1.6: Physical Quantities - Units and Scientific Notation

> **◐ Learning Objectives**
>
> - Express quantities properly using a number and a unit.
> - Recognize the different measurement systems used in chemistry.
> - Describe how prefixes are used in the metric system and identify how the prefixes milli-, centi-, and kilo- compare to the base unit.
> - Understand when and how to use scientific notation to represent measurements.

Measurements

A coffee maker's instructions tell you to fill the coffeepot with 4 cups of water and use 3 scoops of coffee. When you follow these instructions, you are measuring. When you visit a doctor's office, a nurse checks your temperature, height, weight, and perhaps blood pressure (Figure 1.6.1); the nurse is also measuring.
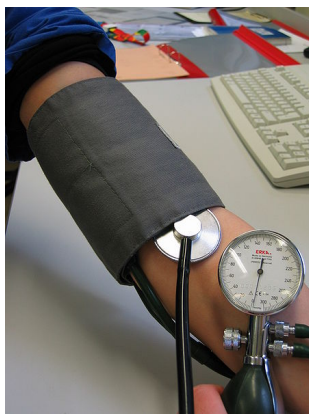

Figure 1.6.1: Measuring Blood Pressure. A nurse or a doctor measuring a patient's blood pressure is taking a measurement. (GNU Free Documentation License; Pia von Lützau via Wikipedia).

Chemists measure the properties of matter using a variety of devices or measuring tools, many of which are similar to those used in everyday life. Rulers are used to measure length, balances (scales) are used to measure mass (weight), and graduated cylinders or pipettes are used to measure volume. Measurements made using these devices are expressed as quantities. A quantity is an amount of something and consists of a **number** and a **unit**. The number tells us *how many* (or how much), and the unit tells us what the scale of measurement is. For example, when a distance is reported as "5.2 kilometers," we know that the quantity has been expressed in units of kilometers and that the number of kilometers is 5.2.

$$\underbrace{5.2}_{\text{number}}\ \underbrace{\text{kilometers}}_{\text{unit}}$$

If you ask a friend how far he or she walks from home to school, and the friend answers "12" without specifying a unit, you do not know whether your friend walks—for example, 12 miles, 12 kilometers, 12 furlongs, or 12 yards.

Without units, a number can be meaningless, confusing, or possibly life threatening. Suppose a doctor prescribes phenobarbital to control a patient's seizures and states a dosage of "100" without specifying units. Not only will this be confusing to the medical professional giving the dose, but the consequences can be dire: 100 mg given three times per day can be effective as an anticonvulsant, but a single dose of 100 g is more than 10 times the lethal amount.

> *Both a number and a unit must be included to express a quantity properly.*

To understand chemistry, we need a clear understanding of the units chemists work with and the rules they follow for expressing numbers.

> ✔ **Example 1.6.1**
>
> Identify the **number** and the **unit** in each quantity.
>
> a. one dozen eggs
> b. 2.54 centimeters
> c. a box of pencils
> d. 88 meters per second
>
> **Answers**
> a. The number is one, and the unit is dozen.
> b. The number is 2.54, and the unit is centimeter.
> c. The number 1 is implied because the quantity is only a box. The unit is box of pencils.
> d. The number is 88, and the unit is meters per second. Note that in this case the unit is actually a combination of two units: meters and seconds.

> ❓ **Exercise 1.6.2**
>
> Identify the **number** and the **unit** in each quantity.
>
> a. 99 bottles of soda
> b. 60 miles per hour
> c. 32 fluid ounces
> d. 98.6 degrees Fahrenheit
>
> **Answer a**
>
> The number is 99, and the unit is bottles of soda.
>
> **Answer b**
>
> The number is 60, and the unit is miles per hour.
>
> **Answer c**
>
> The number 32, and the unit is fluid ounces
>
> **Answer d**
>
> The number is 98.6, and the unit is degrees Fahrenheit

The International System of Units

How long is a yard? It depends on whom you ask and when you asked the question. Today we have a standard definition of the yard, which you can see marked on every football field. If you move the ball ten yards, you get a first down and it doesn't matter whether you are playing in Los Angeles, Dallas, or Green Bay. But at one time that yard was arbitrarily defined as the distance from the tip of the king's nose to the end of his outstretched hand. Of course, the problem there is simple: new king, new distance (and then you have to remark all those football fields).
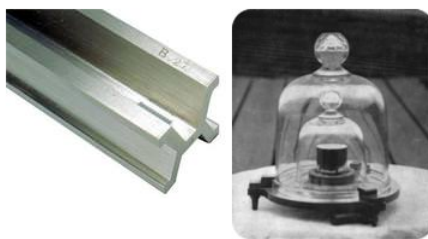

Figure 1.6.2: Meter standard (left) and Kilogram standard (right).

