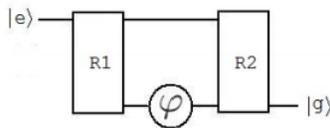


7.23: The Ramsey Atomic Interferometer

The Ramsey interferometer, which closely resembles the Mach-Zehnder interferometer, is constructed using two $\pi/2$ Rabi pulses (R1 and R2) separated by a phase shifter in the lower arm, as shown below.



where

$$|e\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|g\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The matrix representations of the phase shifter and the Rabi elements are as follows:

$$PhaseShift(\phi) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

$$Rabi(\phi) = \begin{pmatrix} \cos(\frac{\phi}{2}) & -\sin(\frac{\phi}{2}) \\ \sin(\frac{\phi}{2}) & \cos(\frac{\phi}{2}) \end{pmatrix}$$

$$Rabi\left(\frac{\pi}{2}\right) = \begin{pmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{pmatrix}$$

The input to the interferometer is the upper state $|e\rangle$ of a two-state atom. The first pulse behaves like a Hadamard gate creating a coherent superposition of $|e\rangle$ and the lower state of the atom, $|g\rangle$.

$$Rabi\left(\frac{\pi}{2}\right) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{2}2^{\frac{1}{2}} \\ \frac{1}{2}2^{\frac{1}{2}} \end{pmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

The phase shifter alters the superposition by adding a phase to $|g\rangle$.

$$PhaseShift(\phi)Rabi\left(\frac{\pi}{2}\right) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{2}2^{\frac{1}{2}} \\ \frac{1}{2}e^{i\phi}2^{\frac{1}{2}} \end{pmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ e^{i\phi} \end{pmatrix}$$

In the absence of a phase shift ($\phi = 0$) the two $\pi/2$ pulses behave like a not gate yielding $|g\rangle$ at the output channel.

$$Rabi\left(\frac{\pi}{2}\right) PhaseShift(0)Rabi\left(\frac{\pi}{2}\right) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

However if $\phi = \pi$, the interferometer is equivalent to the identity operator and the output is $|e\rangle$.

$$Rabi\left(\frac{\pi}{2}\right) PhaseShift(\pi)Rabi\left(\frac{\pi}{2}\right) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

In general, the result is a superposition of $|e\rangle$ and $|g\rangle$.

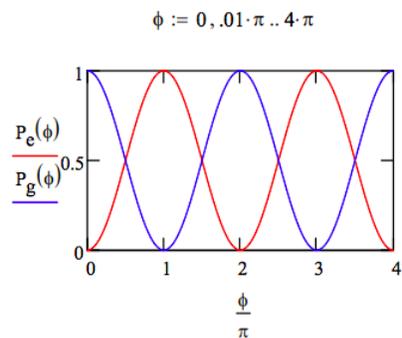
$$Rabi\left(\frac{\pi}{2}\right)PhaseShift(\phi)Rabi\frac{\phi}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{2} - \frac{1}{2}e^{i\phi} \\ \frac{1}{2} + \frac{1}{2}e^{i\phi} \end{pmatrix}$$

$$\begin{pmatrix} e \\ g \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 - e^{i\phi} \\ 1 + e^{i\phi} \end{pmatrix}$$

The probabilities of detecting $|e\rangle$ and $|g\rangle$ at the output channel depend on the phase ϕ , exhibiting interference effects as in the Mach-Zehnder interferometer with a phase shifter in the lower arm. This is shown below both algebraically and graphically.

$$P_e(\phi) = \left[\left| \begin{pmatrix} 1 \\ 0 \end{pmatrix}^T Rabi\left(\frac{\pi}{2}\right)PhaseShift(\phi)Rabi\left(\frac{\pi}{2}\right)\begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|^2 \right] \text{ simplify } \rightarrow \frac{1}{2} - \frac{1}{2}\cos\phi$$

$$P_g(\phi) = \left[\left| \begin{pmatrix} 0 \\ 1 \end{pmatrix}^T Rabi\left(\frac{\pi}{2}\right)PhaseShift(\phi)Rabi\left(\frac{\pi}{2}\right)\begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|^2 \right] \text{ simplify } \rightarrow \frac{1}{2} + \frac{1}{2}\cos\phi$$



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