FUNDAMENTALS OF ATMOSPHERIC SCIENCE

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Book: Fundamentals of Atmospheric Science (Brune)

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Licensing

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CHAPTER OVERVIEW

1: Getting Started

The atmosphere is amazing, awe-inspiring, frightening, deadly, powerful, boring, strange, beautiful, and uplifting – just a few of thousands of descriptions. So much of our lives depend on the atmosphere, yet we often take it for granted. Atmospheric science attempts to describe the atmosphere with physical descriptions using words, but also with mathematics. The goal is to be able to write down mathematical equations that capture the atmosphere’s important physical properties (predictability) and to use these equations to determine the atmosphere’s evolution with time (prediction). Predicting the weather has long been a primary focus, but, increasingly, we are interested in predicting climate.

1.1: The atmosphere is …

We know quite a lot about the atmosphere. It has taken decades, if not centuries, of careful observation and insightful theory that is based on solid physical and chemical laws. We have more to learn. You could help to advance the understanding of the atmosphere, but you must first understand the physical concepts and mathematics that are already well known. That is a primary purpose of this course – to give you that understanding.

Clouds over the Arctic Ocean at sunrise. Credit: W. Brune

What follows, below, is a series of pictures and graphical images. Each one depicts some atmospheric process that will be covered in this course. Look at these images; you will see them again, each in one of the next ten lessons. Of course, in each observation there are many processes going on simultaneously. In the last lesson, you will have the opportunity to look at an observation and attach the physical principles and the mathematics that describe several processes that are causing the phenomena that you are observing.

1.2: You will not believe what you can do with math!

You’ve been told many times that meteorology is a math-intensive field. It is. But for this course, you already know much of the math, and what you haven’t seen, you will see in vector calculus. To get ready for the meteorology and atmospheric science in this course, you will need to refresh your ability to solve simple math problems, including solving simple problems in differential and integral calculus. At the same time, we will remind you about the importance of correctly specifying significant figures and units in your answers to the problems. The goal of this first lesson is to boost your confidence in the math you already know.

How many figures should be in my answer?

Suppose you are asked to solve the following word problem:

In the radar loop, a squall line is oriented in the north-south direction and is heading northeast at 57 km hr^{-1}. In the last frame of the loop, the line is 17 km west of the Penn State campus. You are out running and know that you can make it back to your apartment in 25 minutes. Will you get back to your apartment before you get soaked?

You reason that the line is moving northeast, and thus, at an angle of 45°relative to the east. Therefore, the eastward motion of the squall line is just the velocity times the cosine of 45°. That gives you the eastward speed. You decide to divide the distance by the eastward speed to get the amount of time before the line hits campus. You plug the numbers into your calculator and get the following result:

$$\begin{aligned} \text{time} &= \frac{17\,\text{km}}{(57\,\text{km/h}) \cdot \cos(45^\circ)} \\ &= 0.42178\,\text{hours} \\ &= 25.3070\,\text{minutes} \end{aligned}$$

According to your calculation, you will make it back with 0.3 minutes (18 seconds) to spare. But can you really be sure that the squall line will strike in 25.3070 minutes? Maybe you should figure out how many significant figures your answer really has. To do that, you need to remember the rules:

Significant Figures Rules

1. Non-zero numbers (1,2,3,4,5,6,7,8,9) are ALWAYS significant.
2. Zeroes are ALWAYS significant:
    1. between non-zero numbers
    2. SIMULTANEOUSLY to the right of the decimal point AND at the end of the number
    3. to the left of a written decimal point and part of a number ≥ 10
3. In a calculation involving multiplication or division, multiply numbers as you see them. Then the answer should have the same number of significant figures as the number with the fewest significant figures.
4. In a calculation involving addition and subtraction, the number of significant figures in the answer depends on the number of significant figures to the right of the decimal point when all the added or subtracted numbers are put in terms of the same power-of-ten. Add or subtract all the numbers. The answer has the same number of significant figures as the number with the least number of significant figures to the right of the decimal point.
5. The number of significant figures is unchanged by trigonometric functions, logarithms, exponentiation, and other related functions.
6. Exact numbers never limit the number of significant figures in the result of a calculation and therefore can be considered to have an infinite number of significant figures. Common examples of exact numbers are whole numbers and conversion factors. For example, there are exactly 4 sides to a square and exactly 1000 m in a km.
7. For multi-step calculations, any intermediate results should keep at least one extra significant figure to prevent round-off error. Calculators and spreadsheets will typically keep these extra significant figures automatically.
8. When rounding, numbers ending with the last digit > 5 are rounded up; numbers ending with the last digit < 5 are rounded down; numbers ending in 5 are rounded up if the preceding digit is odd and down if it is even.

Examples

Number(s)	Answer	Number of Significant Figures	Reason
25+.3	25	2	25 has only 2 significant figures
25·0.325·0.3	8	1	25·0.3=7.525·0.3=7.5 , round to 8 because 0.3 has only 1 significant figure
$1.5\left(10^3\right)+3.24\left(10^2\right)$	$1.8(10^3)$	2	$1.5\left(10^3\right)+0.324\left(10^3\right)$ $=1.824\left(10^3\right)$,then drop 2 to get $1.8(10^3)$
$1.5\left(10^3\right)+3.86\left(10^2\right)$	$1.9(10^3)$	2	$1.5\left(10^3\right)+3.86\left(10^2\right)$ $=1.886\left(10^3\right)$,round up then drop 2 to get $1.9(10^3)$
$\frac{(57.3+6.41)}{15.6}$	4.08	3	$\frac{63.71}{15.6}=4.0840$, trim to 3 significant,figures to get 4.08
200(3.142)	600	1	200. has 3 significant figures; 200 (no decimal point) has 1 but is ambiguous
$152(e^{-.52})$	90	2	number in exponent has only 2 significant figures

Check out this video (11:23): Unit Conversions & Significant Figures for a brief (1 minute) explanation of those rules! Start watching at 9:14 for the most relevant information.

Unit Conversions and Significant Figures

Click Answer for transcript of the Significant Figures video.

Answer

Now to the magic of figuring out how many sig figs your answer should have. There are two simple rules for this. If it's addition or subtraction it's only the number of figures after the decimal point that matters. The number with the fewest figures after the decimal point decides how many figures you can have after the decimal in your answer. So 1,495.2+1.9903 you do the math. First you get 1,497.1903 and then you round to the first decimal, because that first number only had one figure after the decimal. So you get 1,497.2. And for multiplication, just make sure the answer has the same sig figs as your least precise measurement. So 60 x 5.0839 = 305.034, but we only know two sig figs so everything after those first two numbers is zeroes: 300. Of course then we'd have to point out to everyone that the second zero but not the third is significant so we'd write it out with scientific notation: 3.0 * 10^2. Because science! Now I know it feels counterintuitive

> not to show all of the numbers that you have at your fingertips, but you've got to realize: all of those numbers beyond the number of sig figs you have? They're lies. They're big lying numbers. You don't know those numbers. And if you write them down people will assume that you do know those numbers. And you will have lied to them. And do you know what we do with liars in chemistry? We kill them! Thank you for watching this episode of Crash Course Chemistry. Today you learned some keys to understanding the mathematics of chemistry, and you want to remember this episode in case you get caught up later down the road: How to convert between units is a skill that you'll use even when you're not doing chemistry. Scientific notation will always make you look like you know what you're talking about. Being able to chastise people for using the wrong number of significant digits is basically math's equivalent of being a grammar Nazi. So enjoy these new powers I have bestowed upon you, and we'll see you next time. Crash Course Chemistry was filmed, edited, and directed by Nick Jenkins. This episode was written by me, Michael Aranda is our sound designer, and our graphics team is Thought Bubble. If you have any questions, comments or ideas for us, we are always down in the comments. Thank you for watching Crash Course Chemistry.

Credit:* *Crash Course

What are the typical types of variables?

There are two types of variables – scalars and vectors. Scalars are amount only; vectors also have direction.

Dimensions and units are your friends.

Most variables have dimensions. The ones used in meteorology are:

- **L**, length
- **T**, time
- **Θ**, temperature
- **M**, mass
- **I**, electric current

Some constants such as π have no units, but most do.

The numbers associated with most variables have units. The system of units we will use is the International System (SI, from the French Système International), also known as the MKS (meter-kilogram-second) system, even though English units are used in some parts of meteorology.

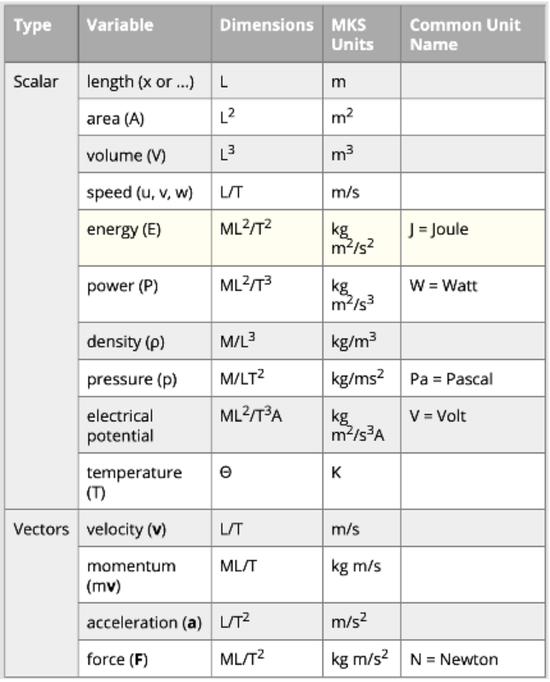
We will use the following temperature conversions:

$$K =^o C + 273.15$$

$$\left(\frac{5}{9}\right)(^oF - 32) =^o C$$

We will use the following variables frequently. Note the dimensions of the variables and the MKS units that go with their numbers.

Variables With Associated Dimensions and MKS Units



Type	Variable	Dimensions	MKS Units	Common Unit Name
Scalar	length (x or ...)	L	m	
	area (A)	L^2	m^2	
	volume (V)	L^3	m^3	
	speed (u, v, w)	L/T	m/s	
	energy (E)	ML^2/T^2	kg m^2/s^2	J = Joule
	power (P)	ML^2/T^3	kg m^2/s^3	W = Watt
	density (ρ)	M/L^3	kg/m^3	
	pressure (p)	M/LT^2	kg/ms^2	Pa = Pascal
	electrical potential	ML^2/T^3A	kg m^2/s^3A	V = Volt
	temperature (T)	Θ	K	
Vectors	velocity (**v**)	L/T	m/s	
	momentum (m**v**)	ML/T	kg m/s	
	acceleration (**a**)	L/T^2	m/s^2	
	force (**F**)	ML/T^2	kg m/s^2	N = Newton

Pressure is used for many applications.

$p=(\text{ normal force})/\text{area } = (\text{ mass } x \text{ acceleration })/\text{ area } = ML/T^2L^2 =$

$1\text{Pa} = 1\text{kgm}^{-1}\text{s}^{-2}; 1\text{hPa} = 100\text{Pa} = 1\text{mb} = 10^{-3}\text{bar}(\text{hPa} = \text{hectoPascal})$

$1013.25\text{hPa} = 1.01325 \times 10^5\text{Pa} = 1$ standard atmospheric pressure $=1$ atm

Wind speed is another frequently used variable.

The knot (kt) is equal to one nautical mile (approximately one minute of latitude) per hour or exactly 1.852 km/hr. The mile is nominally equal to 5280 ft and has been standardized to be exactly 1,609.344 m.

Thus, **1 m/s = 3.6 km/hr ≈ 1.944 kt** and **1 kt ≈ 1.151 mph**.

surface winds are typically 10 kts ~ 5 m/s

500 mb winds are ~50 kts ~ 25 m/s

250 mb winds are ~100 kts ~ 50 m/s

Temperature is a third frequently used variable.

Kelvin (K) must be used in all physical and dynamical meteorology calculations. **Surface temperature is reported in °F or (°C for METARS) and in °C for upper air soundings**.

Water vapor mixing ratio is another frequently used variable.

$$w=\frac{\text{mass } \mathrm{H_2O}}{\text{mass dry air}} \tag{1.2.1}$$

Usually the units for water vapor mixing ratio are $g\text{kg}^{-1}$. In the summer w can be 10 gkg^{-1} in the winter, it can be 1.2 gkg^{-1}

Dimensions truly are your friend. Let me give you an example. Suppose you have an equation **ax + b = cT**, and you know the dimension of **b**, **x**(a distance), and **T** (a temperature), but not **a** and **c**. You also know that each term in the equation – the two on the left-hand side and the one on the right-hand side must all have the same units. Therefore, if you know **b**, you know that the dimensions of **a** must be the same as the dimensions of **b** divided by **L** (length) and the dimensions of **c** must be the same as the dimensions of **b** divided by **Θ**.

Also, if you invert a messy equation and you're not sure that you didn't make a mistake, you can check the dimensions of the individual terms and if they don't match up, it's time to look for your mistake. Or, if you have variables multiplied or divided in an exponential or a logarithm, the resulting product must have no units.

Always write units down and always check dimensions if you aren’t sure. That way, you won’t crash your spacecraft on the back side of Mars. View the following video (2:42).

When NASA Lost a Spacecraft Because it Didn't Use Metric

Click Answer for transcript of the NASA video.

Answer

Remember when NASA lost a spacecraft because it's simultaneously used Imperial and metric measurements on the same mission? The Mars Climate Orbiter disappeared 15 years ago this month and here's a very brief recap of exactly what went wrong. The Mars Climate Orbiter launched on December 11, 1998 on a mission to orbit Mars. This first interplanetary weather satellite was designed to gather data on Mars' climate and also serve as a relay station for the Mars Polar Lander, a mission that launched a few weeks later. But you can't just launched a spacecraft towards Mars and trust that it's going to get where it's going. You to have to monitor its progress. Many spacecraft have reaction wheels to keep them oriented properly and navigation teams behind interplanetary spacecraft that constantly monitor the angular momentum and adjust trajectory to make sure it gets exactly where it needs to go. In the case of the Mars Climate Orbiter, monitoring its trajectory and angular momentum involved a few steps. First, data from the spacecraft was transferred to the ground by telemetry. There it was processed by a software program and stored in an angular momentum desaturation file that process data was what scientists used to adjust the trajectory. Adjustments that were made by firing the spacecraft's thrusters. Every time the thrusters were fired, the resulting change in velocity was measured twice once by software program on the spacecraft and once by software program off the ground. And here's where the problem comes in. It turned out that the two systems the processing software on the spacecraft and the software on the ground we're using two different units of measurements. The software on the spacecraft measured impulse, or the changes by thrusters in newton seconds a commonly accepted metric unit of measurement, while the processing software on the ground use the Imperial pound seconds. And it was unfortunately the ground computer's data that scientists used to update the spacecraft trajectory and because one pound of force is equal to 4.45 Newton's every adjustment was off by a factor of 4.45. For a spacecraft traveling tens of millions of miles to destination a number of seemingly small errors really add up. During the Mars Climate Orbiters nine-month cruise to Mars seven errors were introduced into its trajectory that meant that when it reached the red planet it was 105 miles closer to the Martian surface than expected. This turned out to be an unsurvivably low altitude for its Mars encounter when the spacecraft fire its main engine for the orbit insertion burn that was designed to put it into an elliptical orbit nothing happened. NASA lost contact quite abruptly with the spacecraft. So while we know the root cause of just what went wrong we'll never know exactly what happened to the Mars Climate Orbiter. The loss of the Mars Climate Orbiter very sadly happened in space. Leave your spacey questions and comments below, and don't forget to subscribe.

Credit: Scientific American Space Lab

Quiz 1-1: Significant figures, dimensions, and units.

Now it's time to to take a quiz. I *highly* recommend that you begin by taking the Practice Quiz before completing the graded Quiz. Practice Quizzes are *not* graded and *do not affect your grade* in any way (except to make you more competent and confident to take the graded Quizzes :).

1. In Canvas, find **Practice Quiz 1-1**. You may complete this practice quiz as many times as you want. It is not graded, but it allows you to check your level of preparedness before taking the graded quiz.
2. When you feel you are ready, take **Quiz 1-1**. You will be allowed to take this quiz only **once**. This quiz is timed, so after you start, you will have a limited amount of time to complete it and submit it. Good luck!

1.3: If you thought practice makes perfect, you could be right

Calculus is an integral part of a meteorologist's training. The ability to solve problems with calculus differentiates meteorologists from weather readers. You should know how to perform both indefinite and definite integrals. Brush up on the derivatives for variables raised to powers, logarithms, and exponentials. We will take many derivatives with respect to time and to distance.

Need Extra Practice?

Visit the Khan Academy website that explains calculus with lots of examples, practice problems, and videos. You can start with single variable calculus, but may find it useful for more complicated calculus problems.

Simple Integrals and Derivatives That are Frequently Used to Describe the Behavior of Atmospheric Phenomena

1. $\frac{da}{dt} = -ka$

$\frac{da}{a} = -kdt$

$\int_{a_o}^{a_1} \frac{da}{a} = -\int_{t_o}^{t_1} kdt$

$\ln(a_1) - \ln(a_0) = -k(t_1 - t_0)$

$\ln(a_1/a_0) = -k(t_1 - t_0)$

$a_1/a_0 = e^{(-k(t_1-t_0))} = \exp(-k(t_1 - t_0))$

$a_1 = a_0 e^{(-k(t_1-t_0))} = a_0 \exp(-k(t_1 - t_0))$

2. $p = p_o e^{(-z/H)} \quad ; \quad \int_0^\infty pdz = ?$ (Do the definite integral.)

$\int_0^\infty pdz = -Hp_o e^{-2IH}\big|_0^\infty = -Hp_o(0-1) = p_o H$

3. $p = p_0 e^{\left(-\frac{z}{H}\right)}; \frac{1}{p}\frac{dp}{dz} = ?$

$\frac{dp}{dz} = -\frac{1}{H} p_0 e^{\frac{-z}{H}} = -\frac{1}{H} p; \frac{1}{p}\frac{dp}{dz} = -\frac{1}{H}$

4. $\frac{d\ln(ax)}{dt} = ? \quad \frac{d\ln(ax)}{dt} = \frac{1}{ax}\frac{d(ax)}{dt} = \frac{1}{ax}\frac{adx}{dt} = \frac{1}{x}u$, where $u =$ velocity

5. $d(\cos(x)) = ? \quad d(\cos(x)) = -\sin(x)dx$

You have the power.

Often in meteorology and atmospheric science you will need to manipulate equations that have variables raised to powers. Sometimes, you will need to multiply variables at different powers together and then rearrange your answer to simplify it and make it more useful. In addition, it is very likely that you will need to invert an expression to solve for a variable. The following rules should remind you about powers of variables.

Laws of Exponents

$$a^x a^y = a^{x+y}$$
$$(ab)^x = a^x b^y$$
$$(a^x)^y = a^{xy}$$
$$a^{-x} = \frac{1}{a^x}$$
$$\frac{a^x}{a^y} = a^{x-y}$$
$$a^0 = 1$$

$$\left(\frac{a}{b}\right)^x = a^x \left(\frac{1}{b}\right)^x = \left(\frac{1}{a}\right)^{-x} b^{-x} = \left(\frac{b}{a}\right)^{-x}$$

If $a=b^{x}$, then raise both sides to the exponent $\frac{1}{x}$ to move the exponent to the other side: $a^{\frac{1}{x}}=\left(b^{x}\right)^{\frac{1}{x}}=b^{\frac{x}{x}}=b$

If $a^{x}b^{y}$, and you want to get an equation with a raised to no power, then raise both sides to the exponent $\frac{1}{x}$:

$$
\left(a^{x} b^{y}\right)^{\dfrac{1}{x}}=\left(a^{x}\right)^{\dfrac{1}{x}}\left(b^{y}\right)^{\dfrac{1}{x}}=a b^{\dfrac{y}{x}}=\text { new constant }

This brief video (7:42) sums up these important rules:

Rules of Exponents

Click Answer for transcript of the Rules of Exponents.

Answer

In this video we're going to be talking about all of the basic rules of exponents. And remember, when we're talking about exponents we can have an exponent here like X to the fourth where x is the base what we call the base and four is the exponent this small number in the upper right hand corner. It means that we're going to multiply X by itself four times or it means we have four factors of X multiplied together. So, if we expand this out its x times X times X times X. if we collapse it its X to the fourth. So, what happens when we do addition, subtraction, multiplication, and division of exponents? Well, in all cases we have to be really careful about like terms. For example, when we add terms that have exponents in them together both the bases and the exponents have to be the same in order for us to add them together. So, if we look at this first example 3x squared plus 2x squared the base here is X and the base here is X so the bases are the same which is good because we need that. and the exponents we have 2 and 2 which is good because we also need the exponents to be the same in order to add these together. So basically we have 3x squared added to 2x squared is going to give us five of them, 5x squared. So, if you're going to do addition and subtraction the bases and the exponents have to be the same. In this case we have X to the third plus x squared our bases X are the same but our exponents are different we have three and two. These are not like terms, so we can't add these together we can't simplify this at all. What happens when we do subtraction well again we're looking for similar basis so we have X and X for our base and then we have exponents of four and four. So because the bases and the exponents of the scene we can combine these like terms. We have six of them were subtracting and applied one of them which is going to leave us with five of them. So 5 times X to the fourth, but in this problem despite having the same base they will have a base of X we have different exponents we have a 4 and a 3 and because we're doing subtraction we can't combine these. We can't simplify this at all. What happens when we multiply two values together where exponents are involved? Well, here in order to simplify all we care about is that the bases are the same. The exponents do not have to be the same. So here we have base X and base X and we know already that's all we need to multiply these together it doesn't matter that the exponents are also the same we just add them. So we have three times to these are coefficients on our x squared terms. We multiply those together. So three times two is six, so that's going to be the first part and then we have x squared times x squared. And if we look at that x squared times x squared what we're going to do is add the exponents together. And the reason is because if we expand these out we know that x squared is two factors of X multiplied together. We're multiplying that by another x squared, so we're multiplying that by two more factors of X multiplied together. All together this is X to the fourth. Which we know because this essentially becomes the rule x to the a x x to the B is X to the a plus B. We just add the exponents together. So two plus two is four we get X to the fourth. Here's another example we have X to the third times x squared remember there's an implied one coefficient in front of both of these when we multiply 1 x 1 we get one so there will be a implied one coefficient on our final answer. x cubed + x squared. We just care that the bases are the same and they both have a base X so we know will be able to multiply them together. We have X to the third times x squared and remember that is going to be X to the three plus two so when we simplify we get X to the fifth and that should make sense because we have 3 factors of x x 2 factors of X adding them all up we get five factors of X so X to the fifth. The quotient rule for exponents tells us that in the same way as when we multiplied we didn't have to have the same exponent. When we divide we also don't have to have the same exponent we only care about the bases so here we have like basis. We have base X for both of these the exponents happened to be the same but that doesn't matter we're just going to leave this six and our final answer, so we'll get six here. And then what we're going to do is subtract the exponent in the denominator from the exponent in the numerator so the result is going to be X to the 4 minus 4. This is the four from the numerator this is the four from the denominator. 4-4 is 0 so we get 6 x 20 x to the 0 is 1 so this is 6 times 1 or just six. Even if we have different numbers again we only care about the bases both of these have the same base of X so again we'll just keep our two and our final answer and then we'll have X to the 4-3 because we say numerator exponent minus denominator exponent. That's going to give us 2 times X to the 4 minus 3 is 1. so X to the first which is of course just equal to 2x. What about a power raised to another power or an exponent raised to another exponent? Well, just like before in this example here when we said X to the fourth means multiply X by itself four times here we're saying multiply x squared by itself three times. So this is going to be equal to x squared times x squared times x squared and now we're really just back at this right here for multiplying like bases together and we add the exponents. So, this is just the same as X to the two plus two plus two. Two plus two plus two is six so we get x to the sixth power. What we realize then is that we can expand this and then add the exponents together using this rule over here or we can just multiply these two exponents together. Two times three gives us six and so we can do it that way as well. We can even do this when we have a negative base. So this problem here is telling us multiply 3 factors of negative x squared together so this is going to be negative x squared times negative x squared times negative x squared. We can deal with the negatives separately. Remember we can cancel every two negatives and they become a positive so negative and negative become a positive we're just left with this single negative sign here. so our answer will be negative and then x squared times x squared times x squared we know is X to the sixth. You can also think about it this way when you have this negative sign inside the parentheses. It's the same thing as saying negative 1 times x squared all raised to the third power and then you can apply this exponent to the negative 1 negative 1 times negative 1 times negative 1 is going to give you negative 1 which is this part right here. And then x squared to the third is going to be X to the 60 you get this X to the sixth and when you multiply them together you get negative x to the sixth. So those are just some of the most basic exponent rules that you need to know. ***Credit:*** ***Krista King***

Are you ready to give it a try? Solve the following problem on your own. After arriving at your own answer, click on the link to check your work. Here we go:

? Exercise

$x = ay^b$

What does y equal?

Answer

$$x^{1/b} = \left(ay^b\right)^{1/b} = a^{1/b}\left(y^b\right)^{1/b} = a^{1/b}y$$

$$y = x^{1/b}/a^{1/b} = \left(\frac{x}{a}\right)^{1/b}$$

Quiz 1-2: Solving integrals and differentials.

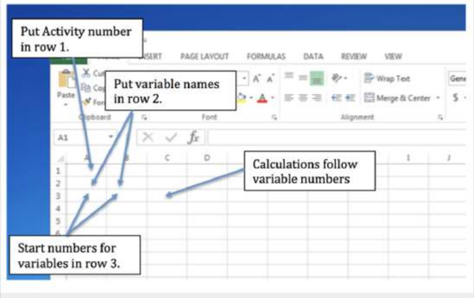
Now it's time to to take another quiz. Again, I *highly* recommend that you begin by taking the Practice Quiz before completing the graded Quiz, since it will make you more competent and confident to take the graded Quiz :).

1. Go to the Canvas and find **Practice Quiz 1-2**. You may complete this practice quiz as many times as you want. It is not graded, but it allows you to check your level of preparedness before taking the graded quiz.
2. When you feel you are ready, take **Quiz 1-2**. You will be allowed to take this quiz only **once**. This quiz is timed, so after you start, you will have a limited amount of time to complete it and submit it. Good luck!

1.4: Are you ready to get with the programming?

Meteorologists and atmospheric scientists spend much of their time thinking deep thoughts about the atmosphere, the weather, and weather forecasts. But to really figure out what is happening, they all have to dig into data, solve simple relationships they uncover, and develop new ways to look at the data. Much of this work is now done by programming a computer. Many of you haven’t done any computer programming yet, and for those of you who have, congratulations – put it to good use in this class. For those who are programming novices, we can introduce you to a few of the concepts of programming by getting you to use Excel or another similar spreadsheet program.

To help you learn and retain the concepts and skills that you will learn in this course, you will solve many word problems and simple math problems. For several activities, we give you the opportunity to practice solving particular types of problems enough times until you gain confidence that you can solve those same types of problems on a quiz. That means that you will be solving some types of problems several times and only the numbers for the variables will change. The simplest way for you to solve these problems is to program a spreadsheet to do that repetitive math for you.



Spreadsheet Screenshot

Click for a text description of the spreadsheet screenshot.

Screenshot shows an Excel Spreadsheet

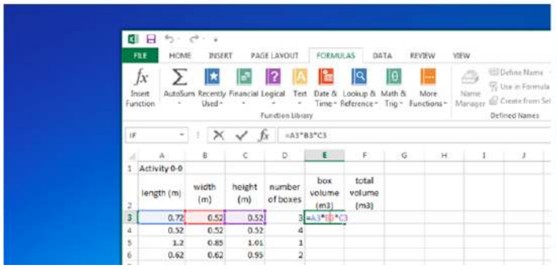
A text box says "put activity number in row 1" and an arrow points to cell A1.

A second text box says "put variable names in row 2" with an arrow pointing to cells A2 and B2

A third text box says "start numbers for variables in row 3" with an arrow pointing to cells A3 and B3

A final text box says "calculations follow variable numbers" with an arrow pointing to C3.

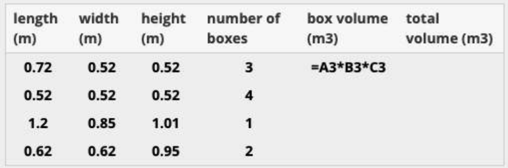
Let’s do a simple example. Suppose we have several boxes, some with different shapes and sizes, and we want to calculate the volume of the boxes and find the total volume. I have put in the names of the variables (with units!) and then the numbers for the length, width, and height of each box type and the total number of each box. To calculate the volume of each box, click on E3 and put an “= a3*b3*c3” in the equation line. Hit enter and it will do the calculation and put the answer in E3. A small square will appear in the lower right corner of E3. Click on this square with the mouse and pull down over the next three rows. Excel will automatically do the calculations for those rows. To calculate the total volume, go to F3 and enter “=d3*e3,” and hit “enter.” Grab the small box and pull down to get the total volume of each type of box. To get the total volume, click on F7, click on “Formulas,” and then “AutoSum,” and finally “Sum.” Excel will show you which cells it intends to sum. You can change this by adjusting the edges of the box it shows.



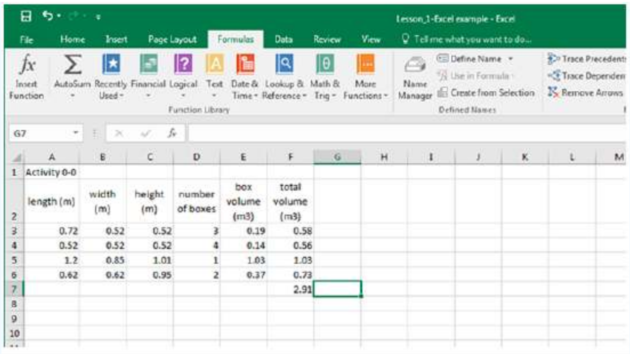
=A3*B3*C3

	A	B	C	D	E	F
1	Activity 0-0					
2	length (m)	width (m)	height (m)	number of boxes	box volume (m3)	total volume (m3)
3	0.72	0.52	0.52	3	=A3*B3*C3	
4	0.52	0.52	0.52	4		
5	1.2	0.85	1.01	1		
6	0.62	0.62	0.95	2		

Click for a text description of the spreadsheet example part 1.



length (m)	width (m)	height (m)	number of boxes	box volume (m3)	total volume (m3)
0.72	0.52	0.52	3	=A3*B3*C3	
0.52	0.52	0.52	4		
1.2	0.85	1.01	1		
0.62	0.62	0.95	2		



	A	B	C	D	E	F	G
1	Activity 0-0						
2	length (m)	width (m)	height (m)	number of boxes	box volume (m3)	total volume (m3)	
3	0.72	0.52	0.52	3	0.19	0.58	
4	0.52	0.52	0.52	4	0.14	0.56	
5	1.2	0.85	1.01	1	1.03	1.03	
6	0.62	0.62	0.95	2	0.37	0.73	
7						2.91	

Click for a text description of the spreadsheet example part 2.

length (m)	width (m)	height (m)	number of boxes	box volume (m3)	total volume (m3)
0.72	0.52	0.52	3	0.19	0.58
0.52	0.52	0.52	4	0.14	0.56
1.2	0.85	1.01	1	1.03	1.03
0.62	0.62	0.95	2	0.37	0.73

Hopefully this example is a refresher for most of you. For those who are totally unfamiliar with Excel, please click on the question mark in the upper right of the screen and type in the box “creating your first workbook.” You can also visit Microsoft's help page for additional step-by-step instructions for how to Use Excel as Your Calculator. The best way to learn, after the introduction, is by doing. The Keynote Support website also lists helpful summaries of instructions.

Activity 1-3: Setting up your Meteo 300 Excel workbook.

Please follow the instructions above for setting up an Excel workbook. You will be using this workbook to do calculations, plot graphs, and answer questions on quizzes and problems for the rest of the course.

This assignment is worth 15 points. Your grade will mostly depend upon showing that you set up the workbook, but some additional points will be assigned contingent upon how well you follow the instructions. When your Excel workbook is complete, please do the following:

1. Make sure that the file for your workbook follows this naming convention: **Workbook_your last name (i.e., Smith)_your first name_(i.e., Eileen).xlsx**. So mine would be Workbook_Brune_William.xlsx
2. In Canvas, find **Activity 1-3: Setting up your Meteo 300 Excel workbook**. Upload your Excel workbook there.

1.5: Summary and Final Tasks

Summary

There is a very good reason that you are taking this class and I am teaching it – all of us are fascinated by the weather, awed by the atmosphere's power, and passionate about learning more about it. Quite honestly, I can't imagine a more rewarding career than the one that you are embarking upon or the one that I have. Nothing could be more rewarding than saving lives by making the atmosphere more predictable or by making the perfect prediction. Nothing.

But, do you know what? The best forecasters are the ones who can not only read weather maps, but who also know physically what the atmosphere is doing. The best forecasters know how to translate the physics into mathematics so that hand-waving can be turned into usable numbers. This course will start to make all of these connections between observations and physical cause-and-effect and help us find numerical solutions to questions.

For those of you who are in related dsiciplines, this course will give you a solid basic understanding of the atmosphere that you can apply in your studies and career, whether it be civil engineeering, mechanical engineering, environmental engineering, chemistry, hydrology, or many other fields.

