22.5.3: iii. Exercise Solutions

Q1

a. Evaluate the z-componenet of μ_{fi}:

$$\mu_{fi} = \langle 2p_z|erCos\theta|1s\rangle, \text{ where } \psi_{1s} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{\frac{3}{2}} e^{\frac{-Zr}{a_0}}, \text{ and } \psi_{2p_z} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{\frac{5}{2}} rCos\theta e^{\frac{-Zr}{2a_0}} \tag{22.5.3.1}$$

$$\mu_{fi} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{\frac{5}{2}}\frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{\frac{3}{2}}\langle rCos\theta e^{\frac{-Zr}{2a_0}}|erCos\theta|e^{\frac{-Zr}{a_0}}\rangle \tag{22.5.3.2}$$

$$= \frac{e}{4\pi\sqrt{2}}\left(\frac{Z}{a_0}\right)^4\int_0^\infty r^2dr\int_0^\pi Sin\theta d\theta\int_0^{2\pi} d\varphi\left(r^2 e^{\frac{-Zr}{2a_0}} e^{\frac{-Zr}{a_0}}\right)Cos^2\theta \tag{22.5.3.3}$$

$$= \frac{e}{4\pi\sqrt{2}}2\pi\left(\frac{Z}{a_0}\right)^4\int_0^\infty\left(r^4 e^{\frac{-3Zr}{2a_0}}\right)dr\int_0^\pi Sin\theta Cos^2\theta d\theta \tag{22.5.3.4}$$

Using integral equation 4 to integrate over r and equation 17 to integrate over θ we obtain:

$$= \frac{e}{4\pi\sqrt{2}}2\pi\left(\frac{Z}{a_0}\right)^4\frac{4}{\left(\frac{3Z}{2a_0}\right)^5}\left(\frac{-1}{3}\right)Cos^3\theta\Big|_0^\pi \tag{22.5.3.5}$$

$$= \frac{e}{4\pi\sqrt{2}}2\pi\left(\frac{Z}{a_0}\right)^4\frac{2^5a_0^5 4}{3^5Z^5}\left(\frac{-1}{3}\right)((-1)^3-(1)^3) \tag{22.5.3.6}$$

$$= \frac{e}{\sqrt{2}}\frac{2^8a_0}{3^5Z} = \frac{ea_0}{Z}\frac{2^8}{\sqrt{2}3^5} = 0.7499\frac{ea_0}{Z} \tag{22.5.3.7}$$

b. Examine the symmetry of the integrands for $\langle 2p_z|ex|1s\rangle$ and $\langle 2p_z|ey|1s\rangle$.

Function	Symmetry
$2p_z$	- 1
x	+ 1
1s	+ 1
y	+ 1

Under this operation the integrand of $\langle 2p_z|ex|1s\rangle$ is (-1)(1)(1) = -1 (it is antisymmetric) and hence $\langle 2p_z|ex|1s\rangle = 0$.
Similarly, under this operation the integrand of $\langle 2p_z|ey|1s\rangle$ is (-1)(1)(1) = -1 (it is antisymmetric) and hence $\langle 2p_z|ey|1s\rangle = 0$.

c.

$$\tau_R = \frac{3\hbar^4c^3}{4\left(\frac{3}{8}\left(\frac{e^2}{a_0}\right)Z^2\right)^3\left(\left(\frac{ea_0}{Z}\right)\frac{2^8}{\sqrt{2}3^5}\right)^2}, \tag{22.5.3.8}$$

$$= \frac{3\hbar^4c^3}{4\frac{3^3}{8^3}\left(\frac{e^6}{a_0^3}\right)Z^6\left(\frac{e^2a_0^2}{Z^2}\right)\frac{2^{16}}{(2)3^{10}}} \tag{22.5.3.9}$$

$$= \frac{\hbar^4c^33^8a_0}{e^8Z^42^8} \tag{22.5.3.10}$$

$$\text{Inserting } e^2 = \frac{\hbar^2}{m_ea_0} \text{ we obtain :} \tag{22.5.3.11}$$

$$\tau_R = \frac{\hbar^4c^33^8a_0m_e^4a_0^4}{\hbar^8Z^42^8} = \frac{3^8}{2^8}\frac{c^3a_0^5m_e^4}{\hbar^4Z^4} \tag{22.5.3.12}$$

