2.2: The de Broglie-Bohr Model for the Hydrogen Atom - Version 3

$\lambda = \frac{h}{mv}$ de Broglie's hypothesis that matter has wave-like properties.

$n\lambda = 2\pi r$ The consequence of de Broglie's hypothesis; an integral number of wavelengths must fit within the circumference of the orbit. This introduces the quantum number, n, which can have values 1, 2, 3...

$mv = \frac{nh}{2\pi r}$ Substitution of the first equation into the second equation reveals that linear momentum is quantized.

$T = \frac{1}{2}mv^2 = \frac{n^2h^2}{8\pi^2 m_e r}$ If momentum is quantized, so is kinetic energy.

$E = T + V = \frac{n^2h^2}{8\pi^2 m_e r^2} - \frac{q^2}{4\pi\varepsilon o_0 r}$ Which means that total energy is quantized.

Below the ground state energy and orbit radius of the electron in the hydrogen atom is found by plotting the energy as a function of the orbital radius. The ground state is the minimum in the curve.

Fundamental constants: electron charge, electron mass, Planck's constant, vacuum permitivity.

$$q = 1.6021777(10)^{-19}\text{coul} \qquad m_e = 9.10939(10)^{-31}\text{kg}$$
$$h = 6.62608(10)^{-34}\text{joule sec} \quad \varepsilon_0 = 8.85419(10)^{-12}\frac{\text{coul}^2}{\text{joule m}}$$

Conversion factors between meters and picometers and joules and atto joules.

$$pm = 10^{-12}m \quad \text{ajoule} = 10^{-18}\text{joule} \quad eV = 1.602177(10)^{-19}\text{joule}$$

Setting the first derivative of the energy with respect to r equal to zero, yields the optimum value of r.

$$\frac{d}{dr}\left(\frac{n^2h^2}{8\pi^2 m_e r^2} - \frac{q^2}{4\pi\varepsilon_0 r}\right) = 0 \quad \text{has solution(s)} \quad n^2h^2\frac{\varepsilon_0}{q^2\pi m_e}$$

Substitution of this value of r back into the energy expression yields the energy gives the energy of the hydrogen atom in terms of the quantum number, n, and the fundamental constants.

$$E = \frac{n^2h^2}{8\pi^2 m_e r^2} - \frac{q^2}{4\pi\varepsilon_0 r} \quad \text{by substitution, yields} \quad E = \frac{-1}{8n^2h^2}\frac{m_e}{\varepsilon_0^2}q^4$$

Calculate the allowed energy levels for the hydrogen atom: n = 1 .. 5

$$E_n = \frac{-1}{8n^2h^2}\frac{m_e}{\varepsilon_0^2}q^4 \quad \frac{E_n}{\text{ajoule}} = \begin{pmatrix} -2.18 \\ -0.545 \\ -0.242 \\ -0.136 \\ -0.087 \end{pmatrix} \quad \frac{E_n}{eV} = \begin{pmatrix} -13.606 \\ -3.401 \\ -1.512 \\ -0.85 \\ -0.544 \end{pmatrix}$$

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