12.T: Character Tables

Nonaxial Groups

These groups are characterized by a lack of a proper rotation axis.

C_1	E
A	1

C_s	E	σ_h		
A'	1	1	x, y, R_z	x^2, y^2, z^2, xy
A"	1	-1	z, R_x, R_y	yz, xz

C_i	E	i		
Ag	1	1	R_x, R_y, R_z	x^2, y^2, z^2, xy, yz, zx
A_u	1	-1	x,y,z	

Cyclic C_n Groups

These groups are characterized by an *n*-fold proper rotation axis C_n.

C_2	E	C_2		
A	1	1	z, R_z	x^2, y^2, z^2, xy
B	1	-1	x, y, R_x, Ry	yz,xz

C_3	E	C_3	$C_3{}^2$		ε=exp(2π/3)
A	1	1	1	z, R_z	x^2+y^2, z^2
E	$\left\{\begin{matrix}1\\1\end{matrix}\right.$	$\begin{matrix}\epsilon\\\epsilon^*\end{matrix}$	$\left.\begin{matrix}\epsilon^*\\\epsilon\end{matrix}\right\}$	(x,y),(R_x,R_y)	(x^2-y^2, xy), (xz, yz)

C_4	E	C_4	C_2	$C_4{}^3$		
A	1	1	1	1	z, R_z	x^2+y^2, z^2
B	1	-1	1	-1		x^2-y^2, xy
E	$\left\{\begin{matrix}1\\1\end{matrix}\right.$	$\begin{matrix}i\\-i\end{matrix}$	$\begin{matrix}-1\\-1\end{matrix}$	$\left.\begin{matrix}-i\\i\end{matrix}\right\}$	(x,y), (R_x,R_y)	(xz, yz)

C_5	E	C_5	$C_5{}^2$	$C_5{}^3$	$C_5{}^4$		ε=exp(i2π/5)
A	1	1	1	1	1	Z, R_z	x^2+y^2, z^2
E_1	$\left\{\begin{matrix}1\\1\end{matrix}\right.$	$\begin{matrix}\epsilon\\\epsilon^*\end{matrix}$	$\begin{matrix}\epsilon^2\\\epsilon^{*2}\end{matrix}$	$\begin{matrix}\epsilon^{*2}\\\epsilon^2\end{matrix}$	$\left.\begin{matrix}\epsilon^*\\\epsilon\end{matrix}\right\}$	(x, y), (R_x,R_y)	(xz, yz)

E_2	1 1	ϵ^2 ϵ^{*2}	ϵ^* ϵ	ϵ ϵ^*	ϵ^{*2} ϵ^2		(x^2-y^2, xy)

C_6	E	C_6	C_3	C_2	C_3^2	C_6^5		$\varepsilon = \exp(i2\pi/6)$
A	1	1	1	1	1	1	z, R_z	x^2+y^2, z^2
B	1	-1	1	-1	1	-1		
E_1	1 1	ϵ ϵ^*	$-\epsilon^*$ $-\epsilon$	-1 -1	$-\epsilon$ $-\epsilon^*$	ϵ^* ϵ	$(R_x,R_y), (x,y)$	(xz, yz)
E_2	1 1	$-\epsilon^*$ $-\epsilon$	$-\epsilon$ $-\epsilon^*$	1 1	$-\epsilon^*$ $-\epsilon$	$-\epsilon$ $-\epsilon^*$		(x^2-y^2, xy)

C_7	E	C_7	C_7^2	C_7^3	C_7^4	C_7^5	C_7^6		$\varepsilon = \exp(i2\pi/7)$
A	1	1	1	1	1	1	1	z, R_z	x^2+y^2, z^2
E_1	1 1	ϵ ϵ^*	ϵ^2 ϵ^{*2}	ϵ^3 ϵ^{*3}	ϵ^{*3} ϵ^3	ϵ^{*2} ϵ^2	ϵ^* ϵ	$(R_x,R_y), (x,y)$	(xz, yz)
E_2	1 1	ϵ^2 ϵ^{*2}	ϵ^{*3} ϵ^3	ϵ^* ϵ	ϵ ϵ^*	ϵ^3 ϵ^{*3}	ϵ^{*2} ϵ^2		(x^2-y^2, xy)
E_3	1 1	ϵ^3 ϵ^{*3}	ϵ^* ϵ	ϵ^2 ϵ^{*2}	ϵ^{*2} ϵ^2	ϵ ϵ^*	ϵ^{*3} ϵ^3		