$$= 25.6289\frac{c^3a_0^5m_e^4}{\hbar^4Z^4} \tag{22.5.3.13}$$

$$= 256,289\left(\frac{1}{Z^4}\right)x\frac{(2.998x10^{10}\text{cm sec}^{-1})^3(0.529177x10^{-8}\text{cm})^5(9.109x10^{-28}\text{ g})^4}{(1.0546x10^{-27}\text{ g cm}^2\text{ sec}^{-1})^4} \tag{22.5.3.14}$$

$$= 1.595x10^{-9}\text{ sec }x\left(\frac{1}{Z^4}\right) \tag{22.5.3.15}$$

So, for example:

Atom	τ_R
H	1.595 ns
He^{+}	99.7 ps
Li^{+2}	19.7 ps
Be^{+3}	6.23 ps
Ne^{+9}	159 fs

Q2

a. $H = H_0 + \lambda H'(t), H'(t) = V\theta(t), H_0\varphi_k = E_k\varphi_k, \omega_k = \dfrac{E_k}{\hbar}, i\hbar\dfrac{\partial\psi}{\partial t} = H\psi$
let $\psi(r,t) = i\hbar\sum_j c_j(t)\varphi_j e^{-i\omega_j t}$ and insert into the above expression:

$$i\hbar\sum_j \left[\dot{c}_j - i\omega_j c_j\right] e^{-i\omega_j t}\varphi_j = i\hbar\sum c_j(t)e^{-i\omega_j t}(H_0 + \lambda H'(t))\varphi_j \tag{22.5.3.16}$$

$$\sum_j \left[i\hbar dot c_j + E_j c_j - c_j E_j - c_j\lambda H'\right] e^{-i\omega_j t}\varphi_j = 0 \tag{22.5.3.17}$$

$$\sum_j \left[i\hbar\dot{c}_j\langle m|j\rangle - c_j\lambda\langle m|H'|j\rangle\right] e^{-i\omega_j t} = 0 \tag{22.5.3.18}$$

$$i\hbar\dot{c}_m e^{-i\omega_m t} = \sum_j c_j\lambda H'_{mj} e^{-i\omega_j t} \tag{22.5.3.19}$$

$$So, \tag{22.5.3.20}$$

$$\dot{c}_m = \frac{1}{i\hbar}\sum_j c_j\lambda H'_{mj} e^{-i(\omega_{jm})} \tag{22.5.3.21}$$

Going back a few equations and multiplying from the left by φ_k instead of φ_m we obtain:

$$\sum_j \left[i\hbar\dot{c}_j\langle k|j\rangle - c_j\lambda\langle k|H'|j\rangle\right] e^{-i\omega_j t} = 0 \tag{22.5.3.22}$$

$$i\hbar\dot{c}_k e^{-i\omega_k t} = \sum_j c_j\lambda H'_{kj} e^{-i\omega_j t} \tag{22.5.3.23}$$

$$So, \tag{22.5.3.24}$$

$$\dot{c}_k = \frac{1}{i\hbar}\sum_j c_j\lambda H'_{kj} e^{-i\omega_j t} \tag{22.5.3.25}$$

$$\text{Now, let:} \tag{22.5.3.26}$$

$$c_m = c_m^{(0)} + c_m^{(1)}\lambda + c_m^{(2)}\lambda^2 + \ldots \tag{22.5.3.27}$$

$$c_k = c_k^{(0)} + c_k^{(1)}\lambda + c_k^{(2)}\lambda^2 + \ldots \tag{22.5.3.28}$$

$$\text{and substituting into above we obtain:} \tag{22.5.3.29}$$

$$\dot{c}_m^{(0)} + \dot{c}_m^{(1)}\lambda + \dot{c}_m^{(2)}\lambda^2 + \ldots = \frac{1}{i\hbar}\sum_j \left[c_j^{(0)} + c_j^{(1)}\lambda + c_j^{(2)}\lambda^2 + \ldots\right]\lambda H'_{mj} e^{-i(\omega_{jm})t} \tag{22.5.3.30}$$

$$\text{first order:} \tag{22.5.3.31}$$

$$\dot{c}_m^{(0)} = 0 \Rightarrow c_m^{(0)} = 1 \tag{22.5.3.32}$$

$$\text{second order: } \dot{c}_m^{(1)} = \frac{1}{i\hbar}\sum_j c_j^{(0)} H'_{mj} e^{-i(\omega_{jm})t} \tag{22.5.3.33}$$