We have now reviewed some important concepts like significant figures and dimensions and units. You will continue to gain confidence in using the differential and integral calculus that you already know. As you go through the course, I want you to look back at the pictures of the atmosphere and imagine which equations are governing the processes that are causing your observations.

Reminder - Complete all of the Lesson 1 tasks!

You have reached the end of Lesson 1! Make sure that you have completed all of the tasks in Canvas.

CHAPTER OVERVIEW

2: Thermodynamics

2.1: Gas Laws

Understanding atmospheric thermodynamics begins with the gas laws that you learned in chemistry. Because these laws are so important, we will review them again here and put them in forms that are particularly useful for atmospheric science. **You will want to memorize these laws** because they will be used again and again in many other areas of atmospheric science, including cloud physics, atmospheric structure, dynamics, radiation, boundary layer, and even forecasting.

A constant pressure balloon stays aloft for weeks at an altitude of 100,000 ft so that the instruments in the attached gondola can make long-term measurements. Credit: National Scientific Balloon Facility, Palestine TX

Looking Ahead

Before you begin this lesson's reading, I would like to remind you of the discussion activity for this lesson. This week's discussion activity will ask you to take what you learn throughout the lesson to answer an atmospheric problem. You will not need to post your discussion response until you have read the whole lesson, but keep the question in mind as you read:

This week's topic is a hypothetical question involving stability. The troposphere always has a capping temperature inversion—it's called the stratosphere. The tropopause is about 16 km high in the tropics and lowers to about 10 km at high latitudes. The stratosphere exists because solar ultraviolet light makes ozone and then a few percent of the solar radiation is absorbed by stratospheric ozone, heating the air and causing the inversion. Suppose that there was no ozone layer and hence no stratosphere caused by solar UV heating of ozone.

Would storms in the troposphere be different if there was no stratosphere to act like a capping inversion? And if so, how?

You will use what you have learned in this lesson about the atmosphere's pressure structure and stability to help you to think about this problem and to formulate your answer and discussions. So, think about this question as you read through the lesson. You'll have a chance to submit your response in 2.6!

Ideal Gas Law

The atmosphere is a mixture of gases that can be compressed or expanded in a way that obeys the Ideal Gas Law:

$$pV = NR^{*}T \tag{2.1.1}$$

where p is pressure $\left(Pa = kgm^{-1}s^{-2}\right)$, V is the volume (m^3), N is the number of moles, R^* is the gas constant $(8.314)\mathrm{K}^{-1}\mathrm{mole}^{-1})$, and T is the temperature (K). Note also that both sides of the Ideal Gas Law equation have the dimension of energy $(J = kgm^2s^{-2})$.

Recall that a mole is 6.02×10^{23} molecules (Avagodro's Number). Equation 2.1 is a form of the ideal gas law that is independent of the type of molecule or mixture of molecules. A mole is a mole no matter its type. The video below (6:17) provides a brief review of the Ideal Gas Law. Note that the notation in the video differs slightly from our notation by using n for N, P for p, and R for R^*.

Ideal Gas Law Introducti [illegible]

Click here for transcript [illegible]

So here I have a tank [illegible] would be in this tank. The arrows I put in here b [illegible] unch of hyperactive little kids, running into eac [illegible] we've got this tank of gas. Let's think about the [illegible] could do is we could say what its temperature [illegible] are moving around, so temperature is very i [illegible] eported in Kelvin. So we could say, for exampl [illegible] as particles in the sample are. When you talk about gas, another important characteristic is pressure. How hard are these gas particles bouncing against the sides of the tank? How much pressure are they exerting on them? And we could measure these with a pressure gauge or something like that on the top of this tank. We could say, the pressure for this is 3.18 atm. That might be a pressure. And another thing that we spend a lot of time talking about when it comes to gas is volume. And again, I have these letters here that are how each one of these things are abbreviated. Volume, V, volume of this tank might be something like 95.2 liters. And finally, look at these particles that I've drawn. There is a certain amount of gas that's in here. And the amount of gas, which is abbreviated by the little letter n, is usually reported in moles, which is a convenient measure of how much of something we have. So we could say that the amount of gas in this tank is 7.5 moles. Now, whenever we have a sample of gas like this, if it's a tank or it's in a balloon or wherever it is, we can describe-- we can give it these various characteristics. And it turns out that also, for any sample of gas, if we know three of these characteristics, we can figure out what the fourth is. All we need to do is know three. And in order to do that, we use an equation that's a representation of the Ideal Gas Law. And it's written as P times V, pressure times volume, equals n, the amount of gas, times R times T, temperature. I'll get to R in a second. Don't worry about it for right now. It's going to be a number that we know. So let's say, for example, that we didn't know what pressure was, but we still knew the temperature, volume, and the amount of gas. No big deal. We could take the equation, PV equals nRT, and rearrange it. Divide both sides by V. Get rid of the V. And then we'd have P equals nRT divided by V. Plug these values in, and we could figure out what the pressure was. Or let's say that we knew what the pressure was of a particular gas sample. We know what the temperature was in a volume. But we didn't know what the amount of gas was. We don't know how much we had. We could figure out that fourth characteristic by rearranging the Ideal Gas Law for n, canceling out R and T on one side, rearranging it to solve for n. And then we could plug in the pressure, the volume, and the temperature, and we could figure out the amount of gas. So in other words, if we know three of these characteristics, we can always figure out what the fourth is. So you may be asking yourself, so R-- what's R? R is what we call a constant. It's a number that we know ahead of time that doesn't depend on the variables in our problem. The R that I'm going to be using most of the time for the videos is 0.0821 liters times atm divided by Kelvin times moles. Now notice that this is a fraction. It has both a top and a bottom. And it also is not just a number, but it has units. And check this out-- the units on R match the units in my problem. They match the characteristics that I'd be using. So I have liters here, liters here, atm, atm, Kelvin, Kelvin, and moles, moles. You always want the units on R to match the units of the characteristics in your Ideal Gas Problem. So because you always want the units to match, there are also different values of R, although I'm going to be using this mostly for the videos I'm doing. For example, let's say that instead of atm, I was using a pressure that was in millimeters of mercury. In this case, I wouldn't want to use this R here. I'd want to use this R here, so that the units match-- millimeters of mercury here, millimeters of mercury here, and the number's different-- 62.4. So again, that's what I use here. Let's say that instead of millimeters of mercury, my pressure was given to me in kPa. I would then use this value of R so that the units match. I've got kPa here, kPa here, and all the others are the same, so 8.31 for that. Now as I keep saying, in most of the videos that I'm going to be doing, I'm going to be using this top R with atm. But you may be asked by your teacher to use a different R. It's no big deal. That's probably just because they're giving you problems that have different pressure units, and they want the pressure units to match. So don't worry at all if you're using one of these other R's. Setting up and solving the Ideal Gas Law is exactly the same. No matter which of these R's you use, it's just a matter of plugging a different R in at the very end. So no matter which one you're using, you should be able to follow all these lessons, and it should all make sense.

Credit: ***Tyler DeWitt***

Usually in the atmosphere we do not know the exact volume of an air parcel or air mass. To solve this problem, we can rewrite the Ideal Gas Law in a different useful form if we divide N by V and then multiply by the average mass per mole of air to get the **mass density**:

$$\rho = \frac{NM}{V} \tag{2.1.2}$$

where M is the molar mass (kg mol^{-1}). Density has SI units of kg m^{-3}. The Greek symbol ρ (rho) is used for density and should not be confused with the symbol for pressure, p.

Thus we can put density in the Ideal Gas Law:

$$p = \frac{\rho R^* T}{M} \tag{2.1.3}$$

or

$$\rho = \frac{Mp}{R^* T} \tag{2.1.4}$$

Density is an incredibly important quantity in meteorology. Air that is more dense than its surroundings (often called its environment) sinks, while air that is less dense than its surroundings rises. Note that density depends on temperature, pressure, and the average molar mass of the air parcel. The average molar mass depends on the atmospheric composition and is just the sum of the fraction of each type of molecule times the molar mass of each molecular constituent:

$$M_{\text{arenge}} = \frac{\sum_i N_i M_i}{\sum_i N_i} = \frac{\sum_i N_i M_i}{-N} = \sum_i \frac{N_i}{N} M_i = \sum_i f_i M_i \tag{2.1.5}$$

where the i subscript represents atmospheric components, N is the number of moles, and M is the molar mass. This video (3:19) shows you how to find the gas density using the Ideal Gas Law. You will note that the person uses pressure in kPa and molar mass in g/mol. Since kPa = 1000 Pa and g = 1/1000 kg, the two factors of 1000 cancel when he multiplies them together and he can get away with using these units. I recommend always converting to SI units to avoid confusion. Also, note that the symbol for density used in the video is d, which is different from what we have used (ρ, the convention in atmospheric science).

Find the Density of a Gas

Click here for transcript of the Find the Density of a Gas video.

> Hey guys how do you solve ideal gas law questions involving density? The key is to have a formula or know how to derive the formula on your own. Remember density is mass over volume. Now, the way that mass is found in the ideal gas law equation is in n because the number of moles is the same as mass over molar mass. So, check this set. I'm going to replace n

with mass over molar mass, and then I'm going to rearrange for m over v. I'm going to undo division by molar mass on the other side and then I'm going to undo multiplication by RT and bring my V over. Here's a what I mean. P times the molar mass divided by RT gives me mass over volume. Mass over volume is density and so my equation is density equals pressure times molar mass divided by RT. We can now use this equation to find the density of oxygen at 55 Celsius and a hundred and three kilopascals. So let's do it. The density is pressure that's 103 kilopascals times molar mass for oxygen. That's 32 grams per mole. R, now, I'm going to put my volume in liters and I'm going to put my pressure in kilopascals which means the relevant are that I want is 8.314 liters, kilopascals per mole Kelvin and my temperature in Kelvin is the temperature in Celsius plus 273, which gives me 328 Kelvin. And all of these units should cancel out to give me a density unit. Kelvin cancels of Kelvin per moles cancel / moles kilopascals canceled the scale pascals and left with grams per liter. Let's do this on the calculator 103 times 32 divided 8.314 divided 328. That's 1.21 grams per liter. That may not seem like a lot, but remember you're dealing with the gas here. If you have a 1-liter balloon how much is it actually going to weigh? Probably the amount of the rubber plus like a gram or so. This here is the density of oxygen gas at 55 and 103 kilopascals. This is your density formula in terms of the ideal gas law. Be able to use it. Best of luck!

Credit: ChemistNate

Solve the following problem on your own. After arriving at your own answer, click on the link to check your work.

Example

Let’s calculate the density of dry air where you live. We will use the Ideal Gas Law and account for the three most abundant gases in the atmosphere: nitrogen, oxygen, and argon. *M* is the molar \text { mass of air; } M=0.029 \mathrm{kg} \mathrm{mol}^{-1} \text { , which is just an average accounting for the fractions of different } gases:

$$M = 0.78M_{N2} + 0.21M_{O2} + 0.01M_{Ar}$$
$$= 0.78 \cdot 0.028 + 0.21 \cdot .032 + 0.01 \cdot 0.040 = 0.029\text{kgmol}^{-1}$$

$R^* = 8.314\text{JK}^{-1}\text{mol}^{-1}$. Here, $p = 960\text{hPa} = 9.6 \times 10^4\text{Pa}$ and $T = 20^\circ\text{C} = 293\text{K}$

Click for answer.

Putting these values into the equation 2.3, we get that the dry air density is 1.1kgm^{-3}.

What would the density be if the room were filled with helium and not dry air at the same pressure and temperature?

Click for answer.

Helium density = (pressure times molar mass of helium)/(Ideal Gas Law constant in SI units times temperature in K) = 0.16 kg m^{-3}

Dry Air

Often in meteorology we use mass-specific gas laws so that we must specify the gas that we are talking about, usually only dry air (N_2 + O_2 + Ar + CO_2 +…) or water vapor (gaseous H_2O). We can divide R^* by M_i to get a mass-specific gas constant, such as $R_d = R^*/M_{dry\ air}$.

Thus, we will use the following form of the Ideal Gas Law **for dry air**:

$$p_d = \rho_d R_d T \tag{2.1.6}$$

where:

$$R_d = \frac{R^*}{M_{d\gamma air}} = \frac{8.314\text{K}^{-1}\text{mol}^{-1}}{0.02897\text{kgmol}^{-1}} = 287\text{m}^2\text{s}^{-2}\text{K}^{-1} = 287\text{Jkg}^{-1}\text{K}^{-1}$$

$M_{dry\ air}$ is 0.02897 kg mol^{-1}, which is the average of the molar masses of the gases in a dry atmosphere computed to four significant figures.

Note that p must be in Pascals (Pa), which is 1/100th of a mb (a.k.a, hPa), and T must be in Kelvin (K).

Water Vapor

We can do the same procedure **for water vapor**:

$$p_v \equiv e = \rho_v R_v T \tag{2.1.7}$$

where

$$R_v = \frac{R^*}{M_{\text{watenapor}}} = \frac{8.314\text{K}^{-1}\text{mol}^{-1}}{0.01802\text{kgmol}^{-1}} = 461\text{m}^2\text{s}^{-2}\text{K}^{-1} = 461\text{Jkg}^{-1}\text{K}^{-1} \tag{2.1.8}$$

Typically e is used to denote the water vapor pressure, which is also called the water vapor partial pressure.

Dalton’s Law

John Dalton. *Frontispiece of John Dalton and the Rise of Modern Chemistry by Henry Roscoe. Licensed under* *Public Domain via Wikimedia Commons*

This gas law is used often in meteorology. Applied to the atmosphere, it says that the total pressure is the sum of the partial pressures for dry air and water vapor:

$$p = p_d + p_{H2O} = p_d + e \tag{2.1.9}$$

Imagine that we put moist air and an absorbent in a jar and screw the lid on the jar. If we keep the temperature constant as the absorbent pulls water vapor out of the air, the pressure inside the jar will drop to p_d. Always keep in mind that when we measure pressure in the atmosphere, we are measuring the ***total*** pressure, which includes the partial pressures of dry air and water vapor.

So it follows that the **density of dry air and water vapor** also add:

$$\rho = \rho_d + \rho_v \tag{2.1.10}$$

Solve the following problem on your own. After arriving at your own answer, click on the link to check your work.

Exercise

Suppose we have two air parcels that are the same size and have the same pressure and temperature, but one is dry and the other is moist air. Which one is less dense?

Click for answer.

We can solve this one without knowing the pressure, temperature, or volume. Let’s assume that 98% of the molecules are dry air, which means the remaining 2% are dry air in the first case and water vapor in second case. Dry air is 0.029 kg mol^{-1} and water vapor is 0.018 kg mol^{-1}, so 2% of the moist air is lighter than the 2% of dry air, and when we consider the total air, this means that *for the same temperature and pressure, moist air is always less dense than dry air*.

Virtual Temperature

Suppose there are two air parcels with different temperatures and water vapor amounts but the same pressure. Which one has a lower density? We can calculate the density to determine which one is lighter, but there is another way to do this comparison. Virtual temperature, T_v, is defined as the temperature dry air must have so that its density equals that of ambient moist air. Thus, virtual temperature is a property of the ambient moist air. Because the air density depends on the amount of moisture (for the same pressure and temperature), we have a hard time determining if the air parcel is more or less dense relative to its surroundings, which may have a different temperature and amount of water vapor. It is useful to pretend that the moist parcel is a dry parcel and to account for the difference in density by determining the temperature that the dry parcel would need to have in order to have the same density as the moist air parcel.

We can define the amount of moisture in the air by a quantity called **specific humidity, *q***:

$$q = \frac{\rho_v}{\rho_d + \rho_v} \tag{2.1.11}$$

We see that q is just the fraction of water vapor density relative to the total moist air density. Usually q is given in units of g of water vapor per kg of dry air, or g kg^{-1}.

Using the Ideal Gas Law and Dalton's Law, we can derive the **equation for virtual temperature**:

$$T_v = T[1 + 0.61q] \tag{2.1.12}$$

where T and T_v have units of Kelvin (not ^oC and certainly not ^oF!) and q must be unitless (e.g., kg kg^{-1}).

Note that moist air always has a higher virtual temperature than dry air that has the same temperature as the moist air because, as noted above, moist air is always less dense than dry air for the same temperature and pressure. Note also that for dry air, $q = 0$ and the virtual temperature is the same as the temperature.

Solve the following problem on your own. After arriving at your own answer, click on the link to check your work.

? Exercise

Consider a blob of air ($T_{blob} = 25$ ^oC, $q_{blob} = 10$ g kg^{-1}) at the same pressure level as a surrounding environment ($T_{env} = 26$ ^oC and $q_{env} = 1$ g kg^{-1}). If the blob has a lower density than its environment, then it will rise. Does it rise?

Click for an answer.

We will use equation (2.10). Remember to convert T from ^oC to K and q from g kg^{-1} to kg kg^{-1}!

$$T_{vblob} = (25 + 273)[1 + 0.61 \cdot .010] = 299.8K = 26.8^\circ C$$
$$T_{venv} = (26 + 273)[1 + 0.61 \cdot .001] = 299.2K = 26.2^\circ C$$

We see that the blob is less dense than its environment and so will rise. This difference of 0.6 ^oC may seem small, but it makes a huge difference in upward motion.

The following are some mistakes that are commonly made in the above calculations:

- not converting from ^oC to K:

$$T_{vblob} = (25)[1 + 0.61 \cdot .010] = 25.15^\circ\text{C} \tag{2.1.13}$$

$$T_{venv} = (26)[1 + 0.61 \cdot .001] = 26.02^\circ\text{C} \tag{2.1.14}$$

We calculate that $T_{vblob} < T_{venv}$, which is the wrong answer.

- not converting q from g/kg to kg/kg:

$$T_{vblob} = (25 + 273)[1 + 0.61 \cdot 10] = 2115K = 1842^\circ C \tag{2.1.15}$$

$$T_{venv} = (26 + 273)[1 + 0.61 \cdot 1] = 481K = 208^\circ C \tag{2.1.16}$$

We calculate that $T_{vblob} > T_{venv}$, which is correct in this case, but the numbers are crazy! After you complete your calculations, if the numbers you get just don't seem right—like these—then you know that you have made a mistake in the calculation. Go looking for the mistake. Don't submit an answer that makes no sense.

Once we find T_v, we can easily find the density of a moist parcel by using equation [2.5], in which we substitute T_v for T. Thus,

$$\rho_d = \frac{p_d}{R_d T_v} \tag{2.1.17}$$

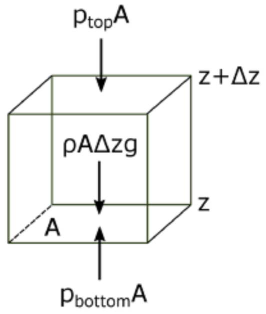
Quiz 2-1: What will that air parcel do?

This quiz will give you practice calculating the virtual temperature and density using the Excel workbook that you set up in the last lesson.

1. Go to Canvas and find **Practice Quiz 2-1**. You may complete this practice quiz as many times as you want. It is not graded, but it allows you to check your level of preparedness before taking the graded quiz. ***I strongly suggest that you enter the equations for density and for virtual temperature in your Excel worksheet and use them to do all your calculations of density and virtual temperature on both the practice quiz and the quiz.***
2. When you feel you are ready, take **Quiz 2-1**. You will be allowed to take this quiz only **once**. This quiz is timed, so after you start, you will have a limited amount of time to complete it and submit it. Good luck!

2.2: The Atmosphere’s Pressure Structure - Hydrostatic Equilibrium

The atmosphere’s vertical pressure structure plays a critical role in weather and climate. We all know that pressure decreases with height, but do you know why?



Air parcel at rest with three forces in balance

The atmosphere’s basic pressure structure is determined by the hydrostatic balance of forces. To a good approximation, every air parcel is acted on by three forces that are in balance, leading to no net force. Since they are in balance for any air parcel, the air can be assumed to be static or moving at a constant velocity.

There are **3 forces that determine hydrostatic balance:**

1. One force is downwards (negative) onto the top of the cuboid from the pressure, *p*, of the fluid above it. It is, from the definition of pressure,

$$F_{top} = -p_{top}A \tag{2.2.1}$$

2. Similarly, the force on the volume element from the pressure of the fluid below pushing upwards (positive) is:

$$F_{\text{bottom}} = p_{\text{bottom}}A \tag{2.2.2}$$

3. Finally, the weight of the volume element causes a force downwards. If the density is ρ, the volume is *V*, which is simply the horizontal area *A*times the vertical height, *Δz*, and *g* the standard gravity, then:

$$F_{\text{weight}} = -\rho V g = -\rho g A\Delta z \tag{2.2.3}$$

By balancing these forces, the **total force on the fluid** is:

$$\sum F = F_{bottom} + F_{top} + F_{weight} = p_{bottom}A - p_{top}A - \rho \text{gA}\Delta z \tag{2.2.4}$$

This sum equals zero if the air's velocity is constant or zero. Dividing by *A*,

$$0 = p_{bottom} - p_{top} - \rho g\Delta z \tag{2.2.5}$$

or:

$$p_{top} - p_{bottom} = -\rho g\Delta z \tag{2.2.6}$$

$P_{top} - P_{bottom}$ is a change in pressure, and Δz is the height of the volume element – a change in the distance above the ground. By saying these changes are infinitesimally small, the equation can be written in differential form, where *dp* is top pressure minus bottom pressure just as dz is top altitude minus bottom altitude.

$$dp = -\rho g dz \tag{2.2.7}$$

The result is the equation:

$$\frac{dp}{dz} = -\rho g \tag{2.2.8}$$

This equation is called the **Hydrostatic Equation**. See the video below (1:18) for further explanation:

Click here for transcrip

Consider an air parcel ich is pressure times area
in the parcel's top, an of gravity actually on the
parcel's mass, which i lume. The volume equals
the parcel's cross sect them equal to 0 since the
parcel's at rest. Notice pressure difference on the
left hand side. And t s the pressure difference
infinitesimally small. ith an equation that's the
derivative of the press s gravity. This equation is
the hydrostatic equation, which describes a change of atmospheric pressure with height.

Using the Ideal Gas Law, we can replace ρ and get the equation for dry air:

$$\frac{dp}{dz} = -g\frac{p}{R_dT} \tag{2.2.9}$$

or

$$\frac{dp}{p} = -\frac{g}{R_dT}dz = -\frac{Mg}{R^*T}dz \tag{2.2.10}$$

We could integrate both sides to get the altitude dependence of p, but we can only do that if T is constant with height. It is not, but it does not vary by more than about ±20%. So, doing the integral,

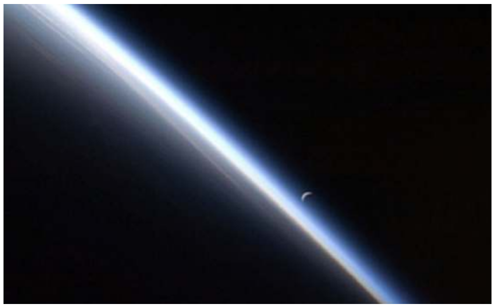
$$p = p_o e^{-z/H} \tag{2.2.11}$$

where p_o is the surface pressure and

$$H = \frac{R^*\overline{T}}{M_{\text{air}}g} \tag{2.2.12}$$

H is called a scale height because when $z = H$, we have $p = p_o e^{-1}$. If we use an average T of 250 K, with M_{air} = 0.029 kg mol^{-1}, then H = 7.2 km. The pressure at this height is about 360 hPa, close to the 300 mb surface that you have seen on the weather maps. Of course the forces are not always in hydrostatic balance and the pressure depends on temperature, thus the pressure changes from one location to another on a constant height surface.

From the hydrostatic equation, the atmospheric pressure falls off exponentially with height, which means that about every 7 km, the atmospheric pressure is about 1/3 less. At 40 km, the pressure is only a few tenths of a percent of the surface pressure. Similarly, the concentration of molecules is only a few tenths of a percent, and since molecules scatter sunlight, you can see in the picture below that the scattering is much greater near Earth's surface than it is high in the atmosphere.



Scattered light near Earth's surface. Credit: NASA

2.3: First Law of Thermodynamics

Weather involves heating and cooling, rising air parcels and falling rain, thunderstorms and snow, freezing and thawing. All of this weather occurs according to the three laws of Thermodynamics. The First Law of Thermodynamics tells us how to account for energy in any molecular system, including the atmosphere. As we will see, the concept of temperature is tightly tied to the concept of energy, namely thermal energy, but they are not the same because there are other forms of energy that can be exchanged with thermal energy, such as mechanical energy or electrical energy. Each air parcel contains molecules that have internal energy, which when thinking about the atmosphere, is just the kinetic energy of the molecules (associated with molecular rotations and, in some cases, vibrations) and the potential energy of the molecules (associated with the attractive and repulsive forces between the molecules). Internal energy does not consider their chemical bonds nor the nuclear energy of the nucleus because these do not change during collisions between air molecules. Doing work on an air parcel involves either expanding it by increasing its volume or contracting it. In the atmosphere, as in any system of molecules, energy is not created or destroyed, but instead, it is conserved. We just need to keep track of where the energy comes from and where it goes.

Floating molecules. Credit: Ivana Vasilj via flickr

Let U be an air parcel's internal energy, Q be the heating rate of that air parcel, and W be the rate that work is done on the air parcel. Then:

$$\frac{dU}{dt} = Q + W \tag{2.3.1}$$

The dimensions of energy are M L^2 T^{-2} so the dimensions of this equation are M L^2 T^{-3}.

To give more meaning to this energy budget equation, we need to relate U, Q, and W to variables that we can measure. Once we do that, we can put this equation to work. To do this, we resort to the Ideal Gas Law.

For processes like those that occur in the atmosphere, we can relate working,W, to a change in volume because work is force times distance. Imagine a cylinder with a gas in it. The cross-sectional area of the piston is A. If the piston compresses the gas by moving a distance dx, the amount of work being done by the piston on the gas is the force (pA) multiplied by the distance (dx). W is then $pAdx/dt$. But the volume change is simply $-Adx/dt$ and so:

$$W = -p\frac{dV}{dt} \tag{2.3.2}$$

Reducing a volume of gas ($dV/dt < 0$) takes energy, so working on an air parcel is positive when the volume is reduced, or $dV/dt < 0$. Thus:

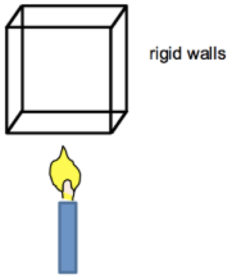
$$\frac{dU}{dt} = Q - p\frac{dV}{dt} \tag{2.3.3}$$

Heat Capacity

The heat capacity C is the amount of energy needed to raise the temperature of a substance by a certain amount. Thus, $C = \frac{Q}{\frac{dT}{dt}}$ and has SI units of J/K. C depends on the substance itself, the mass of the substance, and the conditions under which the energy is added. We will consider two special conditions: constant volume and constant pressure.

Heat Capacity at Constant Volume

Consider a box with rigid walls and thus constant volume: $\frac{dV}{dT}=0$. No work is being done and only internal energy can change due to heating.



Heating a box with rigid walls. Credit: W. Brune, after Verlinde

The candle supplies energy to the box, so $Q > 0$ and $dU/dt > 0$. The internal energy can increase via increases in molecular kinetic and potential energy. However, for an ideal gas, the attractive and repulsive forces between the molecules (and hence the molecular potential energy) can be ignored. Thus, the molecular kinetic energy and, hence, the temperature, must increase:

$$\frac{dT}{dt}>0 \tag{2.3.4}$$

So,

$$Q_{\text{ const }}=\frac{dU}{dt}=C_V\frac{dT}{dt} \tag{2.3.5}$$

C_V, the constant relating Q to temperature change, is called the **heat capacity at constant volume**. Heat capacity has units of J K^{-1}.

Remember that $C_v\frac{dT}{dt}$ is the change in the air parcel's internal energy.

The heat capacity, C_V, depends on the mass and the type of material. So we can write C_V as:

$$C_V=\text{mass}\cdot c_V \tag{2.3.6}$$

where c_V is called the specific heat capacity. The adjective "specific" means the amount of something per unit mass. *The greater the heat capacity, the smaller the temperature change for a given amount of heating.*

Some specific heat capacity values are included in the table below:

Specific Heat Capacity Values

gas	c_V (@ 0°C) J kg^{-1} K^{-1}
dry air	718
water vapor	1390
carbon dioxide	820

Solve the following problem on your own. After arriving at your own answer, click on the link to check your work.

? Exercise

Consider a sealed vault with an internal volume of 10 m^3 filled with dry air (p = 1013 hPa; T = 273 K). If the vault is being heated at a constant rate from the outside at a rate of 1 kW (1,000 J s^{-1}), how long will it take for the temperature to climb by 30 °C?

Click for answer

The 1st Law can be rewritten as:

$$Q = \frac{dU}{dt} + p\frac{dV}{dt} = C_V\frac{dT}{dt} + p\frac{dV}{dt} = \text{mass}\cdot c_V\frac{dT}{dt} + p\frac{dV^{2.27]}}{dt}$$

However, *dV/dt* = 0 because the vault’s volume isn’t changing. So, we can use the equation, rearrange it and integrate it:

$$Q = \text{mass}\cdot c_V\frac{dT}{dt}$$

How do we find the mass of the air inside the vault? Use the Ideal Gas Law to find the number of moles and then multiply by the mass per mole!

$$\text{mass} = M_{\text{dryair}}\cdot n = M_{\text{dryair}}\cdot\frac{pV}{R{*}T} = 0.029\cdot\frac{1.013\times10^5\cdot10}{8.314\cdot273} = 12.9\text{kg}$$

$$\Delta t = \frac{\text{mass}\cdot c_V\cdot\Delta T}{O} = \frac{12.9\cdot718\cdot30}{10^3} = 278\text{s}(\sim 5\text{min})$$

Often we do not have a well-defined volume, but instead just an air mass. We can easily measure the air mass’s pressure and temperature, but we cannot easily measure its volume. Often we can figure out the heating rate per volume (or mass) of air. Thus:

$$\frac{Q}{mass} \equiv q = \frac{mass\cdot c_V\frac{dT}{dt}}{mass} = c_V\frac{dT}{dt};\quad q = c_V\frac{dT}{dt} \tag{2.3.7}$$

where q is the specific heating rate (SI units: J kg^{-1} s^{-1}).

Heat Capacity Constant Pressure

The atmosphere is not a sealed box and when air is heated it can expand. We can no longer ignore the volume change. On the other hand, as the volume changes, any pressure changes are rapidly damped out, causing the pressure in an air parcel to be roughly constant even as the temperature and volume change. This constant-pressure process is called isobaric.

$$Q = \frac{dU}{dt} + p\frac{dV}{dt} \tag{2.3.8}$$

Now the change in the internal energy could be due to changes in temperature *or* changes in volume. It turns out that *internal energy does not change with changes in volume. It only changes due to changes in temperature*. But we already know how changes in internal energy are related to changes in temperature from the example of heating the closed box. That is, the internal energy changes are related by the heat capacity constant volume, C_V. Thus:

$$Q = C_V\frac{dT}{dt} + p\frac{dV}{dt} \tag{2.3.9}$$

Note that when volume is constant, we get the expression of heating a constant volume.

Suppose we pop the lid off the box and now the air parcel is open to the rest of the atmosphere. What happens when we heat the air parcel? How much does the temperature rise?

It’s hard to say because it is possible that the air parcel’s volume can change in addition to the temperature rise. So we might suspect that, for a fixed heating rate *Q*, the temperature rise in the open box will be less than the temperature rise in the sealed box where the volume is constant because the volume can change as well as the temperature.

Enthalpy

Enthalpy (*H*) is an energy quantity that accounts not only for internal energy but also the energy associated with working. It is a useful way to take into consideration *both ways that energy can change* in a collection of molecules – by *internal energy changes* and by *volume changes that result in work being done*

$$enthalpy \equiv H = U + pV$$

Enthalpy is the total energy of the air parcel including effects of volume changes. We can do some algebra and use the **Chain Rule** to write the First Law of Thermodynamics in terms of the enthalpy:

$$Q = \frac{dU}{dt} + p\frac{dV}{dt} = \frac{dU}{dt} + \frac{d(pV)}{dt} - V\frac{dp}{dt} = \frac{d(U+pV)}{dt} - V\frac{dp}{dt} = \frac{dH}{dt} - V\frac{dp}{dt} \tag{2.3.10}$$

If the pressure is constant, which is true for many air parcel processes, then *dp/dt* = 0 and:

$$Q = \frac{dH}{dt} \tag{2.3.11}$$

Summary

- In a constant volume process, heating changes only the internal energy, *U*.
- In a constant pressure process, heating changes enthalpy, *H* (both internal energy and working).

In analogy with constant volume process, for a constant pressure process, we can write:

$$Q_{\text{ constant }} = \frac{dH}{dt} = C_p \frac{dT}{dt}$$

where C_p is the heat capacity at constant pressure and c_p is the specific heat capacity at constant pressure.

Note that c_p takes into account the energy required to increase the volume as well as to increase the internal energy and thus temperature.

