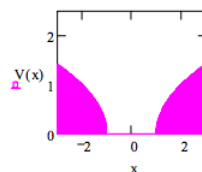# 10.6: Variation Method for a Particle in a Symmetric 1D Potential Well

Definite potential energy: $V(x) := if[(x \geq -1) \cdot (x \leq 1), 0, \sqrt{|x| - 1}]$

Display potential energy:

Choose trial wave function: $\Psi(x, \beta) := \left(\frac{2 \cdot \beta}{\pi}\right)^{\frac{1}{4}} \cdot exp(-\beta \cdot x^2)$
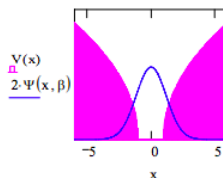
Evaluate the variational integral:

$$E(\beta) := \int_{-\infty}^{\infty} \Psi(x, \beta) \cdot -\frac{1}{2} \cdot \frac{d^2}{dx^2} \Psi(x, \beta) dx + \int_{-\infty}^{\infty} V(x) \cdot \Psi(x, \beta)^2 dx$$

Minimize the energy integral with respect to the variational parameter, β.

$$\beta := .2 \quad \beta := \text{Minimize}(E, \beta) \quad \beta := 0.363 \quad E(\beta) = 0.313$$

Display wave function in the potential well.

Calculate the probability that the particle is in the potential barrier.

$$2 \cdot \int_{1}^{\infty} \Psi(x, \beta)^2 dx = 0.228$$

Define quantum mechanical tunneling.

> Tunneling occurs when a quon (a quantum mchanical particle) has probability of being in a nonclassical region. In other words, a region in which the total energy is less than the potential energy.

Calculate the probability that tunneling is occurring.

$$|x| - 1 = 0.313^2 \Big|_{solve,\, x}^{float,\, 4} \rightarrow \begin{pmatrix} 1.098 \\ -1.098 \end{pmatrix}$$

$$2 \cdot \int_{1.098}^{\infty} \Psi(x, \beta)^2 dx = 0.186$$

Calculate the kinetic and potential energy contributions to the total energy.

Kinetic energy:

$$\int_{-\infty}^{\infty} \Psi(x, \beta) \cdot -\frac{1}{2} \cdot \frac{d^2}{dx^2} \Psi(x, \beta) dx = 0.182$$

Potential energy:

$$\int_{-\infty}^{\infty} V(x) \cdot \Psi(x, \beta)^2 dx = 0.131$$