1.4: Measurement and Chemical Problem Solving

Learning Objectives

- To identify the basic units of measurement of the seven fundamental properties
- Describe the names and abbreviations of the *SI base units* and the *SI decimal prefixes*.
- Define the *liter* and the *metric ton* in these units.
- Explain the meaning and use of *unit dimensions*; state the dimensions of *volume*.
- State the quantities that are needed to define a *temperature scale*, and show how these apply to the *Celsius*, *Kelvin*, and *Fahrenheit* temperature scales.
- Explain how a Torricellian *barometer* works.

Have you ever estimated a distance by "stepping it off"— that is, by counting the number of steps required to take you a certain distance? Or perhaps you have used the width of your hand, or the distance from your elbow to a fingertip to compare two dimensions. If so, you have engaged in what is probably the first kind of measurement ever undertaken by primitive mankind. The results of a measurement are always expressed on some kind of a *scale* that is defined in terms of a particular kind of *unit*. The first scales of distance were likely related to the human body, either directly (the length of a limb) or indirectly (the distance a man could walk in a day).

Ångstrom	1E–6 cm
Astronomical unit	1.495E8 km
Bolt (U.S., cloth)	120 ft
Cable length	720 ft
Chain (engineer's)	100 ft
Chain (surveyor's)	0.1 furlong
Cubit	18 in
Ell	45 in
Foot (U.S.)	12 in
Foot (British)	0.4 pace
Furlong	1/8 mile
Hand	4 in
Inch (U.S.)	1/12 ft
Inch (British)	1/36 yard
League	3 miles
Light year	9.46E12 km
Mile (U.S., staute)	5280 ft
Mile (nautical)	1.853 km
Nail (British)	2.25 in
Pace (British)	30 in
Parsec	3.084E13 km
Point (printer's)	1/72 in
Rope (British)	20 ft
Span	9 in

Figure 1.4.1: Current and past units of distance

Table showing various forms of units for measuring distanced used in the past and the present. Left column shows unit and right column shows value of unit measurement in terms of more commonly used units.

As civilization developed, a wide variety of measuring scales came into existence, many for the same quantity (such as length), but adapted to particular activities or trades. Eventually, it became apparent that in order for trade and commerce to be possible, these scales had to be defined in terms of standards that would allow measures to be verified, and, when expressed in different units (bushels and pecks, for example), to be correlated or converted.

History of Units

Over the centuries, hundreds of measurement units and scales have developed in the many civilizations that achieved some literate means of recording them. Some, such as those used by the Aztecs, fell out of use and were largely forgotten as these civilizations died out. Other units, such as the various systems of measurement that developed in England, achieved prominence through extension of the Empire and widespread trade; many of these were confined to specific trades or industries. The examples shown here are only some of those that have been used to measure length or distance. The history of measuring units provides a fascinating reflection on the history of industrial development.

The most influential event in the history of measurement was undoubtedly the French Revolution and the Age of Rationality that followed. This led directly to the metric system that attempted to do away with the confusing multiplicity of measurement scales by reducing them to a few fundamental ones that could be combined in order to express any kind of quantity. The metric system spread rapidly over much of the world, and eventually even to England and the rest of the U.K. when that country established closer economic ties with Europe in the latter part of the 20th Century. The United States is presently the only major country in which "metrication" has made little progress within its own society, probably because of its relative geographical isolation and its vibrant internal economy.

Science, being a truly international endeavor, adopted metric measurement very early on; engineering and related technologies have been slower to make this change, but are gradually doing so. Even the within the metric system, however, a variety of units were employed to measure the same fundamental quantity; for example, energy could be expressed within the metric system in units of ergs, electron-volts, joules, and two kinds of calories. This led, in the mid-1960s, to the adoption of a more basic set of units, the *Systeme Internationale* (*SI*) units that are now recognized as the standard for science and, increasingly, for technology of all kinds.

The Seven *SI* Base Units and Decimal Prefixes

In principle, any physical quantity can be expressed in terms of only seven base units (Table 1.4.1), with each base unit defined by a standard described in the NIST Web site.

Table 1.4.1: The Seven Base Units

Property	Unit	Symbol
length	meter	m
mass	kilogram	kg
time	second	s
temperature (absolute)	kelvin	K
amount of substance	mole	mol
electric current	ampere	A
luminous intensity	candela	cd

A few special points about some of these units are worth noting:

- The base unit of **mass** is unique in that a decimal prefix (Table 1.4.2) is built into it; i.e., the base SI unit is not the *gram*.
- The base unit of **time** is the only one that is not metric. Numerous attempts to make it so have never garnered any success; we are still stuck with the 24:60:60 system that we inherited from ancient times. The ancient Egyptians of around 1500 BC invented the 12-hour day, and the 60:60 part is a remnant of the base-60 system that the Sumerians used for their astronomical calculations around 100 BC.
- Of special interest to Chemistry is the *mole*, the base unit for expressing the **quantity of matter**. Although the number is not explicitly mentioned in the official definition, chemists define the mole as Avogadro's number (approximately 6.02×10^{23}) of anything.