What is the difference between c_p and c_v? You will see the derivation of the relationship, but I will just present the results:

- by mole:c(p,m)=$c_{(p,m)} = c_{(V,m)} + R^*$
- by mass for dry air: $c_{pd} = c_{Vd} + R_d$
- by mass for water vapor: $c_{pv} = c_{Vv} + R_v$

gas	**c_V (@ 0^oC) J kg^{-1} K^{-1}**	**c_p (@ 0^oC) J kg^{-1} K^{-1}**
dry air	718	1005
water vapor	1390	1858

Since $c_p > c_v$, the temperature change at constant pressure will be less than the temperature change at constant volume because some of the energy goes to increasing the volume as well as to increasing the temperature.

Summary of Forms of the First Law of Thermodynamics

$$Q = C_V \frac{dT}{dt} + p \frac{dV}{dt}$$

$$Q = C_p \frac{dT}{dt} - V \frac{dp}{dt}$$

$$Q = \frac{dU}{dt} + p \frac{dV}{dt}$$

$$Q = \frac{dH}{dt} - V \frac{dp}{dt}$$

and $c_p = c_V + R, C_p = c_p m = c_p \rho V, C_v = c_v m = c_v \rho V; a = V/M(\text{ specific volume})$

We can look at specific quantities, where we divide variables by mass.

$$q = c_V \frac{dT}{dt} + p \frac{d\alpha}{dt}$$

$$q = c_p \frac{dT}{dt} - \alpha \frac{dp}{dt}$$

$$q = \frac{du}{dt} + p \frac{d\alpha}{dt}$$

You can figure out which form to use by following three steps:

1. Define the system. (i.e., what is the air parcel and what are its characteristics?)
2. Determine the process(es) (i.e., constant pressure, constant volume, heating, cooling?). Choose the form of the equation by making a term with a conserved quantity go away (i.e., *dp/dt* = 0 or *dV/dt* = 0) because then you have a simpler equation to deal with.
3. Look at which variables you have and then choose the equation that has those variables.

Exercise

Consider the atmospheric surface layer that is 100 m deep and has an average density of 1.2 kg m^{-3}. The early morning sun heats the surface, which heats the air with a heating rate of F = 50 W m^{-2}. How fast does the temperature in the layer increase? Why is this increase important?

1. What is the system? Air layer. Since we know the heating per unit area, work the problem per unit area.
2. What is the process? Constant pressure and heating by the sun.
3. Which variables do we have? $Q = C_p \frac{dT}{dt}$

Click for answer.

$$Q = FA = 50Wm^{-2}A$$

$$C_p = c_p \rho V = c_p \rho \Delta z A$$

$$Q = FA = c_p \rho \Delta z A \frac{dT}{dt}$$

$$\frac{dT}{dt} = \frac{FA}{c_p \rho \Delta z A} = \frac{F}{c_p \rho \Delta z} = \frac{50}{10051.2100} = 4.2 \times 10^{-4} Ks^{-1} = 1.5Khr^{-1}$$

This temperature increase is important because it is one of the most important factors in determining whether convection will occur later in the day. We will talk more about instability soon.

Here is a video (1:30) explanation of the above problem:

Dry Air Heating

Click here for transcript of the Dry Air Heating Video.

Let's go through this problem considering the heating of air in the lowest part of the atmosphere, which is called the atmospheric boundary. The sun heats the earth, and then the earth heats the air in contact with it. To see how fast the air will heat up, we need to know the heating rate, but we also need to know the air parcel's capacity. Heating rate is given in watts per meter squared, then we can multiply by some arbitrary area to get the total heating rate. Almost always, atmospheric heating and cooling occurs at constant pressure. Heat capacity, then, depends on the specific heat capacity at constant pressure. But it also depends on the air parcel's mass, which is density times volume. So we need to find the density, if it isn't given to us. You can use the ideal gas law for that. The volume is just the height times the area. So we put the heating rate on the left hand side and the effect of the heating on the parcel on the right. We are assuming a fairly uniform air parcel, so we see we really didn't need to multiply by area at all, since it just cancels out. We can rearrange this equation to get the temperature change for time on the left and all the known variables on the right. And then we can put the numbers in, and we can find out what the change in temperature with time is.

? Exercise

Consider the atmospheric surface layer that is 100 m deep and has an average density of 1.2 kg m^{-3}. It is night and dark and the land in contact with the air is cooling at 50 W m^{-2}. If the temperature at the start of the night was 25 ^{o}C, what is the temperature 8 hours later?

1. What is the system? Air layer. Since we know the cooling per unit area, work the problem per unit area.
2. What is the process? Constant pressure and cooling by the land radiating energy to space and the air cooling by being in contact with the land.
3. Which variables do we have? $Q = C_p \frac{dT}{dt}$

Click for answer.

$$Q = FA = -50Wm^{-2}A$$

$$C_p = c_p \rho V = c_p \rho \Delta z A$$

$$Q = FA = c_p \rho \Delta z A \frac{dT}{dt}$$

$$\frac{dT}{dt} = \frac{FA}{c_p \rho \Delta z A} = \frac{F}{c_p \rho \Delta z} = \frac{-50}{10051.2100} = -4.2x10^{-4} Ks^{-1} = -1.5 Khr^{-1}$$

Since the cooling continues for 8 hours, the total amount of cooling is –1.5 K/hr x 8 hr = 12 K or 12 ^{o}C. Thus, the temperature 8 hours later will be 13 ^{o}C.

This cooling near the surface creates a layer of cold air near the surface with a layer of warmer air above it. The layering of warm air over colder air creates an temperature inversion, which suppresses convection and lock pollutants into the air layer near Earth's surface.

2.4: The higher the temperature, the thicker the layer

A surprising way to relate the distance between two pressure surfaces to the temperature of the layer between them.

Consider a column of air between two pressure surfaces. If the mass in the column is conserved, then the column with the greater average temperature will be less dense and occupy more volume and thus be higher. But the pressure is related to the weight of the air above the column and so the upper pressure surface rises. If the temperature of the column is lower, then the pressure surface at the top of the column will be lower.

We can look at this behavior from the point-of-view of hydrostatic equilibrium.

$$\frac{\mathrm{d}p}{dz}=-\rho g=-\frac{pg}{R_dT} \tag{2.4.1}$$

If the temperature is greater, then the change in p with height is less, which means that any given pressure surface is going to be higher.

The difference between any two pressure surfaces is called the **thickness**. We can show that the thickness depends only on temperature:

$$dz=-\frac{dp}{p}\frac{R_d}{g}T \tag{2.4.2}$$

Integrate both sides:

$$\int_{z_1}^{z_2} dz=-\int_{p_1}^{p_2}\frac{dp}{p}\frac{R_d}{g}T \tag{2.4.3}$$

or

$$z_2-z_1=\frac{R_d}{g}\ln\left(\frac{p_1}{p_2}\right)\overline{T} \tag{2.4.4}$$

where T is the average temperature of the layer between p_1 and p_2. So, the thickness is actually a measure of the average temperature in the layer.

To Learn More

As some of you already know, you can use the thickness between different pressure surfaces to estimate the type of precipitation that will fall - snow, rain, or a mixture. You can check out these resources for some more information and example problems:

Weather Forecasting Online Review Questions

Discussion of Thickness and its Uses

? Exercise

Suppose that the 500 mb surface is at 560 dam (decameters, 10s of meters) and the 1000 mb surface is at 0 dam. What is the average temperature of the layer between 1000 mb and 500 mb?

Click for answer

Rearrange equation 2.4 to get an expression in terms of the average temperature and then put all the numbers into the equation to find the average temperature of the layer. Make sure that all the units are correct.

$$\overline{\mathrm{T}}=\frac{10^{*}\left(z_2-z_1\right)}{(287/9.8)\ln\left(p_1/p_2\right)}=\frac{10^{*}(560-0)}{(287/9.8)\ln(1000/500)}=276K$$

2.5: Adiabatic Processes - The Path of Least Resistance

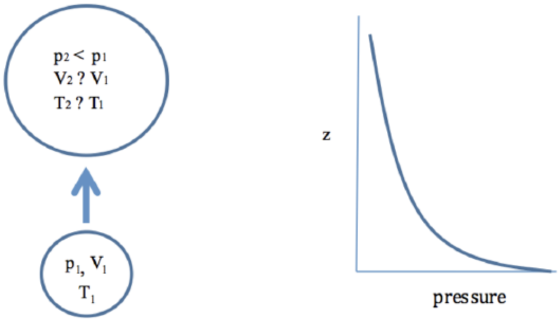
Adiabatic Process

So far, we have covered **constant volume (isochoric)** and **constant pressure (isobaric)** processes. There is a third process that is very important in the atmosphere—the **adiabatic process**. Adiabatic means *no energy exchange* between the air parcel and its environment: $Q = 0$. **Note: adiabatic is not the same as isothermal.**

Consider the Ideal Gas Law:

$$pV = nR^{*}T \tag{2.5.1}$$

If an air parcel rises, the pressure changes, but how does the temperature change? Note that the volume can change as well as the pressure and temperature, and thus, if we specify a pressure change, we cannot find the temperature change unless we know how the volume changed. Without some other equation, we cannot say how much the temperature will rise for a pressure change.



Changes in volume and temperature as an air parcel ascends and the pressure decreases. Credit: W. Brune, after Lamb and Verlinde

However, we can use the First Law of Thermodynamics to relate changes in temperature to changes in pressure and volume for adiabatic processes.

Derivation of the Poisson Relations

I do not expect you to be able to do this derivation, but you should go through it to make sure that you understand all the steps as a way to continue to improve your math skills. Start with the following specific form of the 1st Law for dry air:

$$q = c_p\frac{dT}{dt} - \alpha\frac{dp}{dt} \tag{2.5.2}$$

$$q = 0 = c_p\frac{dT}{dt} - \alpha\frac{dp}{dt} \tag{2.5.3}$$

Divide both sides by T and note that This equation is not rendering properly due to an incompatible browser. See Technical Requirements in the Orientation for a list of compatible browsers. (α is called the **specific volume**):

$$\frac{c_p}{T}\frac{dT}{dt} - \frac{R_d T}{pT}\frac{dp}{dt} = 0 = \frac{c_p}{T}\frac{dT}{dt} - \frac{R_d}{p}\frac{dp}{dt} = c_p\frac{d\ln(T)}{dt} - R_d\frac{d\ln(p)}{dt} \tag{2.5.4}$$

where we figured out the two terms were just derivatives of the natural log of T and p.

$$\frac{d}{dt}\left(c_p\ln(T) - R_d\ln(p)\right) = 0 \tag{2.5.5}$$

But *d/dt* = 0 just means that the value is constant:

$$\left(c_p\ln(T) - R_d\ln(p)\right) = \text{ constant} \tag{2.5.6}$$

Divide by c_p:

$$\ln(T) - \frac{R_d}{c_p}\ln(p) = \ln(T) + \ln\left(p^{(-R_d/c_p)}\right) = \ln\left(Tp^{(-R_d/c_p)}\right) = \text{constant} \tag{2.5.7}$$

If the natural log of a variable is constant then the variable itself must be constant:

$$Tp^{(-R_d/c_p)} = \text{ constant} \tag{2.5.8}$$

We can rewrite R_d/c_p as a new term denoted by the Greek letter gamma, γ

$$\gamma \equiv \frac{c_p}{c_v} = \frac{c_v + R_d}{c_v} = 1.4 \tag{2.5.9}$$

$$\frac{R_d}{c_p} = \frac{\gamma - 1}{\gamma} = 0.286 \tag{2.5.10}$$

We can use the Ideal Gas Law to get relations among p, V, and T, called the Poisson's Relations:

$$Tp^{(-R_d/\sigma_p)} = Tp^{((1-n)/r)} = \text{ constant} \tag{2.5.11}$$

$$\alpha^{\gamma}p = \text{ constant} \tag{2.5.12}$$

$$T\alpha^{\gamma-1} = \text{constant} \tag{2.5.13}$$

Potential Temperature

The Poisson Relation that we use the most is the relation of pressure and temperature because these are two variables that we can measure easily without having to define a volume of air:

$$\frac{T}{\theta} = \left(\frac{p}{p_o}\right)^{(-R_d/c_p)} \tag{2.5.14}$$

or

$$\theta = T\left(\frac{p_o}{p}\right)^{R_d/c_p} = T\left(\frac{1000}{p}\right)^{0.286} \tag{2.5.15}$$

We call θ the potential temperature, which is the temperature that an air parcel would have if the air is brought to a pressure of p_o = 1000 hPa. Potential temperature is one of the most important thermodynamic quantities in meteorology.

Adiabatic processes are common in the atmosphere, especially the dry atmosphere. Also, adiabatic processes are often the same as isentropic processes (no change in entropy).

? Exercise

Air coming over the Laurel Highlands descends from about 700 m (p ~ 932 hPa) to 300 m (p ~ 977 hPa) at State College. Assume that the temperature in the Laurel Highlands is 20 ^{o}C. What is the temperature in State College?

Click for answer.

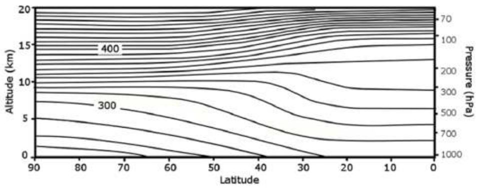
We can find the temperature in State College due only to adiabatic changes by the following equation:

$$T_{700}p_{700} - 0.286 = T_{300}p_{300} - 0.286$$

$$T_{300} = T_{700}\left(\frac{p_{300}}{p_{700}}\right)^{0.286} = (273 + 20)\left(\frac{977}{932}\right)^{0.286} = 297K = 24C$$

This temperature change is 4 ^{o}C, or 7 ^{o}F just from adiabatic compression.

We can plot adiabatic **(isentropic)** surfaces in the atmosphere. An air parcel needs no energy to move along an adiabatic surface. Also, it takes energy for an air parcel to move from potential surface to another potential energy surface.



Potential temperature (solid lines, K) as a function of latitude and altitude. Note that the decrease in potential temperature with height is small in the troposphere and large in the stratosphere. Credit: W. Brune, after Andews, Holton, and Leovy

? Exercise

Suppose an air parcel has p = 300 hPa and T = 230 K. How much heating per unit volume of dry air would be needed to increase the potential temperature by 10 K?

Click for answer.

The heating raises the temperature, and the amount of heating required depends on the heat capacity, constant pressure, which depends on the mass of air, or the density times the volume. Let's do the calculation for a volume of air; that way we can use the density.

First we need to find the temperature rise that is the same as a potential temperature rise of 10 K at a pressure of 300 hPa.

$$d\theta = dT\left(\frac{1000}{p}\right)^{0.286} \qquad dT = 10\left(\frac{300}{1000}\right)^{0.286} = 7.1K$$

Then we need to find the density so that we can calculate the heat capacity:

$$\rho = \frac{p}{R_d T} = \frac{3\times 10^4}{287\cdot 230} = 0.45\text{kg}m^{-3}$$

Now we can put it all together:

$$\frac{Q\Delta t}{V} = \frac{\rho\cdot V\cdot c_p\cdot \Delta T}{V} \Rightarrow \frac{Q\Delta t}{V} = \rho\cdot c_p\cdot \Delta T = 0.45\cdot 1005\cdot 7.1 = 3.2x10^3 Jm^{-3}$$

Note that we were asked to provide the total heating per unit volume, which is just the heating rate times time divided by the unit volume. So the quantity on the left is what we want. Is this heating large? Yes! So it takes a lot of heating or cooling the raise or lower an air parcel potential temperature just 10 K.

Dry Adiabatic Lapse Rate

The temperature change with change in pressure (and thus change in altitude) is a major reason for weather. For dry air, the main effect is buoyancy. So because the pressure change generally follows the hydrostatic equation, the *change in height translates into a change in pressure which translates into a change in temperature due to adiabatic expansion*. Note that as the air parcel rises, its pressure quickly adjusts to the pressure of the surrounding air. Thus we can determine the dry adiabatic lapse rate by starting with the Poisson relation between pressure and temperature:

$$Tp^{\left(-R_d/c_p\right)} = constant \tag{2.5.16}$$

Take the derivative w.r.t. z:

$$\frac{dT}{dz}p^{\left(-R_d/c_p\right)} + T\left(-R_d/c_p\right)p^{\left(-R_d/c_p\right)-1}\frac{dp}{dz} = \frac{dT}{dz} + T\left(-R_d/c_p\right)p^{-1}\frac{dp}{dz} = 0 \tag{2.5.17}$$

But we also know from the hydrostatic equation that:

$$\frac{dp}{dz} = -\rho g \tag{2.5.18}$$

Substituting –ρg for dp/dz into the equation and rearranging the terms:

$$\frac{dT}{dz} = T\left(^{-R_d}/c_p\right)p^{-1}\rho g = -T\left(R_d/c_p\right)p^{-1}g\left(\frac{p}{R_d T}\right) \tag{2.5.19}$$

$$-\frac{dT}{dz} \equiv \Gamma_d = \frac{g}{c_p} = \frac{9.8\frac{m}{s}}{1005Jkg^{-1}K^{-1}} = 9.8Kkm^{-1} \tag{2.5.20}$$

Γ_d is called the **dry adiabatic lapse rate**. Note that the temperature decreases with height, but the dry adiabatic lapse rate is defined as being positive.

? Exercise

Air coming over the Laurel Highlands descends from about 700 m to 300 m at State College. Assume that the temperature in the Laurel Highlands is 20 ^{o}C. What is the temperature in State College?

Click for answer.

We can find the temperature in State College due only to adiabatic changes by using the dry adiabatic lapse rate multiplied by the height change:

$$dT = -\Gamma_d(300 - 700) = 9.8Kkm^{-1} \cdot 0.4 = 4°C$$

This temperature change is 4.0 ^{o}C, or 7 ^{o}F just from adiabatic compression. This answer is very similar to the answer we obtained using the change in potential temperature.

2.6: Stability and Buoyancy

We know that an air parcel will rise relative to the surrounding air at the same pressure if the air parcel’s density is less than that of the surrounding air. The difference in density can be calculated using the virtual temperature, which takes into account the differences in specific humidity in the air parcel and the surrounding air as well as the temperature differences.

Stability

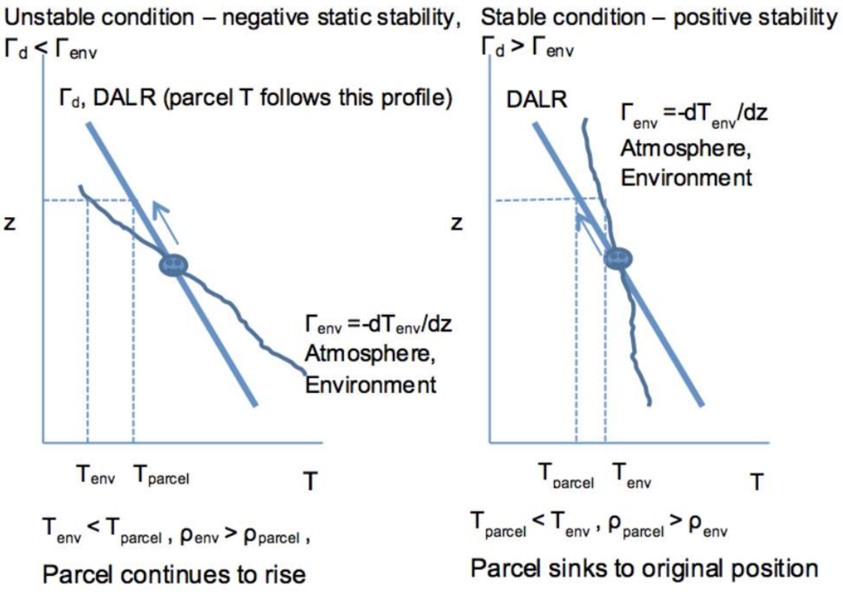
In equilibrium, the sum of forces are in balance and the air parcel will not move. The question is, what happens to the parcel if there is a slight perturbation in its vertical position?

Examples of instability (left) and stability (right). Credit: W. Brune, after Verlinde

For the figure on the left, if the ball is displaced a tiny bit to the left or the right, it will be pulled by gravity and will continue to roll down the slope. That position is unstable. For the figure on the right, if the ball is displaced a little bit, it will be higher than the central position and gravity will pull it back down. It may rock back and forth a little bit, but eventually it will settle down into its original position.

To assess instability of air parcels in the atmosphere, we need to find out if moving the air parcel a small amount up or down causes the parcel to continue to rise or to fall (instability) or if the air parcel returns to its original position (stability).

Now look at some atmospheric temperature profiles. **Important: a dry air parcel that is pushed from its equilibrium position always moves along the dry adiabatic lapse rate (DALR) line.**

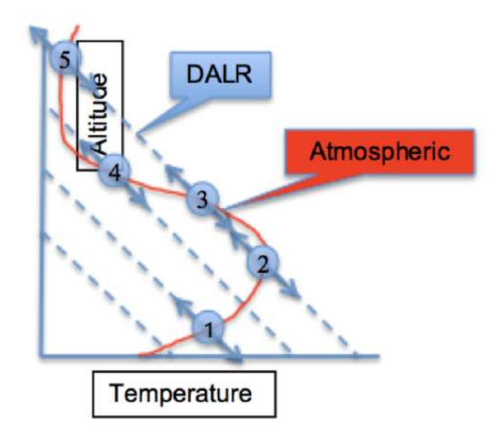


Determining instability (left) and stability (right) of an air parcel by moving it on the DALR slope and seeing if the parcel’s temperature is greater or less than the environmental temperature. Credit: W. Brune

Note that we can also show that if the air parcel is pushed down, it will keep going if the atmospheric (environmental) profile looks like the one on the left and will return to the original position if it looks like the one on the right.

? Example

Use the image above to determine the following:



Credit: W. Brune

Is the air parcel stable or unstable at each of the points, 1-5?

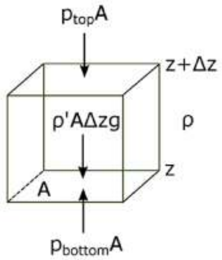
Click for answer.

The red line is the atmospheric temperature profile; the dashed lines are the dry adiabatic lapse rate lines (-9.8 K/km). Consider points 1-5. When an air parcel is pushed up the DALR and its temperature is greater than the atmospheric temperature at the new level, it is warmer and thus less dense. It will continue to rise. When an air parcel is pushed down the DALR and its temperature is less than the atmospheric temperature at that new level, it is colder and thus more dense. It will continue to fall. Both cases are unstable. However, if when an air parcel is pushed up the DALR and its temperature is less than the atmospheric temperature at that new level, it is colder and thus more dense. It will sink back down to its original position and is stable.

Using this thinking, air parcels at points 1, 2, and 5 are stable and at points 3 and 4 are unstable.

Buoyancy

We can calculate the acceleration an unstable air parcel will have and, from this, can determine the parcel's velocity at some later point in time. This acceleration is called buoyancy (B).



Forces on an air parcel that has a different density from its environment. Credit: W. Brune

$$\sum F = F_{\text{bottom}} + F_{\text{top}} + F_{\text{weight}} = p_{\text{bottom}}A - p_{\text{top}}A - \rho' gAh \tag{2.6.1}$$

Let's look at the forces on an air parcel again, like we did to derive the hydrostatic equilibrium. But this time, let's assume that the parcel has a different density than the surrounding air. We will designate quantities associated with the air parcel with an apostrophe ('); environmental parameters will have no superscript.

If the forces are not in balance, then we need to keep the acceleration that we set to zero in the hydrostatic equilibrium case. We can also divide both sides of the equation by the mass of the air parcel:

$$\frac{\sum F}{\rho' V} = \frac{-\frac{p_{\text{ top }} - p_{\text{botton}}}{\Delta z} - \rho' g}{\rho'} \rightarrow \frac{-(-\rho g) - \rho' g}{\rho'} \tag{2.6.2}$$

$$a \equiv B = \frac{(\rho - \rho')\, g}{\rho'} \tag{2.6.3}$$

where we have used the hydrostatic equilibrium of the environment to replace the expression for the pressure change as a function of height with the density times the acceleration due to gravity.

We can then use the Ideal Gas Law to replace densities with virtual temperatures because the pressure of the parcel and its surrounding air is the same:

$$B = \frac{\left({}^{1}/\tau_v - 1/\tau_v'\right) g}{1/T_v'} = \frac{(T_v' - T_v)}{T_v} g \tag{2.6.4}$$

If B>0, then the parcel rises; if B<0, then the parcel descends.

We look at the instability at each point in the environmental temperature profile and can determine Γ_{env} for each point.

Thus,

$$T_{env} = T_0 + \left.\frac{\partial T}{\partial z}\right|_{env} \Delta z = T_0 - \Gamma_{env} \Delta z; T_{\text{parcel}} = T_0 + \left.\frac{\partial T}{\partial z}\right|_{\text{parcel}} \Delta z = T_0 - \Gamma_d \Delta z \tag{2.6.5}$$

so that:

$$B = \frac{(T_0 - \Gamma_d \Delta z - T_0 + \Gamma_{en} \Delta z)}{T_v} g = \frac{(\Gamma_{env} - \Gamma_d)}{T_v} g \Delta z \tag{2.6.6}$$

- If $\Gamma_{env} < \Gamma_d$, the parcel accelerates downward for positive Δz (positive stability).
- If $\Gamma_{env} > \Gamma_d$, the parcel accelerates upward for positive Δz (negative static stability).

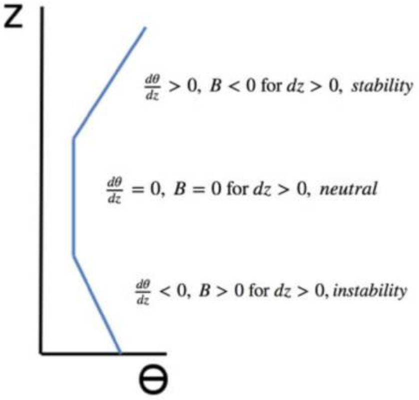
We can put this idea of buoyancy in terms of potential temperature.

$$\theta = T \left(\frac{p_o}{p}\right)^{R_d} / c_p \tag{2.6.7}$$

We want to find $d\theta/dz$. Taking the log of both sides of the equation and replacing a dp/dz term with $-g\rho$, we are able to find the following expression for buoyancy in terms of potential temperature:

$$B = -g \Delta z \frac{1}{\theta} \frac{d\theta}{dz} \tag{2.6.8}$$

Remember that no matter what the environmental temperature or potential temperature profiles, a change in height of an air parcel will result in a temperature that changes along the dry adiabat and a potential temperature that does not change at all. As you can see below, the stability of a layer depends on the change in environmental potential temperature with height. Air parcels try to move vertically with constant potential temperature.



Credit: W. Brune

Parcels will move to an altitude (and air density) for which B = 0. However, if they still have a velocity when they reach that altitude, they will overshoot, experience a negative acceleration, and then descend, overshooting the neutral level again. In this way, the air parcel will oscillate until its oscillation is finally damped out by friction and dissipation of the air parcel. Note that in the neutral section of vertical profile where potential temperature does not change, it is not possible to determine if an air parcel will be stable or unstable. For instance, if the air parcel in the neutral region is given a small upward push, it will continue to rise until it reaches a stable region.

Discussion Activity: Storms in the troposphere

(3 discussion points)

This week's discussion topic is a hypothetical question involving stability. The troposphere always has a capping temperature inversion - it's called the stratosphere. The tropopause is about 16 km high in the tropics and lowers to about 10 km at high latitudes. The stratosphere exists because solar ultraviolet light makes ozone and then a few percent of the solar radiation is absorbed by stratospheric ozone, heating the air and causing the inversion. Suppose that there was no ozone layer and hence no stratosphere caused by solar UV heating of ozone.

Would storms in the troposphere be different if there were no stratosphere to act like a capping inversion? And if so, how?

Use what you have learned in this lesson about the atmosphere's pressure structure and stability to help you to think about this problem and to formulate your answer and discussions. It's OK to be wrong, as long as you have some solid reasoning to back up your ideas. My goal is to get you all to communicate with each other and think hard about atmospheric science.

1. You can access the **Storms in the Troposphere Discussion Forum** in Canvas.
2. Post a response that answers the question above in a thoughtful manner that draws upon course material and outside sources.
3. Keep the conversation going! **Comment on at least one other person's post**. Your comment should include follow-up questions and/or analysis that might offer further evidence or reveal flaws.

This discussion will be worth 3 discussion points. I will use the following rubric to grade your participation:

Discussion Activity Grading Rubric

Evaluation	Explanation	Available Points
Not Completed	Student did not complete the assignment by the due date.	0
Student completed the activity with adequate thoroughness.	Posting answers the discussion question in a thoughtful manner, including some integration of course material.	1
Student completed the activity with additional attention to defending his/her position.	Posting thoroughly answers the discussion question and is backed up by references to course content as well as outside sources.	2
Student completed a well-defended presentation of his/her position, and provided thoughtful analysis of at least one other student’s post.	In addition to a well-crafted and defended post, the student has also engaged in thoughtful analysis/commentary on at least one other student’s post as well.	3

CHAPTER OVERVIEW

3: Moist Processes

Learning Objectives

By the end of this chapter, you should be able to:

- differentiate among the different ways that moisture can be expressed and choose the correct one for finding an answer
- explain the meaning of the lines and spaces on a water vapor phase diagram
- calculate relative humidity using the Clausius–Clapeyron Equation
- solve energy problems related to temperature and phase changes
- demonstrate proficiency with using the skew-T to find the lifting condensation level (LCL), potential temperature, relative humidity, wetbulb temperature, dry and moist adiabats, and equivalent potential temperature

The atmosphere's most abundant chemicals are molecular nitrogen (N_2), molecular oxygen (O_2), and Argon (Ar). These are all only in the gas phase. Water vapor, the next most abundant, can exist as vapor, liquid, or solid. The phase changes of water have a major role in weather and in climate. In the atmosphere, water is always trying to achieve a balance between evaporation and condensation while never really succeeding. In this lesson, you will discover the conditions under which the phases of water are in balance and will see that they depend on only two quantities—the amount of water and the temperature. Equilibrium conditions, often called saturation, are expressed mathematically by the Clausius–Clapeyron Equation. We will see that phase changes of water create weather, including severe weather, and that we can use the 1st Law of Thermodynamics to do many calculations involving situations where there are phase and temperature changes. Combining the Clausius–Clapeyron equation with the equations of thermodynamics, we can construct a diagram called the skew-T. The skew-T is useful in helping us understand both the atmosphere's temperature structure and the location and behavior of clouds.

3.1: Ways to Specify Water Vapor

Up to now, we have dealt with water vapor only as the specific humidity in order to determine the virtual temperature. But there are many ways for us to quantify the amount of water vapor in the atmosphere. The most common are specific humidity, water vapor mixing ratio, relative humidity, and dewpoint temperature.

Student using a sling psychrometer to measure the dewpoint temperature, which is one way to specify atmospheric water vapor. Credit: W. Brune

Specific humidity (*q*) is the density of water vapor (mass per unit volume) divided by the density of all air, including the water vapor:

'_' allowed only in math mode

We have already seen that specific humidity is used to calculate virtual temperature. Specific humidity is unitless, but often we put it in g kg^{-1}.

Water vapor mixing ratio (*w*) is the density of water vapor divided by the density of dry air without the water vapor:

$$\begin{aligned} w &= \frac{m_{\text{water\_vapor}}}{m_{dry\_air}} \\ &= \frac{\rho_{\text{water\_,vapor}}}{\rho_{dv\_,\text{air}}} \\ &= \frac{\rho_v}{\rho_d} \end{aligned}$$

Water vapor mixing ratio is widely used to calculate the amount of water vapor. It is also the quantity used on the skew-T diagram, which we will discuss later in this lesson. Water vapor mixing ratio is unitless, but often we put it in g kg^{-1}.

Since

$$\rho_d = \rho - \rho)v \tag{3.1.1}$$

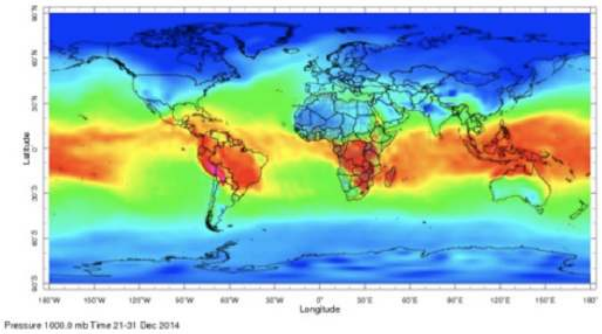
we can rearrange the equations to get the relationship between w (water vapor mixing ratio) and q (specific humidity):

$$q = \frac{w}{1+w} \tag{3.1.2}$$

$$w = \frac{q}{1-q} \tag{3.1.3}$$

The water vapor mixing ratio, *w*, is typically at most about 40 g kg^{-1} or 0.04 kg kg^{-1}, so even for this much water vapor, *q* = 0.040/(1 + 0.040) = 0.038 or 38 g kg^{-1}.

Thus, *water vapor mixing ratio and specific humidity are the same to within a few percent*. But specific humidity is less than the water vapor mixing ratio if the humidity is more than zero. Here is one example of global specific humidity.