$$(n+1)^{st} \text{ order:} \tag{22.5.3.34}$$

$$\dot{c}_m^{(n)} = \frac{1}{i\hbar}\sum_j c_j^{(n-1)} H'_{mj} e^{-i(\omega_{jm})t} \tag{22.5.3.35}$$

$$\text{Similarly:} \tag{22.5.3.36}$$

$$\text{first order:} \tag{22.5.3.37}$$

$$\dot{c}_k^{(0)} = 0 \Rightarrow c_{k\neq m}^{(0)} = 0 \tag{22.5.3.38}$$

$$\text{second order:} \tag{22.5.3.39}$$

$$\dot{c}_k^{(1)} = \frac{1}{i\hbar}\sum_j c_j^{(0)} H'_{kj} e^{-i(\omega_{jk})} \tag{22.5.3.40}$$

$$(\mathrm{n}+1)^{st} \text{ order:} \tag{22.5.3.41}$$

$$\dot{c}_k^{(n)} = \frac{1}{i\hbar}\sum_j c_j^{(n-1)} H'_{kj} e^{-i(\omega_{jk})t} \tag{22.5.3.42}$$

$$\text{So,} \tag{22.5.3.43}$$

$$\dot{c}_m^{(1)} = \frac{1}{i\hbar} c_m^{(0)} H'_{mm} e^{-i(\omega_{mm})t} = \frac{1}{i\hbar} H'_{mm} \tag{22.5.3.44}$$

$$c_m^{(1)}(t) = \frac{1}{i\hbar}\int_0^t dt' V_{mm} = \frac{V_{mm}t}{i\hbar} \tag{22.5.3.45}$$

$$\text{and similarly,} \tag{22.5.3.46}$$

$$\dot{c}_k^{(1)} = \frac{1}{i\hbar} c_m^{(0)} H'_{km} e^{-i(\omega_{mk})t} = \frac{1}{i\hbar} H'_{km} e^{-i(\omega_{mk})t} \tag{22.5.3.47}$$

$$c_k^{(1)}(t) = \frac{1}{i\hbar} V_{km}\int_0^t dt' e^{-i(\omega_{mk})t'} = \frac{V_{km}}{\hbar\omega_{mk}}\left[e^{-i(\omega_{mk})t} - 1\right] \tag{22.5.3.48}$$

$$\dot{c}_m^{(2)} = \frac{1}{i\hbar}\sum_j c_j^{(1)} H'_{mj} e^{-i(\omega_{jm})t} \tag{22.5.3.49}$$

$$\dot{c}_m^{(2)} = \sum_{j\neq m}\frac{1}{i\hbar}\frac{V_{jm}}{\hbar\omega_{mj}}\left[e^{-i(\omega_{mj})t} - 1\right] H'_{mj} e^{-i(\omega_{jm})t} + \frac{1}{i\hbar}\frac{V_{mm}t}{i\hbar} H'_{mm} \tag{22.5.3.50}$$

$$c_m^{(2)} = \sum_{j\neq m}\frac{1}{i\hbar}\frac{V_{jm}V_{mj}}{\hbar\omega_{mj}}\int_0^t dt' e^{-i(\omega_{jm})t'}\left[e^{-i(\omega_{mj})t'} - 1\right] - \frac{V_{mm}V_{mm}}{\hbar^2}\int_0^t t' dt' \tag{22.5.3.51}$$

$$= \sum_{j\neq m}\frac{V_{jm}V_{mj}}{i\hbar^2\omega_{mj}}\int_o^t dt'\left[1 - e^{-i(\omega_{jm})t'}\right] - \frac{|V_{mm}|^2}{\hbar^2}\frac{t^2}{2} \tag{22.5.3.52}$$

$$= \sum_{j\neq m}\frac{V_{jm}V_{mj}}{i\hbar^2\omega_{mj}}\left(t - \frac{e^{-i(\omega_{jm})t} - 1}{-i\omega_{jm}}\right) - \frac{|V_{mm}|^2}{\hbar^2}\frac{t^2}{2} \tag{22.5.3.53}$$