C_8	E	C_8	C_4	C_8^3	C_2	C_8^5	C_4^3	C_8^7		$\varepsilon = \exp(i2\pi/8)$
A	1	1	1	1	1	1	1	1	z, R_z	x^2+y^2, z^2
B	1	-1	1	-1	1	-1	1	-1		
E_1	1 1	ϵ ϵ^*	i $-i$	$-\epsilon^*$ $-\epsilon$	-1 -1	$-\epsilon$ $-\epsilon^*$	$-i$ i	ϵ^* ϵ	$(R_x,R_y), (x,y)$	(xz, yz)
E_2	1 1	i $-i$	-1 -1	$-i$ i	1 1	i $-i$	-1 -1	$-i$ i		(x^2-y^2, xy)
E_3	1 1	$-\epsilon$ $-\epsilon^*$	i $-i$	ϵ^* ϵ	-1 -1	ϵ ϵ^*	$-i$ i	$-\epsilon^*$ $-\epsilon$		

Pyramidal C_{nv} Groups

These groups are characterized by an n-fold proper rotation axis C_n and n mirror planes σ_v which contain C_n

C_{2v}	E	C_2	$\sigma_{(xz)}$	$\sigma_{(yz)}$		
A_1	1	1	1	1	z	x^2, y^2, z^2
A_2	1	1	-1	-1	R_z	xy
B_1	1	-1	1	-1	x, R_y	xz
B_2	1	-1	-1	1	y, R_x	yz

C_{3v}	E	$2C_3$	$3\sigma_v$		
A_1	1	1	1	z	x^2+y^2, z^2
A_2	1	1	-1	R_z	
E	2	-1	0	(R_x, R_y), (x,y)	(xz, yz) $(x^2-y^2$, xy)

C_{4v}	E	$2C_4$	C_2	$2\sigma_v$	$2\sigma_d$		
A_1	1	1	1	1	1	z	x^2+y^2, z^2
A_2	1	1	1	-1	-1	R_z	
B_1	1	-1	1	1	-1		x^2-y^2
B_2	1	-1	1	-1	1		xy
E	2	0	-2	0	0	(R_x, R_y), (x,y)	(xz, yz)

C_{5v}	E	$2C_5$	$2C_5^2$	$5\sigma_v$		
A_1	1	1	1	1	z	x^2+y^2, z^2
A_2	1	1	1	-1	R_z	
E_1	2	$2\cos 72^\circ$	$2\cos 144^\circ$	0	(R_x, R_y), (x,y)	(xz, yz)
E_2	2	$2\cos 144^\circ$	$2\cos 72^\circ$	0		$(x^2-y^2$, xy)

C_{6v}	E	$2C_6$	$2C_3$	C_2	$3\sigma_v$	$3\sigma_d$		
A_1	1	1	1	1	1	1	z	x^2+y^2, z^2
A_2	1	1	1	1	-1	-1	R_z	
B_1	1	-1	1	-1	1	-1		
B_2	1	-1	1	-1	-1	1		
E_1	2	1	-1	-2	0	0	(R_x, R_y), (x,y)	(xz, yz)
E_2	2	-1	-1	2	0	0		$(x^2-y^2$, xy)

$C_{\infty v}$	E	$2C_\infty$	...	$\infty\sigma_v$		
A_1	1	1	...	1	z	x^2+y^2, z^2
A_2	1	1	...	-1	R_z	
E_1	2	$2\cos\phi$	...	0	(x,y), (R_x, R_y)	(xz, yz)
E_2	2	$2\cos 2\phi$	...	0		$(x^2-y^2$, xy)
E_3	2	$2\cos 3\phi$	...	0		
...	...	...	...	...		

Reflection C_{nh} Groups

These groups are characterized by an n-fold proper rotation axis C_n and a mirror plane σ_h *normal* to C_n.

C_{2h}	E	C_2	i	σ_h		
A_g	1	1	1	1	R_z	x^2, y^2, z^2
B_g	1	-1	1	-1	R_x, R_y	xz, yz
A_u	1	1	-1	-1	z	
B_u	1	-1	-1	1	x,y	

C_{3h}	E	C_3	C_3^2	σ_h	S_3	S_3^5		ε=exp(i2π/3)
A'	1	1	1	1	1	1	R_z	x^2+y^2, z^2
E'	1 1	ϵ ϵ^*	ϵ^* ϵ	1 1	ϵ ϵ^*	ϵ^* ϵ	(x,y)	(x^2-y^2, xy)
A''	1	1	1	-1	-1	-1	z	
E''	1 1	ϵ ϵ^*	ϵ^* ϵ	-1 -1	$-\epsilon$ $-\epsilon^*$	$-\epsilon^*$ $-\epsilon$	(R_x, R_y)	(xz, yz)