Owing to the wide range of values that quantities can have, it has long been the practice to employ prefixes such as milli and mega to indicate decimal fractions and multiples of metric units. As part of the SI standard, this system has been extended and formalized (Table 1.4.2).

Table 1.4.2: Prefixes used to scale up or down base units

Prefix	Abbreviation	Multiplier	Prefix	Abbreviation	Multiplier

Prefix	Abbreviation	Multiplier	Prefix	Abbreviation	Multiplier
peta	P	10^{15}	deci	d	10^{-1}
tera	T	10^{12}	centi	c	10^{-2}
giga	G	10^{9}	milli	m	10^{-3}
mega	M	10^{6}	micro	μ	10^{-6}
kilo	k	10^{3}	nano	n	10^{-9}
hecto	h	10^{2}	pico	p	10^{-12}
deca	da	10	femto	f	10^{-15}

Pseudo-Si Units

There is a category of units that are "honorary" members of the SI in the sense that it is acceptable to use them along with the base units defined above. These include such mundane units as the hour, minute, and degree (of angle), etc., but the three shown here are of particular interest to chemistry, and you will need to know them.

Pseudo-Si Units

liter (litre)	L	1 L = 1 dm^3 = 10^{-3} m^3
metric ton	t	1 t = 10^3 kg
united atomic mass unit (amu)	u	1 u = 1.66054×10^{-27} kg

Derived Units and Dimensions

Most of the physical quantities we actually deal with in science and also in our daily lives, have units of their own: volume, pressure, energy and electrical resistance are only a few of hundreds of possible examples. It is important to understand, however, that all of these can be expressed in terms of the SI base units; they are consequently known as *derived units*. In fact, most physical quantities can be expressed in terms of one or more of the following five fundamental units:

- mass (M)
- length (L)
- time (T)
- electric charge (Q)
- temperature (Θ theta)

Consider, for example, the unit of *volume*, which we denote as V. To measure the volume of a rectangular box, we need to multiply the lengths as measured along the three coordinates:

$$V = x \cdot y \cdot z$$

We say, therefore, that volume has the dimensions of length-cubed:

$$dim\{V\} = L^3$$

Thus the units of volume will be m^3 (in the SI) or cm^3, ft^3 (English), etc. Moreover, any formula that calculates a volume must contain within it the L^3 dimension; thus the volume of a sphere is $4/3\pi r^3$. The *dimensions* of a unit are the powers which M, L, t, Q and Q must be given in order to express the unit. Thus,

$$dim\{V\} = M^0 L^3 T^0 Q^0 \Theta^0$$

as given above.

There are several reasons why it is worthwhile to consider the dimensions of a unit.

1. Perhaps the most important use of dimensions is to help us understand the relations between various units of measure and thereby get a better understanding of their physical meaning. For example, a look at the dimensions of the frequently confused electrical terms resistance and resistivity should enable you to explain, in plain words, the difference between them.
2. By the same token, the dimensions essentially tell you how to calculate any of these quantities, using whatever specific units you wish. (Note here the distinction between dimensions and units.)
3. Just as you cannot add apples to oranges, an expression such as $a = b + cx^2$ is meaningless unless the dimensions of each side are identical. (Of course, the two sides should work out to the same units as well.)
4. Many quantities must be dimensionless— for example, the variable x in expressions such as $\log x$, e^x, and $\sin x$. Checking through the dimensions of such a quantity can help avoid errors.

The formal, detailed study of dimensions is known as *dimensional analysis* and is a topic in any basic physics course.

Example 1.4.1

Find the dimensions of energy.

Solution

When mechanical work is performed on a body, its energy increases by the amount of work done, so the two quantities are equivalent and we can concentrate on work. The latter is the product of the force applied to the object and the distance it is displaced. From Newton's law, force is the product of mass and acceleration, and the latter is the rate of change of velocity, typically expressed in meters per second per second. Combining these quantities and their dimensions yields the result shown in Table 1.4.1.

Table 1.4.3: Dimensions of units commonly used in Chemistry

Q	M	L	t	quantity	SI unit, other typical units
1	-	-	-	electric charge	coulomb
-	1	-	-	mass	kilogram, gram, metric ton, pound
-	-	1	-	length	meter, foot, mile
-	-	-	1	time	second, day, year
-	-	3	-	volume	liter, cm^3, quart, fluidounce
-	1	–3	-	density	kg m^{-3}, g cm^{-3}
-	1	1	–2	force	newton, dyne
-	1	–1	–2	pressure	pascal, atmosphere, torr

Q	M	L	t	quantity	SI unit, other typical units
-	1	2	–2	energy	joule, erg, calorie, electron-volt
-	1	2	–3	power	watt
1	1	2	–2	electric potential	volt
1	-	-	–1	electric current	ampere
1	1	1	–2	electric field intensity	volt m^{-1}
–2	1	2	–1	electric resistance	ohm
2	1	3	–1	electric resistivity	-
2	–1	–2	1	electric conductance	siemens, mho