SI Base Units

All measurements depend on the use of units that are well known and understood. The **English system** of measurement units (inches, feet, ounces, etc.) are not used in science because of the difficulty in converting from one unit to another. The **metric system** is used because all metric units are based on multiples of 10, making conversions very simple. The metric system was originally established in France in 1795. **The International System of Units** is a system of measurement based on the metric system. The acronym **SI** is commonly used to refer to this system and stands for the French term, *Le Système International d'Unités*. The SI was adopted by international agreement in 1960 and is composed of seven base units in Table 1.6.1.

Table 1.6.1: SI Base Units of Measurement

Quantity	SI Base Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Temperature	kelvin	K
Time	second	s
Amount of a Substance	mole	mol
Electric Current	ampere	A
Luminous Intensity	candela	cd

The first units are frequently encountered in chemistry. All other measurement quantities, such as volume, force, and energy, can be derived from these seven base units.

📌 The Metric System is Not Ubiquitously Adopted

The map below shows the adoption of the SI units in countries around the world. The United States has legally adopted the metric system for measurements, but does not use it in everyday practice. Great Britain and much of Canada use a combination of metric and imperial units.
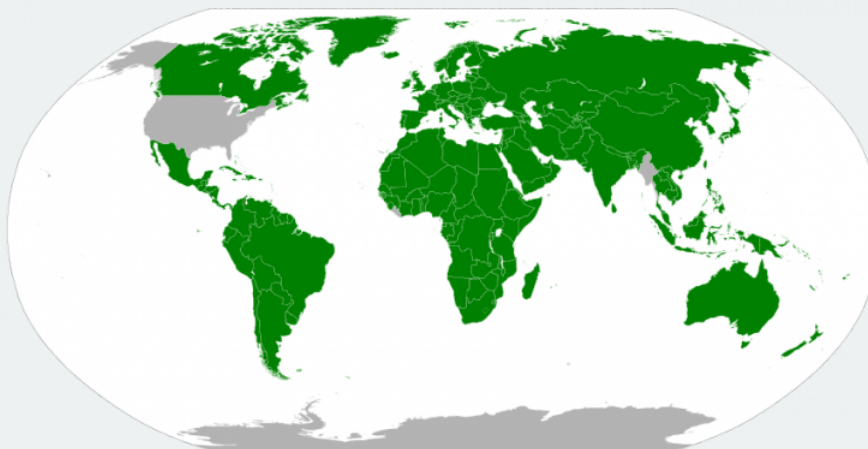

Figure 1.6.2: Areas of world using metric system (in green). Only a few countries are slow or resistant to adoption including the United States.

Metric Prefixes

Conversions between metric system units are straightforward because the system is based on powers of ten. For example, meters, centimeters, and millimeters are all metric units of length. There are 10 millimeters in 1 centimeter and 100 centimeters in 1 meter. **Metric prefixes** are used to distinguish between units of different size. These prefixes all derive from either Latin or Greek terms. For example, *mega* comes from the Greek word $\mu\varepsilon\gamma\alpha\varsigma$, meaning "great". Table 1.6.2 lists the most common metric prefixes and

their relationship to the central unit that has no prefix. Length is used as an example to demonstrate the relative size of each prefixed unit.

Table 1.6.2: SI Prefixes. Commonly used prefixes are bolded.

Prefix	Unit Abbreviation	Meaning	Example
giga	G	1,000,000,000	1 gigameter $(\mathrm{Gm}) = 10^9$ m
mega	M	1,000,000	1 megameter $(\mathrm{Mm}) = 10^6$ m
kilo	k	**1,000**	**1 kilo**meter $(\mathrm{km}) = \mathbf{1,000}$ m
hecto	h	100	1 hectometer $(\mathrm{hm}) = 100$ m
deka	da	10	1 dekameter $(\mathrm{dam}) = 10$ m
		1	1 meter (m)
deci	d	**1/10**	**1 deci**meter $(\mathrm{dm}) = \mathbf{0.1}$ m
centi	c	**1/100**	**1 centi**meter $(\mathrm{cm}) = \mathbf{0.01}$ m
milli	m	**1/1,000**	**1 milli**meter $(\mathrm{mm}) = \mathbf{0.001}$ m
micro	**μ**	**1/1,000,000**	**1 micro**meter $(\mathbf{\mu}\mathrm{m}) = \mathbf{10^{-6}}$ m
nano	n	1/1,000,000,000	1 nanometer $(\mathrm{nm}) = 10^{-9}$ m
pico	p	1/1,000,000,000,000	1 picometer $(\mathrm{pm}) = 10^{-12}$ m