C_{4h}	E	C_4	C_2	C_4^3	i	S_4^3	σ_h	S_4		
A_g	1	1	1	1	1	1	1	1	R_z	x^2+y^2, z^2
B_g	1	-1	1	-1	1	-1	1	-1		x^2-y^2, xy
E_g	1 1	i $-i$	-1 -1	$-i$ i	1 1	i $-i$	-1 -1	$-i$ i	(R_x, R_y)	(xz, yz)
A_u	1	1	1	1	-1	-1	-1	-1	z	
B_u	1	-1	1	-1	-1	1	-1	1		
E_u	1 1	i $-i$	-1 -1	$-i$ i	-1 -1	$-i$ i	1 1	i $-i$	(x,y)	

C_{5h}	E	C_5	C_5^2	C_5^3	C_5^4	σ_h	S_5	S_5^7	S_5^3	S_5^9		ε=exp(i2π/5)
A'	1	1	1	1	1	1	1	1	1	1	R_z	x^2+y^2, z^2
E_1'	1 1	ϵ ϵ^*	ϵ^2 ϵ^{*2}	ϵ^{*2} ϵ^2	ϵ^* ϵ	1 1	ϵ ϵ^*	ϵ^2 ϵ^{*2}	ϵ^{*2} ϵ^2	ϵ^* ϵ	(x, y)	
E_2'	1 1	ϵ^2 ϵ^{*2}	ϵ^* ϵ	ϵ ϵ^*	ϵ^{*2} ϵ^2	1 1	ϵ^2 ϵ^{*2}	ϵ^* ϵ	ϵ ϵ^*	ϵ^{*2} ϵ^2		(x^2-y^2, xy)
A''	1	1	1	1	1	-1	-1	-1	-1	-1	z	
E_1''	1 1	ϵ ϵ^*	ϵ^2 ϵ^{*2}	ϵ^{*2} ϵ^2	ϵ^* ϵ	-1 -1	$-\epsilon$ $-\epsilon^*$	$-\epsilon^2$ $-\epsilon^{*2}$	$-\epsilon^{*2}$ $-\epsilon^2$	$-\epsilon^*$ $-\epsilon$	(R_x, R_y)	(xz, yz)

E$_2$''	$\left\{\begin{matrix}1\\1\end{matrix}\right.$	$\begin{matrix}\epsilon^2\\\epsilon^{*2}\end{matrix}$	$\begin{matrix}\epsilon^*\\\epsilon\end{matrix}$	$\begin{matrix}\epsilon\\\epsilon^*\end{matrix}$	$\begin{matrix}\epsilon^{*2}\\\epsilon^2\end{matrix}$	$\begin{matrix}-1\\-1\end{matrix}$	$\begin{matrix}-\epsilon^2\\-\epsilon^{*2}\end{matrix}$	$\begin{matrix}-\epsilon^*\\-\epsilon\end{matrix}$	$\begin{matrix}-\epsilon\\-\epsilon^*\end{matrix}$	$\left.\begin{matrix}-\epsilon^{*2}\\-\epsilon^2\end{matrix}\right\}$		

