1.5: Unit Conversion with the Metric System

Because chemists often deal with measurements that are both very small (as in the size of an atom) and very large (as in numbers of atoms), it is often necessary to convert between units of metric measurement. For example, a mass measured in grams may be more convenient to work with if it was expressed in mg ($10^{-3} \times$ g). Converting between metric units is an exercise in unit analysis (also called dimensional analysis). Unit analysis is a form of proportional reasoning where a given measurement can be multiplied by a known proportion or ratio to give a result having a different unit, or dimension. For example, if you had a sample of a substance with a mass of 0.0034 grams and you wished to express that mass in mg you could use the following unit analysis:

The given quantity in this example is the mass of 0.0034 grams. The quantity that you want to find is the mass in mg, and the known proportion or ratio is given by the definition of the metric prefix, that is one mg is equal to 10^{-3} grams. Expressing this as a proportion or ratio, you could say there is one mg per 10^{-3} grams, or:

Looking at this expression, the numerator, 1 mg, is equivalent to saying 1×10^{-3} g, which is identical to the value in the denominator. This ratio, therefore has a numeric value of one (anything divided by itself is one, by definition). Algebraically, we know that we are allowed to multiply any number by one and that number will be unchanged. If, however, the number has units, and we multiply it by a ratio containing units, the units in the number will multiply and divide by the units of the ratio, giving the original number (remember you are multiplying by one) but with different units. In the present case, if we multiply the given by the known ratio, the unit "g" will cancel, leaving "mg" as the only remaining unit. The original number in grams has therefore been converted to milligrams, the units that you wanted to find.

The method that we used to solve this problem can be generalized as: **given × known ratio = find**. The given is a numerical quantity (with its units), the known ratio is based on the metric prefixes and is set up so that the units in the denominator of the ratio match the units of given and the units in the numerator match those in find. When these are multiplied, the number from given will now have the units of find. In the ratio used in the example, "g" (the units of given) appear in the denominator and "mg" (the units of find) appear in the numerator.

As an example of a case where the units of the known ratio must be inverted, if you wanted to convert 1.3×10^{7} µg into grams, the given would be 1.3×10^{7} µg, the find would be grams and the known ratio would be based on the definition of µg as one µg per 10^{-6} grams. This ratio must be expressed in the solution with µg (the units of given) in the denominator and g (the units of find) in the numerator.

Note that instead of "one µg per 10^{-6} grams", we must invert the known ratio and state it as either "10^{-6} grams per 1 µg" so that the units of given (µg) will cancel. We can do this inversion because the ratio still has a numeric value of one. Simple ratios like these can also be used to convert English measurements in to their metric equivalents. The ratio relating inches to meters is $\left({}^{0.0254\text{ m}}/_{1\text{ inch}}\right)$.

Exercise 1.5.1

Convert the following metric measurements into the indicated units:

a. 9.3×10^{-4} g into ng
b. 278 g into mg

Exercise 1.5.2

Convert the following metric measurements into the indicated units:

a. 2,057 grams - as kg
b. 1.25×10^{-7} meters - as µm
c. 6.58×10^{4} meters - as km
d. 2.78×10^{-1} grams - as mg

In the examples we have done thus far, we have been able to write a known ratio based on the definition of the appropriate metric prefix. But what if we wanted to take a number that was expressed in milligrams and convert it to a number with the units of nanograms? In a case like this, we need to use two known ratios in sequence; the first with the units of given (mg) in the denominator and the second with the units of find (ng) in the numerator. For example, if we were given 0.00602 mg and asked to find ng, we could set up a ratio based on grams per mg. If we solved the problem at this point, we would have a result with the units of grams. To get a final answer in terms of ng, we would need to multiply this intermediate result (the new given) by a ratio based on nanograms per gram.

In the first two terms, the units of “mg” cancel and in the second two terms, “g” cancels leaving only “ng”, the units of find. One of the reassuring pleasures of doing these types of problems is that, if you set up your problem and the units cancel, leaving only the units of find, you know you have set up the problem correctly! All you have to do is to do the sequential calculations and you know your answer is correct!

Exercise 1.5.3

Convert the following metric measurements into the indicated units:

a. 2,057 mg - into kg
b. 1.25×10^{-7} km - into µm
c. 9.3×10^{-4} pg - into ng
d. 6.5×10^{4} mm - into km