Dimensional Analysis

Skills to Develop

- Use dimensional analysis to calculate solutions with the correct units

A **dimension** is any measurable extent, such as length, time, and mass. **Units** help describe the measurement according to certain standards. In the metric system for example, a one-dimensional (1-D) length is measured in meters (m) a two-dimensional (2-D) area is measured in meters squared (m^2), and a three-dimensional (3-D) volume is measured in meters cubed (m^3). Other types of quantities (time, mass, temperature) are measured using different units because they have different dimensions. **Analysis** means to think about something, often focusing on one part at a time. Putting it all together, **dimensional analysis** means thinking about units piece by piece. Dimensional analysis can by to correctly go between different types of units, to catch mistakes in one's calculations, and to make many useful calculations in real life.

Essentially, dimensional analysis means multiplying by one. You collect a set of "conversion factors" or ratios that equal one, and then multiply a quantity that you are interested in by those "ones." For example, if you want to know how many seconds it would take to get from New York to Philadelphia, you'd do it like this:

First, using the express train it takes 2.5 hours to get to Philadelphia from a station in New York. Then, we know that 1 hour = 60 minutes and 1 minute = 60 seconds, so (1h / 60 min) = 1, and (1 min / 60 s) = 1. Now, all we have to do it multiply our starting number (2.5 h) by "one" twice, making sure that the units cancel correctly so that we have only seconds at the end.

$$(2.5\ \cancel{h})\left(\dfrac{60\ \cancel{min}}{1\ \cancel{h}}\right)\left(\dfrac{60 s}{1\ \cancel{min}}\right) = 9.0 \times 10^3 s \tag{1}$$

If each part is not put in the right place, the units will come out wrong. For example:

$$\left(\dfrac{1}{2.5\ \cancel{h}}\right)\left(\dfrac{1\ \cancel{h}}{60\, m\ \cancel{min}}\right)\left(\dfrac{1\ \cancel{min}}{60\, s}\right) = 1.1 \times 10^{-4} s^{-1} \tag{2}$$

In this case, we put the starting quantity on the bottom, so we got s^{-1} when the units are canceled out. Here is an example of not being able to cancel out the units correctly:

\[(2.5 h)\left(\dfrac{1 h}{60 min}\right)\left(\dfrac{60 s}{1 min}\right) = 2.5 s\cdot h^2\cdot min^{-2}$$

The important part is that if you check the units to make sure that they come out right, you can be pretty sure you set the calculation up right!

Here is an example of how dimensional analysis can help. A student was calculating initial velocity (v_0) from this equation:

$$d = (v_0)t + \dfrac{at^2}{2}$$

But the student had derived the equation incorrectly, and used this equation instead:

$$v_0 = \dfrac{d}{t} - \dfrac{at^2}{2}$$

So the student had the wrong answer, but didn't know that because he just put the numbers for d, t, and a into his calculator using the wrong equation. If he had checked the units, he would have seen that (d/t) has units of meters per second (m/s) while (at^2)/2 has units of meters (m).

Dimensional analysis is often useful when you want to estimate some quantity in the real world. For instance, maybe you want to know how much money you spend on coffee each month. If you spend $5 per cup and have 2 cups per day, and there are approximately 30 days in a month, than you can set up a calculation just like those above to calculate dollars per month spent on coffee. This works for many important, less obvious situations, for instance in business, to get an approximate idea of some quantity.

Contributors and Attributions

- Emily V Eames (City College of San Francisco)