C_{6h}	E	C_6	C_3	C_2	C_3^2	C_6^5	i	S_3^5	S_6^5	σ_h	S_6	S_3		ε=exp(i2π/6)
A_g	1	1	1	1	1	1	1	1	1	1	1	1	R_z	x^2+y^2, z^2
B_g	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1		
E_{1g}	$\left\{\begin{matrix}1\\1\end{matrix}\right.$	$\begin{matrix}\epsilon\\\epsilon^*\end{matrix}$	$\begin{matrix}-\epsilon^*\\-\epsilon\end{matrix}$	$\begin{matrix}-1\\-1\end{matrix}$	$\begin{matrix}-\epsilon\\-\epsilon^*\end{matrix}$	$\begin{matrix}\epsilon^*\\\epsilon\end{matrix}$	$\begin{matrix}1\\1\end{matrix}$	$\begin{matrix}\epsilon\\\epsilon^*\end{matrix}$	$\begin{matrix}-\epsilon^*\\-\epsilon\end{matrix}$	$\begin{matrix}-1\\-1\end{matrix}$	$\begin{matrix}-\epsilon\\-\epsilon^*\end{matrix}$	$\left.\begin{matrix}\epsilon^*\\\epsilon\end{matrix}\right\}$	(R_x, R_y)	(xz, yz)
E_{2g}	$\left\{\begin{matrix}1\\1\end{matrix}\right.$	$\begin{matrix}-\epsilon^*\\-\epsilon\end{matrix}$	$\begin{matrix}-\epsilon\\-\epsilon^*\end{matrix}$	$\begin{matrix}1\\1\end{matrix}$	$\begin{matrix}-\epsilon^*\\-\epsilon\end{matrix}$	$\begin{matrix}-\epsilon\\-\epsilon^*\end{matrix}$	$\begin{matrix}1\\1\end{matrix}$	$\begin{matrix}-\epsilon^*\\-\epsilon\end{matrix}$	$\begin{matrix}-\epsilon\\-\epsilon^*\end{matrix}$	$\begin{matrix}1\\1\end{matrix}$	$\begin{matrix}-\epsilon^*\\-\epsilon\end{matrix}$	$\left.\begin{matrix}-\epsilon\\-\epsilon^*\end{matrix}\right\}$		(x^2-y^2, xy)
A_u	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	z	
B_u	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1		
E_{1u}	$\left\{\begin{matrix}1\\1\end{matrix}\right.$	$\begin{matrix}\epsilon\\\epsilon^*\end{matrix}$	$\begin{matrix}-\epsilon^*\\-\epsilon\end{matrix}$	$\begin{matrix}-1\\-1\end{matrix}$	$\begin{matrix}-\epsilon\\-\epsilon^*\end{matrix}$	$\begin{matrix}\epsilon^*\\\epsilon\end{matrix}$	$\begin{matrix}-1\\-1\end{matrix}$	$\begin{matrix}-\epsilon\\-\epsilon^*\end{matrix}$	$\begin{matrix}\epsilon^*\\\epsilon\end{matrix}$	$\begin{matrix}1\\1\end{matrix}$	$\begin{matrix}\epsilon\\\epsilon^*\end{matrix}$	$\left.\begin{matrix}-\epsilon^*\\-\epsilon\end{matrix}\right\}$	(x, y)	
E_{2u}	$\left\{\begin{matrix}1\\1\end{matrix}\right.$	$\begin{matrix}-\epsilon^*\\-\epsilon\end{matrix}$	$\begin{matrix}-\epsilon\\-\epsilon^*\end{matrix}$	$\begin{matrix}1\\1\end{matrix}$	$\begin{matrix}-\epsilon^*\\-\epsilon\end{matrix}$	$\begin{matrix}-\epsilon\\-\epsilon^*\end{matrix}$	$\begin{matrix}-1\\-1\end{matrix}$	$\begin{matrix}\epsilon^*\\\epsilon\end{matrix}$	$\begin{matrix}\epsilon\\\epsilon^*\end{matrix}$	$\begin{matrix}-1\\-1\end{matrix}$	$\begin{matrix}\epsilon^*\\\epsilon\end{matrix}$	$\left.\begin{matrix}\epsilon\\\epsilon^*\end{matrix}\right\}$		

Dihedral D_n Groups

D_2	E	C_2(z)	C_2(y)	C_2(x)		
A	1	1	1	1		x^2, y^2, z^2
B_1	1	1	-1	-1	z, R_z	xy
B_2	1	-1	1	-1	y, R_y	zx
B_3	1	-1	-1	1	x, R_x	yz

D_3	E	$2C_3$	$3C_2$		
A_1	1	1	1		x^2+y^2, z^2
A_2	1	1	-1	z, R_z	
E	2	-1	0	(R_x, R_y), (x,y)	(x^2-y^2, xy) (xz, yz)

D_4	E	$2C_4$	$C_2(C_4^2)$	$2C_2$'	$2C_2$''		
A_1	1	1	1	1	1		x^2+y^2, z^2
A_2	1	1	1	-1	-1	z, R_z	
B_1	1	-1	1	1	-1		x^2-y^2
B_2	1	-1	1	-1	1		xy
E	2	0	-2	0	0	(R_x, R_y), (x,y)	(xz, yz)

D_5	E	$2C_5$	$2C_5^2$	$5C_2$		
A_1	1	1	1	1		x^2+y^2, z^2
A_2	1	1	1	-1	z, R_z	
E_1	2	$2\cos 72^\circ$	$2\cos 144^\circ$		$(R_x, R_y), (x,y)$	(xz, yz)
E_2	2	$2\cos 144^\circ$	$2\cos 72^\circ$			(x^2-y^2, xy)

D_6	E	$2C_6$	$2C_3$	C_2	$2C_2'$	$3C_2''$		
A_1	1	1	1	1	1	1		x^2+y^2, z^2
A_2	1	1	1	1	-1	-1	z, R_z	
B_1	1	-1	1	-1	1	-1		
B_2	1	-1	1	-1	-1	1		
E_1	2	1	-1	-2	0	0	$(R_x, R_y), (x,y)$	(xz, yz)
E_2	2	-1	-1	2	0	0		(x^2-y^2, xy)

Prismatic D_{nh} Groups

These groups are characterized by

i. an *n*-fold proper rotation axis C_n
ii. *n* 2-fold proper rotation axes C_2 normal to C_n
iii. a mirror plane σ_h normal to C_n and containing the C_2 axes.

