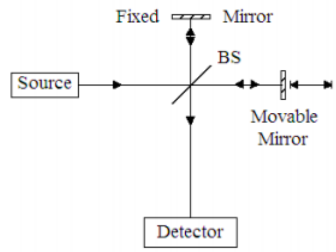
7.21: Two Analyses of the Michelson Interferometer



Path difference between interferometer arms is δ. Phase accumulated due to path difference: $exp\left(i\frac{2\pi\delta}{\lambda}\right)$

S stands for source, **D** for detector, **T** for transmitted and **R** for reflected. The evolution of the photon wave function at various stages is given below.

$$S = \frac{1}{\sqrt{2}}(T + iR)$$

$$T = \frac{exp\left(i\frac{2\pi\delta}{\lambda}\right)}{\sqrt{2}}(iD + S)$$

$$R = \frac{1}{\sqrt{2}}(D + iS)$$

$$S = \frac{1}{\sqrt{2}}(T + iR)\Big|^{substitute,\ T = \frac{exp\left(i\frac{2\pi\delta}{\lambda}\right)}{\sqrt{2}}(iD+S)}_{substitute,\ R = \frac{1}{\sqrt{2}}(D+iS)} \rightarrow S = -\frac{S}{2} + \frac{Se^{\frac{2i\pi\delta}{\lambda}}}{2} + \frac{De^{\frac{2i\pi\delta}{\lambda}}}{2} + \frac{Di}{2}$$

The probability the photon will arrive at the detector is the square of the absolute magnitude of the coefficient of D.

$$\frac{-1}{2}\left(e^{\frac{-2i\pi\delta}{\lambda}}\right)\frac{1}{2}\left(e^{\frac{-2i\pi\delta}{\lambda}}\right) simplify \rightarrow \sin\left(\frac{\pi\delta}{\lambda}\right)^2$$

The same results are now illustrated using a matrix mechanics approach. Horizontal and vertical motion of the photon are represented by vectors. The source emits a horizontal photon, the detector receives a vertical photon. The beam splitter and the phase shift due to path length difference are represented by matrices. A matrix representation for the mirrors is unnecessary because the simply return the photon to the beam splitter.

Horizontal motion: $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Vertical motion: $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Beam splitter:

$$BS = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Phase shift:

$$A(\delta) = \begin{pmatrix} e^{2i\pi\frac{\delta}{\lambda}} & 0 \\ 0 & 1 \end{pmatrix}$$

Calculate the probability amplitude and probability that the photon will arrive at the detector

$$\begin{pmatrix} 0 & 1 \end{pmatrix} BS \begin{pmatrix} e^{2i\pi\frac{\delta}{\lambda}} & 0 \\ 0 & 1 \end{pmatrix} BS \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \frac{e^{\frac{2i\pi\delta}{\lambda}}}{2} + \frac{1}{2}i$$

$$\left(\frac{e^{\frac{-2i\pi\delta}{\lambda}}(-i)}{2} + \frac{1}{2}(-i)\right)\left(\frac{e^{\frac{2i\pi\delta}{\lambda}}(i)}{2} + \frac{1}{2}i\right) simplify \rightarrow \cos\left(\frac{\pi\delta}{\lambda}\right)^2$$

Calculate the probability amplitude and probability that the photon will be returned to the source.

$$\begin{pmatrix} 1 & 0 \end{pmatrix} BS \begin{pmatrix} e^{2i\pi\frac{\delta}{\lambda}} & 0 \\ 0 & 1 \end{pmatrix} BS \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \frac{e^{\frac{2i\pi\delta}{\lambda}}}{2} - \frac{1}{2}$$

$$\left(\frac{e^{\frac{-2i\pi\delta}{\lambda}}}{2} - \frac{1}{2}\right)\left(\frac{e^{\frac{2i\pi\delta}{\lambda}}}{2} - \frac{1}{2}\right) simplify \rightarrow \sin\left(\frac{\pi\delta}{\lambda}\right)^2$$

Plotting these results in units of λ yields:

$$\delta = 0, .01 .. 1$$