2.55: The Wigner Distribution for the 1s State of the 1D Hydrogen Atom

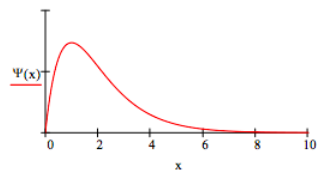
This tutorial presents three pictures of the 1s state of the one-dimensional hydrogen atom using its position, momentum and phase-space representations.

The energy operator for the one-dimensional hydrogen atom in atomic units is:

$$\frac{-1}{2}\frac{d^2}{dx^2}\blacksquare-\frac{1}{x}\blacksquare$$

The ground state eigenstate is:

$$\Psi(x)=2x\exp(-x) \qquad \int_0^\infty \Psi(x)^2 dx=1$$

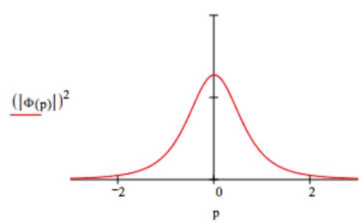


The ground state energy is -0.5 E_h.

$$\frac{-1}{2}\frac{d^2}{x^2}\Psi(x)-\frac{1}{x}\Psi(x)=E\Psi(x)\text{ solve, E}\rightarrow\frac{-1}{2}$$

The momentum wave function is generated by the following Fourier transform of the coordinate space wave function.

$$\Phi(p)=\frac{1}{\sqrt{2\pi}}\int_0^\infty \exp(\text{-i p x})\Psi(x)dx\rightarrow\frac{2^{\frac{1}{2}}}{(\text{i p}+1)\pi^{\frac{1}{2}}}$$

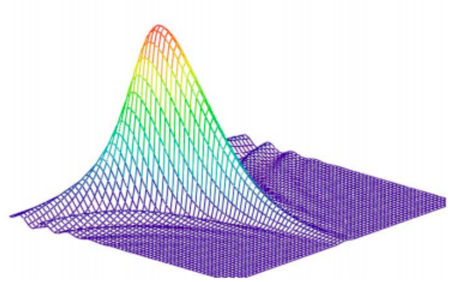


The Wigner function for the hydrogen atom ground state is generated using the momentum wave function.

$$\text{W}(\text{x, p})=\frac{1}{2\pi}\int_{-\infty}^{\infty}\overline{\Phi\left(\text{p}+\frac{\text{s}}{2}\right)}\exp(\text{-i s x})\Phi\left(\text{p}-\frac{\text{s}}{2}\right)\text{ds}$$

The Wigner distribution is displayed graphically.

$$N=60 \quad i=0..N \quad x_i=\frac{6i}{N} \quad j=0..N \quad p_j=-5+\frac{10j}{N} \quad \text{Wigner}_{i,j}=\text{W}(x_i,p_j)$$



Wigner