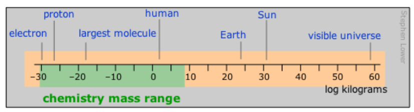
Dimensional analysis is widely employed when it is necessary to convert one kind of unit into another, and chemistry students often use it in "chemical arithmetic" calculations, in which context it is also known as the "Factor-Label" method. In this section, we will look at some of the quantities that are widely encountered in Chemistry, and at the units in which they are commonly expressed. In doing so, we will also consider the actual range of values these quantities can assume, both in nature in general, and also within the subset of nature that chemistry normally addresses. In looking over the various units of measure, it is interesting to note that their unit values are set close to those encountered in everyday human experience

Mass is not weight

These two quantities are widely confused. Although they are often used synonymously in informal speech and writing, they have different dimensions: *weight* is the *force* exerted on a mass by the local gravational field:

$$f = ma = mg \tag{1.4.1}$$

where g is the acceleration of gravity. While the nominal value of the latter quantity is 9.80 m s^{-2} at the Earth's surface, its exact value varies locally. Because it is a force, the SI unit of weight is properly the *newton*, but it is common practice (except in physics classes!) to use the terms "weight" and "mass" interchangeably, so the units *kilograms* and *grams* are acceptable in almost all ordinary laboratory contexts.



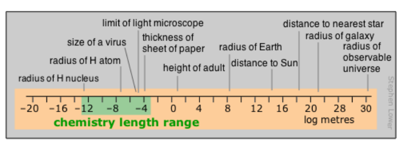
Numeric scale in terms of log kilogram. Range is negative 30 to 60 with intervals of 10. The range from negative 30 to 8 is highlighted in green to show chemistry mass range.

Please note that in this diagram and in those that follow, the numeric scale represents the *logarithm* of the number shown. For example, the mass of the electron is 10^{-30} kg.

The range of masses spans 90 orders of magnitude, more than any other unit. The range that chemistry ordinarily deals with has greatly expanded since the days when a microgram was an almost inconceivably small amount of material to handle in the laboratory; this lower limit has now fallen to the atomic level with the development of tools for directly manipulating these particles. The upper level reflects the largest masses that are handled in industrial operations, but in the recently developed fields of geochemistry and enivonmental chemistry, the range can be extended indefinitely. Flows of elements between the various regions of the environment (atmosphere to oceans, for example) are often quoted in teragrams.

Length

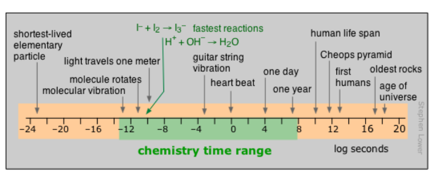
Chemists tend to work mostly in the moderately-small part of the distance range. Those who live in the lilliputian world of crystal- and molecular structures and atomic radii find the *picometer* a convenient currency, but one still sees the older non-SI unit called the *Ångstrom* used in this context; 1Å = 10^{-10} m = 100pm. Nanotechnology, the rage of the present era, also resides in this realm. The largest polymeric molecules and colloids define the top end of the particulate range; beyond that, in the normal world of doing things in the lab, the *centimeter* and occasionally the *millimeter* commonly rule.



Numeric scale in terms of log meters. Range is negative 20 to 30 with intervals of 4. The range from negative 12 to negative 4 is highlighted in green to show chemistry length range.

Time

For humans, time moves by the heartbeat; beyond that, it is the motions of our planet that count out the hours, days, and years that eventually define our lifetimes. Beyond the few thousands of years of history behind us, those years-to-the-powers-of-tens that are the fare for such fields as evolutionary biology, geology, and cosmology, cease to convey any real meaning for us. Perhaps this is why so many people are not very inclined to accept their validity.

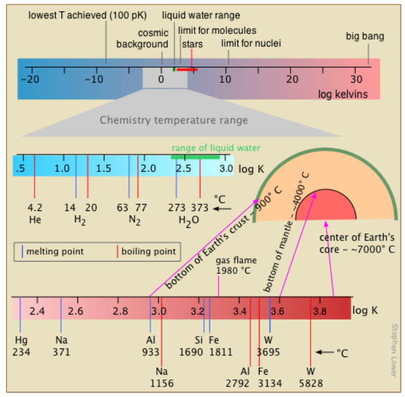


Numeric scale in terms of log seconds. Range is negative 24 to 20 with intervals of 4. The range from negative 12 to 8 is highlighted in green to show chemistry time range.

Most of what actually takes place in the chemist's test tube operates on a far shorter time scale, although there is no limit to how slow a reaction can be; the upper limits of those we can directly study in the lab are in part determined by how long a graduate student can wait around before moving on to gainful employment. Looking at the microscopic world of atoms and molecules themselves, the time scale again shifts us into an unreal world where numbers tend to lose their meaning. You can gain some appreciation of the duration of a nanosecond by noting that this is about how long it takes a beam of light to travel between your two outstretched hands. In a sense, the material foundations of chemistry itself are defined by time: neither a new element nor a molecule can be recognized as such unless it lasts long enough to have its "picture" taken through measurement of its distinguishing properties.