Specific humidity (*q*) for December 2014. The color scale goes from 0 at blue to 30 g kg^{-1} for pink. Green is about 10 g kg^{-1} and red is about 20 g kg^{-1}. Credit: IRI

The greatest absolute specific humidity is in the tropics with maximum values approaching 30 g kg^{-1}. The smallest values are at the high latitudes and are close to zero. Why is specific humidity distributed over the globe in this way?

Relative humidity (*RH*) is another measure of water vapor in the atmosphere, although we must be careful when using it because a low relative humidity may not mean a low water vapor mixing ratio (i.e., at high temperatures) and a high relative humidity might still be quite dry air (i.e., at low temperatures).

According to the World Meteorological Organization (WMO) definition,

$$RH = \frac{w}{w_s} \tag{3.1.4}$$

where **w_s is the saturation mixing ratio** (the mixing ratio at which RH = 100%). w and w_s can both have units of g kg^{-1} or kg kg^{-1}, as long as they are consistent. Relative humidty is usually expressed as a percent. Thus, when $w = w_s$, $RH = 1 = 100\%$. In most problems involving RH, it is important to keep in mind conversions between decimal fractions and percent.

A more physically based definition of the relative amount of moisture in the air is the saturation ratio, S:

$$S = \frac{e}{e_s} \tag{3.1.5}$$

where **e is the vapor pressure (hPa)** and **e_s is the saturation vapor pressure**. The saturation ratio is used extensively in cloud physics (Lesson 5). To see how RH and S are related, start with the Ideal Gas Law and then do some algebra:

$$e = \rho_v R_v T \tag{3.1.6}$$

$$p_d = \rho_d R_d T \tag{3.1.7}$$

$$\varepsilon \equiv R_d/_R \tag{3.1.8}$$

so

$$w = \frac{\varepsilon e}{p_d} = \frac{\varepsilon e}{p - e} \tag{3.1.9}$$

$$\frac{w}{w_s} = \frac{e}{e_s}\left(\frac{p - e_s}{p - e}\right) \tag{3.1.10}$$

$$RH = S\left(\frac{p - e_s}{p - e}\right) \tag{3.1.11}$$

where $\varepsilon = 0.622$ is just the molar mass of water (18.02 kg mol^{-1}) divided by the mass of dry air (28.97 kg mol^{-1}). e and e_s are typically less than 7% of p, and since e is usually 20%–80% of e_s, the difference between the two definitions is usually less than a few percent.

Note that at saturation, you can replace w with w_s and e with e_s in the equation that relates w to e.

Some processes depend upon the absolute amount of water vapor, which is given by the specific humidity, water vapor mixing ratio, and water vapor pressure, and other processes depend on the relative humidity. For example, the density of a moist air parcel depends on the absolute amount of water vapor. So does the absorption and emission of infrared atmospheric radiation. On the other hand, cloud formation depends on the relative humidity, although the cloud might be kind of wimpy if the absolute humidity is small.

One of the most common indicators of absolute humidity is the **dewpoint temperature**. We will postpone the discussion of it until after we learn about the relationship between temperature and saturation vapor pressure, e_s.

Exercise

If the density of water vapor is 10.0 g m^{-3} and the density of dry air is 1.10 kg m^{-3}, what is the water vapor mixing ratio and what is the specific humidity?

Click for answer

$$w = \frac{10.0 \mathrm{g m^{-3}}}{1.10 \mathrm{kg m^{-3}}} = 9.09 \frac{\mathrm{g}}{\mathrm{kg}}$$

$$q = \frac{10.0 \mathrm{g m^{-3}}}{1.10 \mathrm{kg m^{-3}} + \left(10.0 \mathrm{g m^{-3}}/1000 \mathrm{g kg^{-1}}\right)} = 9.01 \frac{\mathrm{g}}{\mathrm{kg}}$$

Exercise

If the water vapor mixing ratio is 21 g kg^{-1} and the relative humidity is 84%, what is the saturation water vapor mixing ratio?

Click for answer

$$RH = \frac{w}{w_s} \quad \rightarrow \quad w_s = \frac{w}{RH} = \frac{21 \mathrm{g kg^{-1}}}{0.84} = 25 \mathrm{g kg^{-1}}$$

3.2: Condensation and Evaporation

What is **vapor pressure**? Because of the Ideal Gas Law (Equation 2.1), we can think of vapor pressure *e* (SI units = hPa or Pa) as being related to the concentration of water vapor molecules in the atmosphere,

$$eV = NR^*T \tag{3.2.1}$$

and

$$e = nR^*T \tag{3.2.2}$$

where *n* is the number of moles per unit volume (*n* = *N*/*V*).

What makes liquid water different from ice or water vapor? It is actually the weak bonds between water molecules that are called hydrogen bonds. These bonds are 20 times weaker than the bonds between hydrogen and oxygen in the same molecule and can be broken by collisions with other molecules if they are traveling fast enough and have enough kinetic energy to break the bonds. So the differences between vapor, liquid, and ice are related to the number of hydrogen bonds. In vapor, there are essentially no hydrogen bonds between molecules. In ice, each water molecule is hydrogen bonded to four other water molecules. And in liquid, only some of those hydrogen bonds are made and they are constantly changing as the water molecules and clusters of water molecules bump into and slide past each other.

Water vapor condensed into liquid drops on a spider web overnight. As the sun rises and the air heats up, there will be net evaporation and the beads will shrink and disappear during the day. Credit: devra via flickr

Think about a liquid water surface on a molecular scale. What is happening all the time is that some water molecules in the gas phase are hitting the surface and sticking (i.e., making hydrogen bonds), while at the same time other water molecules are breaking free from the hydrogen bonds that tie them to other molecules in the liquid and are becoming water vapor. The water vapor surface is like a Starbucks, but even busier. We can easily calculate the flux of molecules that are hitting the surface using simple physical principles, although it is harder to calculate the number that are leaving the liquid. Both are happening all the time, although usually the amount of condensation and evaporation aren't the same, so that we usually have **net evaporation or net condensation**.

In **equilibrium**, the flux of molecules leaving the surface exactly balances the flux of molecules that are hitting the surface. This condition is called equilibrium, or **saturation**. We can show that:

$$\frac{\text{condensation}}{\text{evaporation}} = \frac{e}{e_s} = S \cong RH = \frac{w}{w_s} \tag{3.2.3}$$

Thus, when $S = 1$, $e = e_s$, RH is approximately 100%, and w is approximately w_s. Condensation and evaporation are in balance. These two processes are going on all the time, but sometimes there can be more evaporation than condensation, or more condensation than evaporation, or evaporation equaling condensation. However, **water is always trying to come into equilibrium**.

So we know that the amount of water in vapor phase determines the condensation rate and thus e. So what determines e_s? We will see next that **e_s depends on only one variable: temperature!**

3.3: Phase Diagram for Water Vapor - Clausius Clapeyron Equation

The Clausius–Clapeyron Equation

We can derive the equation for e_s using two concepts you may have heard of and will learn about later: **entropy** and **Gibbs free energy**, which we will not go into here. Instead, we will quote the result, which is called the **Clausius–Clapeyron Equation**,

$$\frac{1}{e_s}\frac{de_s}{dT} = \frac{l_v}{R_v T^2} \tag{3.3.1}$$

where l_v is the **enthalpy of vaporization** (often called the latent heat of vaporization, about 2.5 x 10^6 J kg^{-1}), $\boldsymbol{R_v}$ is the **gas constant for water vapor** (461.5 J kg^{-1} K^{-1}), and T is the absolute temperature. The enthalpy of vaporization (i.e., latent heat of vaporization) is just the amount of energy required to evaporate a certain mass of liquid water.

What is the physical meaning? The right-hand side of Equation 3.3.1 is always positive, which means that the saturation vapor pressure always increases with temperature (i.e., $de_s/dT > 0$). This positive slope makes sense because we know that as water temperature goes up, evaporation is faster (because water molecules have more energy and thus a greater chance to break the bonds that hold them to other water molecules in a liquid or in ice). At saturation, condensation equals evaporation, and since evaporation is greater, condensation must be greater as well. Much of the higher condensation comes from having more water vapor molecules hitting the liquid surface, which according to the Ideal Gas Law, means that the water vapor pressure is higher.

This expression can be integrated, assuming that l_v is a constant with temperature (it is not quite constant!) to give the equation:

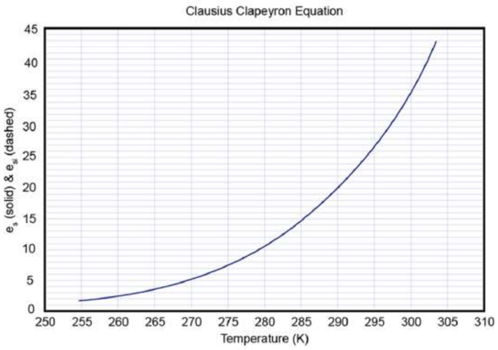
$$e_s = e_{so}\exp\left(\frac{l_v}{R_v T_o}\right)\exp\left(\frac{-l_v}{R_v T}\right) \tag{3.3.2}$$

Generally T_o is taken to be 273 K and e_{so} is then 6.11 hPa.

Notes:

- e_s depends only on T, the absolute temperature. It is essentially independent of the atmospheric pressure, or any other factors.
- l_v is not constant with temperature but instead changes slightly (from 2.501 x 10^6 J kg^{-1} at 0 °C to 2.257 x 10^6 J kg^{-1} at 100 °C).
- Thus, the most accurate forms of the integrated Clausius–Clapeyron Equation are more complicated but easy to deal with when using a computer.

What does the plot of this equation look like?



The equilibrium vapor pressure between water vapor (to the right of the line) and liquid water (to the left of the line) as calculated by the Clausius–Clapeyron Equation. The line for liquid water can be extended below 273 K, the freezing point, because water can remain liquid at those low temperatures and become a "supercooled" liquid.

What happens between vapor and ice? The same methods can be applied and the same basic equations are obtained, except with a different constant:

$$e_{si} = e_{so}\exp\left(\frac{l_s}{R_v T_o}\right)\exp\left(\frac{-l_s}{R_v T}\right) \tag{3.3.3}$$

where e_{si} is the saturation vapor pressure for the ice vapor equilibrium and l_s is the enthalpy of sublimation (direct exchange between solid water and vapor = 2.834 x 10^6 J kg^{-1}).

Equations for e_s and e_{si} that account for variations with temperature of l_v and l_s, respectively, can be found in Bohren and Albrecht (Atmospheric Thermodynamics, Oxford University Press, New York, 1998, ISBN 0-19-509904-4):

$$e_s = e_{so} \exp\left[(6808\text{K})\left(\frac{1}{T_o} - \frac{1}{T}\right) - 5.09 \ln \frac{T}{T_o}\right] \tag{3.3.4}$$

$$e_{si} = e_{so} \exp\left[(6293\text{K})\left(\frac{1}{T_o} - \frac{1}{T}\right) - 0.555 \ln \frac{T}{T_o}\right] \tag{3.3.5}$$

where $T_o = 273\text{K}, \quad e_{so} = 6.11\text{hPa}$

Note that e_{so} is the saturation vapor pressure at T_o and that e_{so} = 6.11 hPa and T_o = 273 K. Note how the constants are slightly different because the latent heat of vaporization for liquid water is different from the latent heat of vaporization for ice. Note that T in these equations must be in Kelvin.

Dewpoint Temperature as a Measure of Water Vapor

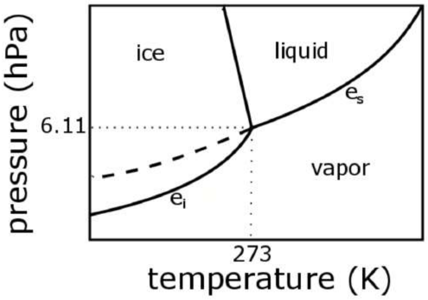
Simply put, the dewpoint temperature is the temperature at which the atmosphere's water vapor would be saturated. It is always less than or equal to the actual temperature. Mathematically,

$$w(T) = w_s(T_d) \tag{3.3.6}$$

which means that the water vapor pressure at some temperature T (not multiplied by T) equals the water vapor saturation pressure at the dewpoint temperature, T_d. So we see that because w_s depends only on T_d at a given pressure, T_d is a good method for designating the absolute amount of water vapor.

The Phase Diagram for Water

We can draw the phase diagram for water. There are three equilibrium lines that meet at the triple point, where all three phases exist (e_s = 6.1 hPa; T = 273.14 K). Along the line for e_s, vapor and liquid are in equilibrium, and evaporation balances condensation. Along the line for e_{si}, vapor and ice are in equilibrium and sublimation equals deposition. Along the line for e_{sm}, liquid and ice are in equilibrium and melting balances fusion.



Phase diagram for water for most water pressures and temperatures that are relevant to the atmosphere. e_s is the saturation pressure (i.e. equilibrium pressure) between liquid and vapor and e_{si} (mislabeld as e_i in the diagram) is the saturation pressure between vapor and ice. The dashed line is a continuation of e_s into the ice phase and represents supercooled water. All three phases meet at the triple point. This diagram is not to scale. Credit: W. Brune

Is it possible to have water in just one phase? Yes!

The simplest case is when all the water is vapor, which occurs when the water vapor pressure is low enough and the temperature (and thus saturation vapor pressure) is high enough that all the water in the system is evaporated and in the vapor phase.

Let's think about what it would take to have all the water in the liquid phase. Suppose we have a cylinder closed on one end and a sealed piston in the other that is in a temperature bath so that we can hold the cylinder and its contents at a fixed temperature (i.e., isothermal). Initially, we fill the cylinder with liquid water and have a small volume of pure water vapor at the top. If we set the bath temperature to, say, 280 K and let the system sit for a while, the vapor will become saturated, which is on the e_s line. For isothermal compression, in which energy is removed from the system by the bath in order to keep the temperature constant, a push on the piston will slightly raise the vapor pressure above e_s and there will be net condensation until equilibrium is obtained again. If we continue to slowly push in the piston, eventually all the cylinder's volume will be filled with liquid water and the cylinder will contain only one phase: liquid. If we continue to push the piston and the bath keeps the temperature constant, then the water pressure will increase.

In the atmosphere, ice or liquid almost always has a surface that is exposed to the atmosphere and thus there is the possibility that water can sublimate or evaporate into this large volume. Note that the presence or absence of dry air has little effect on the condensation and evaporation of water, so it is not the presence of air that is important, but instead, it is the large volume for water vapor that is important.

Conditions can exist in the atmosphere for which the water pressure and temperature are in the liquid or sometimes solid part of the phase diagram. But these conditions are unstable and there will be condensation or deposition until the condensation and evaporation or sublimation and deposition come into equilibrium, just as in the case of the piston above. Thus, more water will go into the liquid or ice phase so that the water vapor pressure drops down to the saturation value. When the water pressure increases at a given temperature to put the system into the liquid region of the water phase diagram, the water vapor is said to be supersaturated. This condition will not last long, but it is essential in cloud formation, as we will see in the lesson on cloud physics.

Note also that the equilibrium line for ice and vapor lies below the equilibrium line for supercooled liquid and vapor for every temperature. Thus $e_{si} < e_s$ for every temperature below 0 ^{o}C because $l_s > l_v$ in the Clausius–Clapeyron Equation. This small difference between e_{si} and e_s can be very important in clouds, as we will also see in the lesson on cloud physics.

3.4: Solving Energy Problems Involving Phase Changes and Temperature Changes

When a cloud drop evaporates, the energy to evaporate it must come from somewhere because energy is conserved according to the 1st Law of Thermodynamics. It can come from some external source, such as the sun, from chemical reactions, or from the air, which loses some energy and thus cools. Thus, temperature changes and phase changes are related, although we can think of phase changes as occurring at a constant temperature. The energy associated with phase changes drives much of our weather, especially our severe weather, such as hurricanes and deep convection. We can quantify the temperature changes that result from phase changes if we have a little information on the mass of the air and the mass and phases of the water.

In the previous lesson, we said that all changes of internal energy were associated with a temperature change. But the phase changes of water represent another way to change the energy of a system that contains the phase-shifter water. So **often we need to consider both temperature change and phase change when we are trying to figure out what happens with heating or cooling.**

For atmospheric processes, we saw that we must use the specific heat at constant pressure to figure out what the temperature change is when an air mass is heated or cooled. Thus the heating equals the temperature change times the specific heat capacity, constant pressure times the mass of the air. For **dry air**, we designate this **specific heat constant pressure as c_{pd}**. For **water vapor**, we designate this **specific heat constant pressure as c_{pv}**. So for example, the energy required to change temperature for a dry air parcel is

$$c_{pd} m \Delta T = c_{pd} \rho V \Delta T \tag{3.4.1}$$

where c_{pd} is the specific heat capacity for dry air at constant pressure. If we have moist air, then we need to know the mass of dry air and the mass of water vapor, calculate the heat capacity of each of them, and then add those heat capacities together.

For liquids or solids, specific heat, constant volume, and specific heat, constant pressure, are about the same, so we have only one for each type of material, including liquid water (c_w) and ice (c_i).

For phase changes, there is no temperature change. Phase changes occur at a constant temperature. So to figure out the energy that must be added or removed to cause a phase change, we only need to know what the phase change is (melting/freezing, sublimating/depositing, evaporating/condensing) and the mass of water that is changing phase. So, for example, the energy needed to melt ice is $l_f m_{ice}$.

Icicles melting. The energy for the phase change from ice to liquid water comes from the air, which must be warmer than freezing. Credit: Liz West via flickr

The following tables provide numbers and summarize all the possible processes involving dry air and water in its three forms.

Specific Heat Capacity at 0 oC (units: J kg^{-1} K^{-1})

Dry air	Water vapor	Liquid water	Ice
c_{pd}	c_{pv}	c_w	c_i
1005	1850	4218	2106

Latent Heat (units: J kg^{-1})

Vaporization @ 0 °C	Vaporization @ 100 °C	Fusion @ 0 °C	Sublimation @ 0 °C
l_v	l_v	l_f	l_s
2.501×10^6	2.257×10^6	0.334×10^6	2.834×10^6

Temperature Change

Dry air	Water vapor	Liquid water	Ice
$c_{pd}\, m_d = c_{pd}\, \rho_d V$	$c_{pv}\, m_v = c_{pv}\, \rho_v V$	$c_w\, m_{liquid}$	$c_i\, m_{ice}$

Phase Change

vapor → liquid	liquid → vapor	vapor → ice	ice → vapor	liquid → ice	ice → liquid
$l_v\, m_{vapor}$	$l_v\, m_{liquid}$	$l_s\, m_{vapor}$	$l_s\, m_{ice}$	$l_f\, m_{liquid}$	$l_f\, m_{ice}$

Note

To solve energy problems you can generally follow these steps:

1. Identify the energy source and write it on the left-hand side of the equation.
2. Identify all the changes in temperature and in phase and put them on the right-hand side.
3. You should know all of the variables in the equation except one. Rewrite the equation so that the variable of interest is on the left-hand side and all the rest are on the right-hand side.

Knowing how to perform simple energy calculations helps you to understand atmospheric processes that you are observing, and to predict future events. Why is the air chilled in the downdraft of the thunderstorm? When will the fog dissipate? When might the sun warm the surface enough to overcome a near-surface temperature inversion and lead to thunderstorms? We can see that evaporating, subliming, and melting can take up a lot of energy and that condensing, depositing, and freezing can give up a lot of energy. In fact, by playing with these numbers and equations, you will see how powerful phase changes are and what a major role they play in many processes, particularly convection.

With the elements in the tables above, you should be able to take a word problem concerning energy and construct an equation that will allow you to solve for an unknown, whether the unknown be a time or a temperature or a total mass.

In the atmosphere, these problems can be fairly complex and involve many processes. For example, when thinking about solar energy melting a frozen pond, we would need to think about not only the solar energy needed to change the pond from ice to liquid water, but we would also need to consider the warming of the land in which the pond rests and the warming of the air above the pond. Further, the land and the ice might absorb energy at different rates, so we would need to factor in the rates of energy transfer among the land and the pond and the air.

So we can make these problems quite complex, or we can greatly simplify them so that you will understand the basic concepts of energy required for temperature and phase changes. In this course, we are going to solve fairly simple problems and progress to slightly more complicated ones. Let’s look at a few examples. I will give you some examples and then you can do more for Quiz 3-3.

? Example Problems

A small puddle is frozen and its temperature is 0 °C. How much solar energy is needed to melt all the ice? Assume that m_{ice} = 10.0 kg.

1. The heating source is the sun and we are trying to calculate the total solar energy. Put this on the left-hand side.
2. The change that we want is the melting of the ice. We know the mass and the latent heat. We write those on the right-hand side.
3. The equation already has the unknown variable on the left-hand side.

$$\int Qdt = l_f m_{ice} = \left(0.334\times 10^6 \mathrm{Jkg^{-1}}\right)(10.0\mathrm{kg}) = 3.34\times 10^6 \mathrm{J}$$

To put this amount of energy into perspective, this energy is equivalent to a normal person walking at about 4 mph for 2 hours (assuming the person burns 400 calories per hour, which is really 400 kilocalories per hour in scientific units).

Now let's assume that the ice is originally at –20.0 °C. Now we have to both raise the temperature and melt the ice. If we don't warm the ice, some of it will simply refreeze. Our equation now becomes:

$$\int Qdt = l_f m_{ice} + c_i m_{ice}\Delta T$$

$$= \left(0.334\times 10^6 \mathrm{Jkg^{-1}}\right)(10.0\mathrm{kg}) + \left(2106\mathrm{Jkg^{-1}K^{-1}}\right)(10.0\mathrm{kg})(0.0 - -20.0)\mathrm{K}$$

$$= 3.76\times 10^6 \mathrm{J}$$

We see that the amount of energy required increased by about 25%. Most of the energy is still required to melt the ice, not change the temperature.

Now let's assume that the solar heating rate is constant at 191 W m^{-2}and that the area of the puddle is 2.09 m^2. How long does it take the sun to raise the temperature of the ice and then melt it?

$$\int Qdt = \frac{\overline{\Delta Q}}{\Delta A} A\Delta t = l_f m_{ice} + c_i m_{ice}\Delta T$$

\Delta t=\dfrac{l_{f} m_{i c e}+c_{i} m_{i c e} \Delta T}{\dfrac{\overline{\Delta Q}}{\Delta A} A}=\dfrac{3.76 \times 10^{6} \mathrm{J}}{\left(191 \mathrm{Jm}^{-2} \mathrm{s}^{-2}\right)\left(2.09 \mathrm{m}^{2}\right)}=9.42 \times 10^{3} \mathrm{s}=2.6 \mathrm{h}

We could now assume that the source of heating is not the sun but instead is warm air passing over the puddle. If the temperature of the air is 20.0 °C and we assume that its temperature drops to 0.0 °C after contacting the ice, what is the mass of air that is required to warm the ice and then melt it?

$$\int Qdt = c_{pd} m_{aii} T_{air} = l_f m_{ice} + c_i m_{ice}\Delta T_{ice}$$

$$m_{air} = \frac{l_f m_{ice} + c_i m_{ice}\Delta T_{ice}}{c_{pd}\Delta T_{air}} = \frac{3.76\times 10^6 \mathrm{J}}{\left(1005\mathrm{Jkg^{-1}K^{-1}}\right)(20.0\mathrm{K})} = 187\mathrm{kg}$$

See this video (2:28) for further explanation:

Melting Ice

Click here for transcript of the Melting Ice Video.

> Let's work a problem about melting a small frozen pond. When we set up the equations, we'll always put our source of heating or cooling on the left, and the things changing temperature or phase on the right. We'll start with the simplest case, and then introduce more information see how to solve the problems. The energy required to melt all the ice is simply the integral of the heating rate over time. That goes on the left side, because it's a source. The change that we're observing is the melting of the ice into liquid water. For this first part, there is no temperature change, just a phase change. We need to know the mass of the ice, and the latent heat of fusion, which tells us about the amount of energy required to convert ice to liquid water. The energy required is about three million joules. Now, let's complicate the problem a little more. Let's start with the ice at a temperature of minus 20 degrees C, but we are still interested in the energy required to melt the ice. Now, we need more energy. Well, we'll need to first raise the temperature of the ice to 0 degrees from minus 20 degrees C, and then we can melt it. So only two terms in the equation. So we add the second term, which accounts for the energy required to raise 10 kilograms of ice by 20 degrees C. Takes another million joules. Now we'll specify that the solar heating rate was 200 watts per meter squared. And now we know what the area of the puddle is. So the integral of the heating rate is just the average heating rate times time, if the heating rate is constant. And so that gives us a way to figure out how long it would take to melt all the ice. We write down the equation for heating on the left, and the changes on the right. The changes, remember, include both the melting of the ice, and the raising of the ice temperature from minus 20 degrees C to 0. Then we rearrange the equation so that only the times remain on the left. When we do this, the answer is that it takes about three hours

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3.5: The Skew-T Diagram- A Wonderful Tool!

The skew-T is widely used in meteorology to examine the vertical structure of the atmosphere as well as to determine which processes are likely to happen.

Need a refresher?

Check out this video (1:23):

Skew-T Basics

Click here for transcript of the Skew-T Basics video.

> PRESENTER: Let's go over some Skew-T basics just to make sure everyone's on the same page. What is a Skew-T provided by UCAR. Lines will be the same as those on Skew-Ts produced by other organizations. But the colors may be different. The horizontal blue lines are pressure levels in millibar [INAUDIBLE]. The temperature lines are tilted 45 degrees to the right and are in degrees C. Dry adiabats, which indicate constant potential temperature, are the red lines, curving upward to the left. And they are marked in degrees Kelvin. The moist adiabats, which indicate the temperature change of the saturated air parcel, are the green dot dash lines, curving to the left and eventually become parallel to the dry adiabats. They are marked in degrees C. The saturated water vapor mixing ratio is plotted with golden dash lines and has units of grams per kilogramme of dry air. Of course, this is just a water vapor mixing ratio, it's the temperature is higher than the dew point. Off to the right is the wind speed and direction at different altitudes. Plotted on this diagram are the temperature, the red solid line, and the dew point, the green solid line. Both taken from a radio [INAUDIBLE].

You know a little about the skew-T from your previous study, but for those who did not take a previous course or who need a refresher, there are many useful websites that can help you understand the skew-T and how to use it. Two useful resources are the following:

Weatherprediction.com Review of Skew-T Parameters

Introduction to Mastering the Skew-T Diagram Video

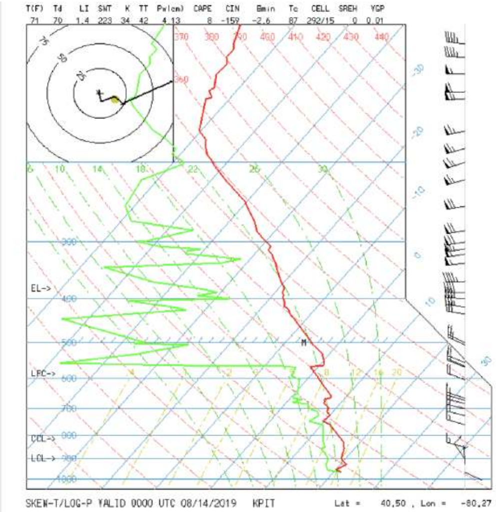
In this video (1:24) I will show you how the skew-T relates to a cumulus cloud:

Skew-T and Cloud video

Click here for transcript of the Skew-T and Cloud Video.

> Here's a picture of a mature cumulus cloud over the ocean. We can see the cloud base here, the vertical growth, and the cloud top here. Above and below the cloud is clear air. We can imagine what temperature and dew point are radius on record if you were to launch one from below the cloud. Initially we would see a temperature decrease, probably close to the dry adiabatic lapse rate of 10 degrees c per kilometer. We will see the dew point decrease slightly relative to the temperature which is skewed to 45 degrees on the Skew-T diagram. At cloud base temperature and dew point are about the same. Inside the cloud the temperature and dew point stay together along the moist adiabat, which is a temperature decrease of about six degrees c per kilometer. Remember that the relative humidity is about 100% in the clouds. The air above the cloud is likely stable, which is why the cloud's height is limited. Stable air has a lapse rate that is less than the adiabatic lapse rate. In addition the dew point likely drops off because the middle to upper troposphere tends to be drier than the lower troposphere. When you look at an upper air sounding you can often pick out where the clouds are by looking at where the temperature and dew point get close together.

First, familiarize yourself with all of the lines. Look at a radiosonde ascent, such as the one from the National Center for Atmospheric Research Research Applications Laboratory (type of plot: GIF of skew-T). The atmospheric sounding line to the right (higher temperature) is the atmospheric temperature. The line to the left (lower temperature) is the dewpoint temperature and at the same time is the water vapor mixing ratio, since $w = w_s(T_d)$. If you are unsure about all the other lines, refer back to your notes or look it up online.



Skew-T diagram for Pittsburgh PA on April 28, 2015 at 0000 UTC.

Credit: NOAA

Please also note the following:

- The dry adiabat is the same line as an isentrope (curved red dash lines tilting to the upper left).
- The water vapor mixing ratio is the saturation water vapor mixing ratio at the dewpoint temperature, T_d, for each pressure level (gold dot-dash lines tilting to the upper right).
- In clear air, for air parcels moving vertically:
    - air parcels move along the dry adiabat and the potential temperature remains constant, even if they contain moisture;
    - the water vapor mixing ratio is constant (but notice that $T_d$changes!);
    - T_d of an air parcel moving vertically (and adiabatically) is decreasing, but not as fast as T if that air parcel is decreasing.
- Eventually, an air parcel moving vertically (along the dry adiabat) will have a temperature and dewpoint temperature that are the same, thus saturated.
- At this altitude level, called the Lifting Condensation Level (LCL), the relative humidity = 100%, $T = T_d$, and $w = w_s$, and $e = e_s$. At this pressure and temperature, a cloud forms. Actually, the formation of a cloud requires a relative humidity that exceeds 100% by a few tenths of a percent, but generally use 100% for the skew-T calculations. We will see why this extra relative humidity is necessary in the next lesson.

? See the video below (1:19) for further explanation:

Finding LCL

Click here for transcript of the Finding LCL Video.

> Let's see how to find the lifting condensation level, the LCL. The LCL's the level where a cloud will form if the air mass near a surface is pushed upward. We have two quantities that are conserved when lifted. The potential temperature, or theta, and the water vapor mixing ratio, w. As the air parcel is pushed up, then w goes up the constant w line and theta goes up the dry adiabat, which is the constant theta line. Note that both the temperature and the dew point temperature are changing and getting closer to each other as the air parcel ascends. When the two lines meet, the relative humidity is 100%, a cloud forms, and this is the lifting condensation level. Once the LCL has been reached and the cloud is formed, any further ascent will be in the cloud. The air parcel temperature will follow the moist adiabat, which is less than the dry adiabat. Because as water condenses it gives up its energy to warm the air a little bit. If the air were pushed down, its temperature would follow the moist adiabat, as long as it was above the LCL. But below the LCL, it will follow the dry adiabat. And the water vapor mixing ratio will follow the constant w line.

Moist Adiabat

When the air parcel is in a cloud, ascent causes a temperature decrease while the air remains saturated (i.e., $w=w_s$, RH=100%). Since $w_s$decreases, the amount of water in the vapor phase decreases while the amount in the liquid or solid phase increases, but the total amount of water is constant (unless it rains!). As water vapor condenses, energy is released into the air and warms it a little bit. Thus, the lapse rate of the moist adiabat (curved dot-long-dash green lines tilting toward the upper left) is less than the lapse rate of the dry adiabat (9.8 K/km).

As long as it doesn’t rain or snow, an air parcel will move up and down a moist adiabat as long as it is in a cloud and will move up and down a dry adiabat when $w < w_s$ below the LCL.

- Once a cloud forms, any further rise of the air parcel will follow the moist adiabat (condensation of water vapor heats the air so that the temperature decrease with height is less than the dry adiabat). As long as the ascent is in the cloud, the relative humidity will remain near 100% and $\mathrm{w} = \mathrm{w_s(T)}$. Since T decreases on ascent, $\mathrm{w_s}$ decreases, and more water goes into the liquid or ice phase.
- If the air parcel descends, it will descend along the moist adiabat until it reaches the LCL in temperature and more of the water evaporates or sublimates into the vapor phase. Just below the LCL, all of the water will be vapor and the air parcel temperature will descend down the dry adiabat and the water vapor mixing ratio will be constant.

The following is a summary for air parcel ascent and descent:

- Find initial p, T (or w_s), and T_d (or w).
- Move the parcel up the dry adiabat that intercepts T.
- Move w up the constant w line. Note that T_d is continually changing, so use w.
- Where the two lines intercept is the Lifting Condensation Level (LCL).
- A cloud will form.

- If the air parcel continues to rise inside the cloud,
    - w will always equal w_s.
    - the air parcel will follow the moist adiabat.

- If the parcel then descends,
    - it will follow the moist adiabat down to the LCL.
    - it will follow the dry adiabat below that.
    - w will follow the w line below that.

The following video (1:43) discusses The process of adiabatic cooling and heating.

Adiabatic Heating and Cooling

Click here for transcript of the Adiabatic heating and cooling video.