D_{2h}	E	$C_2(z)$	$C_2(y)$	$C_2(x)$	i	$\sigma(xy)$	$\sigma(xz)$	$\sigma(yz)$		
A_g	1	1	1	1	1	1	1	1		x^2, y^2, z^2
B_{1g}	1	1	-1	-1	1	1	-1	-1	R_z	xy
B_{2g}	1	-1	1	-1	1	-1	1	-1	R_y	xz
B_{3g}	1	-1	-1	1	1	-1	-1	1	R_x	yz
A_u	1	1	1	1	-1	-1	-1	-1		
B_{1u}	1	1	-1	-1	-1	-1	1	1	z	
B_{2u}	1	-1	1	-1	-1	1	-1	1	y	
B_{3u}	1	-1	-1	1	-1	1	1	-1	x	

D_{3h}	E	$2C_3$	$3C_2$	σ_h	$2S_3$	$3\sigma_v$		
A_1'	1	1	1	1	1	1		x^2+y^2, z^2
A_2'	1	1	-1	1	1	-1	R_z	
E'	2	-1	0	2	-1	0	(x,y)	(x^2-y^2, xy)
A_1''	1	1	1	-1	-1	-1		
A_2''	1	1	-1	-1	-1	1	z	

E''	2	-1	0	-2	1	0	(R_x, R_y)	(xz, yz)

D_{4h}	E	$2C_4$	C_2	$2C_2'$	$2C_2''$	i	$2S_4$	σ_h	$2\sigma_v$	$2\sigma_d$		
A_{1g}	1	1	1	1	1	1	1	1	1	1		x^2+y^2, z^2
A_{2g}	1	1	1	-1	-1	1	1	1	-1	-1	R_z	
B_{1g}	1	-1	1	1	-1	1	-1	1	1	-1		x^2-y^2
B_{2g}	1	-1	1	-1	1	1	-1	1	-1	1		xy
E_g	2	0	-2	0	0	2	0	-2	0	0	(R_x, R_y)	(xz, yz)
A_{1u}	1	1	1	1	1	-1	-1	-1	-1	-1		
A_{2u}	1	1	1	-1	-1	-1	-1	-1	1	1	z	
B_{1u}	1	-1	1	1	-1	-1	1	-1	-1	1		
B_{2u}	1	-1	1	-1	1	-1	1	-1	1	-1		
E_u	2	0	-2	0	0	-2	0	2	0	0	(x,y)	

D_{5h}	E	$2C_5$	$2C_5^2$	$5C_2$	σ_h	$2S_5$	$2S_5^3$	$5\sigma_v$		
A_1'	1	1	1	1	1	1	1	1		x^2+y^2, z^2
A_2'	1	1	1	-1	1	1	1	-1	R_z	
E_1'	2	$2\cos 72^\circ$	$2\cos 144^\circ$	0	2	$2\cos 72^\circ$	$2\cos 144^\circ$		(x,y)	
E_2'	2	$2\cos 144^\circ$	$2\cos 72^\circ$	0	2	$2\cos 144^\circ$	$2\cos 72^\circ$			(x^2-y^2, xy)
A_1''	1	1	1	1	-1	-1	-1	-1		
A_2''	1	1	1	-1	-1	-1	-1	1	z	
E_1''	2	$2\cos 72^\circ$	$2\cos 144^\circ$	0	-2	$-2\cos 72^\circ$	$-2\cos 144^\circ$	0	(R_x, R_y)	(xz, yz)
E_2''	2	$2\cos 144^\circ$	$2\cos 72^\circ$	0	-2	$-2\cos 144^\circ$	$-2\cos 72^\circ$	0		

D_{6h}	E	$2C_6$	$2C_3$	C_2	$3C_2'$	$3C_2''$	i	$2S_3$	$2S_6$	σ_h	$3\sigma_d$	$3\sigma_v$		
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1		x^2+y^2, z^2
A_{2g}	1	1	1	1	-1	-1	1	1	1	1	-1	-1	R_z	
B_{1g}	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1		
B_{2g}	1	-1	1	-1	-1	1	1	-1	1	-1	-1	1		
E_{1g}	2	1	-1	-2	0	0	2	1	-1	-2	0	0	(R_x, R_y)	(xz, yz)
E_{2g}	2	-1	-1	2	0	0	2	-1	-1	2	0	0		(x^2-y^2, xy)
A_{1u}	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1		
A_{2u}	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	z	
B_{1u}	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1		