Temperature

Temperature, the measure of thermal intensity, spans the narrowest range of any of the base units of the chemist's measurement toolbox. The reason for this is tied into temperature's meaning as a measure of the intensity of thermal kinetic energy. Chemical change occurs when atoms are jostled into new arrangements, and the weakness of these motions brings most chemistry to a halt as absolute zero is approached. At the upper end of the scale, thermal motions become sufficiently vigorous to shake molecules into atoms, and eventually, as in stars, strip off the electrons, leaving an essentially reaction-less gaseous fluid, or plasma, of bare nuclei (ions) and electrons.



Numeric scale in terms of log Kelvins. Range is negative 20 to 30 with intervals of 10. The range from negative 3 to 4 is highlighted to show chemistry temperature range. This range is magnified further to show two numeric scales, one highlighting the lower range and the other the upper range. Melting and boiling points of different compounds are also shown in the scale.

The degree is really an *increment* of temperature, a fixed fraction of the distance between two defined reference points on a *temperature scale*.

> Although rough means of estimating and comparing temperatures have been around since AD 170, the first mercury thermometer and temperature scale were introduced in Holland in 1714 by Gabriel Daniel Fahrenheit. Fahrenheit established three fixed points on his thermometer. Zero degrees was the temperature of an ice, water, and salt mixture, which was about the coldest temperature that could be reproduced in a laboratory of the time. When he omitted salt from the slurry, he reached his second fixed point when the water-ice combination stabilized at "the thirty-second degree." His third fixed point was "found at the ninety-sixth degree, and the spirit expands to this degree when the thermometer is held in the mouth or under the armpit of a living man in good health."
>
> After Fahrenheit died in 1736, his thermometer was recalibrated using 212 degrees, the temperature at which water boils, as the upper fixed point. Normal human body temperature registered 98.6 rather than 96. In 1743, the Swedish astronomer Anders Celsius devised the aptly-named *centigrade* scale that places exactly 100 degrees between the two reference points defined by the freezing and boiling points of water.

When we say that the temperature is so many degrees, we must specify the particular scale on which we are expressing that temperature. A temperature scale has two defining characteristics, both of which can be chosen arbitrarily:

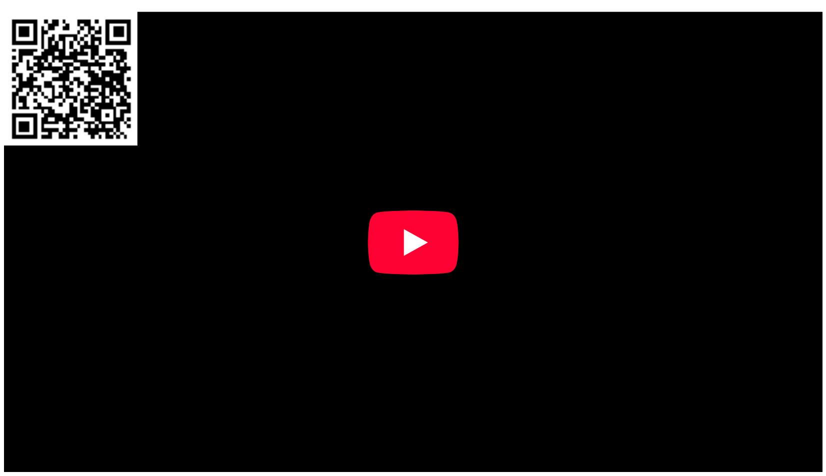
- The temperature that corresponds to 0° on the scale;
- The magnitude of the *unit increment* of temperature– that is, the size of the *degree*.

To express a temperature given on one scale in terms of another, it is necessary to take both of these factors into account. The key to temperature conversions is easy if you bear in mind that between the so-called ice- and steam-points of water there are 180 Fahrenheit degrees, but only 100 Celsius degrees, making the F° 100/180 = 5/9 the magnitude of the C°. Note the distinction between "°C" (a *temperature*) and "C°" (a temperature *increment*). Because the ice point is at 32°F, the two scales are offset by this amount. If you remember this, there is no need to memorize a conversion formula; you can work it out whenever you need it.

Near the end of the 19th Century when the physical significance of temperature began to be understood, the need was felt for a temperature scale whose zero really means zero— that is, the complete absence of thermal motion. This gave rise to the *absolute temperature scale* whose zero point is –273.15 °C, but which retains the same degree magnitude as the Celsius scale. This eventually got renamed after Lord Kelvin (William Thompson); thus the Celsius degree became the *kelvin*. Thus we can now express an increment such as five C° as "five kelvins"

> **The "other" Absolute Scale**
>
> In 1859 the Scottish engineer and physicist William J. M. Rankine proposed an absolute temperature scale based on the Fahrenheit degree. Absolute zero (0° Ra) corresponds to –459.67°F. The Rankine scale has been used extensively by those same American and English engineers who delight in expressing heat capacities in units of BTUs per pound per F°.

The importance of absolute temperature scales is that absolute temperatures can be entered directly in all the fundamental formulas of physics and chemistry in which temperature is a variable.