We've all seen clouds build up on one side of the mountain and then on the other side just dissipate into blue sky. Maybe you want to know why that happens. Like most natural events, this one has an impressive scientific term attached to it. It's called adiabatic cooling and heating, and occurs because of changes in air pressure. Here's some time lapse video that shows what happens. Basically, as a parcel of air encounters a mountain, it is forced upward. As air pressure decreases with

> altitude, the air parcel expands. Expansion causes air to cool. When the air cools to its so-called dew point, the water vapor in the air condenses and becomes visible as a cloud. If there's enough moisture and the adiabatic cooling is strong enough, it rains or snows. Essentially the opposite occurs on the other side of the mountain. The cool air sinks and compresses. Compression results in increased temperature. When temperature rises above the dew point, the cloud dissipates into invisible water vapor. In Wyoming, especially in winter, most of the moisture-laden air masses come from the Pacific, approaching our mountains from the west. So as adiabatic cooling occurs, more rain and snow is dumped on west-facing slopes. As warmer, drier air descends on the eastern slopes, it accounts for another famous phenomenon of the plains, the so-called Chinook winds. So we've looked at clouds from both sides now. Knowing why they form and disappear does not diminish their beauty. But if it weren't for our mountains and the dynamic processes that occur, one, we would be a much drier place, and frankly, much less interesting. I'm Tom Hill from the University of Wyoming Cooperative Extension Service exploring the nature of Wyoming.
>
> *Credit: UWyo Extension*

Other Potential Temperatures

There are other potential temperatures that are useful because they are conserved in certain situations and therefore can help you understand what the atmosphere is doing and what an air parcel is likely to do.

Virtual Potential Temperature

Virtual potential temperature is the potential temperature of virtual temperature, where density differences caused by water vapor are taken into account in the virtual temperature by figuring out the temperature of dry air that would have the same density:

$$\theta_v=\left(\frac{p_o}{p}\right)^{(v-1)/r}T_v=\left(\frac{p_o}{p}\right)^{(v-1)/r}T(1+0.61q) \tag{3.5.1}$$

This quantity is useful when comparing the potential temperatures (and thus densities) of air parcels at different pressures.

Wet Bulb Potential Temperature

The wet-bulb temperature is the temperature a volume of air would have if it were cooled adiabatically while maintaining saturation by liquid water; all the latent heat is supplied by the air parcel so that the air parcel temperature when it descends to 1000 hPa is less than its temperature would be had it descended down the dry adiabat.

The wet bulb temperature at any given pressure level is found by finding the LCL and then bringing the parcel up or down to the desired pressure level on the moist adiabat.

The wet bulb potential temperature, Θw, is the wet bulb temperature at p=1000 hPa.

How can we use the wet bulb potential temperature? The wet bulb potential temperature is conserved, meaning it does not change, when an air mass undergoes an adiabatic process, such as adiabatic uplift or descent. If we consider large air masses that acquire similar temperature and humidity, then this entire air mass can take on the same wet bulb potential temperature. Colder, drier air masses will have a lower Θw. The Θw of this air mass can change if a diabatic process occurs, such as a cold air mass moving over warm land and warming, or the air mass cooling by radiating to space during the night, but these processes can sometimes take days. So an 850-mb map of Θw is one indicator of air masses and the fronts between air masses.

> See the video below (:32) for further explanation:

Finding Wetbulb Θ

Click here for transcript of the Finding Wetbulb Θ Video.

> Let's see how to find the wet-bulb potential temperature on the Skew-T. The first step is to find the LCL. Once we find the LCL, then we have a saturated air parcel. And it's temperature is the wet-bulb temperature. To find the wet-bulb potential temperature, we simply follow the moist adiabat down to a pressure of 1,000 millibar. We see that the wet-bulb potential temperature is about 19 C, while the potential temperature is about 34 C.

Equivalent Potential Temperature

The equivalent potential temperature is potential temperature that an air parcel would have if it were lifted to the LCL, then lifted along the moist adiabat all the way to the stratosphere so that all the water vapor condensed into liquid, and then lost all the condensed water, and returned down to 1000 hPa along a dry adiabat. Equivalent potential temperature accounts for the effects of condensation or evaporation on the change in the air parcel temperature.

Every 1 g/kg (g water vapor to kg of dry air) causes Θ_e to increase about 2.5K. So, a moist air parcel with w = 10 g kg^{-1}, which is not uncommon, will have Θ_e that is 25K greater than Θ.

Approximately,

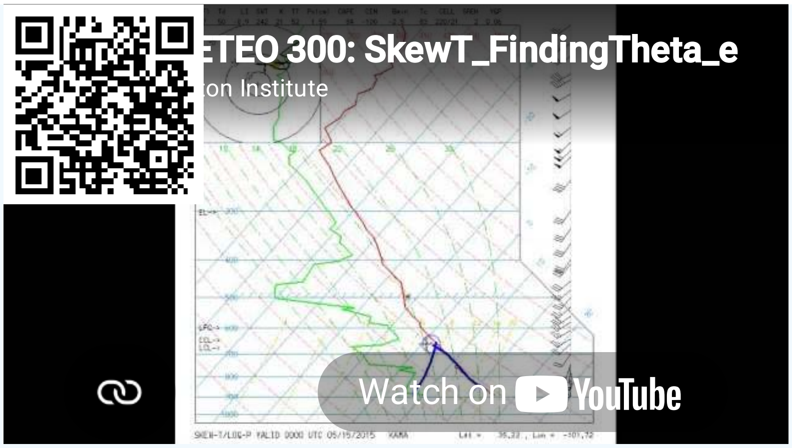
$$\theta_e \approx \theta + \frac{l_v w}{c_p} \tag{3.5.2}$$

Where Θ is the potential temperature, l_v is the latent heat of vaporization, w is the water vapor mixing ratio, and c_p is the specific heat capacity, constant pressure.

How can we use the equivalent potential temperature? The equivalent potential temperature, Θ_e is conserved when an air parcel or air mass undergoes an adiabatic process, just like the wet bulb potential temperature, Θw, is. Note also the total amount of water in vapor, liquid, and ice form is also conserved during adiabatic processes. So, if we look at Θ_e and total water, we can learn a lot about the history of an air parcel. These conserved quantities are very useful to understand the history of air parcels around clouds. For example, if Θ_e changes but the total water mixing ratio is constant, then the air parcel was either heated or cooled by a non-adiabatic process. On the other hand, if both $\mathrm{\Theta_e}$ and $\mathrm{w_t}$ change proportionally, then two air parcels with different initial values for Θ_e and $\mathrm{w_t}$ have mixed. On a larger, more synoptic scale, gradients in Θ_e can be used to indicate the presence of fronts.

Another use of Θ_e is as an indicator of unstable air. Air parcels that have higher Θ_e tend to be unstable. Thus regions of high Θ_e air are regions where thunderstorms might form if the surface heating is great enough to erase a temperature inversion.

See the video (1:01) below for further explanation:



Finding Θ_e

Click here for transcript of the Finding Θ_e video.

> Let's see how to find the equivalent potential temperature, called theta-e, on this Skew-T. The equivalent potential temperature is the potential temperature that an air parcel would have if all this water vapor were converted to liquid water, thus warming the air. And then the liquid water was removed. To find theta-e, we find the LCL. Go up to the moist adiabat until it's parallel with the dry adiabat. And then go down the dry adiabat that matches the moist adiabat until we reach the pressure of 1,000 millibar. In this case, theta-e is about 330 Kelvin, or 57 degrees C. Note that the lines aren't marked with such high temperatures. But we can determine which temperature this line represents by looking at the 360 Kelvin dry adiabat. And then counting one, two, three lines over, where the lines are in intervals of 10 K.

Quiz 3-4: Using the skew-T.

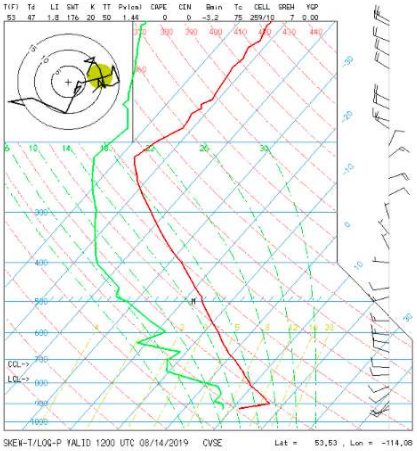
1. Find **Practice Quiz 3-4** in Canvas. You may complete this practice quiz as many times as you want. It is not graded, but it allows you to check your level of preparedness before taking the graded quiz.
2. When you feel you are ready, take **Quiz 3-4**. You will be allowed to take this quiz only **once**. Good luck!

3.6: Understanding the Atmosphere’s Temperature Profile

Now we can begin to understand the reasons for the troposphere’s typical temperature profile. The atmosphere is mostly transparent to the incoming solar visible radiation, so Earth’s surface warms, and thus warms and moistens the air above it. This warm, moist air initially rises dry adiabatically, and then moist adiabatically once a cloud forms. Different air masses with different histories and different amounts of water mix and the result is a typical tropospheric temperature profile that has a lapse rate of (5-8) K km^{-1}.

If atmospheric temperature profiles were determined only by atmospheric moisture, drier air masses would have lapse rates that are more like the dry adiabatic lapse rate, in which case we would expect that the skies would have fewer, thinner clouds. Moister air masses would have lapse rates that are closer to the moist adiabatic lapse rate, resulting in a sky filled with clouds at many altitudes.

But many processes affect the temperature of air at different altitudes, including mixing of air parcels, sometimes even from the stratosphere, and rain and evaporation of rain. Exchange of infrared radiation between Earth’s surface, clouds, and IR-absorbing gases (i.e., water vapor and carbon dioxide) also plays a major role in determining the atmosphere’s temperature profile, as we will show in the lesson on atmospheric radiation.The resulting atmospheric profiles can have local lapse rates that can be anywhere from less than the moist adiabatic lapse rate to greater than the dry adiabatic lapse rate. Look carefully at the temperature profile below. You will see evidence of many of these processes combining to make the temperature profile what it is.



Skew-T diagram at Edmonton, AB, Canada, on April 28, 2015 at 0000 UTC. Diagram from NOAA public data.

Credit: NCAR

If we average together all of these profiles over the whole year, we can come up with a typical tropospheric temperature profile. According to the International Civil Aviation Organization (Doc 7488-CD, 1993), the standard atmosphere has a temperature of 15^{o}C at the surface, a lapse rate of -6.5^{o}C from 0 km to 11 km, is constant from 11 km to 20 km, and then has a positive lapse rate of 1^{o}C from 20 km to 32 km in the stratosphere. Even though this standard profile is a good representation of a globally averaged profile, it is unlikely that such a temperature profile was ever seen with a radiosonde.

Combining knowledge of stability along with the knowledge of moist processes enables us to understand the behavior of clouds in the atmosphere. The following picture of water vapor released from a cooling tower at the Three-Mile Island nuclear reactor near Harrisburg, PA shows the water vapor quickly condensing to form a cloud. The cloud ascends, but then reaches a level at which its density matches the density of the surrounding air. The cloud then stops ascending and begins to spread out.

Water vapor plume rising from the Three-Mile Island nuclear power plant near Harrisburg, PA. The mushroom shape is due to the temperature profile in the lowest part of the troposphere.

Credit: W. Brune

3.7: Summary and Final Tasks

Water vapor is a key atmospheric constituent that is essential for weather. There are many ways to express and measure the amount of atmospheric water vapor – specific humidity, water vapor mixing ratio, partial pressure, relative humidity, and dewpoint temperature – and these are all related and can be used interchangeably, although some provide more physical insight than others depending on the question being asked. Water’s most important characteristic in the atmosphere is that it can change phases between vapor, liquid, and ice. In the atmosphere, water is either in the vapor phase or trying to establish an equilibrium between vapor and liquid or vapor and ice. The equilibria conditions are given by the Clausius-Clapeyron equation, which shows that the equilibrium (a.k.a saturation) water vapor pressure depends only upon the temperature. Water phase changes pack a big energy punch and drive weather events. We can calculate the atmospheric temperature changes resulting from phase changes and then see that these temperature changes greatly affect the buoyancy of air parcels and therefore their vertical motion.

A good way to visualize atmospheric vertical structure and behavior is the skew-T diagram. With it, we can readily deduce atmospheric properties and predict what weather is likely to happen if solar heating causes some air near the surface to ascend. Some of the most important properties found using the soundings on the skew-T are the lifting condensation level, the potential temperature, and the equivalent potential temperature. The behavior of a typical sounding on the skew-T shows that the troposphere’s thermal structure is caused in large part by adiabatic ascent and descent, although we will see later that absorption and emission of infrared radiation by water vapor and carbon dioxide also have a hand in shaping the temperature vertical profile.

Reminder - Complete all of the Lesson 3 tasks!

You have reached the end of Lesson 3! Make sure you have completed all of the activities before you begin Lesson 4.

CHAPTER OVERVIEW

4: Atmospheric Composition

Learning Objectives

By the end of this chapter, you should be able to:

- explain the role that each atmospheric constituent plays in atmospheric structure and weather
- identify changes in minor and trace gas amounts and the impacts these changes have on the atmosphere
- explain how the atmosphere cleanses itself using methane as an example
- use chemical equations to show how ozone is formed in the stratosphere and the troposphere and how they differ
- diagram the lifecycle of aerosol particles with an emphasis on their role in weather

The atmosphere consists mostly of dry air - mostly molecular nitrogen (78%), molecular oxygen (21%), and Argon (0.9%) - and highly variable amounts of water vapor (from parts per million in air to a few percent). Now we will consider gases and particles in the atmosphere at trace levels. The most abundant of the trace gases in the global atmosphere is carbon dioxide (~400 parts per million, or 400 x 10^{-6}), but there are thousands of trace gases with fractions much less than a few parts per million. Some, particularly the reactive hydroxyl (OH) radical, are important even though their abundance is less than 1 part per trillion (10^{-12}). The atmosphere also contains small particles with sizes from nanometers (10^{-9} m) to microns (10^{-6} m) coming from many sources. These trace gases and particles are as important to atmospheric structure and weather as are nitrogen, oxygen, and water vapor and they also play a huge role in human and ecological health and global climate. In this chapter, we will examine the atmosphere's composition and its changes over time. The atmosphere is continually inundated with surface emissions of gases and particles (and some from space) but it has chemical mechanisms to clean itself. We will see how two atmospheric pollutants - ozone and small particles - are produced. In later lessons, we will see that without these chemical processes and particles, there would be no clouds and, thus, no real weather.

4.1: Atmospheric Composition

The major gases that comprise today's atmosphere are in Table 4.1.1. The mixing ratio of a gas X is defined as the fraction of total moles that are the moles of gas X. For instance, 78 moles of every 100 total moles of air is nitrogen, so nitrogen's mixing ratio is 0.78. Note that in atmospheric composition, the mixing ratio is the *moles* of the gas divided by the *total moles of air*. In contrast, the water vapor mixing ratio is the *mass* of water vapor divided by the *mass of dry air*.

Table 4.1.1: Major Constituents in Earth’s Present Atmosphere

Constituent	Molecular Mass (g/mol)	Mixing Ratio (mol mol^{-1})	Role in the Atmosphere
nitrogen (N_2)	28.013	0.7808	transparent; provides heat capacity and momentum; exchanged with biomass; decomposed in combustion
oxygen (O_2)	31.998	0.2095	transparent except for in extreme ultraviolet; provides some heat capacity and momentum; exchanged with life; source of important reactive gases like ozone
argon (Ar)	39.948	0.0093	no role
carbon dioxide (CO_2)	44.010	0.000385 (385 ppmv)	transparent in visible; absorbs infrared light (i.e., contributes to global warming); exchanged with life; product of combustion
neon (Ne)	20.183	0.0000182	no role, but makes colorful glowing signs
water vapor (H_2O)	18.015	$2\text{x}10^{-6}$ to 0.05	gas transparent in visible; absorbs infrared light (i.e., contributes to global warming); exists as vapor, liquid, and solid; exchanged with life; product of combustion
aerosol particles	varies	0-500 ug m^{-3} (note different units)	essential for cloud formation; interact with visible and infrared light; exchanged with surfaces and life
methane (CH_4)	16.04	0.00000182 (1820 ppbv)	transparent in visible; absorbs in infrared (i.e. contributes to global warming); exchanged with life; source of CO_2 and H_2O
ozone (O_3)	48.00	0.01 – 10 ppm	transparent in visible; absorbs in UV and infrared; reactive and source of more reactive gases
particles	varies	0-100’s μg m^{-3} of air	absorbs and scatters light; acts as CCN and IN (see below)

Key features of the gases include their compressibility (i.e., ability to expand or shrink in volume), their transparency in the visible, their momentum, and their heat capacity. Water vapor has the additional important feature of existing in the vapor, liquid, and solid phases in the atmosphere and on Earth’s surface. The most important properties of small particles include their ability to dissolve in water in order to be **Cloud Condensation Nuclei (CCN)** or to maintain a lattice structure similar to ice in order to be Ice Nuclei (IN), as well as their ability to absorb and scatter sunlight. These properties depend completely on the particle size and composition. Most atmospheric gases participate in the atmosphere's chemistry, which is initiated by sunlight, as you will soon see.

Units used when quantifying atmospheric composition

Three different units are typically used when specifying the amounts of gases. One is the mass mixing ratio, which is the mass of a chemical species divided by the total mass of air. You have already encountered this with the specific humidity of water vapor. A second is the volume mixing ratio, which is just the number of molecules of a chemical species in a unit volume divided by the total number of all molecules in a unit volume. For gases with relatively large fractions like nitrogen, oxygen, and argon, we use percent to indicate this fraction. For minor gases like carbon dioxide and ozone, we use parts per million (10^{-6}) ppmv or parts per billion (10^{-9}) ppbv by volume (meaning by number not mass). Lastly we need to use the concentration, or number per unit volume, to calculate reaction rates and lifetimes.

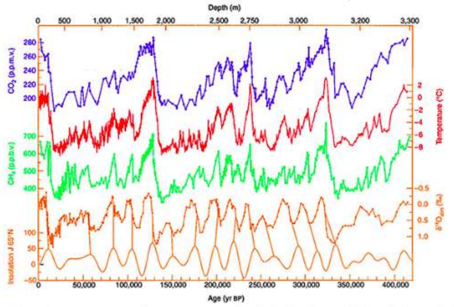
To convert between volume mixing ratios and concentrations, use the following procedure. For a species X, to convert from a mixing ratio, notated χ_X, to a concentration, notated [X], use the Ideal Gas Law to find the number of total molecules in a cm^3 and then multiply by χ_X, expressed as a fraction. Suppose p = 960 hPa (or mb) and T = 296K, and X = 60 ppbv, then

$$\begin{aligned}[X] = \frac{p}{kT}\chi_X = \frac{96000 Pa}{\left(1.38\times 10^{-23} JK^{-1} \text{ molecule }^{-1}\right)(296K)}\left(\frac{1m^3}{10^6 cm^3}\right) 60x10^{-9} \\ = 1.4\times 10^{12}\, moleculescm^{-3}\end{aligned} \tag{4.1.1}$$

4.2: Changes in Atmospheric Composition

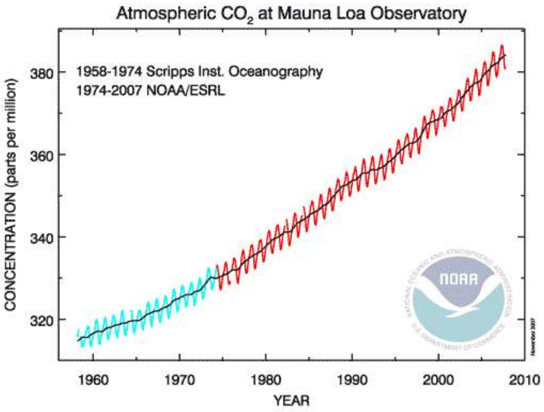
Since the rise of oxygen, 2 billion years ago, the nitrogen and oxygen fractions in the atmosphere have been stable. Water vapor is highly variable but, on average, appears to also have been fairly stable. Recent data from satellites and sondes indicate that perceptible water (the total amount of water that is in a column from the surface to space) has increased 1.3 ± 0.3% per decade over the oceans in the past 25 years (Trenberth et al., Climate Dynamics, 2005).

Historical (up to 500 years before the present) changes in CO_2 and CH_4 show large, rapid variations. Note that the historical range for CO_2 is 200-300 parts per million (10^{-6}), and for methane is 350-400 parts per billion (ppbv)**.** These changes in gas amounts have been driven by changes in Earth's temperature, which come from changes in Earth's orbit, the axis of Earth's rotation, and volcanoes. Until recently, changes in Earth's temperature caused changes in these gas amounts, which then reinforced the warming. In the past century, changes in the gas amounts have been driving the observed change in Earth's temperature.



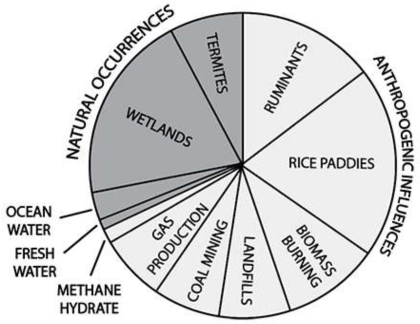
Changes in carbon dioxide, methane, and temperature over the past half million years. As measured from gases trapped in an ice core at Vostok, Antarctica. Credit: "Vostok 420ky curves insolation". Licensed under Public Domain via Wikimedia Commons

The recent changes in carbon dioxide show a fairly constant increase over the past 50 years. There is a smaller seasonal cycle imposed on this trend. This seasonal behavior occurs because CO_2 is taken up by plants in the northern hemisphere in summer, since most of the plants are in the northern hemisphere. Note that the current increase to above 400 ppm now extends well above any other time in the past half million years. Much of this CO_2increase can be linked to fossil fuel combustion. We will examine the scientific consequences of these CO_2 levels in the lesson on applications of radiation.

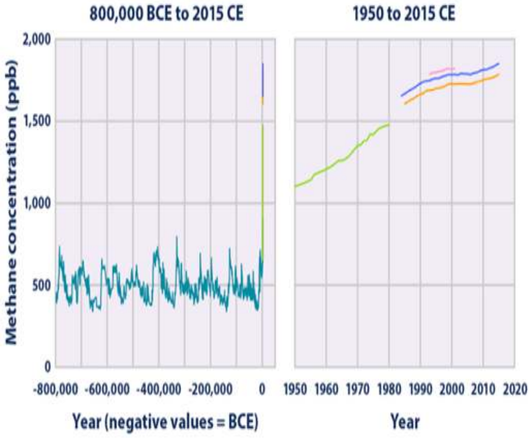


Atmospheric CO_2 mixing ratios measured at Mauna Loa, Hawaii for more than 50 years. Credit: NOAA

Another important trace gas is methane (CH_4), which is often called natural gas when it is used to produce power and heat. Methane has many sources, some of them natural and some of them anthropogenic (meaning human-made). Its atmospheric mixing ratio is now greater than 1800 parts per billion (ppb, or a fraction of 10^{-9} of air). We see from the figures below that almost 2/3 of atmospheric methane sources are anthropogenic and that methane measured at Hawaii, like all other locations, has been increasing except for the early to mid 2000s. This slowdown is not understood, but since the late 2000s, methane has been increasing again.



Sources of atmospheric methane: Freshwater, ocean water, wetlands, termites, ruminants, rice paddies, biomass burning, landfills, coal mining, gas production, methane hydrate. Ruminants are cattle, sheep, goats, etc. 2/3 of the total is due to human activities.



Changes in methane. Left, since 800,000 years ago. Right, since 1950. Note the tripling in the past 100 years, well above any levels since 800,000 years ago. The decade-long slow-down in the methane increase from 2000 to 2010 is not well understood. Credit: EPA

There are trends and variations in many of the other trace gases as well. Some others, like nitrous oxide (N_2O), are increasing, while others, like human-made chlorofluorocarbons (CCl_2F_2, CCl_3F) are decreasing. There are other trace gases that increase as the sun rises and decrease as it sets and are heavily involved in atmospheric chemistry. We will talk about these gases next.

4.3: Other Trace Gases

Hundreds of different trace gases have been measured in the atmosphere and perhaps thousands more have yet to be measured. Many of these are **volatile organic compounds (VOCs)**. *Volatile* means that the compound may exist in the liquid or solid phase but that it easily evaporates. *Organic* means that the compound contains carbon but is not carbon dioxide, carbon monoxide, or carbides and carbonates found in rocks. There are also other chemicals like the nitrogen oxides (e.g., nitric oxide (NO), nitrogen dioxide (NO_2), nitric acid (HNO_3)), sulfur compounds (e.g., sulfur dioxide (SO_2), sulfuric acid (H_2SO_4)) and halogen compounds (e.g., natural methyl chloride (CH_3Cl), human-made chlorofluorocarbons (CCl_2F_2)). If we pay attention, we can often smell and identify many of these chemicals, even at trace levels, although some, like methane, carbon monoxide (CO), and chlorofluorocarbons are odorless. We enjoy smelling the VOCs emitted by trees in a forest – aah, that fresh pine smell – but we hold our nose to escape the smells of a stagnant swamp.

In addition to these thousands of chemicals that are emitted into the atmosphere every day, there are also some very reactive compounds that are created by atmospheric chemistry and play the important role of cleaning the atmosphere of many gases. The most important reactive gases are ozone (O_3) and hydroxyl (OH). We will focus the discussion of atmospheric chemistry on these two.

The Atmosphere’s Oxidizing Capacity

Earth’s atmosphere is an *oxidizing environment*. This term means what you think it would: gases that are emitted into the atmosphere react in a way that increases their oxygen content. Gases that contain oxygen tend to be “stickier” on surfaces and more water soluble, which means that they stick when they hit a surface or they can be readily taken up in clouds and rain drops and be deposited on Earth’s surface. We call gases hitting the surface and sticking “dry deposition” and gases being taken up in precipitation and rained out “wet deposition.”

Let’s consider a natural gas that is very important in our lives – methane (a.k.a., natural gas). More and more methane is being extracted from below Earth’s surface and used to run our electrical power plants, heat our homes, cook our food, and, increasingly, to run our transportation vehicles. Methane is a simple molecule – CH_4 – in which each of carbon’s four bonds is made with a hydrogen atom. Energy comes from heating methane to high enough temperatures that cause it to react, giving off energy as more stable molecules are formed. In complete combustion, each methane molecule is converted into CO_2 and two H_2O. In the process, four oxygen atoms or two oxygen molecules are consumed.

This same process occurs in the atmosphere, but at much lower temperatures and at a much slower rate. In both cases, the first step in the methane oxidation sequence is the reaction with the hydroxyl radical (OH). In water, hydroxyl loses an electron and is ionized (OH^-), but in the atmosphere, hydroxyl is not ionized. We call OH a free radical because it has an odd number of electrons (eight for oxygen and one for hydrogen). Any gas with an odd number of electrons is reactive because the electrons want to be paired up in molecules because that makes them more stable.

Often, combustion is inefficient, resulting in the formation of carbon monoxide (CO). Examples include forest fires, humans burning fields to clear them for planting, poorly tuned vehicles, inefficient industrial processes, and other human-caused processes. The primary way that CO is removed from the atmosphere is by reacting with atmospheric OH. It takes a while for CO to be removed from the atmosphere by the reaction with OH, so that satellite instruments can track CO plumes as they emerge from their sources and flow around the world.

Where does OH come from?

Before we tackle this question, let’s first look at where ozone (O_3) comes from. We will start with the stratosphere (a.k.a, good ozone because it blocks solar UV that harms humans, other animals, agriculture, and ecosystems) and then eventually we will consider tropospheric ozone (a.k.a., bad ozone, which is the ozone that hurts our health when we breathe it and that damages plants and their fruit).

Discussion Activity: Trace Gases

I would like you to think about which trace gas is the most important and why. By trace gas I mean a gas with a mixing ratio of less than 20 ppm in the atmosphere. Defend your choice. Use information from this lesson as well as other sources (credit them, please!) to describe the qualities of this gas that make you think that it is the most important trace gas. Then read the choices of your classmates and respond to their choices and follow-up with further questions and/or analysis

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4.4: Stratospheric Ozone Formation

Ozone is ozone no matter where it is in the atmosphere. Good ozone is good only because it is in the stratosphere where we cannot breathe it (Figure 4.4.1). Bad ozone also absorbs solar ultraviolet light, but it is down near Earth's surface where we can breathe it. For UV protection, we are interested in the total number of ozone molecules between us and the Sun. 90% of ozone molecules are in the stratosphere and 10% are in the troposphere - some down near Earth's surface where we can breathe them. There are important issues affecting human and ecological health for both good ozone and bad ozone. For good ozone, the most important issues are the reduction of ozone globally, the **Antarctic Ozone Hole**, and Arctic ozone loss that is caused by chlorofluorocarbons. Reduced ozone means more solar UV gets to the ground causing more skin cancer. For bad ozone, the most important issues include the production of too much ozone in cities and nearby regions that is caused by too many pollutants from traffic, industrial processes, power generation, and other human activities. Increased ozone means more people have respiratory and heart problems. Let's look at both the good and the bad, starting with the stratospheric ozone.

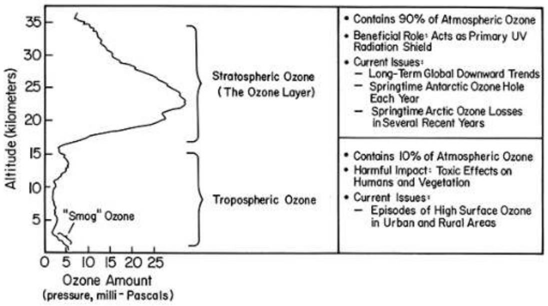


Figure 4.4.1: A cross-section of the typical vertical ozone profile for the tropics. Credit: World Meteorological Organization

Click for a text description of the Cross Section image.

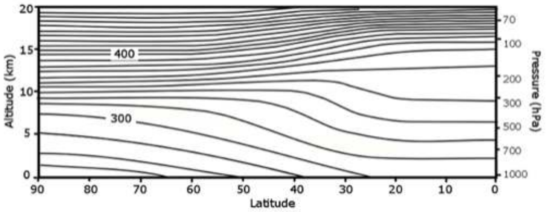
Explanation diagram of the typical vertical ozone profile for the tropics

Stratospheric Ozone (the ozone layer, ~15-35 km) contains 90% of atmospheric ozone beneficial role: acts as primary UV radiation shield Current issues

- long-term global downward trends
- springtime Antarctic ozone hole each year
- springtime arctic ozone losses in several recent years
- episodes of high surface ozone in urban and rural areas

Tropospheric Ozone (0-15 km) contains 10% of atmospheric ozone harmful impact: toxic effects on humans and vegetation current issues

To get the total amount of ozone between us and the Sun, we simply add up the ozone amount starting at the surface and going up to the top of the ozone layer. Note how much more ozone there is in the stratosphere. At higher latitudes, the bottom of the stratospheric ozone layer is at approximately 10-12 km. Recall the following image from Lesson 2:



Potential temperature (solid lines, K) as a function of latitude and altitude. Note that the decrease in potential temperature with height is small in the troposphere and large in the stratosphere. Credit: W. Brune, after Andews, Holton, and Leovy

The process of stratospheric ozone formation starts with ozone (O_3), which is made by ultraviolet sunlight in the stratosphere (but not the troposphere, as we shall see). The two reactions are:

$$O_2 + \text{hard}\,UV \rightarrow O + O \tag{4.4.1}$$

$$O + O_2 + N_2 \rightarrow O_3 + N_2 \tag{4.4.2}$$

Note that N_2 doesn't really react in this last chemical equation, but instead, simply bumps into the O_3 molecule as it is being formed and stabilizes it by removing some of the energy from O_3. We call O_3 an oxidant because it can react with some compounds and oxidize them.

This O_3 can be broken apart by ultraviolet light to make O_2 and O. Usually O combines with O_2 to form O_3 in this way: $O+O_2+N_2 \rightarrow O_3+N_2$, so nothing really happens, except that the solar energy that breaks apart the O_2 ends up as extra energy for the O_3 and for the colliding N_2 and, as a result, ends up warming the air. Sometimes O collides with O_3 and reacts: $O+O_3 \rightarrow O_2+O_2$. Putting all of the reactions together, we can see the chemical lifecycle of ozone in the stratosphere. This set of reactions was proposed in the 1930s by Chapman:

Stratospheric Ozone and Atomic Oxygen Production, Cycling, and Loss

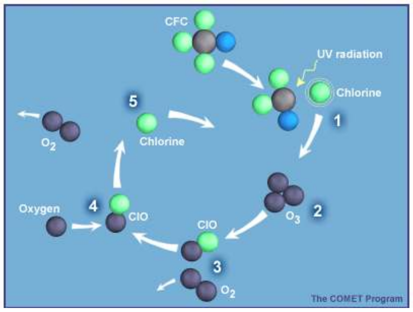
O_2 + hard UV → O + O	production
$2(O + O_2 + N_2 \rightarrow O_3 + N_2)$	cycling
O_3 + UV → O_2 + O	cycling
$O + O_3 \rightarrow O_2 + O_2$	loss
Net: UV → higher T	

These four reactions could produce the basic characteristics of the ozone layer as it was in the 1940s through the 1970s. However, this theory produced peak ozone levels that were 50 milli-Pascals, not the 25-30 milli-Pascals seen in the first figure above. Thus, the measured levels of stratospheric ozone were about half of those predicted by Chapman's theory - it was a real puzzle. However, in the 1970s, scientists proposed new sets of reactions by other gases that accomplished the same results as the loss reaction shown above. A famous example involved chlorine, which comes mostly from human-made chlorofluorocarbons (CFCs):

The Stratospheric Chlorine Catalytic Cycles That Destroy Ozone

CFCs + UV → product + Cl	production
$Cl + O_3 \rightarrow ClO + O_2$	cycling
$ClO + O \rightarrow Cl + O_2$	cycling
$Cl + CH_4 \rightarrow HCl + CH_3$	loss
Net O_3+O: $O_3 + O \rightarrow O_2 + O_2$	

During the cycle, chlorine (Cl) and chlorine monoxide (ClO) aren’t destroyed but instead are just recycled into each other. With each cycle, two ozone molecules are lost (one directly and a second because O almost always reacts with O_2 to form O_3). This cycle can run for hundreds of thousands of times before Cl gets tied up in HCl. So ClO and Cl levels of tens of parts per trillion of air (10^{-12}) are able to destroy several percent of the few parts per million of O_3. Sherry Rowland and Mario Molina figured this cycle out and wrote a paper about it in 1974. They received a Nobel Prize in Chemistry in 1995 for this work. When catalytic cycles involving chlorine, nitrogen oxides, and OH are included with the theory, the agreement between the theory and the measurements gets much better.



