9.5: The Ideal Gas Law

The three gas laws that we covered in Section 9.2, 9.3, 9.4 and 9.5 describe the effect of pressure, temperature and the number of moles of a gas on volume. The three independent gas laws are:

- ***Boyle's law:*** $V \propto 1/P$
- ***Charles's law:*** $V \propto T$
- ***Avogadro's law:*** $V \propto n$

If volume (*V*) is proportion to each of these variables, it must also be proportional to their product:

$$V \propto \frac{nT}{P}$$

If we replace the proportionality symbol with a constant (let's just choose ***R*** to represent our constant), we can re-write the equation as:

$$V = R\left(\frac{nT}{P}\right)$$

or

$$PV = nRT$$

Example 9.5.1:

The value of the proportionality constant *R*, can be calculated from the fact that exactly one mole of a gas at exactly 1 atm and at 0 ˚C (273 K) has a volume of 22.414 L.

Solution

Substituting in the equation:

$$PV = nRT \;\; or \;\; R = \frac{PV}{nT}$$

$$R = \frac{(1\ atm)(22.414\ L)}{(1\ mole)(273\ K)} = 0.082057\ L\ atm\ mol^{-1}\ K^{-1}$$

The uncomfortable and somewhat obnoxious constant is called the **universal gas constant**, and you will need to know it (or look it up) whenever you solve problems using the combined ideal gas law.

Exercise 9.5.1

What volume will 17.5 grams of N_2 occupy at a pressure of 876 mm Hg and at 123 ˚C?

Many of the problems that you will encounter when dealing with the gas laws can be solved by simply using the "two-state" approach. Because ***R*** is a constant, we can equate an initial and a final state as:

$$R = \frac{P_1V_1}{n_1T_1} \;\; for\ the\ initial\ state$$

$$R = \frac{P_2V_2}{n_2T_2} \;\; for\ the\ final\ state$$

$$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$$

Using this equation, you can solve for multiple variables within a single problem.

Exercise 9.5.1

A sample of oxygen occupies 17.5 L at 0.75 atm and 298 K. The temperature is raised to 303 K and the pressure is increased to 0.987 atm. What is the final volume of the sample?

If you noticed, we calculated the value of the proportionality constant *R* based on the fact that *exactly* one mole of a gas at *exactly* 1 atm and at 0 ˚C (273 K) has a volume of 22.414 L. This is one of the "magic numbers in chemistry; *exactly* one mole of *any* gas under these conditions will occupy a volume of 22.414 L. The conditions, 1 atm and 0 ˚C, are called **standard temperature and pressure**, or **STP**. The fact that all gases occupy this same molar volume can be rationalized by realizing that 99.999% of a gas is empty space, so it really doesn't matter what's in there, it all occupies the same volume. This realization is attributed to Amedeo Avogadro and **Avogadro's hypothesis**, published in 1811, suggested that *equal volumes of all gases at the same temperature and pressure contained the same number of molecules.* This is the observation that led to the measurement of Avogadro's number (6.0221415×10^{23}), the number of *things* in a mole. The importance of the "magic number" of 22.414 L per mole (at STP) is that, when combined with the ideal gas laws, any volume of a gas can be easily converted into the number of moles of that gas.

Exercise 9.5.1

1. A sample of methane has a volume of 17.5 L at 100.0 ˚C and 1.72 atm. How many moles of methane are in the sample?
2. A 0.0500 L sample of a gas has a pressure of 745 mm Hg at 26.4˚ C. The temperatureis now raised to 404.4 K and the volume is allowed to expand until a final pressure of 1.06 atm is reached. What is the final volume of the gas?
3. When 128.9 grams of cyclopropane (C_3H_6) are placed into an 8.00 L cylinder at 298 K, the pressure is observed to be 1.24 atm. A piston in the cylinder is now adjusted so that the volume is now 12.00 L and the pressure is 0.88 atm. What is the final temperature of the gas?