B_{2u}	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1		
E_{1u}	2	1	-1	-2	0	0	-2	-1	1	2	0	0	(x,y)	
E_{2u}	2	-1	-1	2	0	0	-2	1	1	-2	0	0		

D_{8h}	E	$2C_8$	$2C^3_8$	$2C_4$	C_2	$4C_2'$	$4C_2''$	i	$2S_8$	$2S^3_8$	$2S_4$	σ_h	$4\sigma_d$	$4\sigma_v$		
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1	1	1		x^2+y^2, z^2
A_{2g}	1	1	1	1	1	-1	-1	1	1	1	1	1	-1	-1	R_z	
B_{1g}	1	-1	-1	1	1	1	-1	1	-1	-1	1	1	1	-1		
B_{2g}	1	-1	-1	1	1	-1	1	1	-1	-1	1	1	-1	1		
E_{1g}	2	$-\sqrt{2}$	$\sqrt{2}$	0	-2	0	0	2	$-\sqrt{2}$	$\sqrt{2}$	0	-2	0	0	(R_x, R_y)	(xz, yz)
E_{2g}	2	0	0	-2	2	0	0	2	0	0	-2	2	0	0		(x^2-y^2, xy)
E_{3g}	2	$\sqrt{2}$	$-\sqrt{2}$	0	-2	0	0	2	$\sqrt{2}$	$-\sqrt{2}$	0	-2	0	0		
A_{1u}	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1		
A_{2u}	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	1	1	z	
B_{1u}	1	-1	-1	1	1	1	-1	-1	1	1	-1	-1	-1	-1	1	
B_{2u}	1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1		
E_{1u}	2	$-\sqrt{2}$	$\sqrt{2}$	0	-2	0	0	-2	$\sqrt{2}$	$-\sqrt{2}$	0	2	0	0	(x,y)	
E_{2u}	2	0	0	-2	2	0	0	-2	0	0	2	-2	0	0		
E_{1u}	2	$\sqrt{2}$	$-\sqrt{2}$	0	-2	0	0	-2	$-\sqrt{2}$	$\sqrt{2}$	0	2	0	0		

$D_{\infty h}$	E	$2C_\infty$	...	$\infty\sigma_v$	i	$2S_\infty$	...	∞C_2		
S_g^+	1	1	...	1	1	1	...	1		x^2+y^2, z^2
S_g^-	1	1	...	-1	1	1	...	-1	R_z	
π_g	2	$2\cos\phi$	...	0	2	$-2\cos\phi$	...	0	(R_x, R_y)	(xz, yz)
D_g	2	$2\cos 2\phi$	...	0	2	$2\cos 2\phi$	...	0		(x^2-y^2, xy)
...	...	...	...	......	...	...	...	...		
S_u^+	1	1	...	1	-1	-1	...	-1	z	
S_u^-	1	1	...	-1	-1	-1	...	1		
π_u	2	$2\cos\phi$	...	0	-2	$2\cos\phi$	...	0	(x, y)	
D_u	2	$2\cos 2\phi$	...	0	-2	$-2\cos 2\phi$	...	0		
...	...	...	...	...	...	...	...	...		

Antiprismatic D_{nd} Groups

These groups are characterized by

i. an n-fold proper rotation axis C_n
ii. n 2-fold proper rotation axes C_2 normal to C_n
iii. n mirror planes σ_d which contain C_n.

D_{2d}	E	$2S_4$	C_2	$2C_2'$	$2\sigma_d$		
A_1	1	1	1	1	1		x^2+y^2, z^2
A_2	1	1	1	-1	-1	R_z	
B_1	1	-1	1	1	-1		x^2-y^2
B_2	1	-1	1	-1	1	z	xy
E	2	0	-2	0	0	(x, y), (R_x, R_y)	(xz, yz)

D_{3d}	E	$2C_3$	$3C_2$	i	$2S_6$	$3\sigma_d$		
A_{1g}	1	1	1	1	1	1		x^2+y^2, z^2
A_{2g}	1	1	-1	1	1	-1	R_z	
E_g	2	-1	0	2	-1	0	(R_x, R_y)	(x^2-y^2, xy),(xz, yz)
A_{1u}	1	1	1	-1	-1	-1		
A_{2u}	1	1	-1	-1	-1	1	z	
E_u	2	-1	0	-2	1	0	(x, y)	

