t) \right. \\
 & \left. + \frac{1}{\sqrt{2^m \cdot m! \cdot \sqrt{\pi}}} \cdot \text{Her}\left(m, x - \frac{s}{2}\right) \cdot \exp\left[-\frac{\left(x - \frac{s}{2}\right)^2}{2}\right] \cdot \exp(-i \cdot E_m \cdot t) \right] ds
 \end{aligned}$$

Display Wigner distribution:

$$\begin{aligned}
 N = 60 \quad i := 0 \dots N \quad x_i := -2.5 + \frac{5 \cdot i}{N} \\
 j := 0 \dots N \quad p_j := -2.5 + \frac{5j}{N} \quad \text{Wigner}_{i,j} := W(x_i, p_j)
 \end{aligned}$$



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