The chlorine catalytic cycle that destroys ozone. Credit: UCAR

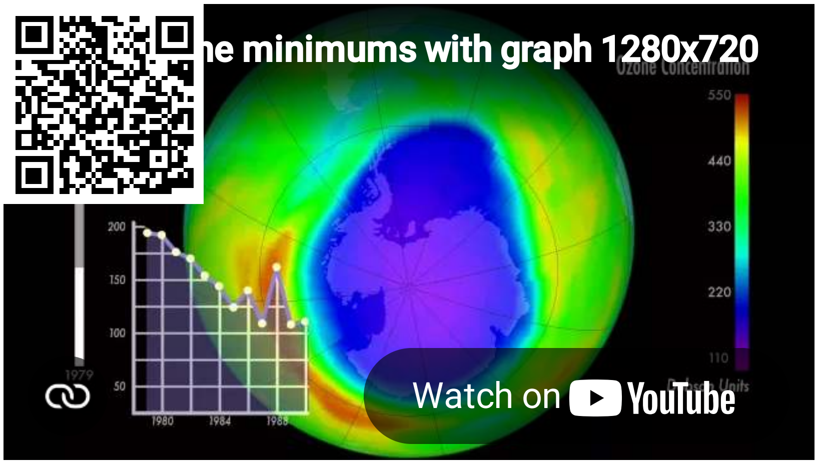
Exercise

Note that the total ozone amount at midlatitudes is greater than the amount in the tropics. This should seem strange to you because the solar UV that is part of the Chapman mechanism is strongest in the tropics. Why do you think that total ozone is distributed this way?

Click for answer.

Answer: The ozone distribution is due to the motion of air through the stratosphere. Air comes from the troposphere into the stratosphere mostly in the tropics and, as it rapidly moves from west to east, it slowly moves from the tropics to near the poles, where it re-enters the troposphere. So most of the ozone is made in the tropics at higher altitudes and then some of this ozone is destroyed by chemical reactions as the ozone is transported poleward and downward. So, while the ozone mixing ratio decreases slightly from tropics to high latitudes, the ozone concentration (moles per volume) increases as it is transported to lower altitudes where the pressure and number of moles is greater, more than twice as large (see the Lesson 2 image above).

The low ozone over Antarctica above is the Antarctic Ozone Hole; the video below (:31) entitled "Ozone Minimums With Graph" (from NASA) shows changes in ozone concentration between 1979 and 2013.



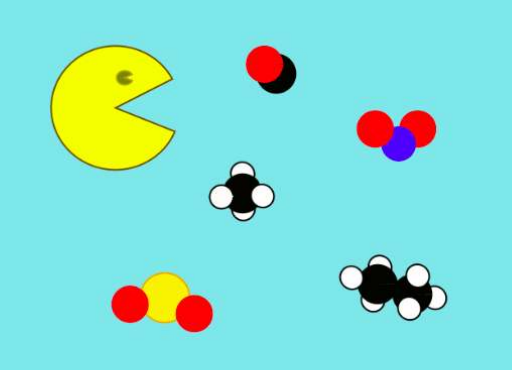
Satellite map of total ozone from September 6, 2004. Note that the total ozone amount is greater at midlatitudes than it is in the tropics, even though the solar UV is most intense in the tropics. The extremely low ozone over Antarctica is the Antarctic Ozone Hole. Credit: NASA GSFC

The Antarctic ozone hole is an extreme example of the destructive power of chlorine catalytic cycles. Different catalytic cycles dominate the ozone destruction over Antarctica and, to a lesser extent, the Arctic. But, when aided by chemistry on the surfaces of naturally occurring polar stratospheric clouds, all the Cl in the form of HCl is liberated so that the polar catalytic cycles are able to destroy a few percent of the ozone per day in a plug the size of Antarctica from an altitude of 12 km all the way up to 20 km.

Fortunately, the amount of chlorine being injected into the stratosphere is decreasing due to the Montreal Protocol, the world’s first international global environmental treaty.

4.5: The Story of the Atmosphere's PAC-MAN

The atmosphere's oxidation capacity is its ability to clean itself of all of the gases that are emitted into it. What does stratospheric ozone have to do with the atmosphere's oxidation capacity, which mostly occurs in the troposphere and mostly by the atmosphere's PAC-MAN, hydroxyl (OH)? It turns out that natural dynamic processes actually pull air down from the stratosphere and mix it into the troposphere, eventually mixing some of this ozone to Earth's surface. This naturally occurring surface ozone provides a baseline value for near-surface ozone, but ozone pollution is more than ten times greater than this baseline in cities. Ozone is both sticky on surfaces and fairly reactive in the atmosphere. It is lost both by depositing on surfaces and through being chemically destroyed by reactions in the atmosphere.



Hydroxyl (OH) is the PAC-MAN™ of the atmosphere. It reacts with thousands of different molecules, including carbon monoxide (CO), nitrogen dioxide (NO_2), methane (CH_4), sulfur dioxide (SO_2), and ethane (C_2H_6). Credit: W. Brune

The following chemical sequences are the humble beginnings of the atmosphere's PAC-MAN. OH is generated throughout the stratosphere and troposphere by a two-step reaction sequence. The first step is:

$$O_3 + UV \rightarrow O_2 + O^* \tag{4.5.1}$$

where O* is an excited-state oxygen atom that has extra chemical energy. O* can lose this extra energy by colliding with N_2 and O_2, but it can also collide with a water molecule to make two OH molecules:

$$O^* + H_2O \rightarrow OH + OH \tag{4.5.2}$$

OH is very reactive. You can think of OH as being water that has had a hydrogen taken away and wants it back. There are other sources for OH, but this one is the most important globally. OH reacts with many other atmospheric constituents. In fact, it is so reactive, that its lifetime in the atmosphere is less than a second.

Another important oxidant is nitric oxide (NO). It comes from combustion (power plants, internal combustion engines, fires) or lightning. In cities, the NO mixing ratio is tens of ppb (10^{-9}, by moles) during morning rush hour and a bit smaller during evening rush hour, but there is typically about a ppb around during the day. In very remote areas, the levels of NO are a hundred times less. NO can react with many other chemicals, but it reacts with O_3:

$$NO + O_3 \rightarrow NO_2 + O_2 \tag{4.5.3}$$

which forms nitrogen dioxide, NO_2. NO_2 is not very stable:

$$NO_2 + \text{nearUV} \rightarrow NO + O \tag{4.5.4}$$

ut the O reacts immediately with O_2 to form ozone:

$$O + O_2 + N_2 \rightarrow O_3 + N_2 \tag{4.5.5}$$

If a NO_2 molecule is produced, then an O_3 molecule will be produced during the day when the sun is out. *Note that if we think of these three reactions as a cycle, no ozone was either created or destroyed because it is destroyed in [4.5] and created in [4.2].*

What happens to all of the methane emitted into the atmosphere?

Methane is a volatile organic compound (VOC). Methane oxidation is a good model for what happens to all of the volatile organic compounds that you smell every day and all the ones that you can't smell. I am not going to show you the entire reaction sequence. Instead, here are just a few steps.

The first step is the reaction between methane and hydroxyl:

$$CH_4 + OH \rightarrow CH_3 + H_2O \tag{4.5.6}$$

Note that water vapor is made and CH_3 is a radical because it has 12+3 = 15 protons and, therefore, electrons. Just as for most other VOCs, and some other trace emissions, the reaction with OH is the main way methane is removed from the atmosphere. Otherwise, it would build up to high abundance.

CH_3 is very reactive. It adds an O_2:

$$\mathrm{CH_3 + O_2 + N_2 \rightarrow CH_3O_2 + N_2} \tag{4.5.7}$$

If there is any NO around, the following reaction happens:

$$\mathrm{CH_3O_2 + NO \rightarrow CH_3O + NO_2} \tag{4.5.8}$$

followed by:

$$CH_3O + O_2 \rightarrow CH_2O + HO_2 \tag{4.5.9}$$

and:

$$HO_2 + NO \rightarrow OH + NO_2 \tag{4.5.10}$$

The chemical CH_2O is formaldehyde. Some of you may have encountered it in high school chemistry or biology and so may be familiar with the smell. You also see that we got the OH molecule back.

Ultimately, formaldehyde gets broken down to CO and the net reaction of methane oxidation is:

$$CH_4 + 2NO + 3O_2 \rightarrow\rightarrow\rightarrow\rightarrow CO_2 + 2H_2O + 2NO_2 \tag{4.5.11}$$

Remember that NO_2 is easily broken apart by the UV sunlight that reaches Earth's surface, so we can take this reaction sequence a step further and show that in the presence of sunlight, reactions [4.6] and [4.2] give:

$$NO_2 + \mathrm{nearUV} + O_2 \rightarrow NO + O_3 \tag{4.5.12}$$

or

$$CH_4 + 5O_2 \rightarrow\rightarrow\rightarrow\rightarrow CO_2 + 2H_2O + 2O_3 \tag{4.5.13}$$

In this final chemical equation, we do not see OH, HO_2, NO, or NO_2, yet they are essential to the formation of ozone. They are catalytic, which means that they are neither created nor destroyed in the reaction sequence, but instead are simply recycled between OH and HO_2 and between NO and NO_2. There are other reactions that destroy these reactive chemicals by producing other chemicals that are much less reactive and sticky, a main one being:

$$OH + NO_2 + N_2 \rightarrow HNO_3 + N_2 \tag{4.5.14}$$

where HNO_3 is nitric acid, a very sticky and water soluble chemical. However, each OH that is produced can typically oxidize more than ten methane molecules before it reacts with NO_2 to form nitric acid. And as reaction [4.13] shows, each time methane is completely oxidized, two O_3 molecules are produced. That's a lot of chemical steps to remember, but I don't want you to necessarily remember them. *I want you to see that the process started with a reaction of OH with a volatile organic compound (in this case methane) and that in the subsequent reactions, the product molecules had more and more oxygens attached to them. This process is why we say that the atmosphere is an oxidizing environment.*

Where does ozone pollution come from?

Ozone is a different sort of pollutant from others because it is not directly emitted by a factory or power plant or vehicle but instead is produced by atmospheric chemistry.

Three ingredients are needed to make ozone pollution: *volatile organic compounds (VOCs)* (like methane); *nitric oxide (NO from combustion)*; and *sunlight*. When we say this, we assume that we already have some ozone and water to provide the OH to get the reactions started. Every VOC goes through an oxidation process that is similar to the methane oxidation reaction sequence. In the methane oxidation sequence, steps [4.9] and [4.11] make NO_2, which in the presence of sunlight makes ozone through step [4.6] followed by step [4.2]. Voila! Ozone is formed from methane oxidation in the presence of nitrogen oxides and sunlight. Now imagine the thousands of volatile organic compounds in the atmosphere and realize that all of them - both anthropogenic and natural - can participate in the production of ozone pollution. Now you have seen the sequence of chemical reactions that produce tropospheric ozone.

Let's look at a video (3:14) entitled "Ground Level Ozone: What Is It?" that explains ozone production without getting into the gory details of the chemistry.

Ground Level Ozone: What is it? Credit: UCARConnect

Click here for transcript of the Ground Level Ozone video.

We're all pretty familiar with what O2 is. I hope so. You need to breathe it to live. Yes, O2 is oxygen, that life-giving gas, but what is O3? O3 is another gas essential to our survival but it's definitely not for breathing. O3 is ozone high up in the stratosphere. It's made naturally and absorbs harmful ultraviolet rays from the Sun. Without it life as we know it wouldn't, couldn't exist. We need the ozone layer in the stratosphere. We want it, we rely on it. But don't get too used to singing ozone's praises. High ozone levels at lower altitudes, what we call the troposphere, where we live and breathe or anything but natural and beneficial. In fact, down here it turns out to be a toxic atmospheric pollutants. Yep, you heard me right. ground-level ozone primarily exists due to human activities that burn fossil fuels. Transportation, our power and industrial plants, and other activities expel nitrogen oxides and hydrocarbons. When those compounds interact with sunlight, voila, ozone is created a contributor to smog. that's why I ozone levels increase during the summer months when sunlight is abundant. Yes, smog love summer just like many of us. We run, bike, hike, fish, play, stroll, oh yeah, and breathe. Yes, the fact that more people are outside when it's warmer makes us particularly vulnerable to Ozone's harmful impacts. Ozone is a harmful oxidant when we inhale it it's like getting a sunburn inside your lungs and it can be particularly serious for the young, old, active, and those with respiratory conditions at any age. And it's not just humans that are vulnerable ozone harms plants, crops, and agricultural yield interfering with pretty important processes like well, photosynthesis and even our economy. To make matters worse ozone production increases with higher temperatures which are occurring more frequently with climate change. The EPA sets national ambient air quality standards for several pollutants in the United States including ground level ozone. When a county is out of compliance they need to know what can be done to improve air quality. and let's not forget that air pollution is a global comments. air pollution is shared from surrounding cities states and also country's halfway around the world. What can we do, what are we willing to do to improve current levels? Drive less, carpool, avoid car idling, set your home's thermostat higher in the summer and lower in the winter, avoid gas powered lawn & garden tools on severe ozone days. There's a lot to do and lots to know about air quality knowing more about the sources and contributors to ozone and other atmospheric pollutants will help us chart our course.

Ozone pollution is bad for the health of people, crops, and forests. Ozone can react with some types of VOCs, including types that make up our lungs, and breathing it can cause serious health problems and even death. Ozone reacts with the VOCs that make up plants and stunts their growth and damages their fruit. The Clean Air Act from the 1970s has dramatically decreased the levels of air pollution in the United States, including ozone. The EPA can take the credit for much of the progress against air pollution in the United States. But there is still a ways to go and the progress may be reversed due to effects of climate change. Since ozone pollution increases at higher temperatures, the increases in global temperatures could actually reverse the steady progress in ozone reduction and ozone pollution could once again increase, unless volatile organic compounds and nitrogen oxides are reduced even more.

Now you can see why OH is called the PAC-MAN of the atmosphere. But how can we tell how long it will take for OH to remove from the atmosphere some trace gas like methane? Let's look at an equation for the budget of methane. It is produced in the atmosphere by all the emissions from cows and wetlands. It is removed from the atmosphere by reactions with OH [4.7]. The rate of removal, that is the change in the methane concentration, is always proportional to the amount of the two reactants, in this case, CH_4 and OH. So, the change in methane is given by the balance between the production and the loss by reaction with OH:

$$\frac{d\,[CH_4]}{dt} = \backslash)production\backslash(- k_{OH+CH4}[OH]\,[CH_4] \tag{4.5.15}$$

where k_{OH+CH4} is the reaction rate coefficient (units: cm^3 $molecule^{-1}$ s^{-1}) and *[OH]* and *[CH_4]* are the concentrations of OH and CH_4 (units: molecules cm^{-3}). Note that the production is positive and increases CH_4 with time while the loss is negative and decreases CH_4 with time.

We use [OH] to indicate the concentration of OH (molecules cm^{-3}), which is quite different from the OH mixing ratio (ppt = 10^{-12}, or ppb = 10^{-9}). 1 ppt ~ $2.4\text{x}10^7$ molecules cm^{-3} and 1 ppb ~ $2.4\text{x}10^{10}$ molecules cm^{-3} for typical surface conditions. See the video below (1:47) entitled "Rate Equation" for further explanation:

Rate Equation

Click here for transcript of the Rate Equation video

Let me explain equation 4.15, which is a rate equation for methane. A rate equation is just a differential equation. The change of something with respect to time equals the production rate of something, minus the fraction of something that is lost each unit of time, multiplied by the amount of something. Note that the loss rate of something is always proportional to something. That something can be anything. It does not have to be a chemical concentration. It could be the amount of milk in your refrigerator, or the number of socks in your drawer, both of which tend to disappear over time. And equation 4.15 is the methane concentration, which has units of molecules per centimeter cubed. The production rate is in units of molecules per centimeter cubed per second. Remember, each term of the equation must have the same units. The last term is the loss rate. The reaction rate coefficient has units of centimeter cubed per molecule per second, but when we multiply it by the OH concentration, we get a product that has units of per second, which is a frequency. Now, OH varies from almost 0 at night, to a peak value at midday. However, we can take an average OH to find the average loss rate of methane. Note that if we assume that the production rate suddenly goes to 0, then we find a very simple equation, which has an exponential

> solution. We designate the time that it takes the exponential factor to go to minus 1 as a lifetime, which is just the inverse of a loss frequency.

How can we find out what the lifetime of methane is? We assume that the production suddenly stops and equals 0. Then [4.15] becomes:

$$\frac{d\left[CH_4\right]}{dt} = -k_{OH+CH4}[OH]\left[CH_4\right] \tag{4.5.16}$$

$$\frac{d\left[CH_4\right]}{\left[CH_A\right]} = -k_{OH+CH4}[OH]dt \tag{4.5.17}$$

k_{OH+CH4} is the reaction rate coefficient for this reaction. Assume that OH is constant. Because OH is generated mostly from sunlight, it follows the sunshine and is greatest near midday and is very small at night. However, we assume that the OH concentration is the average over the day and night in order to assign it a constant value. Now integrate both sides of the equation:

$$\int_{\left[CH_4\right]_0}^{\left[CH_4\right]} \frac{d\left[CH_4\right]}{\left[CH_4\right]} = \int_0^t -k_{OH+CH_4}[OH]dt \tag{4.5.18}$$

$$\ln\left(\left[CH_4\right]\right) - \ln\left(\left[CH_4\right]_0\right) = -k_{OH+CH_4}\overline{[OH]}t \tag{4.5.19}$$

$$\ln\left(\frac{\left[CH_4\right]}{\left[CH_4\right]_0}\right) = -k_{OH+CH_4}\overline{[OH]}t \tag{4.5.20}$$

take exponential of both sides

$$\frac{\left[CH_4\right]}{\left[CH_4\right]_0} = e^{-k_{OH+CH_4}\overline{[OH]}t} \tag{4.5.21}$$

$$\left[CH_4\right] = \left[CH_4\right]_0 e^{-k_{OH+CH_4}\overline{[OH]}t} \tag{4.5.22}$$

So we see that *methane decreases exponentially with time*.

The atmospheric lifetime is defined as the time it takes something to decrease to $e^{-1} = 0.37$ of its initial value. So to find the lifetime of methane in the atmosphere, we see when $k_{OH+CH4}[OH]\ t = 1$, or:

$$\tau = \frac{1}{k_{OH+CH4}[OH]} \tag{4.5.23}$$

where τ indicates the lifetime. $k_{OH+CH4} = 3x10^{-15}$ cm^3 $molecule^{-1}$ s^{-1} and $[OH] \sim 10^6$ molecules cm^{-3}, so:

$$\tau = \frac{1}{3x10^{-15}10^6} = 3 \times 10^8 \text{seconds} \sim 10 \tag{4.5.24}$$

years

This reaction rate coefficient is fairly slow. Other VOCs have reaction rate coefficients that are typically hundreds to hundreds of thousands of times faster, so the lifetimes of most VOCs is hours to days.

The atmospheric lifetime of a gas is very important for determining how far a gas can travel from its source. Some trace gases have lifetimes of hours, so unless they are made by atmospheric chemistry, they can't travel more than a few tens of kilometers from their sources. Other gases have much longer lifetimes - methane is a good example with its 10-year lifetime. In 10 years, it can travel from its sources to most anywhere around the globe, even to the stratosphere. NASA measures the amounts of several gases from space. An excellent NASA website for accessing these satellite data and having it plotted as global maps is the Center for Trace Gas Data & Information Website at the NASA Goddard Space Flight Center's Earth Sciences Distributed Active Archive Center (GES DISC).

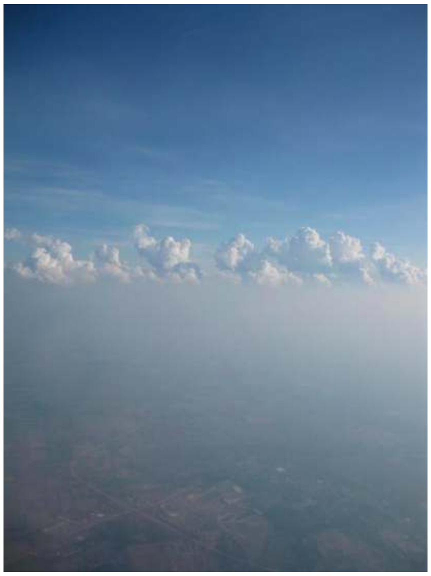
This concept of atmospheric lifetime is pretty important. For instance, what if an industry is spewing a chemical into the atmosphere that is toxic at a certain concentration in the atmosphere? Then it is important to know if that chemical is removed in less time than it takes to become toxic or if it is going to continue to build up at toxic levels and not leave the atmosphere for a long, long time. If its atmospheric lifetime is hundreds to thousands of years, then maybe we shouldn’t let that industry dump that chemical into the air.

4.6: Where do Cloud Condensation Nuclei (CCN) come from?

Now that you know everything about the atmosphere's gas-phase composition, it is time to look at its particle composition. We are interested in atmospheric particles for several reasons:

- smaller ones can get into the lungs and cause serious health problems;
- smaller ones can absorb or scatter sunlight, thus affecting climate;
- some of them are good cloud condensation nuclei, which are essential for the formation of clouds.

Atmospheric aerosol is most obvious to us on warm muggy summer days. Under these conditions, there are lots of aerosol particles and they absorb water and swell up to a size that is quite efficient at scattering sunlight. The following picture was taken over Maryland on a flight between Washington Dulles airport and State College airport. Above the fair weather cumulus clouds is blue sky in the free troposphere. Below the clouds is the atmospheric boundary layer, which is filled with aerosol that has been well mixed by warm, moist air parcels rising and stirring the boundary layer air. The haze is so thick that it is a little hard to see the ground.



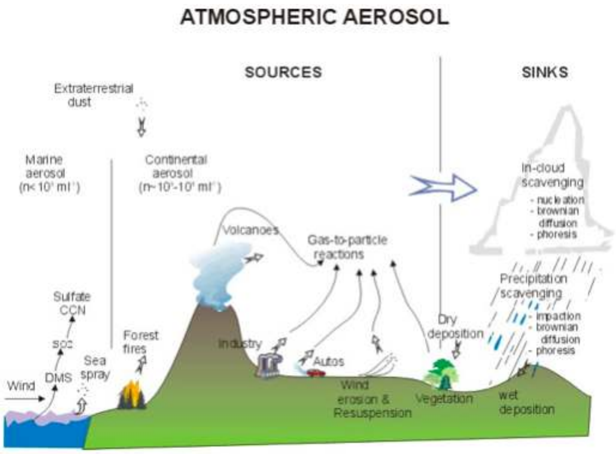
Summertime haze over Maryland. Credit: W. Brune

Atmospheric particles come from many different sources. Good cloud condensation nuclei (CCN) must be *small particles*, so that they do not settle too fast, and must be *hydrophilic*, which means that water can stick. They can be *either soluble* (i.e., dissolvable in water), *or insoluble*, but most are soluble.

Most particles originate from emissions from Earth's surface. **Primary aerosols** are emitted directly from the source, although the smaller ones start off as hot gases that rapidly condense to form particles even before they leave the smokestack or tailpipe. **Secondary aerosols** are gaseous emissions that are converted to aerosol particles by chemical reactions in the atmosphere. Some of these become CCN. This process is often called gas-to-particle conversion. Most CCN are secondary aerosols.

The sources are both natural and anthropogenic (human-made). Seaspray, volcanoes, forests, and forest fires, as well as gas-to-particle conversion of naturally occurring gases such as sulfur dioxide (SO_2) and some naturally occurring VOCs, such as α-pinene (which gives the pine smell) are important natural particle sources. Industry, power plants, using fires to clear cropland, transportation, and gas-to-particle conversion of anthropogenic SO_2 and numerous other gas emissions are important anthropogenic particle sources.

Note that we must pay attention not only to the aerosol sources but also the aerosol sinks, as shown in the diagram below.



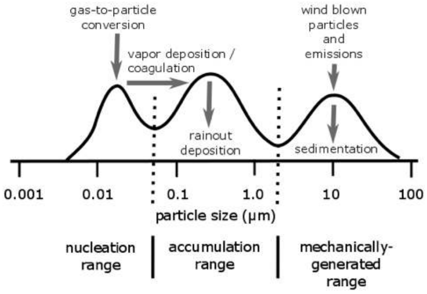
Atmospheric Aerosol Sources and Sinks Sources of Atmospheric Aerosol: forest fires, extra terrestrial dust, continental aerosol, volcanoes, industry, autos, wind erosion and resuspension, gas-to-particle reactions Sources of Atmospheric inks: In-cloud scavenging (nucleation, bornian diffusion, phoresis), precipitation scavenging (impaction, brownian diffusion, phoresis) *Credit: NOAA*

The different sources make particles of different sizes. The typical size distribution (i.e. number of particles in a volume of air, plotted as a function of size) has bumps in it, with more particles at some sizes than at others, as seen in the diagram below. Reading these bumps tells us a lot about how the particles were made.

The **nucleation mode** (there are other designations for this) includes particles that are made by gas-to-particle conversion. A low-volatility vapor is one that will condense onto particles or other surfaces when its vapor pressure exceeds its low saturation vapor pressure. This situation is analogous to water.

Coarse mode includes particles made by mechanical processes. The hydrophilic coarse particles can be CCN, but they settle out pretty fast.

Accumulation mode particles are usually made when nucleation particles collide and stick (called coagulation) or when gases accumulate on a nucleation mode particle. They neither settle fast nor coagulate, so they tend to hang around in the atmosphere for a few weeks. They make pretty good CCN.



Typical aerosol size distributions and their sources and sinks. Nucleation range particles quickly grow into accumulation range particles, which fall slowly to the ground and stay a week or two in the atmosphere. The larger particles, called coarse particles, fall to the ground within hours or sometimes days. Gas-to-particle conversion means that the particles start as gases but are converted by reactions to sticky chemicals that form particles. Credit: W. Brune

PM2.5 - Secondary Particles from Gas-to-Particle Formation

PM2.5 is a particle size designation that means “Particle matter smaller than 2.5 µm in diameter”. Another common term is PM10, which is "particle matter smaller than 10 µm in diameter". PM2.5 particles are the ones that are most important for human health and climate, and, in many cases, cloud formation because of their longer lifetime in the atmosphere.

Secondary particles start with the emission of VOCs or sulfur compounds, which react mainly with OH to start a sequence of reactions. These reactions tend to add oxygen to the molecules, which chemically makes them stickier (with a lower saturation vapor pressure) and more water soluble, which is just what is needed to make them better cloud condensation nuclei.

For particles that start as gaseous sulfur compounds, such as sulfur dioxide (SO_2), the reaction sequence starts with OH and the reaction product is sulfuric acid, a compound that has a very low vapor pressure and is very sticky.

$$SO_2 + OH \to\to H_2SO_4 \tag{4.6.1}$$

Sulfuric acid is easily taken up into cloud drops and raindrops and then can be deposited on Earth’s surface when it rains. The good news is that the rain cleans the atmosphere. The bad news is that the rain is very acidic and has earned the name “acid rain” because of its harmful effects on forests and on buildings, memorials, and statues.

If sulfur sources are upwind of an area, the particles in that area will contain some sulfur. But almost all atmospheric particles also contain some organic compounds and sometimes particles are mainly made up of carbon-containing organic compounds. Some of these organic particles are primary, but most of the small ones are made by gas-to-particle conversion, which is just a simple way to say the volatile organic compounds react in the atmosphere with OH or O_3 to form less volatile organic compounds that become aerosol particles. The chemicals in these particles can continue to oxidize, thus making them even better CCN.

We can demonstrate gas-to-particle conversion of a VOC that is often emitted into the atmosphere by trees. This compound is limonene and also comes from oranges. In the video (4:47) below entitled "Demonstration of Gas to Particle Conversion," I will use orange peel to demonstrate this effect.

Gas to Particle Conversion Demonstration

Click here for transcript of the Gas to Particle Conversion video.

> Today I'm going to show you how particles can come from gases. So what I have here is I have an artificial atmosphere right here, a glass jar. I have a light source in there, which will act like the sun, so here's the sun right here. I'll set the sun. Sun's off right now. And then I have a source of gases, an orange. So let me first peel the orange. Let's peel the orange here. A little bit of orange. Take some peel off the orange. OK, that's enough. So now here we have some orange peel. That smells really good. So what I'm going to do is take some orange peel and squeeze it a little bit, and I'm going to drop it in the atmosphere. Squeeze some more and drop it in the atmosphere. Now I'll put the lid on the atmosphere. OK. There we go. Now you see I have orange peel here and so there's some gases that smell really good, these volatile organic compounds. So I can smell, and they smell great. And I'm going to show you that even though I put that in there, I have a light source. Here's a little laser, just a laser pointer, and you see I'm shining it in there and you don't really see any particles at all in here. So look in here and there are no particles. See that? No particles. OK, now I'm going to turn on the sun, which is this nice ultraviolet light here. So I'll turn on the sun, and we're just going to wait a little bit. So the sun is on. You can see it glowing here and here. And so what that's doing is that's producing lots and lots and lots of OH, the hydroxyl radical. And it's also producing lots of ozone, and so it's doing what the atmosphere does, which is oxidizing the volatile organic compounds that came from the orange peel. Those volatile organic compounds, one of the main ones is called limonene, which is of course also in lemons and limes. And so we are just going to let that cook a little bit and let the day go on a little bit. And then what we can do is we can see if we're making any particles. So let me shake it up a little bit like there's a little bit of a wind. There we go. So now you see we have a very bright light from the lamp. It's really gotten bright. And we can see that it's doing a lot. Heat it up a little bit more. Now we'll just take a quick look and see if we see anything. So the laser

> light here we can't see anything in the atmosphere outside of the chamber, so I'm shining it. Right now this is hitting my hand, as you see here. And there's no sort of scattering. But I've turned off the light now. Now we shine this in here, and we see a tremendous beam. And so all this is small particles that we made which are now scattering that light. See you can see that very strong beam. And all those particles came from the volatile organic compounds, limonene and others, that came out of the orange peel and then they were oxidized in the atmosphere to make less volatile compounds, one that had lower vapor pressure, and then those all stuck and made these nice little particles that you see here. So there's a demonstration of gas to particle conversion.

4.7: Summary and Final Tasks

Atmospheric composition, even of trace gases, has a huge influence on weather and climate. Carbon dioxide is the most abundant trace gas – its mixing ratio is 400 ppm and growing, but other trace gases are also emitted into the atmosphere. The atmosphere cleans itself of these gases by atmospheric chemistry, which oxidizes the gas emissions and produces new chemicals that contain oxygen and so are stickier and more water soluble.

These new chemicals can be removed from the atmosphere either by hitting surfaces and sticking or by being taken up in clouds or rain drops and precipitated to the ground. The main oxidant is hydroxyl (OH), which is made with ozone, UV sunlight, and water vapor and starts the removal sequence by reacting with gas emissions. In these reaction sequences ozone pollution is produced if the pollutant nitric oxide (NO) is also present.

This *pollutant ozone* is nearby and is harmful to human health and agriculture. *Stratospheric ozone*, on the other hand, shields Earth from harmful UV, and is made a completely different way – by the breaking apart of O_2 to produce O, which reacts readily with O_2 to form O_3. Some of this stratospheric ozone is then transported to Earth, but at levels much lower than pollutant levels. Methane oxidation is an example of the VOC reactions that produce ozone and particles.

An important concept is the atmospheric lifetime of gases and particles. This can be determined by solving a simple linear differential equation. The methane lifetime was shown to be about 10 years.

Particles have many natural and anthropogenic sources; some are emitted directly from the sources (primary particles) and some are produced by atmospheric chemistry (secondary particles). Particles affect human health, visibility, scattering and absorption of light, and are essential for cloud formation, as will be seen in the next lesson on cloud physics.

Reminder - Complete all of the Lesson 4 tasks!

You have reached the end of Lesson 4! Double-check that you have completed all of the activities before you begin Lesson 5.