Units of Temperature: Units of Temperature, YouTube(opens in new window) [youtu.be] (opens in new window)

Pressure

Pressure is the measure of the *force* exerted on a unit area of surface. Its SI units are therefore newtons per square meter, but we make such frequent use of pressure that a derived SI unit, the *pascal*, is commonly used:

$$1\ Pa = 1\ N\ m^{-2}$$

The concept of pressure first developed in connection with studies relating to the atmosphere and vacuum that were carried out in the 17th century.

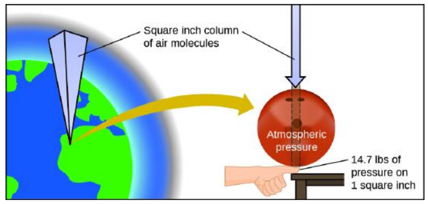


Figure 1.4.1: The atmosphere above us exerts a large pressure on objects at the surface of the earth, roughly equal to the weight of a bowling ball pressing on an area the size of a human thumbnail. Diagram of Earth with a square inch column of air molecules extending to the atmosphere. This column points to an arrow pointing down on a bowling ball resting on a human thumbnail placed on top of a table.

Atmospheric pressure is caused by the weight of the column of air molecules in the atmosphere above an object, such as the tanker car below. At sea level, this pressure is roughly the same as that exerted by a full-grown African elephant standing on a doormat, or a typical bowling ball resting on your thumbnail. These may seem like huge amounts, and they are, but life on earth has evolved under such atmospheric pressure. If you actually perch a bowling ball on your thumbnail, the pressure experienced is twice the usual pressure, and the sensation is unpleasant.

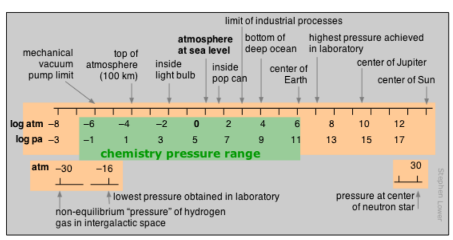
Video 1.4.1: A dramatic illustration of atmospheric pressure is provided in this brief video, which shows a railway tanker car imploding when its internal pressure is decreased. A smaller scale demonstration of this phenomenon is briefly explained.

The molecules of a gas are in a state of constant thermal motion, moving in straight lines until experiencing a collision that exchanges momentum between pairs of molecules and sends them bouncing off in other directions. This leads to a completely random distribution of the molecular velocities both in speed and direction— or it would in the absence of the Earth's gravitational field which exerts a tiny downward force on each molecule, giving motions in that direction a very slight advantage. In an ordinary container this effect is too small to be noticeable, but in a very tall column of air the effect adds up: the molecules in each vertical layer experience more downward-directed hits from those above it. The resulting force is quickly randomized, resulting in an increased pressure in that layer which is then propagated downward into the layers below.

At sea level, the total mass of the sea of air pressing down on each 1-cm^2 of surface is about 1034 g, or 10340 kg m^{-2}. The force (weight) that the Earth's gravitional acceleration g exerts on this mass is

$$f = ma = mg = (10340\ kg)(9.81\ m\ s^{-2}) = 1.013 \times 10^5\ kg\ m\ s^{-2} = 1.013 \times 10^5\ N$$

resulting in a pressure of 1.013×10^5 n m^{-2} = 1.013×10^5 Pa. The actual pressure at sea level varies with atmospheric conditions, so it is customary to define standard atmospheric pressure as 1 atm = 1.01325 x 10^5 Pa or 101.325 kPa. Although the standard atmosphere is not an SI unit, it is still widely employed. In meteorology, the *bar*, exactly 1.000 × 10^5 = 0.967 atm, is often used.



Numeric scale given in terms of log atm and log pascals respectively. Range is negative 8 to 12 with intervals of 2 atm and negative 3 to 17 with intervals of 2 pascals. The range from negative 6 to 6 atm and negative 1 to 11 pascals is highlighted to show chemistry pressure range.

The Barometer

In the early 17th century, the Italian physicist and mathematician Evangalisto Torricelli invented a device to measure atmospheric pressure. The Torricellian *barometer* consists of a vertical glass tube closed at the top and open at the bottom. It is filled with a liquid, traditionally mercury, and is then inverted, with its open end immersed in the container of the same liquid. The liquid level in the tube will fall under its own weight until the downward force is balanced by the vertical force transmitted hydrostatically to the column by the downward force of the atmosphere acting on the liquid surface in the open container. Torricelli was also the first to recognize that the space above the mercury constituted a vacuum, and is credited with being the first to create a vacuum.