$$= \sum_{j\neq m}{}' \frac{V_{jm}V_{mj}}{\hbar^2\omega_{mj}^2}\left(e^{-i(\omega_{jm})t} - 1\right) + \sum_{j\neq m}{}' \frac{V_{jm}V_{mj}}{i\hbar^2\omega_{mj}} t - \frac{|V_{mm}|^2 t^2}{2\hbar^2} \tag{22.5.3.54}$$

$$\text{Similarly,} \tag{22.5.3.55}$$

$$\dot{c}_k^{(2)} = \frac{1}{i\hbar}\sum_j c_j^{(1)} H'_{kj} e^{-i(\omega_{jk})t} \tag{22.5.3.56}$$

$$= \sum_{j\neq m}\frac{1}{i\hbar}\frac{V_{jm}}{\hbar\omega_{mj}}\left[e^{-i(\omega_{mj})t} - 1\right] H'_{kj} e^{-i(\omega_{jk})t} + \frac{1}{i\hbar}\frac{V_{mm}t}{i\hbar} H'_{km} e^{-i(\omega_{mk})} \tag{22.5.3.57}$$

$$c_k^{(2)}(t) = \sum_{j\neq m}{}' \frac{V_{jm}V_{kj}}{i\hbar^2\omega_{mj}}\int_0^t dt' e^{-i(\omega_{jk})t'}\left[e^{-i(\omega_{mj})t'} - 1\right] - \frac{V_{mm}V_{km}}{\hbar^2}\int_0^t t' dt' e^{-i(\omega_{mk})t'} \tag{22.5.3.58}$$

$$= \sum_{j\neq m}{}' \frac{V_{jm}V_{kj}}{i\hbar^2\omega_{mj}}\left(\frac{e^{-i(\omega_{mj}+\omega_{jm})t} - 1}{-i\omega_{mk}} - \frac{e^{-i(\omega_{jk})t} - 1}{-\omega_{jk}}\right) - \frac{V_{mm}V_{km}}{\hbar^2}\left[e^{-i(\omega_{mk})t'}\left(\frac{t'}{-i\omega_{mk}} - \frac{1}{-(i\omega_{mk})^2}\right)\right]_0^t \tag{22.5.3.59}$$

$$= \sum_{j\neq m}{}' \frac{V_{jm}V_{kj}}{\hbar^2\omega_{mj}}\left(\frac{e^{-i(\omega_{mk})t} - 1}{\omega_{mk}} - \frac{e^{-i(\omega_{jk})t} - 1}{\omega_{jk}}\right) + \frac{V_{mm}V_{km}}{\hbar^2\omega_{mk}}\left[e^{-i(\omega_{mk})t'}\left(\frac{t'}{i} - \frac{1}{\omega_{mk}}\right)\right]_0^t \tag{22.5.3.60}$$

$$= \sum_{j\neq m}{}' \frac{V_{jm}V_{kj}}{E_m - E_j}\left(\frac{e^{-i(\omega_{mk})t} - 1}{E_m - E_k} - \frac{e^{-i(\omega_{jk})t}}{E_j - E_k}\right) + \frac{V_{mm}V_{km}}{\hbar(E_m - E_k)}\left[e^{-i(\omega_{mk})t}\left(\frac{t}{i} - \frac{1}{\omega_{mk}}\right) + \frac{1}{\omega_{mk}}\right] \tag{22.5.3.61}$$

So, the overall amplitudes c_m, and c_k, to second order are:

$$c_m(t) = 1 + \frac{V_{mm}t}{i\hbar} + \sum_{j\neq m}{}' \frac{V_{jm}V_{mj}}{i\hbar(E_m - E_j)}t + \sum_{j\neq m}{}' \frac{V_{jm}V_{mj}}{\hbar^2(E_m-E_j)^2}(e^{-i(\omega_{mk})t} - 1) - \frac{|V_{mm}|^2 t^2}{2\hbar^2} \tag{22.5.3.62}$$

$$c_k(t) = \frac{V_{km}}{(E_m - E_k)}\left[e^{-i(\omega_{mk})t} - 1\right] + \frac{V_{mm}V_{km}}{(E_m-E_k)^2}\left[1 - e^{-i(\omega_{mk})t}\right] + \frac{V_{mm}V_{km}}{(E_m-E_k)}\frac{t}{\hbar i}e^{-i(\omega_{mk})t} + \sum_{j\neq m}{}' \frac{V_{jm}V_{kj}}{E_m - E_j}\left(\frac{e^{-i(\omega_{mk})t}-1}{E_m-E_k} - \frac{e^{-i(\omega_{jk})t}-1}{E_j-E_k}\right) \tag{22.5.3.63}$$