D_{4d}	E	$2S_8$	$2C_4$	$2S_8^3$	C_2	$4C_2'$	$4\sigma_d$		
A_1	1	1	1	1	1	1	1		x^2+y^2, z^2
A_2	1	1	1	1	1	-1	-1	R_z	
B_1	1	-1	1	-1	1	1	-1		
B_2	1	-1	1	-1	1	-1	1	z	
E_1	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	0	0	(x, y)	
E_2	2	0	-2	0	2	0	0		(x^2-y^2, xy)
E_3	2	$-\sqrt{2}$	0	$\sqrt{2}$	-2	0	0	(R_x, R_y)	(xz, yz)

D_{5d}	E	$2C_5$	$2C_5^2$	$5C_2$	i	$2S_{10}^3$	$2S_{10}$	$5\sigma_d$		
A_{1g}	1	1	1	1	1	1	1	1		x^2+y^2, z^2
A_{2g}	1	1	1	-1	1	1	1	-1	R_z	
E_{1g}	2	$2\cos 72^\circ$	$2\cos 144^\circ$	0	2	$2\cos 72^\circ$	$2\cos 144^\circ$	0	(R_x, R_y)	(xz, yz)
E_{2g}	2	$2\cos 144^\circ$	$2\cos 72^\circ$	0	2	$2\cos 144^\circ$	$2\cos 72^\circ$	0		(x^2-y^2, xy)
A_{1u}	1	1	1	1	-1	-1	-1	-1		

A_{2u}	1	1	1	-1	-1	1	-1	1	z	
E_{1u}	2	$2\cos 72^\circ$	$2\cos 144^\circ$	0	-2	$-2\cos 72^\circ$	$-2\cos 144^\circ$	0	(x, y)	
E_{2u}	2	$2\cos 144^\circ$	$2\cos 72^\circ$	0	-2	$-2\cos 144^\circ$	$-2\cos 72^\circ$	0		

D_{6d}	E	$2S_{12}$	$2C_6$	$2S_4$	$2C_3$	$2S_{12}^5$	C_2	$6C_2'$	$6\sigma_d$		
A_1	1	1	1	1	1	1	1	1	1		x^2+y^2, z^2
A_2	1	1	1	1	1	1	1	-1	-1	R_z	
B_1	1	-1	1	-1	1	-1	1	1	-1		
B_2	1	-1	1	-1	1	-1	1	-1	1	z	
E_1	2	$\sqrt{3}$	1	0	-1	$-\sqrt{3}$	-2	0	0	(x, y)	
E_2	2	1	-1	-2	-1	1	2	0	0		(x^2-y^2, xy)
E_3	2	0	-2	0	2	0	-2	0	0		
E_4	2	-1	-1	2	-1	-1	2	0	0		
E_5	2	$-\sqrt{3}$	1	0	-1	$\sqrt{3}$	-2	0	0	(R_x, R_y)	(xz, yz)

Improper Rotation S_n Groups

These groups are characterized by an n-fold improper rotation axis S_n, where n is necessarily even

S_4	E	S_4	C_2	S_4^3		
A	1	1	1	1	R_z	x^2+y^2, z^2
B	1	-1	1	-1	z	x^2-y^2, xy
E	$\left\{\begin{matrix}1\\1\end{matrix}\right.$	$\begin{matrix}i\\-i\end{matrix}$	$\begin{matrix}-1\\-1\end{matrix}$	$\left.\begin{matrix}-i\\i\end{matrix}\right\}$	(x, y); (R_x, R_y)	(xz, yz)

S_6	E	C_3	C_3^2	i	S_6^5	S_6		
A_g	1	1	1	1	1	1	R_z	x^2+y^2, z^2
E_g	$\left\{\begin{matrix}1\\1\end{matrix}\right.$	$\begin{matrix}\epsilon\\\epsilon^*\end{matrix}$	$\begin{matrix}\epsilon^*\\\epsilon\end{matrix}$	$\begin{matrix}1\\1\end{matrix}$	$\begin{matrix}\epsilon\\\epsilon^*\end{matrix}$	$\left.\begin{matrix}\epsilon^*\\\epsilon\end{matrix}\right\}$	(R_x, R_y)	(x^2-y^2, xy), (xz, yz)
A_u	1	1	1	-1	-1	-1	z	
E_u	$\left\{\begin{matrix}1\\1\end{matrix}\right.$	$\begin{matrix}\epsilon\\\epsilon^*\end{matrix}$	$\begin{matrix}\epsilon^*\\\epsilon\end{matrix}$	$\begin{matrix}-1\\-1\end{matrix}$	$\begin{matrix}-\epsilon\\-\epsilon^*\end{matrix}$	$\left.\begin{matrix}-\epsilon^*\\-\epsilon\end{matrix}\right\}$	(x, y)	