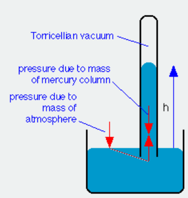
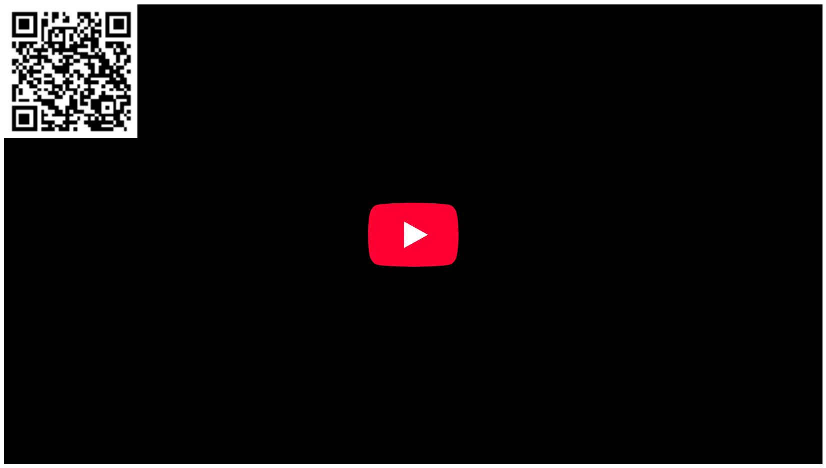


Diagram showing a barometer setup with arrows pointing on the surface of the liquid to indicate pressure due to atmosphere as well as arrow pointing up in the vertical tube to show height of liquid in the tube.

One *standard atmosphere* will support a column of mercury that is 760 mm high, so the “millimeter of mercury”, now more commonly known as the *torr*, has long been a common pressure unit in the sciences: 1 atm = 760 torr.



International System of Units (SI Units): International System of Units (SI Units), YouTube(opens in new window) [youtu.be]

Summary

The natural sciences begin with ***observation,*** and this usually involves ***numerical measurements*** of quantities such as length, volume, density, and temperature. Most of these quantities have ***units*** of some kind associated with them, and these units must be retained when you use them in calculations. Measuring units can be defined in terms of a very small number of fundamental ones that, through "dimensional analysis", provide insight into their derivation and meaning, and must be understood when converting between different unit systems.

Contributions

- Stephen Lower, Professor Emeritus (Simon Fraser U.) Chem1 Virtual Textbook
- Paul Flowers (University of North Carolina - Pembroke), Klaus Theopold (University of Delaware) and Richard Langley (Stephen F. Austin State University) with contributing authors. Textbook content produced by OpenStax College is licensed under a Creative Commons Attribution License 4.0 license. Download for free at http://cnx.org/contents/85abf193-2bd...a7ac8df6@9.110).

Learning Objectives

- To be introduced to the dimensional analysis and how it can be used to aid basic chemistry problem solving.
- To use dimensional analysis to identify whether an equation is set up correctly in a numerical calculation
- To use dimensional analysis to facilitate the conversion of units.

Dimensional analysis is amongst the most valuable tools physical scientists use. Simply put, it is the conversion between an amount in one unit to the corresponding amount in a desired unit using various conversion factors. This is valuable because certain measurements are more accurate or easier to find than others.

A Macroscopic Example: Party Planning

If you have every planned a party, you have used dimensional analysis. The amount of beer and munchies you will need depends on the number of people you expect. For example, if you are planning a Friday night party and expect 30 people you might estimate you need to go out and buy 120 bottles of sodas and 10 large pizza's. How did you arrive at these numbers? The following indicates the type of dimensional analysis solution to party problem:

(30 \; \cancel{humans}) \times \left(\dfrac{\text{4 sodas}}{1 \; \cancel{human}} \right) = 120 \; \text{sodas} \label{Eq1}

$$(30 \;\; \cancel{humans}) \times \left(\dfrac{0.333 \text{ pizzas}}{1 \;\; \cancel{human}} \right) = 10 \text{ pizzas} \tag{1.4.2}$$

Notice that the units that canceled out are lined out and only the desired units are left (discussed more below). Finally, in going to buy the soda, you perform another dimensional analysis: should you buy the sodas in six-packs or in cases?

$$(120 \; sodas) \times \left(\dfrac{1 \text{ six pack}}{6 \; sodas} \right) = 20 \text{ six packs} \tag{1.4.3}$$

$$(120 \; sodas) \times \left(\dfrac{1 \text{ case}}{24 \; sodas} \right) = 5 \text{ cases} \tag{1.4.4}$$

Realizing that carrying around 20 six packs is a real headache, you get 5 cases of soda instead.

In this party problem, we have used dimensional analysis in two different ways:

- In the first application (Equations 1.4.1 and Equation 1.4.2), dimensional analysis was used to calculate how much soda is needed need. This is based on knowing: (1) how much soda we need for one person and (2) how many people we expect; likewise for the pizza.
- In the second application (Equations 1.4.3 and 1.4.4), dimensional analysis was used to ***convert units*** (i.e. from individual sodas to the equivalent amount of six packs or cases)

Using Dimensional Analysis to Convert Units

Consider the conversion in Equation 1.4.3:

$$(120 \; sodas) \times \left(\dfrac{1 \text{ six pack}}{6 \; sodas} \right) = 20 \text{ six packs} \tag{1.4.5}$$

If we ignore the numbers for a moment, and just look at the units (i.e. ***dimensions***), we have:

$$\text{soda} \times \left(\dfrac{\text{six pack}}{\text{sodas}} \right)$$

We can treat the dimensions in a similar fashion as other numerical analyses (i.e. any number divided by itself is 1). Therefore:

$$\text{soda} \times \left(\frac{\text{six pack}}{\text{sodas}}\right) = \cancel{\text{soda}} \times \left(\frac{\text{six pack}}{\cancel{\text{sodas}}}\right)$$

So, the dimensions of the numerical answer will be "six packs".

