1.1: Operators

Each physically measurable quantity has a corresponding operator. The eigenvalues of the operator tell the values of the corresponding physical property that can be observed

In quantum mechanics, any experimentally measurable physical quantity **F** (e.g., energy, dipole moment, orbital angular momentum, spin angular momentum, linear momentum, kinetic energy) whose classical mechanical expression can be written in terms of the cartesian positions {q_i} and momenta {p_i} of the particles that comprise the system of interest is assigned a corresponding quantum mechanical operator **F**. Given **F** in terms of the {q_i} and {p_i}, **F** is formed by replacing p_j by $-i\hbar\frac{\partial}{\partial q_j}$ and leaving q_j untouched. For example, if

$$F=\sum_{1=1}^{N}\left(\frac{p_1^2}{2m_1}+\frac{1}{2}k(q_1-q_1^0)^2+L(q_1-q_1^0)\right)$$

then

$$F=\sum_{1=1}^{N}\left(\frac{-\hbar^2}{2m_1}\frac{\partial^2}{\partial q_1^2}+\frac{1}{2}k(q_1-q_1^0)^2+L(q_1-q_1^0)\right)$$

The x-component of the dipole moment for a collection of N particles has

$$F=\sum_{j=1}^{N}Z_j e x_j$$

and

$$F=\sum_{j=1}^{N}Z_j e x_j$$

where Z_je is the charge on the j^{th} particle.

The mapping from F to **F** is straightforward only in terms of cartesian coordinates. To map a classical function F, given in terms of curvilinear coordinates (even if they are orthogonal), into its quantum operator is not at all straightforward. Interested readers are referred to Kemble's text on quantum mechanics which deals with this matter in detail. The mapping can always be done in terms of cartesian coordinates after which a transformation of the resulting coordinates and differential operators to a curvilinear system can be performed. The corresponding transformation of the kinetic energy operator to spherical coordinates is treated in detail in Appendix A. The text by EWK also covers this topic in considerable detail.

The relationship of these quantum mechanical operators to experimental measurement will be made clear later in this chapter. For now, suffice it to say that these operators define equations whose solutions determine the values of the corresponding physical property that can be observed when a measurement is carried out; only the values so determined can be observed. This should suggest the origins of quantum mechanics' prediction that some measurements will produce **discrete** or **quantized** values of certain variables (e.g., energy, angular momentum, etc.).