b. The perturbation equations still hold:

$$\dot{c}_m^{(n)} = \frac{1}{i\hbar}\sum_j c_j^{(n-1)}H'_{mj}e^{-i(\omega_{jm})t} \; ; \; \dot{c}_k^{(n)} = \frac{1}{i\hbar}\sum_j c_j^{(n-1)}H'_{kj}e^{-i(\omega_{jk})t} \tag{22.5.3.64}$$

$$\text{So, } c_m^{(0)} = 1 \text{ and } c_k^{(0)} = 0 \tag{22.5.3.65}$$

$$\dot{c}_m^{(1)} = \frac{1}{i\hbar}H'_{mm} \tag{22.5.3.66}$$

$$c_m^{(1)} = \frac{1}{i\hbar}V_{mm}\int_{-\infty}^{t} dt' e^{-i(\omega)t'} = \frac{V_{km}}{i\hbar(-i\omega_{mk}+\eta)}\left[e^{-i(\omega_{mk}+\eta)}\right] \tag{22.5.3.67}$$

$$= \frac{V_{km}}{E_m - E_k + i\hbar\eta}\left[e^{-i(\omega_{mk}+\eta)t}\right] \tag{22.5.3.68}$$

$$\dot{c}_m^{(2)} = \sum_{j\neq m}{}' \frac{1}{i\hbar}\frac{V_{jm}}{E_m - E_j + i\hbar\eta}e^{-i(\omega_{mj}+\eta)t}V_{mj}e^{\eta t}e^{-i(\omega_{jm})t} + \frac{1}{i\hbar}\frac{V_{mm}e^{\eta t}}{i\hbar\eta}V_{mm}e^{\eta t} \tag{22.5.3.69}$$

$$c_m^{(2)} = \sum_{j\neq m}{}' \frac{1}{i\hbar}\frac{V_{jm}V_{mj}}{E_m-E_j+i\hbar\eta}\int_{-\infty}^{t} e^{2\eta t'}dt' - \frac{|V_{mm}|^2}{2\hbar^2\eta^2}\int_{-\infty}^{t} e^{2\eta t'}dt' = \sum_{j\neq m}{}' \frac{V_{jm}V_{mj}}{i\hbar 2\eta(E_m - E_j + i\hbar\eta)}e^{2\eta t} - \frac{|V_{mm}|^2}{2\hbar^2\eta^2}e^{2\eta t} \tag{22.5.3.70}$$

$$\dot{c}_k^{(2)} = \sum_{j\neq m}{}' \frac{1}{i\hbar}\frac{V_{jm}}{E_m - E_j + i\hbar\eta}e^{-i(\omega_{mj}+\eta)t}H'_{kj}e^{-i(\omega_{jk})t} \tag{22.5.3.71}$$

c. In part a. the $c^{(2)}(t)$ grow linearly with time (for V_{mm} = 0) while in part b. they remain finite for $\eta\langle 0$. The result in par a. is due to the sudden turning on of the field.

d.

$$|c_k(t)|^2 = \left|\sum_j \frac{V_{jm}V_{kj}e^{-i(\omega_{mk}+2\eta)t}}{(E_m - E_j + i\hbar\eta)(E_m - E_k + 2i\hbar\eta)}\right|^2 \tag{22.5.3.72}$$

$$= \sum_{jj'} \frac{V_{kj}V_{kj'}V_{jm}V_{j'm}e^{4\eta t}}{\left[(E_m-E_j)(E_m-E_{j'}) + i\hbar\eta(E_j - E_{j'}) + \hbar^2\eta^2\right]((E_m-E_k)^2 + 4\hbar^2\eta^2)} \tag{22.5.3.73}$$

$$\text{Now, look at the limit as } \eta \to 0^+ : \tag{22.5.3.74}$$

$$\frac{d}{dt}|c_k(t)|^2 \neq 0 \text{ when } E_m = E_k \tag{22.5.3.75}$$

$$lim_{\eta\to 0^+}\frac{4\eta}{((E_m-E_k)^2+4\hbar^2\eta^2)}\alpha\delta(E_m - E_k) \tag{22.5.3.76}$$

$$\text{So, the final result is the } 2^{\,nd} \text{ order golden rule expression:} \tag{22.5.3.77}$$

$$\frac{d}{dt}|c_k(t)|^2\frac{2\pi}{\hbar}\delta(E_m - E_k)\lim_{\eta\to 0^+}\left|\frac{V_{jm}V_{kj}}{(E_j - E_m - i\hbar\eta)}\right|^2 \tag{22.5.3.78}$$