How can we use dimensional analysis to be sure we have set up our equation correctly? Consider the following alternative way to set up the above unit conversion analysis:

$$120\ \cancel{\text{soda}} \times \left(\frac{6 \text{ sodas}}{\cancel{\text{six pack}}}\right) = 720\ \frac{\text{sodas}^2}{1 \text{ six pack}}$$

- While it is correct that there are 6 sodas in one six pack, the above equation yields a value of 720 with units of ***sodas²/six pack***.
- These rather bizarre units indicate that the equation has been setup ***incorrectly*** (and as a consequence you will have a ton of extra soda at the party).

Using Dimensional Analysis in Calculations

In the above case it was relatively straightforward keeping track of units during the calculation. What if the calculation involves powers, etc? For example, the equation relating kinetic energy to mass and velocity is:

$$E_{kinetics} = \frac{1}{2}\text{mass} \times \text{velocity}^2 \tag{1.4.6}$$

An example of units of mass is kilograms (kg) and velocity might be in meters/second (m/s). What are the dimensions of $E_{kinetic}$?

$$(kg) \times \left(\frac{m}{s}\right)^2 = \frac{kg\ m^2}{s^2}$$

The $\frac{1}{2}$ factor in Equation 1.4.6 is neglected since pure numbers have no units. Since the velocity is squared in Equation 1.4.6, the ***dimensions*** associated with the numerical value of the velocity are also squared. We can double check this by knowing the the Joule (J) is a measure of energy, and as a composite unit can be decomposed thusly:

$$1\ J = kg\frac{m^2}{s^2}$$

> **Units of Pressure**
>
> Pressure (P) is a measure of the Force (F) per unit area (A):
>
> $$P = \frac{F}{A}$$
>
> Force, in turn, is a measure of the acceleration (a) on a mass (m):
>
> $$F = m \times a$$
>
> Thus, pressure (P) can be written as:
>
> $$P = \frac{m \times a}{A}$$
>
> What are the units of pressure from this relationship? (*Note: acceleration is the change in velocity per unit time*)
>
> $$P = \frac{kg \times \frac{\cancel{m}}{s^2}}{m^{\cancel{2}}}$$
>
> We can simplify this description of the units of Pressure by dividing numerator and denominator by m:
>
> $$P = \frac{\frac{kg}{s^2}}{m} = \frac{kg}{m\ s^2}$$
>
> In fact, these are the units of a the composite *Pascal* (*Pa*) unit and is the SI measure of pressure.

Performing Dimensional Analysis

The use of units in a calculation to ensure that we obtain the final proper units is called *dimensional analysis*. For example, if we observe experimentally that an object's potential energy is related to its mass, its height from the ground, and to a gravitational force, then when multiplied, the units of mass, height, and the force of gravity must give us units corresponding to those of energy.

Energy is typically measured in joules, calories, or electron volts (eV), defined by the following expressions:

- 1 J = 1 (kg·m^2)/s^2 = 1 coulomb·volt
- 1 cal = 4.184 J
- 1 eV = 1.602 × 10^{-19} J

Performing dimensional analysis begins with finding the appropriate conversion factors. Then, you simply multiply the values together such that the units cancel by having equal units in the numerator and the denominator. To understand this process, let us walk through a few examples.

> **Example 1.4.1**
>
> Imagine that a chemist wants to measure out 0.214 mL of benzene, but lacks the equipment to accurately measure such a small volume. The chemist, however, is equipped with an analytical balance capable of measuring to $\pm 0.0001\ g$ Looking in a reference table, the chemist learns the density of benzene ($\rho = 0.8765\ g/mL$). How many grams of benzene should the chemist use?
>
> **Solution**
>
> $$0.214\ \cancel{mL} \left(\frac{0.8765\ g}{1\ \cancel{mL}}\right) = 0.187571\ g$$
>
> Notice that the mL are being divided by mL, an equivalent unit. We can cancel these our, which results with the 0.187571 g. However, this is not our final answer, since this result has too many significant figures and must be rounded down to three significant digits. This is because 0.214 mL has three significant digits and the conversion factor had four significant digits. Since 5 is greater than or equal to 5, we must round the preceding 7 up to 8.
>
> Hence, the chemist should weigh out 0.188 g of benzene to have 0.214 mL of benzene.

Example 1.4.2

To illustrate the use of dimensional analysis to solve energy problems, let us calculate the kinetic energy in joules of a 320 g object traveling at 123 cm/s.

