2.57: The Wigner Distribution for the 2p State of the 1D Hydrogen Atom

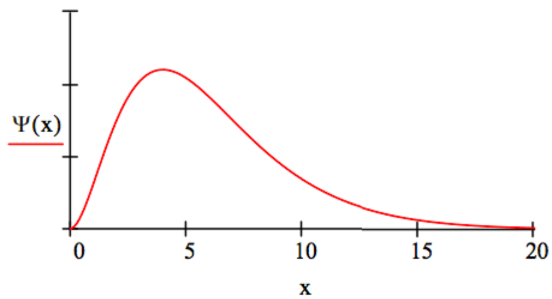
This tutorial presents three pictures of the 2p state of the one-dimensional hydrogen atom using its position, momentum and phase-space representations.

The energy operator for the one-dimensional hydrogen atom in atomic units is:

$$\frac{-1}{2}\frac{d^2}{dx^2}\blacksquare + \frac{\mathrm{L(L+1)}}{2x^2}\blacksquare - \frac{1}{x}\blacksquare$$

The 2p wave function is:

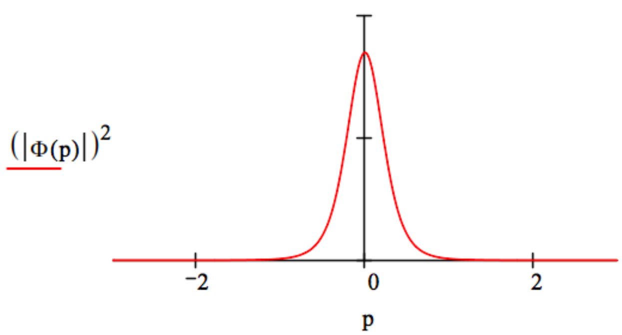
$$\Psi(x) = \frac{1}{\sqrt{24}}x^2\exp\left(-\frac{x}{2}\right) \qquad \int_0^\infty \Psi(x)^2 dx = 1$$



The 2p state energy is -0.125 E_h.

$$\frac{-1}{2}\frac{d^2}{dx^2}\Psi(x) + \frac{1}{x^2}\Psi(x) - \frac{1}{x}\Psi(x) = \mathrm{E}\Psi(x) \text{solve, E} \rightarrow \frac{-1}{8} = -0.125$$

The momentum wave function is generated by the following Fourier transform of the coordinate space wave function.

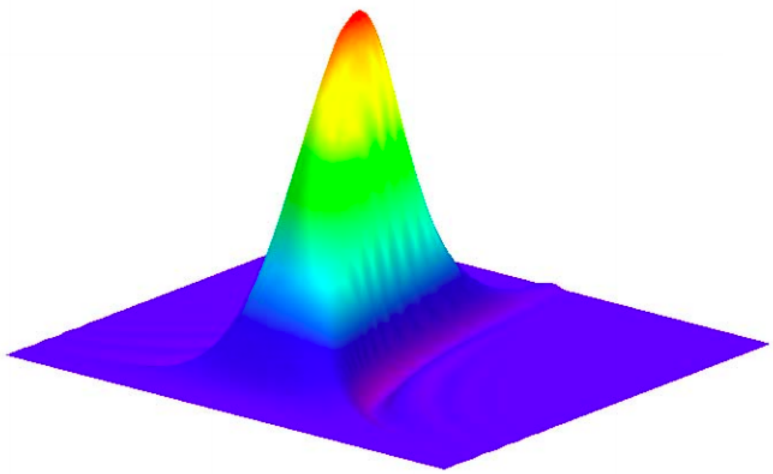


The Wigner function (phase-space representation) for the 2p state is generated using the momentum wave function.

$$\mathrm{W(x, p)} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\overline{\left(\mathrm{p}+\frac{\mathrm{s}}{2}\right)}\exp(\text{-i s x})\Phi\left(\mathrm{p}-\frac{\mathrm{s}}{2}\right)\mathrm{ds}$$

The Wigner distribution is displayed graphically.

$$N = 100 \quad i = 0..N \quad x_i = \frac{15i}{N} \quad j = 0..N \quad p_j = -3 + \frac{6j}{N} \quad \mathrm{Wigner}_{i,\,j} = \mathrm{W}(x_i,\, p_j)$$



Wigner