Just as expressing a quantity without a unit is meaningless, so is using the incorrect format for units and prefixes. As you may have noticed, most metric abbreviations are lowercase. We use "m" for meter and not "M", which you will see later to represent solution concentration, something very different from length. However, when it comes to volume, the base unit "liter" is abbreviated as "L" and not "l". So we would write 3.5 milliliters as $3.5\ \mathrm{mL}$.

As a practical matter, whenever possible you should express the units in a small and manageable number. If you are measuring the weight of a material that weighs 6.5 kg, this is easier than saying it weighs $6500\ \mathrm{g}$ or $0.65\ \mathrm{dag}$. All three are correct, but the kg units in this case make for a small and easily managed number. However, if a specific problem needs grams instead of kilograms, go with the grams for consistency.

> ✔ Example 1.6.2: Unit Abbreviations
>
> Give the abbreviation for each unit and define the abbreviation in terms of the base unit.
>
> a. kiloliter
> b. microsecond
> c. decimeter
> d. nanogram
>
> **Solutions**
>
> Solutions to example explaining unit abbreviations.
>
> | | Explanation | Answer |
> |---|---|---|
> | **a** | The prefix kilo means "1,000 ×," so 1 kL equals 1,000 L | kL |
> | **b** | The prefix micro implies 1/1,000,000th of a unit, so 1 µs equals 0.000001 s. | µs |
> | **c** | The prefix deci means 1/10th, so 1 dm equals 0.1 m. | dm |

	Explanation	Answer
d	The prefix nano means 1/1000000000, so a nanogram is equal to 0.000000001 g	ng

> **? Exercise 1.6.2**
>
> Give the abbreviation for each unit and define the abbreviation in terms of the base unit.
>
> a. kilometer
> b. milligram
> c. nanosecond
> d. centiliter
>
> **Answer a**
>   km
>
> **Answer b**
>   mg
>
> **Answer c**
>   ns
>
> **Answer d**
>   cL

Scientific Notation

Chemists often work with numbers that are exceedingly large or small. For example, entering the mass in grams of a hydrogen atom into a calculator would require a display with at least 24 decimal places. A system called **scientific notation** avoids much of the tedium and awkwardness of manipulating numbers with large or small magnitudes. In scientific notation, these numbers are expressed in the form

$$N \times 10^n \tag{1.6.1}$$

where N is a number greater than or equal to 1 and less than 10 (1 ≤ N < 10), and n is a positive or negative integer ($10^0 = 1$). The number 10 is called the base because it is this number that is raised to the power n. Although a base number may have values other than 10, the base number in scientific notation is always 10.

A simple way to convert numbers to scientific notation is to move the decimal point as many places to the left or right as needed to give a number from 1 to 10 (N). The magnitude of n is then determined as follows:

- If the decimal point is moved to the left n places, n is positive.
- If the decimal point is moved to the right n places, n is negative.

Another way to remember this is to recognize that as the number N decreases in magnitude, the exponent increases and vice versa. The application of this rule is illustrated in Example 1.6.1.

> **✔ Example 1.6.1: Expressing Numbers in Scientific Notation**
>
> Convert each number to scientific notation.
>
> a. 637.8
> b. 0.0479
> c. 7.86
> d. 12,378

e. 0.00032
f. 61.06700
g. 2002.080
h. 0.01020

Solution

Solutions to example explaining how numbers look in scientific notation.

	Explanation	Answer
a	To convert 637.8 to a number from 1 to 10, we move the decimal point two places to the left: 637.8 Because the decimal point was moved two places to the left, n = 2.	6.378×10^2
b	To convert 0.0479 to a number from 1 to 10, we move the decimal point two places to the right: 0.0479 Because the decimal point was moved two places to the right, n = −2.	4.79×10^{-2}
c	This is usually expressed simply as 7.86. (Recall that $10^0 = 1$.)	7.86×10^0
d	Because the decimal point was moved four places to the left, n = 4.	1.2378×10^4
e	Because the decimal point was moved four places to the right, n = −4.	3.2×10^{-4}
f	Because the decimal point was moved one place to the left, n = 1.	6.106700×10^1
g	Because the decimal point was moved three places to the left, n = 3.	2.002080×10^3
h	Because the decimal point was moved two places to the right, n = -2.	1.020×10^{-2}

Scientific Notation: Addition and Subtraction

Before numbers expressed in scientific notation can be added or subtracted, they must be converted to a form in which all the exponents have the same value. The appropriate operation is then carried out on the values of N. Example 1.6.2 illustrates how to do this.