Q3

For the sudden perturbation case:

$$|c_m(t)|^2 = 1 + \sum_j{}' \frac{V_{jm}V_{mj}}{(E_m-E_j)^2}\left[e^{-i(\omega_{jm})t} - 1 + e^{i(\omega_{jm})t} - 1\right] + O(V^3) \tag{22.5.3.79}$$

$$|c_m(t)|^2 = 1 + \sum_j{}' \frac{V_{jm}V_{mj}}{(E_m-E_j)^2}\left[e^{-i(\omega_{jm}t)} + e^{i(\omega_{jm})t} - 2\right] + O(V^3) \tag{22.5.3.80}$$

$$|c_k(t)|^2 = \frac{V_{km}V_{mk}}{(E_m-E_k)^2}\left[-e^{-i(\omega_{mk})t} - e^{i(\omega_{mk})t} + 2\right] + O(V^3) \tag{22.5.3.81}$$

$$1 - \sum_{k\neq m}{}' |c_k(t)|^2 = 1 - \sum_k{}' \frac{V_{km}V_{mk}}{(E_m-E_k)^2}\left[-e^{-i(\omega_{mk})t} - e^{i(\omega_{mk})t} + 2\right] + O(V^3) \tag{22.5.3.82}$$

$$= 1 + \sum_k{}' \frac{V_{km}V_{mk}}{(E_m-E_k)^2}\left[e^{-i(\omega_{mk})t} + e^{i(\omega_{mk})t} - 2\right] + O(V^3) \tag{22.5.3.83}$$

$\therefore$ to order V^2, $|c_m(t)|^2 = 1 - \sum_k{}' |c_k(t)|^2$, with no assumptions made regarding V_{mm}.

For the adiabatic perturbation case:

$$\begin{aligned}
|c_m(t)|^2 &= 1+\sum_{j\neq m}{}' \left[\frac{V_{jm}V_{mj}e^{2\eta t}}{i\hbar 2\eta(E_m-E_j+i\hbar\eta)}+\frac{V_{jm}V_{mj}e^{2\eta t}}{-i\hbar 2\eta(E_m-E_j+i\hbar\eta)}\right]+O(V^3) && (22.5.3.84)\\
&= 1+\sum_{j\neq m}{}' \frac{1}{i\hbar 2\eta}\left[\frac{1}{(E_m-E_j+i\hbar\eta)}-\frac{1}{(E_m-E_j-i\hbar\eta)}\right]V_{jm}V_{mj}e^{2\eta t}+O(V^3) && (22.5.3.85)\\
&= 1+\sum_{j\neq m}{}' \frac{1}{i\hbar 2\eta}\left[\frac{-2i\hbar\eta}{(E_m-E_j)^2+\hbar^2\eta^2}\right]V_{jm}V_{mj}e^{2\eta t}+O(V^3) && (22.5.3.86)\\
&= 1-\sum_{j\neq m}{}' \left[\frac{V_{jm}V_{mj}e^{2\eta t}}{(E_m-E_j)^2+\hbar^2\eta^2}\right]+O(V^3) && (22.5.3.87)\\
|c_k(t)|^2 &= \frac{V_{km}V_{mk}}{(E_m-E_k)^2+\hbar^2\eta^2}e^{2\eta t}+O(V^3) && (22.5.3.88)
\end{aligned}$$

$\therefore$ to order V^2, $|c_m(t)|^2 = 1-\sum_k{}' |c_k(t)|^2$, with no assumptions made regarding V_{mm} for this case as well.

This page titled 22.5.3: iii. Exercise Solutions is shared under a CC BY-NC-SA 4.0 license and was authored, remixed, and/or curated by Jack Simons via source content that was edited to the style and standards of the LibreTexts platform.