S_8	E	S_8	C_4	S_8^3	C_2	S_8^5	C_4^3	S_8^7		$\varepsilon=\exp(i2\pi/8)$
A	1	1	1	1	1	1	1	1	R_z	x^2+y^2, z^2
B	1	-1	1	-1	1	-1	1	-1	z	

E_1	1 1	ϵ ϵ^*	i $-i$	$-\epsilon^*$ $-\epsilon$	-1 -1	$-\epsilon$ $-\epsilon^*$	$-i$ i	ϵ^* ϵ	(R_x, R_y), (x, y)	
E_2	1 1	i $-i$	-1 -1	$-i$ i	1 1	i $-i$	-1 -1	$-i$ i		(x^2-y^2, xy)
E_3	1 1	$-\epsilon^*$ $-\epsilon$	$-i$ i	ϵ ϵ^*	-1 -1	ϵ^* ϵ	i $-i$	$-\epsilon$ $-\epsilon^*$		(xz, yz)

Cubic Groups

These polyhedral groups are characterized by not having a C_5 proper rotation axis.

T	E	$4C_3$	$4C_3^2$	$3C_2$		
A	1	1	1	1		$x^2+y^2+z^2$
E	1 1	ϵ ϵ^*	ϵ^* ϵ	1 1		$(2z^2-x^2-y^2, x^2-y^2)$
T	3	0	0		(R_x, R_y, R_z), (x, y, z)	(xz, yz, xy)

T_h	E	$4C_3$	$4C_3^2$	$3C_2$	i	$4S_6$	$4S_6^5$	$3\sigma_h$		ε=exp(i2π/3)
A_g	1	1	1	1	1	1	1	1		$x^2+y^2+z^2$
E_g	1 1	ϵ ϵ^*	ϵ^* ϵ	1 1	1 1	ϵ ϵ^*	ϵ^* ϵ	1 1		$(2z^2-x^2-y^2, x^2-y^2)$
T_g	3	0	0	-1	1	0	0	-1	(R_x, R_y, R_z)	(xz, yz, xy)
A_u	1	1	1	1	-1	-1	-1	-1		
E_u	1 1	ϵ ϵ^*	ϵ^* ϵ	1 1	-1 -1	$-\epsilon$ $-\epsilon^*$	$-\epsilon^*$ $-\epsilon$	-1 -1		
T_u	3	0	0	-1	-1	0	0	1	(x, y, z)	

T_d	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$		
A_1	1	1	1	1	1		$x^2+y^2+z^2$
A_2	1	1	1	-1	-1		
E	2	-1	2	0	0		$(2z^2-x^2-y^2, x^2-y^2)$
T_1	3	0	-1	1	-1	(R_x, R_y, R_z)	
T_2	3	0	-1	-1	1	(x, y, z)	(xz, yz, xy)

O	E	$8C_3$	$3C_2$	$6C_4$	$6C_2$		
A_1	1	1	1	1	1		$x^2+y^2+z^2$
A_2	1	1	1	-1	-1		
E	2	-1	2	0	0		$(2z^2-x^2-y^2, x^2-y^2)$

T_1	3	0	-1	1	-1	(R_x, R_y, R_z), (x, y, z)	
T_2	3	0	-1	-1	1		(xz, yz, xy)

O_h	E	$8C_2$	$6C_2$	$6C_4$	$3C_2(C_4^2)$	i	$6S_4$	$8S_6$	$3\sigma_h$	$6\sigma_d$		
A_{1g}	1	1	1	1	1	1	1	1	1	1		$x^2+y^2+z^2$
A_{2g}	1	1	-1	-1	1	1	-1	1	1	-1		
E_g	2	-1	0	0	2	2	0	-1	2	0		$(2z^2-x^2-y^2, x^2-y^2)$
T_{1g}	3	0	-1	1	-1	3	1	0	-1	-1	(R_x, R_y, R_z)	
T_{2g}	3	0	1	-1	-1	3	-1	0	-1	1		(xz, yz, xy)
A_{1u}	1	1	1	1	1	-1	-1	-1	-1	-1		
A_{2u}	1	1	-1	-1	1	-1	1	-1	-1	1		
E_u	2	-1	0	0	2	-2	0	1	-2	0		
T_{1u}	3	0	-1	1	-1	-3	-1	0	1	1	(x, y, z)	
T_{2u}	3	0	1	-1	-1	-3	1	0	1	-1		