✔ Example 1.6.2: Expressing Sums and Differences in Scientific Notation

Carry out the appropriate operation and then express the answer in scientific notation.

a. $(1.36 \times 10^2) + (4.73 \times 10^3)$
b. $(6.923 \times 10^{-3}) - (8.756 \times 10^{-4})$

Solution

Solution to example explaining how sums and differences look in scientific notation.

	Explanation	Answer

LibreTexts™

	Explanation	Answer
a	Both exponents must have the same value, so these numbers are converted to either $$(1.36 \times 10^2) + (47.3 \times 10^2) = (1.36 + 47.3) \times 10^2 = 48.66 \times 10^2$$ or $$(0.136 \times 10^3) + (4.73 \times 10^3) = (0.136 + 4.73) \times 10^3) = 4.87 \times 10^3$$. Choosing either alternative gives the same answer, reported to two decimal places: In converting 48.66×10^2 to scientific notation, n has become more positive by 1 because the value of N has decreased.	4.87×10^3
b	Converting the exponents to the same value gives either $$(6.923 \times 10^{-3}) - (0.8756 \times 10^{-3}) = (6.923 - 0.8756) \times 10^{-3}$$ or $$(69.23 \times 10^{-4}) - (8.756 \times 10^{-4}) = (69.23 - 8.756) \times 10^{-4} = 60.474 \times 10^{-4}$$. In converting 60.474×10^{-4} to scientific notation, n has become more positive by 1 because the value of N has decreased.	6.047×10^{-3}

Scientific Notation: Multiplication and Division

When multiplying numbers expressed in scientific notation, we multiply the values of N and add together the values of n. Conversely, when dividing, we divide N in the dividend (the number being divided) by N in the divisor (the number by which we are dividing) and then subtract n in the divisor from n in the dividend. In contrast to addition and subtraction, the exponents do not have to be the same in multiplication and division. Examples of problems involving multiplication and division are shown in Example 1.6.3.

> ✔ Example 1.6.3: Expressing Products and Quotients in Scientific Notation
>
> Perform the appropriate operation and express your answer in scientific notation.
>
> a. $[(6.022 \times 10^{23})(6.42 \times 10^{-2})]$
> b. $\dfrac{1.67 \times 10^{-24}}{9.12 \times 10^{-28}}$
> c. $\dfrac{(6.63 \times 10^{-34})(6.0 \times 10)}{8.52 \times 10^{-2}}$
>
> **Solution**
>
> Solutions to example explaining how products and quotients look in scientific notation.
>
> | | Explanation | Answer |
> |---|---|---|
> | **a** | In multiplication, we add the exponents: $$(6.022 \times 10^{23})(6.42 \times 10^{-2}) = (6.022)(6.42) \times 10^{[23+(-2)]} = 38.7 \times 10^{21}$$ In converting 38.7×10^{21} to scientific notation, n has become more positive by 1 because the value of N has decreased. | 3.87×10^{22} |

1.6.7

	Explanation	Answer
b	In division, we subtract the exponents: $\dfrac{1.67 \times 10^{-24}}{9.12 \times 10^{-28}} = \dfrac{1.67}{9.12} \times 10^{[-24-(-28)]} = 0.183 \times 10^4$ In converting 0.183×10^4 to scientific notation, n has become more negative by 1 because the value of N has increased.	1.83×10^3
c	This problem has both multiplication and division: $\dfrac{(6.63 \times 10^{-34})(6.0 \times 10)}{(8.52 \times 10^{-2})} = \dfrac{39.78}{8.52} \times 10^{[-34+1-(-2)]}$	4.7×10^{-31}

Key Takeaways

- Identifying a quantity properly requires both a number and a unit.
- Metric prefixes derive from Latin or Greek terms. The prefixes are used to make the units manageable.
- The SI system is based on multiples of ten. There are seven basic units in the SI system. Five of these units are commonly used in chemistry.