22.5.1: i. Exercises

Q1

Time dependent perturbation theory provides an expression for the radiative lifetime of an excited electronic state, given by τ_R:

$$\tau_R = \frac{3\hbar^4 c^3}{4(E_i - E_f)^3 |\mu_{fi}|^2},$$

where i refers to the excited state, f refers to the lower state, and μ_{fi} is the transition dipole.
a. Evaluate the z-component of the transition dipole for the $2p_z \rightarrow$ 1s transition in a hydrogenic atom of nuclear charge Z, given:

$$\psi 1s = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{\frac{3}{2}} e^{\frac{-Zr}{a_0}}, \text{ and } \psi_{2p_z} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{\frac{5}{2}} rCos\theta e^{\frac{-Zr}{2a_0}}$$

Express your answer in units of ea_0.
b. Use symmetry to demonstrate that the x- and y-components of μ_{fi} are zero, i.e.

$$\langle 2p_z|ex|1s\rangle = \langle 2p_z|ey|1s\rangle = 0.$$

c. Calculate the radiative lifetime τ_R of a hydrogenlike atom in its $2p_z$ state. Use the relation $e^2 = \dfrac{\hbar^2}{m_e a_0}$ to simplify our results.

Q2

Consider a case in which the complete set of states $\{\phi_k\}$ for a Hamiltonian is known.
a. If the system is initially in the state m at time t=0 when a constant perturbation V is suddenly turned on, find the probability amplitudes $C_k^{(2)}(t)$ and $C_m^{(2)}(t)$, to second order in V, that describe the system being in a different state k or the same state m at time t.
b. If the perturbation is turned on adiabatically, what are $C_k^{(2)}(t)$ and $C_m^{(2)}(t)$?
Here, consider that the initial time is $t_0 \rightarrow -\infty$, and the potential is V e^{\eta}t\), where the positive parameter η is allowed to approach zero $\eta \rightarrow 0$ in order to describe the adiabatically (i.e., slowly) turned on perturbation.
c. Compare the results of parts a. and b. and explain any differences.
d. Ignore first order contributions (assume they vanish) and evaluate the transition rates $\frac{d}{dt}|C_k^{(2)}(t)|^2$ for the results of part b. by taking the limits $\eta \rightarrow 0^+$, to obtain the adiabatic results.

Q3

If a system is initially in a state m, conservation of probability requires that the total probability of transitions out of state m be obtainable from the decrease in the probability of being in state m. Prove this to the lowest order by using the results of exercise 2, i.e. show that: $|C_m|^2 = 1 - \sum_{k \neq m} |C_k|^2$.