CHAPTER OVERVIEW

5: Cloud Physics

Learning Objectives

By the end of this chapter, you should be able to:

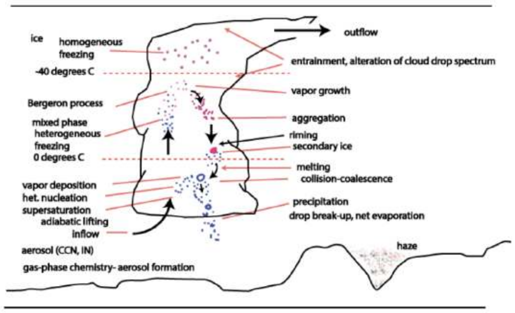
- identify cloud types
- describe the essentials for cloud formation
- on a Koehler curve, explain the behavior of a particle in different supersaturation environments
- explain the lifecycle of cloud formation through precipitation

Clouds and precipitation are integral to weather and can be difficult to forecast accurately. Clouds come in different sizes and shapes that depend on atmospheric motions, their composition, which can be liquid water, ice, or both, and the temperature. While clouds and precipitation are being formed and dissipated over half the globe at any time, their behavior is driven by processes that are occurring on the microscale, where water molecules and small particles collide. We call these microscale processes “cloud microphysics” and microphysics is the focus of this lesson. Three ingredients are required for the formation of clouds: moisture, aerosol, and cooling. If any one of these is missing, a cloud will not form. Over eighty years ago, a simple hypothesis was developed to explain the formation of clouds. This hypothesis has been thoroughly tested and validated and is now called Koehler Theory. We will learn the elements of Koehler Theory and how to use them to determine when clouds will form and when they will not, becoming only haze. Clouds do not automatically precipitate. In fact, most clouds do not. We will learn about the magic required for precipitation to form. Thus, cloud formation through precipitation is a series of microsteps, each of which is necessary, but not sufficient, to achieve precipitation.

5.1: Looking at the Whole Cloud

Putting It All Together

We can put all of the processes from this lesson together to look at the lifecycle of a cloud:



The lifecycle of a cloud. This schematic contains essentially all the processes that we have talked about in this lesson. Credit: W. Brune

The following is a description of convection's stages of development:

- Local perturbation in atmospheric density fields, sometimes driven by uneven surface heating or moisture evaporation, starts relative vertical motion
- Stage 1: **"Developing Stage"** (also called Cumulus Stage)
  - Updraft dominates center of cloud, cloud drops form and grow
  - Release of latent heat provides the energy for vertical motion and growth
- Stage 2: **"Mature Stage"**
  - Downdrafts form in addition to updrafts, causing gust fronts
  - Cloud reaches height so that freezing occurs and precipitation develops
  - Evaporation of precipitation drives downdrafts
- Stage 3: **"Dissipating Stage"**
  - Downdrafts only
  - Water mass is removed by sedimentation/evaporation

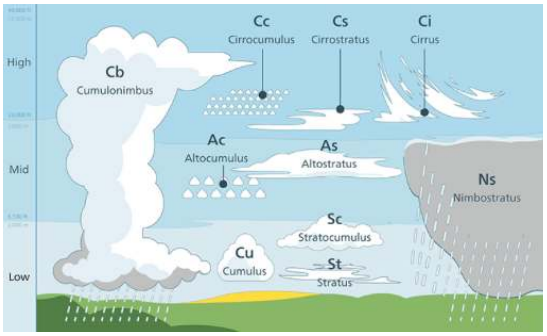
The video below (2 min.) includes some great time-lapse video of clouds forming and disappearing. Check it out:

Quiz 5-4: How precipitation forms.

1. You can take **Practice Quiz 5-4** as many times as you like.
2. When you feel you are ready, take **Quiz 5-4**. You will be allowed to take this quiz only **once**. Good luck!

5.2: Do you recognize these clouds, drops, and snowflakes?

Clouds have fascinated people for millennia, but it wasn't until 1802 that Luke Howard first classified clouds with the terms that are used today. His classification scheme was formalized later in the 19th century and has 10 basic cloud types with many minor variations (see figure below).



Genus classification of clouds by altitude of occurrence.

Credit: Wikipedia

Clouds!

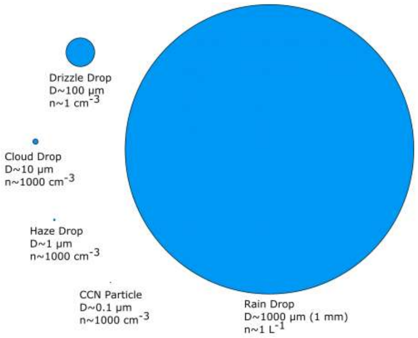
This website contains a nice overview of the cloud types with descriptions and accompanying images. Check out some of their amazing photos!

NOAA and NASA put together this thorough Sky Watcher Chart that describes a wide variety of cloud formations.

Cloud physics goes beyond the classification of clouds to determine the actual physical and chemical mechanisms that create clouds and cause their evolution over time. There are two aspects of cloud physics. One is the physics on the **cloud scale**, which is tens to hundreds of meters in size. This physics is driven in part by behavior in the cloud's environment, such as the wind shear or the location of a front, and determines the evolution of the cloud and the cloud's size and shape. All of this action, however, is not possible without the physics that is occurring on the **microscale**, which is less than a few centimeters in size.

This lesson deals mostly with the physics that occurs on the microscale and is often called **cloud microphysics**. Now that you are familiar with the concepts of thermodynamics and water vapor, we are ready to look at the fundamentals of cloud microphysics. To understand cloud-scale physics will require an understanding of atmospheric dynamics and turbulence, which are introduced in later lessons of this course.

A cloud is defined as a (visible) suspension of small particles in the atmosphere. For a water cloud, there are a number of types of particles that we are interested in.



Cloud drop sizes and characteristics**.** D is the typical diameter; n is the typical number per volume of air. Sizes are almost but not quite to scale.Rain Drop: D~1000μm, n~1L-1; CCN Particle: D~0.1μm, n~1000cm-3; Haze Drop: D~1μm, n~1000cm-3; Cloud Drop: D~10μm, n~1000cm-3; Drizzle Drop: D~100μm, n~1cm-3

Credit: W. Brune (after Lamb and Verlinde)

Note the wide range in size, volume, and number of particles in the figure above. The smallest, the cloud condensation nuclei (CCN), can have rather little water vapor and are made up of substances to which water can attach (called hydrophilic, water loving). The other particles grow by adding water molecules but still contain the original CCN upon which they formed.

We can specify the amount of water that is in liquid form by using the liquid water content (*LWC*). The liquid water content can be defined as:

$$LWC = \omega_L = \frac{\text{mass of liquid water}}{\text{volume of air}}, \quad \backslash)units\backslash(= gm^{-3} \tag{5.2.1}$$

Typical *LWC* are 0.1- 0.9 g m^{-3}, but a few g m^{-3} are possible for wetter conditions.

? Exercise

A cloud drop is typically 10 µm in diameter, while a raindrop, which comes from a collection of cloud drops, is typically 1 mm (1000 µm) in diameter. How many cloud drops does it take to make a raindrop?

Click for answer.

Answer: Find the volume of the cloud drop and the volume of the raindrop and then find out how many times bigger the raindrop is. The answer is the number of cloud drops it takes to make a raindrop.

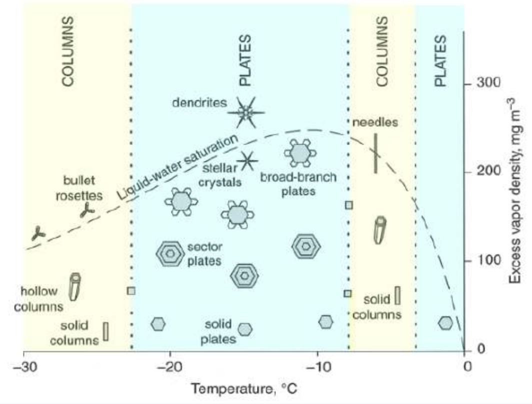
$$n_{\text{cloud}} V_{\text{cloud}} = V_{\text{rain}} \tag{5.2.2}$$

$$n_{\text{cloud}} = \frac{V_{\text{rain}}}{V_{\text{cloud}}} = \frac{4/3\pi \left(r_{\text{rain}}\right)^3}{4/3\pi \left(r_{\text{cloud}}\right)^3} = \left(\frac{r_{\text{rain}}}{r_{\text{cloud}}}\right)^3 = \left(\frac{1000}{10}\right)^3 = 10^6 \tag{5.2.3}$$

So we see that it takes about a million cloud drops to make one raindrop. Thus 10^9 cloud drops per m^3 of cloud should make about 10^3 raindrops per m^3 of cloud. This is about the number per m^3 that are observed.

Quiz 5-1: Cloud drops and liquid mass.

1. Find **Practice Quiz 5-1** in Canvas. You may complete this practice quiz as many times as you want. It is not graded, but it allows you to check your level of preparedness before taking the graded quiz.
2. When you feel you are ready, take **Quiz 5-1**. You will be allowed to take this quiz only **once**. Good luck!



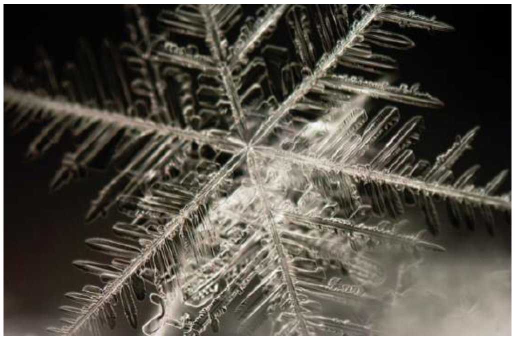
Ice crystal shape for different temperatures and different excess water density levels. Excess water vapor is the amount of water vapor above the saturation water vapor amount. This situation can occur as air is lifted and the temperature (and thus saturation vapor pressure) drops faster than the water vapor can deposit on the ice.

Click for a text description of the ice crystals image.

Ice crystal shapes for different temperatures: Columns: bullet rosettes (above liquid-water saturation line), hollow columns, solid columns Plates: dendrites (above liquid-water saturation line), stellar crystals, broad branch plates, sector plates, solid plates Columns: needles (on liquid-water saturation line), solid columns Plates

Credit: W. Brune (after Lamb and Verlinde)

Ice crystal habits as a function of temperature and excess water vapor (i.e., water vapor greater than saturation water vapor).



A snowflake. Its shape with dendrites indicates that it formed with a lot of excess water vapor and a temperature of about -16^{o}C.

Credit: bkaree1 via flickr

The next time it snows, catch snowflakes on a cold surface and take a good look at them. Their shape will tell you a lot about the environment in which they were formed. In State College, we often see plates with broad branches and sometimes we see dendrites, telling us that the snowflakes were formed at altitudes in the cloud where the temperature was between -22^{o}C and -8^{o}C and the excess water vapor was large.

The following video (3:52) entitled "Snowflake Safari" gives a simple explanation of snowflake formation and shows some nice pictures of different snowflake shapes.

Snowflake Safari

Click here for transcript of the Snowflake Safari video.

FLORA LICHTMAN: Sure there's sledding, snowmen, skiing, but a winter storm can also mean safari. KEN LIBBRECHT: You really just need a snowy day. Take a magnifying glass, go out, there's all sorts of different things you can see. FLORA LICHTMAN: That's Ken Libbrecht, the physicist at Caltech who also happens to be a snowflake expert. He's been hunting flakes for years and documenting them before they melt with this microscope camera rig. KEN LIBBRECHT: My travel with that the hard part is getting it through airport security. FLORA LICHTMAN: Snow crystals come in roughly 35 flavors Libbrecht says. Some more common than others of course. KEN LIBBRECHT: Stellar dendrites are pretty common standard sort of shopping mall snowflake with six branches. FLORA LICHTMAN: Then there's the variant fern-like stellar dendrites. KEN LIBBRECHT: ...and they look like a little bitty ferns. FLORA LICHTMAN: Also common are... KEN LIBBRECHT: ..needles, columns. One of my favorites are capped columns. FLORA LICHTMAN: Which look kind of like a satellite or... KEN LIBBRECHT:...two wheels on axle. Unfortunately the most common thing you'll find is just kind of junky looking snow looks like sand. FLORA LICHTMAN: The least common, the ivory-billed woodpecker of snowflakes is big... KEN LIBBRECHT: ...five millimeters in diameter and nicely symmetrical with lots of intricate markings. Those are really gorgeous and hard to find. FLORA LICHTMAN: But you can increase your chances if you seek out snowflake hotspots. KEN LIBBRECHT: Northern Ontario is a good spot. Vermont and Michigan and I have been there. Northern Japan actually is pretty good. I'm anxious to try to Siberia. FLORA LICHTMAN: See certain conditions breed better crystals. KEN LIBBRECHT: The best temperature is around five degrees Fahrenheit. Sometimes though you can see it's really nice crystals just below freezing. FLORA LICHTMAN: Ok a little review of where snowflakes come from. They're born in the clouds. It all starts with a speck of dust or bacterium. KEN LIBBRECHT: Gunk in the air. FLORA LICHTMAN:...and the gunk floats around the cloud. KEN LIBBRECHT:...for half a mile. FLORA LICHTMAN: Picking up water molecules. KEN LIBBRECHT: Then they shuffle around a little bit until they find the right spot to sit in and that the water molecules themselves are lined up in the hexagonal array. That's where the the order is generated. FLORA LICHTMAN: ...and that order is what makes it a crystal. KEN LIBBRECHT:...and as a grows larger the points of the hexagon stick out a little bit in the air so each of the six corners sprouts and arm and that's one of the things we're trying to understand in details how crystals grow. FLORA LICHTMAN: The details of that growth are determined by the microenvironment, the flake encounters, as it travels through the cloud. KEN LIBBRECHT: Humidity is low the crystals grow slow and humidity is high they go fast. FLORA LICHTMAN: In other words of flakes identity is shaped by the environment it grows up in and because two snow crystals aren't likely to follow the exact same path, you're not likely to find two of the exact same flake. Just how environment affects crystal growth is something Librecht studies in the lab, by growing his own snowflakes. KEN LIBBRECHT: We call these designer snowflakes. You can sort of dial-up what you want. FLORA LICHTMAN: Give it the right environment and something to grow on and it'll build itself. KEN LIBBRECHT: A really nice example of how really complicated structures emerged spontaneously not alive test the DNA or anything like that genetic code. It just happens. To understand more about how works will be able to use it for something or at the very least we'll just understand how it works. FLORA LICHTMAN: Happy new year. For Science Friday, I'm Flora Lichtman.

Credit: SciFri

Discussion Activity: Cloud Identification

(3 discussion points)

It's time to look up at the sky to observe the clouds. During the next week, take pictures of clouds and identify the clouds in the pictures. Try to focus on just one cloud type per image. Submit **an image** that depicts **at least one cloud type**.

You will upload each image in its own post. You should **include the following** in your post:

- **your name**
- **the picture's location**
- **the picture's date and time**
- **your identification of the cloud**
- **your reasoning for the identification in a short sentence or phrase.**

Copy and **paste your picture into the post** box. Instructions for how to embed an image in your post can be accessed here.

1. You can access the **Cloud Identification Discussion Forum** in Canvas.
2. Post your pictures of clouds with their identification using the format described above.
3. Keep the conversation going! **Comment on at least one other person's post**. Your comment should include follow-up questions and/or reasoning for an alternate identification of the clouds in the post.

This discussion will be worth 3 discussion points. I will use the following rubric to grade your participation:

Discussion Activity Grading Rubric

Evaluation	Explanation	Available Points
Not Completed	Student did not complete the assignment by the due date.	0
Student completed the activity with adequate thoroughness.	Posting answers the discussion question in a thoughtful manner, including some integration of course material.	1
Student completed the activity with additional attention to defending his/her position.	Posting thoroughly answers the discussion question and is backed up by references to course content as well as outside sources.	2
Student completed a well-defended presentation of his/her position, and provided thoughtful analysis of at least one other student's post.	In addition to a well-crafted and defended post, the student has also engaged in thoughtful analysis/commentary on at least one other student's post as well.	3

5.3: What are the requirements for forming a cloud drop?

There are three requirements for forming a cloud drop:

1. **M**oisture
2. **A**erosol
3. **C**ooling

If any one of these three is missing, a cloud cannot form. We have talked about moisture and aerosol and now need to consider ways that the air can be cooled. The air needs to be cooled so that the water vapor pressure initially equals and then exceeds the water saturation vapor pressure.

An easy way to remember these key ingredients is to think of a Big **MAC**.

A Big **MAC**. Hungry for some more? Credit: Cleaveland via flickr

Saturation occurs when $e=e_s$, $w=w_s$, and condensation = evaporation. At saturation, $RH = e/e_s \sim w/w_s = 1$, or in terms of percent, 100%. When we find the lifting condensation level (LCL) on a skew-T, we are finding the pressure level at which T (as determined from the dry adiabat) = T_d (as determined from the constant water vapor mixing ratio), or when $w = w_s$.

Let’s define two new variables that are useful in discussing the cloud drop formation.

Often we talk about the **saturation ratio**:

$$s = \frac{e}{e_s} \tag{5.3.1}$$

where e is the water vapor pressure and e_s is the saturation vapor pressure. $S < 1$ for a subsaturated environment, $S = 1$ for saturation (condensation = evaporation), and $S > 1$ for a supersaturated environment.

We also talk about **supersaturation**:

$$s = S - 1 = \frac{e}{e_s} - 1 \tag{5.3.2}$$

$s = 0$ at saturation; $s < 0$ for a subsaturated environment; $s > 0$ for a supersaturated environment. Note that s and S are both unitless.

This equation applies only for a flat surface of pure water. When we get into situations where we have curvature or a solute, we need to think about the supersaturation relative to the equilibrium value of e, e_{eq}, which can be different from e_s. So, depending on the circumstances, e_{eq} can be e_s (flat liquid water), e_i (flat ice), e_{sc} (curved liquid water), e_{sol} (curved solution), or some combination. We will see that a small supersaturation is actually needed to form clouds.

? Exercise

The relative humidity is 85%. What is the saturation ratio? What is the supersaturation?

Answer

S = 0.85 and s = 0.85 - 1 = -0.15

The relative humidity is 102%. What is the saturation ratio? What is the supersaturation?

? Exercise

The relative humidity is 102%. What is the saturation ratio? What is the supersaturation?

Answer

S = 1.02 and s = 0.02

Note that it is possible to have the relative humidity be greater than 100%, which makes the supersaturation positive. This condition can't last long because condensation will exceed evaporation until they become equal. But how can supersaturation happen?

5.4: How can supersaturation be achieved?

Three basic mechanisms for cooling the air are **RUM**: **Radiation**, **Uplift**, and **Mixing**.

Radiation and mixing happen at constant pressure (isobaric); uplift happens at constant energy (adiabatic). Let’s consider these three cases in more detail. A good way to show what is happening is to use the water phase diagram. The video (3:15) entitled "Supersaturation Processes 2" below will explain these three processes in greater detail:

Supersaturation Processes 2

Click here for transcript of the Supersaturation Processes 2 video.

> Clouds will not form unless the air becomes supersaturated, meaning that its relative humidity is slightly greater than 100%. Or put it another way, it's supersaturation is greater than 0%. Let's look at the three ways that supersaturation can be achieved, radiative cooling, mixing, and adiabatic ascent. We can use the water phase diagram of water vapor on the y-axis versus temperature on the x-axis to examine these processes. Supersaturation means that the environment moves from the all-vapor part of the phase diagram into the all-liquid part by crossing the equilibrium line, which is given by the Clausius Clapeyron equation. I will mention only the essentials for each process, what changes and what stays the same. For radiative cooling, the water vapor pressure stays the same, but the temperature drops. And because the saturation vapor pressure depends only on temperature, the saturation vapor pressure also drops. The saturation vapor pressure decreases until it gets equal to and then a little less than the vapor pressure. And then the supersaturation above 0. The next process is mixing. Mixing clouds usually form when unsaturated, warm, moist air from a source is mixed into the unsaturated, colder, drier environmental air. As the warm, moist air mixes with the colder, drier air, the temperature and vapor pressure of the moist air parcel becomes the average of the temperature and vapor pressure of the moist, warm air parcel multiplied by the number of moles and the temperature and vapor pressure of the cold, dry environmental air multiplied by the number of moles, all this divided by the total number of moles. As the air parcel mixes with more environmental air, the parcel's temperature and vapor pressure move along the mixing line between the two initial air parcel states. If this line crosses the equilibrium line and goes into the liquid part of the phase diagram, supersaturation becomes greater than 0 and the cloud forms. If the air parcel continues to entrain the dry air, continues along the mixing line, and it may eventually cross the equilibrium line back into the vapor region, ant the cloud will evaporate. Contrails are one example of a mixing cloud. The contrail length tells you something about what the temperature and environmental pressure of the environmental air must be. The third process is adiabatic ascent. As an air parcel ascends, it's pressure and temperature drop. Because the water vapor mixing ratio is constant until a cloud forms, the drop in the pressure means a drop in the water vapor pressure. At the same time, a drop in the temperature means a drop in the saturation vapor pressure, which depends only on temperature. So vapor pressure and saturation vapor pressure are both dropping. However, in adiabatic ascent, the saturation vapor pressure drops faster than the vapor pressure, and eventually, they become equal. And then supersaturation becomes greater than 0, and the cloud forms.

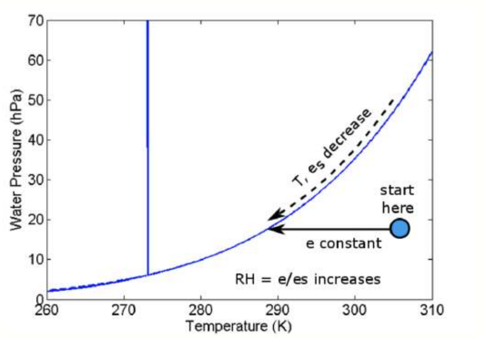
Radiative Cooling

All matter radiates energy as electromagnetic waves, as we will see in the next lesson. When an air mass radiates this energy (mostly in the infrared part of the spectrum), it cools down, but the amount of water vapor does not change.

We can understand this process by using the water phase diagram (see figure below). Initially, the air mass is at the position of the blue dot. As the air parcel cools and the temperature drops, the air parcel temperature moves to the left on the diagram but the water vapor pressure does not change. However, because the temperature drops, e_s drops. When e_s becomes slightly less than e, a cloud forms.

Summary

- e is constant as T decreases.
- Since e_s depends only on T, e_s also decreases until $e_s < e$.
- When e_s becomes slightly less than e, a cloud forms.



Water phase diagram for radiative cooling, with an air parcel starting with e and T marked by the blue dot. The horizontal arrow marks the cooling of the air parcel and the downward pointing arrow marks the change in e_s and T as the parcel cools. When e_s becomes slightly less than e, a cloud forms. Credit: W. Brune

An example of radiative cooling in action is radiation fog, which occurs overnight when Earth's surface and the air near it cool until a fog forms (see figure below).

Radiation fog. Moist air cools overnight by radiation to space while its water vapor mixing ratio remains roughly constant. However, the saturation vapor pressure drops as the temperature drops. When the saturation vapor pressure drops to be the same as the vapor pressure, a radiation fog forms. Credit: Montgomery County Planning Commission via flickr

Mixing

Assume two air parcels with different temperatures and water vapor partial pressures are at the same total pressure. If these two parcels mix, then the temperature and the water vapor partial pressure is going to be a weighted average of the T and e of the two parcels. The weighting is determined by the fraction of moles that each parcel contributes to the mixed parcel. Mathematically, for parcel 1 with e_1, T_1, and N_1 (number of moles) and parcel 2 with e_2, T_2, and N_2, the e and T of the mixed parcel are given by the equations:

$$e = \frac{N_1}{N_1+N_2}e_1 + \frac{N_2}{N_1+N_2}e_2 \tag{5.4.1}$$

$$T=\frac{N_1}{N_1+N_2}T_1+\frac{N_2}{N_1+N_2}T_2 \tag{5.4.2}$$

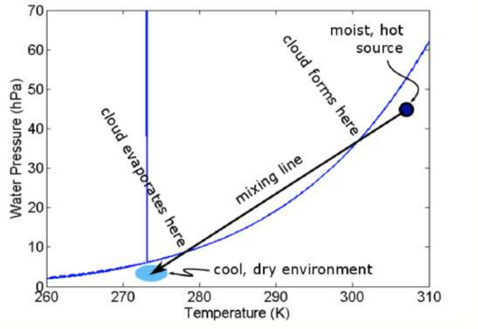
or approximately

$$e=\frac{M_1}{M_1+M_2}e_1+\frac{M_2}{M_1+M_2}e_2 \tag{5.4.3}$$

$$T=\frac{M_1}{M_1+M_2}T_1+\frac{M_2}{M_1+M_2}T_2 \tag{5.4.4}$$

where M_1 and M_2 are the masses of the air parcels. On the phase diagram, these give straight lines for different proportions of the mixed parcel being from parcel 1 (0% to 100%) and parcel 2 (100% to 0%), as in the figure below.

Note that both of these two air parcels are unsaturated. So how does a cloud form? Think about a single warm, moist air parcel mixing into the environment of colder, drier air. As the warm mixes into more and more of the drier air, it gets increasingly diluted but the mixed parcel continues to grow. As the amount of environmental air in the mixture increases, the average e and T of the mixed air parcel decreases to be closer to the environmental values and the mixed parcel's e and T follow a mixing line. Starting in the upper right near the warmer parcel, as the mixed parcel continues to grow, eventually the e and T will hit the Clausius-Clapeyron curve. As it continues to push into the liquid portion of the phase diagram and become supersaturated, a mixing cloud will form. The cloud will stay as long as the mixed e and T put the parcel to the left of the Clausius-Clapeyron curve. However, once the mixed parcel comes to the right of the curve, the cloud will evaporate.



Water phase diagram for mixing, with two air parcels at (T_1,e_1) and (T_2,e_2). When the two air parcels mix, the temperature and vapor pressure of the mixed parcel lies along the mixing line between the two parcels. If a small, warm, moist parcel is mixed into a colder, drier environment, then as the warm moist parcel mixes with more environmental air, the size of the mixed air parcel grows and the temperature and vapor pressure follow the mixing line toward the environmental temperature and vapor pressure (Eq. 5.4). Credit: W. Brune

Example:

Suppose air Parcel 1 has e = 20 hPa, T = 40^{o}C, and N = 40,000 moles; and Parcel 2 has e = 5 hPa, T = 10^{o}C, and N = 80,000 moles. Then using equation 5.4 (top):

$$e=\frac{40,0000}{40,000+80,000}20+\frac{80,000}{40,00+80,000}5=10\mathrm{hPa} \tag{5.4.5}$$

$$T=\frac{40,000_1}{40,000+80,000}40+\frac{80,000}{40,00+80,000}10=20^{\circ}\mathrm{C} \tag{5.4.6}$$

There are many good examples of mixing clouds. One is a jet contrail; a second is your breath on a cold day; a third is a fog that forms when cold air moves over warm moist ground, say just after rain.

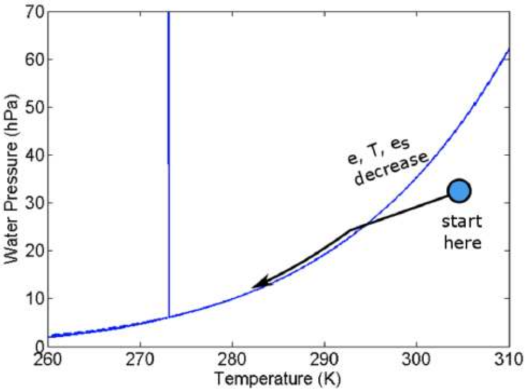
Contrails of different ages. Look at the fresh contrails. They are very thin and consist of water from the jet engines. However, the older contrails look bigger and more spread out, yet we know that the water from the jet engines is not enough to make such big contrails. This extra water vapor must have come from the atmosphere. Credit: Mike Lewinski via flickr

Uplift

The uplift of air can lead to cloud formation, as we know from the skew-T. Uplift is generally the same as adiabatic ascent. This adiabatic ascent can be driven by convection, by a less dense air mass overriding a more dense one, or by air flowing up and over a mountain. The following happens:

- The water vapor mixing ratio remains the same, but e drops as p drops, thus reducing the possibility that $RH = e/e_s$ will reach 100%.
- The temperature drops in accordance with Poisson's relations so that $e_s$also drops.

The question is "Does e or e_s drop faster so that eventually e equals e_s?" It turns out that e_s drops faster. As a result, in uplifted air, e and e_s converge at the lifting condensation level (LCL) and a cloud forms just at that level (see figure below).



Water phase diagram for uplift, with an air parcel starting at the beginning of the arrow. As the parcel ascends both e and T (and thus es) decrease, but es decreases faster than e so that eventually e > es. Once the cloud forms as the line passes into the liquid part of the phase diagram, the water line tries to achieve saturation. Credit: W. Brune

The arrow on the figure above shows the changes in e and T (and thus e_s) as an air parcel rises. Once $e_s <= e$, then $s > 0$ and the air parcel is supersaturated. This supersaturated situation is not stable; the water vapor in excess of e_s forms liquid. As the uplift continues, more water vapor is converted into liquid water and the vapor pressure remains close to e_s. All convective clouds, that is clouds with vertical extent, form this way. An example of adiabatic uplift is a cumulus cloud, as seen in the figure below.

A cumulus cloud over the ocean. Credit: JanneG via pixabay.com

Why is supersaturation required for a cloud drop to form?

I thought that cloud drops formed when $w = w_s$. Why is supersaturation required for a cloud drop to form?

To answer this question, we need to look through a microscope at the nanometer scale, which is the scale of molecules and small particles. You all know that cloud condensation nuclei are needed for clouds to form, but do you know why? Watch the following video (3:16) entitled "Glory: The Cloud Makers."

Glory: The Cloud makers

Click here for transcript of the Glory: The Cloud Makers video.

> [music playing] NARRATOR: Aerosols are suspended throughout Earth's atmosphere, and the tiny, varied particles play a mysterious role in human induced climate change. Just like people, every aerosol particle is unique. Sometimes aerosols occur naturally, from things like volcanoes, but they can also originate from human activity. Aerosols are short-lived, but have an active lifetime! In just a short expanse of time, particles can change their size and composition and even travel across vast oceans. Aerosols are difficult to study, and one important new area of research involves how these particles impact clouds. Without aerosols, clouds could not exist. MICHAEL MISHCHENKO: An aerosol particle can serve as a cloud condensation nucleus. NARRATOR: The introduction of too many aerosols will modify a cloud's natural properties. MICHAEL MISHCHENKO: The more aerosol particles we have in the atmosphere, the more cloud droplets we can have. NARRATOR: Clouds play an important role in regulating Earth's climate; aerosol-rich clouds become bigger, brighter, and longer lasting. Aerosols impact clouds in other ways. Some aerosol particles primarily reflect solar radiation and cool the atmosphere, and others absorb radiation, which warms the air. When aerosols heat the atmosphere, they create an environment where clouds can't thrive. The suppression of clouds leads to further warming of the atmosphere by solar radiation. Researchers are still working to understand the role of these curious particles. MICHAEL MISHCHENKO: We need to study the distribution of particles globally, and the only way to do that is from satellites. NARRATOR: New tools will soon help scientists study aerosols. The Aerosol Polarimetry Sensor, or APS, is among a suite of instruments onboard NASA's upcoming Glory mission. The APS will provide a global dataset of aerosol distribution with unprecedented accuracy and specificity. Unique data from the

> Glory mission, along with NASA's fleet of Earth observing satellites, will help researchers investigate the intricacies of Earth's changing climate. [music playing] [wind blowing] *Credit: NASA*

In the atmosphere, relative humidity rarely rises much above 100% because small aerosol particles act as Cloud Condensation Nuclei (CCN). *Two effects* most strongly determine the amount of supersaturation that each particle must experience in order to accumulate enough water to grow into a cloud drop. The first is a *physical effect* of curvature on increasing the water vapor equilibrium pressure; the second is a *chemical effect* of the aerosol dissolving in the growing water drop and reducing its vapor equilibrium pressure. You will learn about these two effects in the next two sections of this lesson.

5.5: Curvature Effect - Kelvin Effect

Let’s look at the curvature effect first (Figure 5.5.1). Consider the forces that are holding a water drop together for a flat and a curved surface. The forces on the hydrogen bonding in the liquid give a net inward attractive force to the molecules on the boundary between the liquid and the vapor. The net inward force, divided by the distance along the surface, is called surface tension, σ with units of N/m or J/m^2.

Figure 5.5.1: Sketches of the curvature effect. Left is a flat surface of pure water; right is a curved surface of pure water. Credit: W. Brune

If the surface is curved, then the amount of bonding that can go on between any one water molecule on the surface and its neighbors is reduced. As a result, there is a greater probability that any one water molecule can escape from the liquid and enter the vapor phase. Thus, the evaporation rate increases. The greater the curvature, the greater the chance that the surface water molecules can escape. Thus, it takes less energy to remove a molecule from a curved surface than it does from a flat surface.

