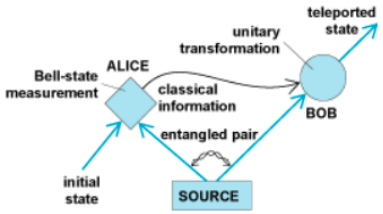
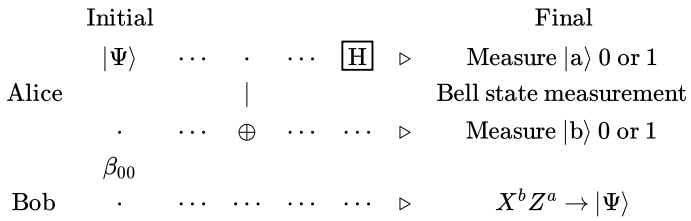
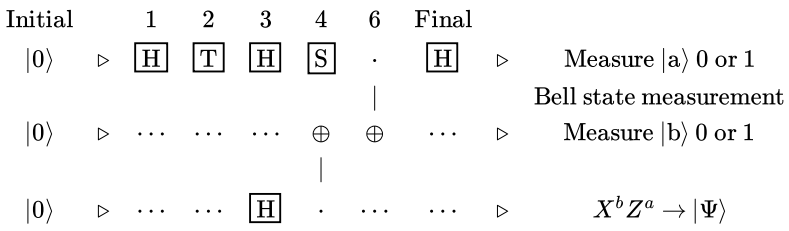
8.9: A Quantum Teleportation Experiment for Undergraduates

This Mathcad document examines the math involved in a teleportation experiment for undergraduates using IBM's 5-qubit quantum processor (IBM Quantum Experience) posted by S. Fedortchenko at arXiv:1607.02398v1. Except for state preparation it is identical to the following teleportation circuit, which can be found in my "Teleportation: Another Look."





Fedortchenko's teleportation circuit is shown below.



Single qubit operators:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix} \quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Two qubit operators:

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{ICNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Demonstrate the generation of the teleportee state held by Alice:

$$\text{SHTH} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.854 + 0.354i \\ 0.354 + 0.146i \end{pmatrix}$$

$$e^{i\frac{\pi}{8}} \left[\cos\left(\frac{\pi}{8}\right) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \sin\left(\frac{\pi}{8}\right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] = \begin{pmatrix} 0.854 + 0.354i \\ 0.354 + 0.146i \end{pmatrix}$$

Demonstrate the creation of the entangled Bell state shared by Alice and Bob:

$$\text{ICNOT}\ (\text{kronecker}(\text{I}, \text{H})) \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.707 \\ 0 \\ 0 \\ 0.707 \end{pmatrix} \quad \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right]$$

Creation of the teleportee state and the entangled Bell state occurs in the first four steps.

$\mathrm{StatePrep} = \mathrm{kronecker\ (S,\ ICNOT)\ kronecker\ (H,\ kronecker(I,\ H))\ kronecker\ (T,\ kronecker(I,\ I))\ kronecker\ (H\ kronecker\ (I,\ I))}$

Teleportation occurs in steps 5 and 6.

$$textTC = kronecker(H, kronecker(I, I))kronecker(CNOT, I)$$

$$\mathrm{TC\ StatePrep} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.427 + 0.177i \\ 0.177 + 0.073i \\ 0.177 + 0.073i \\ 0.427 + 0.177i \\ 0.427 + 0.177i \\ -0.177 - 0.073i \\ -0.177 - 0.073i \\ 0.427 + 0.177i \end{pmatrix}$$

Measurement occurs in the final step on the top two wires with possible outcomes |00>, |01>, |10> and |11>. In other words, Alice makes a Bell state measurement (see first figure above) on the two qubits in her possession and informs Bob of the result through a classical channel. He then performs an operation on his qubit to recover the teleported state.

If Alice observes |00> Bob does nothing (the identity operation) because he has the teleportee on his register. If Alice observes |01> Bob applies the X operator, if she finds |10> he uses the Z operator, and finally if Alice observes |11> Bob applies the X operator followed by the Z operator. Further mathematical detail is provided by showing explicitly the four equally probable measurement outcomes that Alice observes, and Bob's subsequent action on his register.

Computational Details

Measurement operator for |0>:

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Measurement operator for |1>:

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Alice measures |00>:

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.854 + 0.354i \\ 0.354 + 0.146i \end{pmatrix}$$

Bob's action:

$$I \begin{pmatrix} 0.854 + 0.354i \\ 0.354 + 0.146i \end{pmatrix} = \begin{pmatrix} 0.854 + 0.354i \\ 0.354 + 0.146i \end{pmatrix}$$

$$\mathrm{2kronecker} \left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathrm{kronecker} \left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathrm{I} \right] \right] \mathrm{TC\ StatePrep} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.854 + 0.354i \\ 0.354 + 0.146i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Alice measures |01>:

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.354 + 0.146i \\ 0.854 + 0.354i \end{pmatrix}$$

Bob's action:

$$X \begin{pmatrix} 0.354+0.146i \\ 0.854+0.354i \end{pmatrix} = \begin{pmatrix} 0.854+0.354i \\ 0.354+0.146i \end{pmatrix}$$

$$2\text{kronecker}\left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \text{kronecker}\left[\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \text{I}\right]\right] \text{TC StatePrep} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0.354+0.146i \\ 0.854+0.354i \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Alice measures |10>:

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.854+0.354i \\ -0.354-0.146i \end{pmatrix}$$

Bob's action:

$$Z \begin{pmatrix} 0.854+0.354i \\ -0.354-0.146i \end{pmatrix} = \begin{pmatrix} 0.854+0.354i \\ 0.354+0.146i \end{pmatrix}$$

$$2\text{kronecker}\left[\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \text{kronecker}\left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \text{I}\right]\right] \text{TC StatePrep} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.854+0.354i \\ -0.354-0.146i \\ 0 \\ 0 \end{pmatrix}$$

Alice measures |11>:

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -0.354-0.146i \\ 0.854+0.354i \end{pmatrix}$$

Bob's action:

$$ZX \begin{pmatrix} 0.854+0.354i \\ -0.354-0.146i \end{pmatrix} = \begin{pmatrix} 0.854+0.354i \\ 0.354+0.146i \end{pmatrix}$$

$$2\text{kronecker}\left[\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \text{kronecker}\left[\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \text{I}\right]\right] \text{TC StatePrep} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0.354-0.146i \\ 0.854+0.354i \end{pmatrix}$$