Solution

To obtain an answer in joules, we must convert grams to kilograms and centimeters to meters. Using Equation 1.4.6, the calculation may be set up as follows:

\[\begin{align*} KE&=\dfrac{1}{2}mv^2=\dfrac{1}{2}(g) \left(\dfrac{kg}{g}\right) \left[\left(\dfrac{cm}{s}\right)\left(\dfrac{m}{cm}\right) \right]^2 \

4pt] &= (\cancel{g})\left(\dfrac{kg}{\cancel{g}}\right) \left(\dfrac{\cancel{m^2}}{s^2}\right) \left(\dfrac{m^2}{\cancel{cm^2}}\right) = \dfrac{kg·m^2}{s^2} \\[4pt] &=\dfrac{1}{2}320\; \cancel{g} \left(\dfrac{1\; kg

Alternatively, the conversions may be carried out in a stepwise manner:

Step 1: convert g to kg

$$320\ \cancel{g}\left(\frac{1\ kg}{1000\ \cancel{g}}\right)=0.320\ kg$$

Step 2: convert cm to m

$$123\ \cancel{cm}\left(\frac{1\ m}{100\ \cancel{cm}}\right)=1.23\ m$$

Now the natural units for calculating joules is used to get final results

$$\begin{aligned} KE &= \frac{1}{2}0.320\ kg(1.23\ ms)^2 \\ &= \frac{1}{2}0.320\ kg\left(1.513\frac{m^2}{s^2}\right)=0.242\ \frac{kg\cdot m^2}{s^2}=0.242\ J \end{aligned}$$

Of course, steps 1 and 2 can be done in the opposite order with no effect on the final results. However, this second method involves an additional step.

Example 1.4.3

Now suppose you wish to report the number of kilocalories of energy contained in a 7.00 oz piece of chocolate in units of kilojoules per gram.

Solution

To obtain an answer in kilojoules, we must convert 7.00 oz to grams and kilocalories to kilojoules. Food reported to contain a value in Calories actually contains that same value in kilocalories. If the chocolate wrapper lists the caloric content as 120 Calories, the chocolate contains 120 kcal of energy. If we choose to use multiple steps to obtain our answer, we can begin with the conversion of kilocalories to kilojoules:

$$120\ \cancel{kcal}\left(\frac{1000\ \cancel{cal}}{\cancel{kcal}}\right)\left(\frac{4.184\ \cancel{J}}{1\ \cancel{cal}}\right)\left(\frac{1\ kJ}{1000\ \cancel{J}}\right)=502\ kJ$$

We next convert the 7.00 oz of chocolate to grams:

$$7.00\ \cancel{oz}\left(\frac{28.35\ g}{1\ \cancel{oz}}\right)=199\ g$$

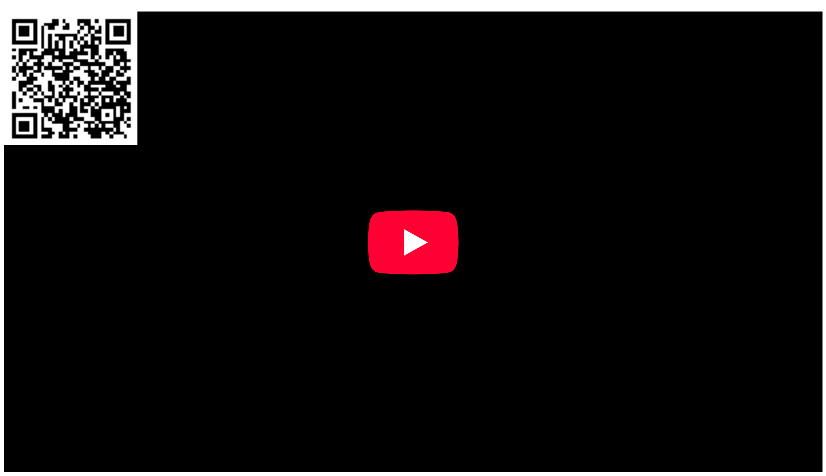
The number of kilojoules per gram is therefore

$$\frac{502\ kJ}{199\ g}=2.52\ kJ/g$$

Alternatively, we could solve the problem in one step with all the conversions included:

$$\left(\frac{120\ \cancel{kcal}}{7.00\ \cancel{oz}}\right)\left(\frac{1000\ \cancel{cal}}{1\ \cancel{kcal}}\right)\left(\frac{4.184\ \cancel{J}}{1\ \cancel{cal}}\right)\left(\frac{1\ kJ}{1000\ \cancel{J}}\right)\left(\frac{1\ \cancel{oz}}{28.35\ g}\right)=2.53\ kJ/g$$

The discrepancy between the two answers is attributable to rounding to the correct number of significant figures for each step when carrying out the calculation in a stepwise manner. Recall that all digits in the calculator should be carried forward when carrying out a calculation using multiple steps. In this problem, we first converted kilocalories to kilojoules and then converted ounces to grams.



Converting Between Units: Converting Between Units, YouTube(opens in new window) [youtu.be]

Summary

Dimensional analysis is used in numerical calculations, and in converting units. It can help us identify whether an equation is set up correctly (i.e. the resulting units should be as expected). Units are treated similarly to the associated numerical values, i.e., if a variable in an equation is supposed to be squared, then the associated dimensions are squared, etc.

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