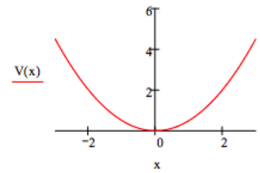
10.40: Variation Method Using the Wigner Function- The Harmonic Oscillator

Define potential energy:

$$V(x)=\frac{x^2}{2}$$

Display potential energy:



Choose trial wave function:

$$\psi(x,\beta=\left(\frac{2\beta}{\pi}\right)^{\frac{1}{4}}exp(-\beta x^2)$$

Calculate the Wigner distribution function:

$$W(x,p,\beta)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\psi\left(x+\frac{s}{2},\beta\right)exp(isp)\psi(\left(x-\frac{s}{2},\beta\right)ds\Big|_{assume,\ \beta>0}^{simplify}\rightarrow\frac{1}{\pi}e^{\frac{-1}{2}\frac{4\beta^2x^2+p^2}{\beta}}$$

Evaluate the variational integral:

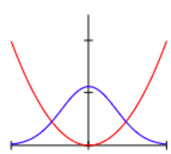
$$E(\beta)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}W(x,p,\beta)\left(\frac{p^2}{2}+V(x)\right)dx\ dp$$

Minimize the energy integral with respect to the variational parameter, β.

β = 1 β = Minimize (E, β) β = 0.5 E(β) = 0.5

Calculate and display the coordinate distribution function:

$$Px(x,\beta)=\int_{-\infty}^{\infty}W(x,p,\beta)dp$$



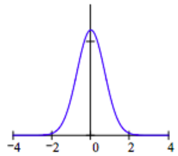
Classical turning point: $x_{cl}=0.5^{\frac{1}{2}}\quad x_{cl}=0.707$

Probability that tunneling is occurring:

$$2\int_{0.707}^{\infty}Px(x,\beta)dx=0.317$$

Calculate and display the momentum distribution function:

$$Pp(p,\beta)=\int_{-\infty}^{\infty}W(x,p,\beta)dx$$



Display the Wigner distribution function:

$$N = 60 \quad i = 0 .. N \quad x_i = -3 + \frac{6i}{N} \quad j = 0 .. N \quad p_j = -5 + \frac{10j}{N} \quad Wigner_{i,j} = W(x_i, p_j, \beta)$$