When we work through the math, we arrive at the **Kelvin Equation**:

$$e_{sc}(T) = e_s(T) \cdot \exp\left(\frac{2_\sigma}{n_L \cdot R \cdot T \cdot r_d}\right) \tag{5.5.1}$$

where e_{sc} is the equilibrium vapor pressure over *a* curved surface of pure water, e_s is the equilibrium vapor pressure over a flat surface of pure water, both of which are functions of temperature, although e_{sc} is also a function of the drop radius, n_L is the number of moles per volume of water (55.5 moles L^{-1}). R^* is the molar gas constant, σ is the water surface tension, and r is the radius of the drop.

$$\left(\frac{2\sigma}{n_L \cdot R^*}\right) = 3.3 \times 10^{-7} \mathrm{m} K^{-1} \tag{5.5.2}$$

Since the evaporation is much, much greater over a curved surface, the condensation must also be much, much greater in order to keep condensation = evaporation, which is required for saturation (i.e., equilibrium). Thus, the saturation vapor pressure over a curved surface is much greater than the saturation vapor pressure over a flat surface of pure water. When we plot this equation, we get the plot in Figure 5.5.2:

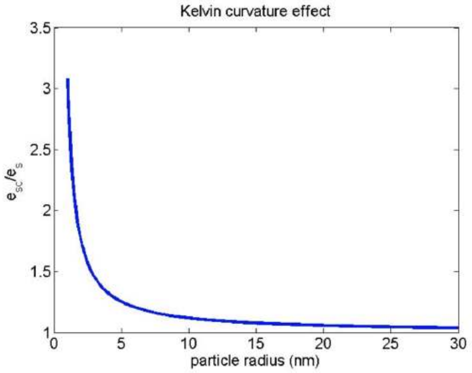


Figure 5.5.2: Dependence of ratio of the saturation vapor pressure over a curved surface to saturation vapor pressure over a flat surface on the drop radius. Credit: W. Brune

Note the rapid increase in equilibrium vapor pressure for particles that have radii less than 10 nm. Of course, all small clusters of water vapor and CCN start out at this small size and grow by adding water.

The Kelvin Equation can be approximated by expanding the exponential into a series:

$$e_{sc}(T) = e_s(T) \cdot \left(1 + \frac{2\sigma}{n_L \cdot R^* \cdot T \cdot r_d}\right) = e_s(T) \cdot \left(1 + \frac{a_K}{r_d}\right), \tag{5.5.3}$$

where

$$a_K = \frac{2\sigma}{n_L \cdot R^* \cdot T} \tag{5.5.4}$$

Summary

Cloud drops start as very small spherical drops, but the vapor pressure required for them to form is much greater than e_s until they get closer to 10^{-2}µm in size. The Kelvin effect is important only for tiny drops; it is important because all drops start out as tiny drops and must go through that stage. As drops gets bigger, their radius increases and e_{sc} approaches e_s.

So, is it possible to form a cloud drop out of pure water? This process is called *homogeneous nucleation.* The only way for this to happen is for two molecules to stick together, then add another, then another, etc. But the radius of the nucleating drop is so small that the vapor pressure must be very large. It turns out that drops probably can nucleate at a reasonable rate when the relative humidity is about 440%. Have you ever heard of such a high relative humidity?

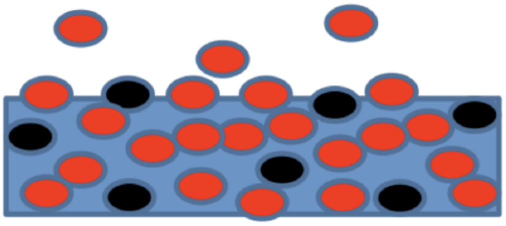
So, the lesson here is that homogeneous nucleation is very unlikely because of the Kelvin effect.

5.6: Solute Effect - Raoult’s Law

On the other hand, the atmosphere is not very clean either. There are all kinds of dirt and other particles in the atmosphere. Some of these are hydrophilic (i.e., they like water) and water soluble (i.e., they dissolve in water). So let’s see what the effect of soluble CCN might be on the water evaporation rate for a flat water surface. We’ll then put the curvature and the solute effects together.

First, here are some important definitions:

- **Solvent:** The chemical that another chemical is being dissolved into. For us, the solvent is H_2O.
- **Solute:** The chemical that is being dissolved in the solvent.



Sketch of a flat liquid surface with a solvent (water, red dots) and a solute (black dots). Credit: W. Brune

The simplest view of this effect is that solute molecules are evenly distributed in the water (solvent). Therefore, some solute molecules occupy surface sites that would otherwise be occupied by water molecules. Thus, the solute prevents water molecules from evaporating from those sites. Adding more solute means that more surface sites would be occupied by solute molecules and water vapor would have even less opportunity to break hydrogen bonds and escape the liquid. The real view is more complicated by the electrostatic interactions between water and solute molecules that cause an attraction between water and solute molecules, but the basic result is the same as the simple view.

Because the evaporation rate is lowered, that means that there will be net condensation until the water vapor flux to the surface matches the water vapor flux leaving the surface. When equilibrium is established between the lower evaporation and condensation, the condensation will be less, which means that the water saturation vapor pressure will be lower. The equilibrium vapor pressure is less than e_s, which, remember, is the saturation vapor pressure over a flat surface of pure water. As the amount of solute is increased, the equilibrium vapor pressure of the solution will be even less.

We can quantify this equilibrium vapor pressure over a solution with a few simple equations.

The mole fraction is defined as:

$$\chi_s = \frac{n_s}{n_w + n_s} \approx \frac{n_s}{n_L} \tag{5.6.1}$$

if $n_w > n_s$. n_w is the number of moles of water in a liter of solution, n_s is the number of moles of solute in a liter of solution, and n_L is the number of moles per liter of pure water.

As a result, the evaporation rate of the water molecules from the surface decreases. This can be written as **Raoult’s Law**:

$$e_{sol} = e_s \cdot (1 - \chi_s) \tag{5.6.2}$$

We can approximate Raoult’s Law for a reasonably dilute solution by writing:

$$\chi_s \approx \frac{n_s}{n_L} = \frac{iN_s}{n_L V_{drop}} = \frac{iN_s}{n_L\left(4\pi r_d^3/3\right)} \equiv \frac{BiN_s}{r_d^3}, \tag{5.6.3}$$

where

$$B = \frac{3}{4\pi n_L} \tag{5.6.4}$$

N_s is the total moles of solute. Note that an i was added. This factor is called the **Van ’t Hoff factor** and it accounts for the splitting of some solutes into components when they dissolve. An example is salt, NaCl, which splits into two ions in solution, Na^+ and Cl-, which means that $i = 2$ in this case.

5.7: Vapor Deposition

The growth of the cloud drop depends initially on vapor deposition, where water vapor diffuses to the cloud drop, sticks, and thus makes it grow. The supersaturation of the environment, s_{env}, must be greater than s_k for this to happen, but as the drop continues to grow, s_k approaches 0 (i.e., e_{eq} approaches e_s), so smaller amounts of supersaturation still allow the cloud drop to grow. Deriving the actual equation for growth is complex, but the physical concepts are straightforward.

- The growth rate (dm_d/dt, where m_d is the mass of the drop) is proportional to $s_{env} - s_k$. Physically, this statement means that *the greater the difference between the supersaturation in the environment and supersaturation at the particle's surface, the faster water vapor will diffuse and stick on the surface*. For instance, if s_{env} equaled s_k, then the evaporation and condensation of water on the particle's surface would be equal and there would be no mass growth.
- As water vapor diffuses to the drop and forms water, energy is released (i.e., latent heat of condensation) and this raises the temperature of the cloud drop surface, T_{sfc}, so that $T_{sfc} > T_{env}$. But an outward energy flow occurs and is proportional to $T_{sfc} - T_{env}$. Physically, this statement means that the particle and the air molecules around it are warmed by latent heat release. These warmer molecules lose some of this energy by colliding with the cooler molecules further away from the particle and warm them by increasing their kinetic energy (Figure 5.7.1).

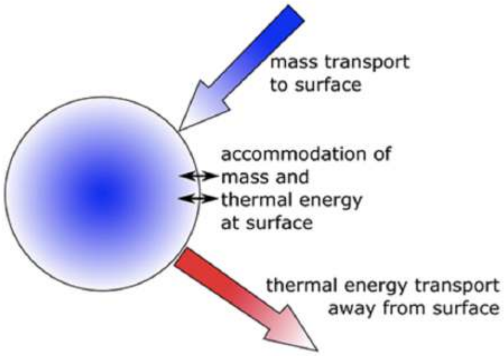


Figure 5.7.1: Schematic of the two physical processes in the growth of a cloud drop by vapor deposition. One is vapor deposition and the other is the transfer of condensational heating to the atmosphere; Credit: W. Brune (after Lamb and Verlinde)

When we account for both the flow of water molecules to the cloud drop surface and the flow of energy away from the surface, we can show that:

$$\frac{dm_d}{dt} = 4\pi r_d \rho_l G(T,p)\left(s_{env} - s_k\right) \tag{5.7.1}$$

where G is a coefficient that is a function of T and p, ρ_L is the density of liquid water, and the other variables have already been defined. G incorporates the effects of the mass transport of water vapor molecules to the surface and the transport of heat generated on condensation away from the particle surface. As a result, the drop radius grows as the square root of a constant times time (Figure 5.7.2).

$$r_d = (\text{C time })^{1/2} \tag{5.7.2}$$

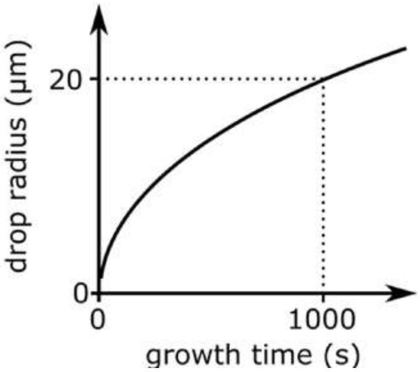


Figure 5.7.2: Growth of a cloud drop by vapor deposition as a function of time. Dashed lines indicate drop size after the typical cloud lifetime. Credit: W. Brune

Physical Explanation

- The nucleated cloud drop radius increases fairly rapidly at the beginning, but within minutes slows down because of the square root dependence on time.
- So, cloud drops can grow to 10–20 μm in 15 or so minutes, but then grow bigger much more slowly.
- Since a typical cloud only lasts 10s of minutes, it is not possible for cloud drops to grow into rain drops by vapor deposition alone.
- CCN nucleation followed by vapor deposition can make clouds, but it can’t make them rain.
- We can develop a similar expression for vapor deposition on ice, but the vapor depositional growth on ice is a little faster than on liquid.

Conclusion

We need other processes to get cloud drops big enough to form precipitation, either liquid or solid.

5.8: Did you know most precipitation comes from collision-coalescence?

There are two types of processes for growth into precipitation drops: **warm cloud processes** and **cold cloud processes**. In warm clouds, the processes all involve only liquid drops. In cold clouds, the processes can involve only solid particles, as well as mixed phases (both supercooled liquid and ice). Some of the most important processes involve collisions between drops, whether they be liquid or solid.

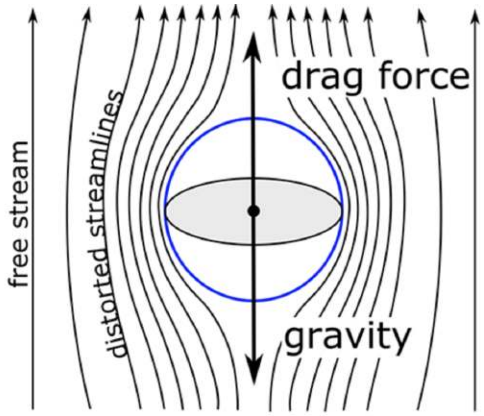
Collisions

Collisions occur in both cold and warm clouds and can involve either liquid drops or solid particles or both.

- **Collision–Coalescence:** Large liquid drop scavenges smaller liquid drops as it falls.
- **Riming:** Falling ice collects liquid water, which freezes on its surface.
- **Capture Nucleation:** Large liquid drop captures small ice particle, which acts as an ice nucleus and causes the large drop to freeze. The particle that is collected can be either an ice nucleus (IN) or a piece of ice, which also is a good ice nucleus. In either case, the supercooled liquid drop freezes on contact with the IN.
- **Aggregation:** Falling snowflake scavenges other snowflakes that aggregate to make a larger snowflake bundle.

For a cloud drop at rest, gravity is the only external force. Once the cloud drop accelerates, then the air resistance forms another force called *drag*, which is a function of the velocity.

In less than a second, the particle reaches a fall speed such that the drag force exactly balances the gravitational force and the velocity becomes constant. This velocity is called the *terminal velocity*. For instance, the terminal velocity of a 10 μm radius cloud drop is about 1 mm s^{-1}, while the terminal velocity for a 100 μm drop is about 1 m s^{-1}.



Air flow around a falling particle. The shaded area is the cross sectional area of the particle. Note the movement of air around the particle. Only the air in innermost streamline collides with the particle; the rest goes around it. Credit: W. Brune (after Lamb and Verlinde)

The growth of a cloud drop into a precipitation drop by collision–coalescence is given by the equation:

$$\frac{dm_L}{dt} = \tag{5.8.1}$$

Area swept out*efficiency of collection*velocity difference*liquid water content

$$\frac{dm_L}{dt} = A_g \cdot E_c \cdot (v_L - v_s) \cdot LWC \tag{5.8.2}$$

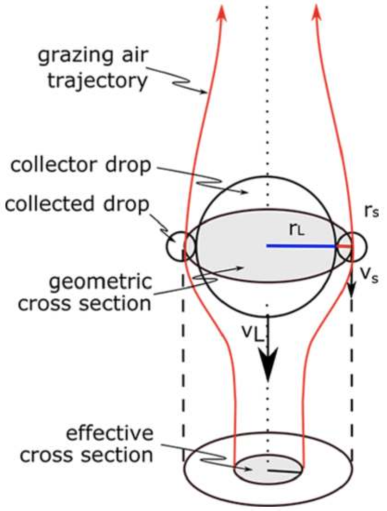
$$\frac{dm_L}{dt} = \pi\,(r_L + r_s)^2 \cdot E_c \cdot (v_L - v_s) \cdot LWC \tag{5.8.3}$$

- m_L is the mass of the large drop that is falling,
- A_g is the geometric cross-sectional area for which collisions between the falling large drop and the many drops below is possible,
- E_c is the collision-coalescence efficiency (i.e., a collection efficiency), which is the fraction of the actual cross sectional area that is swept out compared to the cross sectional area that is geometrically possible (smaller drops can follow air streamlines

and go around the big drop) (see the figure below),

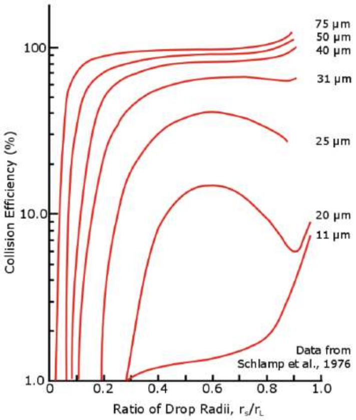
- v_L is the velocity of the large drop and v_s is the velocity of the smaller, slower falling drops below,
- and *LWC* is the liquid water content.

The figure below provides a good conceptual picture of collision–coalescence. The collector drop must be falling faster than the smaller collected drop so that the two of them can collide. As the air streamlines bow out around the drop, they carry the smaller drops with them around the drop, and the effective cross-section becomes less than the actual cross-section, which is simply the cross-sectional area of a disk with a radius that is the sum of the radii of the large collector drop and the smaller collected drops. As drops get bigger, they have too much inertia to follow the air streamlines, thus making the collision more likely.



Schematic of the maximum possible geometric cross-section of a large and small drop and the actual cross-section due to particles following air streamlines around the big particle. Credit: W. Brune (after Lamb and Verlinde)

E_c is small for 10 μm drops, so by a random process, some drops become bigger than others and begin collecting smaller drops (see figure below). $E_c$increases as the radius of the falling drop increases. When the larger falling drop gains a radius of more than 100 μm, its collision–coalescence efficiency is very good for all smaller drops down to sizes of about 10–20 μm.



Collision–collection efficiencies for two drops, with the percent collision efficiency on the vertical axis; the ratio of the radius of the small drop, r_s, to the radius of the large drop, r_L, on the horizontal axis; and lines for individual large drop radii. Credit: W. Brune (after Rogers and Yau)

Once a collecting drop has reached a radius of a few hundred μm, it is falling fast and its collision–coalescence efficiency is close to 100%. The growth of its radius then has the approximate form:

$$r_d \propto \exp(\text{time}) \tag{5.8.4}$$

So, the activated cloud drops grow to 10–20 μm by the slow growth of vapor deposition (square root of time). Then when collision–coalescence starts and produces a few big drops, they can grow exponentially with time.

Smaller drops are typically spherical. Once these drops get to be above a mm in radius, they become increasingly distorted, with a flattened bottom due to drag forces, and they look a little like the top half of a hamburger bun. They can be further distorted so that the middle of the bun-shape gets pushed up by the drag forces so that the drop takes on a shape resembling an upside down bowl.

Eventually they break up, either by getting thin enough in the middle that they break into pieces or by colliding with other drops so hard that filaments or sheets of liquid break off to form other drops. These processes create a whole range of sizes of particles. Thus rain consists of drops that have a wide spectrum of sizes. The following video (2:50) entitled "How Raindrops are Formed" starts with a simplified view of the atmosphere's water cycle, but then shows examples of a falling drop, collision–coalescence, and cloud drop break-up.

How Raindrops are Formed

Click here for transcript of the How Raindrops are Formed.

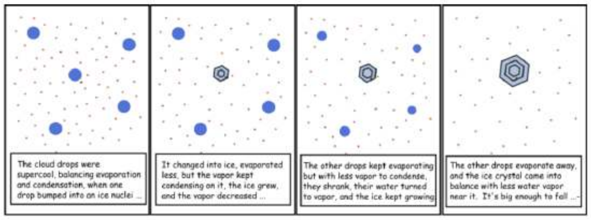
> Now this is a familiar scene. The sun's heat causes water from plants, lakes, and oceans to turn from a liquid to a vapor. High in the atmosphere the water vapor then cools down and condenses from a gas back into a liquid. The liquid water then falls back to the surface in the form of rain, snow, ice, or hail. Water runs off into streams lakes and oceans or is stored in the ground or in snow path. This is the water cycle and it describes our most vital resource moves through the whole earth system, but like most things in our world when we look at the tiny parts that make up the whole we can learn a lot more about the phenomenon. Take the shape of a single raindrop. Small droplets of water in the atmosphere are spherical in shape due to the surface tension or skin of the water molecules. As these droplets grow they become heavier and start to fall through the air. As they fall, the raindrop collides with other drops and continues to get bigger. These larger raindrops fall through the air faster the wind resistance on the underside of the drop causes the bottom of the drop to flatten resulting in a drop looking like a hamburger bun. As the drop continues to fall and grow at some point it becomes too large for the surface tension to hold it together, so the raindrop breaks apart into smaller spiracle drops. Investigating the processes we can't see with the naked eye is nothing new. Science and technology drive each other forward and often lead to insights and discoveries along the way. With the invention of high-speed photography we finally saw the most basic elements of our watery planet in action. Understanding how a tiny raindrop falls through the atmosphere does more than debunk the myth that a raindrop Falls like a teardrop. It actually makes a difference when it comes to measuring precipitation in particular for ground radars. Ground radars look at the sides of the raindrops and then estimate the vertical and horizontal sighs. A heavier, flatter drop allows radars to identify heavier precipitation. In fact the two radars on board the GPM satellite can also measure drop sizes from space and so a more accurate look at rain drops gives us a more accurate look at how global rainfall is shaping up. *Credit*

> For riming, capture nucleation, and aggregation, there are similar equations with terms similar to those in Equation 5.16—an area swept out, a collection efficiency, the relative velocity, and the liquid or solid mass concentration of the smaller drops or ice. These are typically a bit more complicated if the ice is not spherical, but the concepts are the same. These ice collision–coalescence processes are able to produce ice particles big enough to fall, and if these particles warm as they pass through the warm part of the cloud, they can turn into liquid rain. A significant fraction of rain in the summer can come from ice collision–coalescence processes above the freezing line in the clouds.

5.9: An Unusual Way to Make Precipitation in Mixed-Phase Clouds

Recall that water can exist in liquid form even when T < 273 K. This supercooled liquid needs ice nuclei (IN) in order to become ice, although at a temperature of about –40^{o}C, it can convert to ice homogeneously.

Recall from Chapter 4 that the vapor pressure over supercooled liquid water is greater than the vapor pressure over ice at the same temperature. So, if an ice particle is introduced into air that contains liquid water with T < 273K, the ambient vapor pressure in equilibrium with the liquid will be greater than the saturation vapor pressure of the ice. The ice will grow, but this uptake of water vapor will cause the ambient water vapor pressure to be less than the saturation vapor pressure for the liquid drops and the liquid drops will have net evaporation. This process will continue so that the ice grows at the expense of the liquid drops, which will shrink. The transfer of water is not by the liquid drops colliding with the ice crystal; the transfer of water comes from the liquid drops evaporating water to make water vapor and then that water vapor diffusing over to the ice, where it condenses. This process is called the **Bergeron–Findeisen Process**, and is a way that precipitation-sized drops can be formed in about 40 min in mixed-phase clouds (see figure below).



Cartoon of ice growth in the presence of supercooled liquid drops

Click for a text description of the cartoon.

Frame 1: The cloud drops were supercool, balancing evaporation and condensation, when one drop bumped into an ice nuclei...

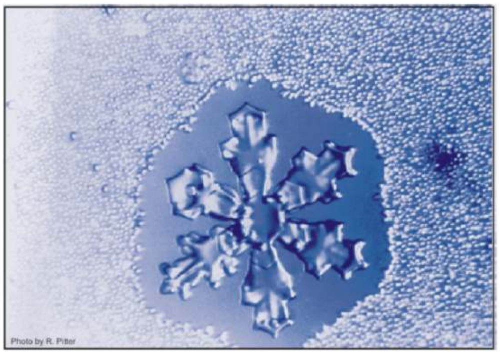
Frame 2: It changed into ice, evaporated less, but the vapor kept condensing on it, the ice grew, and the vapor decreased...

Frame 3: The other drops kept evaporating but with less vapor to condense, they shrank, their water turned to vapor, and the ice kept growing

Frame 4: The other drops evaporate away, and the ice crystal came into balance with less water vapor near it. It's big enough to fall...

Credit: W. Brune

This process, as unusual as it seems, actually works, as can be seen in the figure below!



Photograph of an ice crystal growing by the Bergeron effect in a field of small supercooled liquid drops. *Credit: R. Pitter*

5.10: Summary and Final Tasks

Summary

Clouds are shaped and sized by atmospheric motions and mixing with the surrounding air and are composed of either liquid or ice drops depending on their environmental temperature. The basic shapes are stratus or cumulus or mixtures of the two; the altitudes define low, middle, and high clouds.

Understanding clouds requires looking at individual cloud drops through a microscope. Cloud drops form when there is sufficient moisture, aerosol to act as Cloud Condensation Nuclei (CCN), and cooling air. This cooling air becomes supersaturated with water vapor by radiative cooling (e.g., valley fog), uplift (e.g., cumulus convection), or mixing (e.g., contrail). Each CCN particle requires supersaturation to grow into a cloud drop as a competition takes place between a curvature effect (tiny particles have higher saturation vapor pressure than flat surfaces) that inhibits water uptake, while a solute effect (the particle dissolving in liquid water) enhances water uptake. Once the atmosphere has cooled enough to achieve supersaturation greater than the critical supersaturation for a CCN particle, that particle can take on enough water to continue growing large enough to become a cloud drop.

Initially, the drop grows by vapor deposition, but this process slows down as the square root of time, so that the formation of raindrops is not possible within the typical 30-minute lifetime of a cloud. Other processes are at work. In a warm cloud, where all the drops are liquid, collisions and coalescence of drops, with occasional breakup, exponentially increases the size of the drops as they fall. In a cold cloud, precipitation drops can grow either by riming of ice with supercooled liquid drops or by collisions and aggregation of ice particles or by vapor deposition from supercooled liquid to ice.

Reminder - Complete all of the Lesson 5 tasks!

You have reached the end of Lesson 5! Double-check that you have completed all of the activities before you begin Lesson 6.

CHAPTER OVERVIEW

6: Atmospheric Radiation

Learning Objectives

By the end of this chapter, you should be able to:

- identify the causes of changing solar radiation on Earth
- calculate properties of the spectrum of solar and earth radiation in terms of the Planck function
- calculate the absorption between you and a light source
- explain why the sky looks blue and hazy in the summer

Atmospheric radiation plays a critical role in life on Earth and in weather. Without solar heating, Earth would be a dead frozen ball hurtling through space. Luckily, the energy that Earth receives from solar radiation is sufficient to produce liquid water on its surface, thus enabling life to thrive. In this chapter, we will look at solar radiation and its changes over time. Radiation is just another form of energy and can be readily converted into other forms, especially thermal energy, which is sometimes called "heat." In this chapter, we will use the word "radiation" to mean all electromagnetic waves, including ultraviolet, visible, and infrared. We will introduce some unfamiliar terms like "radiance" and "irradiance" and will be careful with our language to prevent confusion.

6.1: Prelude to Atmospheric Radiation

An important concept in studying atmospheric radiation is that all objects emit and absorb radiation. For a perfect emitter, the radiation emitted by an object, called the irradiance, is determined by the Planck function, which depends only on temperature and wavelength. The higher the temperature, the greater the radiation emitted at all wavelengths and the shorter the wavelength of the peak energy. The Sun emits in the visible while Earth and its atmosphere emit in the infrared. No object is really a perfect emitter at every wavelength; the unitless number emissivity measures how good or poor an emitter is. At each wavelength, a good emitter is a good absorber.

How well an object absorbs at different wavelengths of radiation, called its absorptivity, depends on its chemical composition and the rules of quantum mechanics. As a result, some absorption is strong and some is weak; some is in sharp lines while some is in broad features in the wavelength spectrum; some is in the UV, particularly due to O_2 and O_3, little is in the visible, and much absorption, including broad bands and sharp lines, occurs in the infrared, particularly by H_2O, CO_2, and O_3.

The radiation that is not absorbed by a gas, liquid or solid is either transmitted or scattered. The amount of transmitted radiation depends on the absorption and scattering cross-sections of the gas, liquid, or solid components of matter, so that the larger the cross-section and the distance through the matter, the less radiation passes through the matter. The decay of the transmitted light with distance through the matter is exponential, as is described by Beer’s Law.

Earth’s surface and atmospheric gases can emit and absorb radiation at the same wavelengths. Most of Earth’s emissions are at infrared wavelengths, whether the emission be from the surface, clouds, or atmospheric gases.

Scattering of atmospheric radiation complements absorption and is even more difficult to track through the atmosphere than absorption is. The wavelength of the radiation and the size, shape, and composition of the scattering particle together determine the scattering efficiency and scattering pattern. Many of the skies that we remember best are due to the scattering and absorption of sunlight.

6.2: Atmospheric Radiation - Why does it matter?

Everything radiates—the Sun, the Earth, the atmosphere, and you. The energy provided by the Sun is reused in the Earth system to provide the energy that drives weather and climate. But ultimately, the infrared radiation radiated by Earth into space must balance the solar visible radiation coming into the Earth system. From the point-of-view of the Earth system, we are most concerned about how atmospheric radiation interacts with matter. Matter is simply molecules and atoms and the structures that they build, such as the air, the clouds, the Earth, and the Sun.

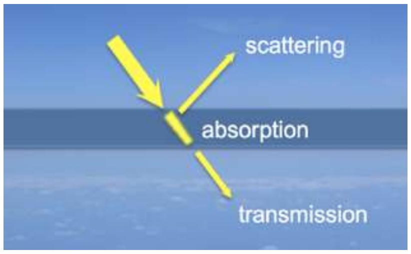
When radiation encounters matter, three things can happen. The radiation can be *transmitted through* matter; it can be *absorbed by* the matter; it can be *scattered by* the matter. One of these three things must happen, so we can sum them up to one:

$$a+\tau+s=1 \tag{6.2.1}$$

where τ is the transmissivity, the fraction transmitted; a is the absorptivity, the fraction absorbed; and s is the reflectivity, the fraction that is scattered or reflected.

Scattering vs. reflecting

Scattering and reflecting are related but are different because reflection is scattering *in a particular direction* whereas scattering tends to go *in a range of directions*.



What can happen when radiation meets matter. The sum of radiant energy that is scattered, absorbed, and transmitted must equal the amount of incoming radiant energy.Credit: W. Brune

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6.3: Start at the Source - Earth Rotating Around the Sun

Solar radiation drives the Earth system and makes life possible. Solar radiation is absorbed and then put to use to increase the surface temperature, to change the phase of water, and to fuel atmospheric chemistry. The uneven distribution of solar radiation on Earth’s surface drives atmospheric dynamics.

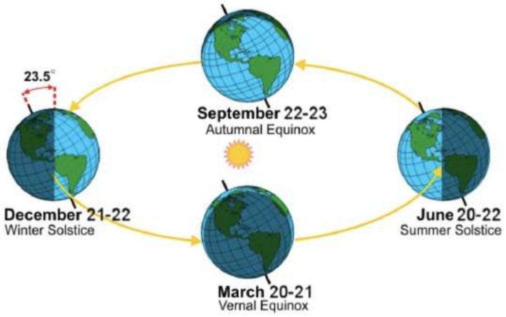

Figure 6.3.1: Earth’s orbit around the sun. *Credit: National Weather Service*

The total amount of solar energy per unit time and unit area, also called the solar irradiance, is 1361 W m^{-2} at the top of the atmosphere (Stephens et al., 2012, Nature Geoscience 5, p. 691). It is distributed unevenly over Earth’s surface. That distribution changes over the course of the seasons (see next two figures). The seasons result primarily from the Earth's rotation axis not being perpendicular to the plane of the Earth's orbit around the Sun.

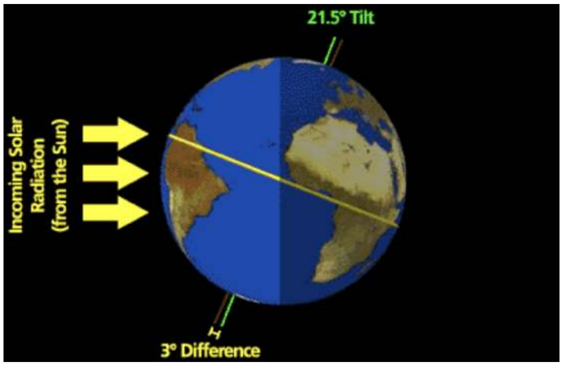

Figure 6.3.2: The current tilt of Earth’s orbit. More solar radiation is absorbed by Earth's surface near the equator than at high latitudes as a result of the curvature of the Earth. Credit: National Park Service

Earth’s spin and its orbit around the sun are not constant, but instead, change with time, like a spinning top. The orbit’s eccentricity (i.e., how different it is from circular) varies with a 100,000-year period. The tilt of Earth’s rotation axis with respect to the perpendicular from its orbit, which is called its obliquity, varies from 22.1° to 24.5° during a 41,000-year cycle. It is currently 23.4° and decreasing. Finally, the precession of Earth’s orbit, which is the orientation of the rotation axis with respect to Earth’s orbital position, also varies with a period of about 26,000 years, although the eccentricity of the orbit is also rotating around the sun, so that the effective period of precession is about 21,000 years. These motions, when taken together, slowly and periodically change the distribution of solar irradiance on Earth’s surface and are described by the **Milankovitch Theory** (see next two figures).

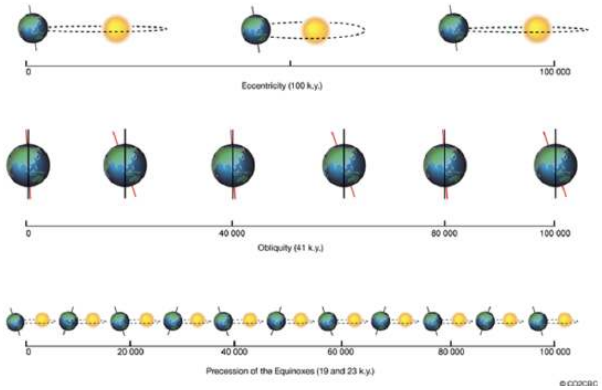

Figure 6.3.3: Changes in Earth's orbit that occur over time (top). Eccentricity (non-circular shape) varies over 100,000 years (middle). Obliquity (tilt of Earth's rotation axis) varies every 41,000 years (bottom). Precession (obliquity relative to position in Earth's orbit) varies over a cycle with a period of approximately 20,000 years. Credit: CO2CRC

Changes in Earth's orbit and spin are not the only ways that solar irradiance changes—the Sun's energy output also changes. It has been increasing slightly (0.05–0.10%) over the past 300 years and varies by another ~0.1% over the course of the 11-year solar cycle. The ultraviolet (200–300 nm) irradiance has increased about 3% in the past 300 years and varies by ~1.5% between solar maximum and solar minimum. This increased UV leads to greater stratospheric ozone production, which increases stratospheric heating, leading to poleward displacement of the stratospheric meridional wind.

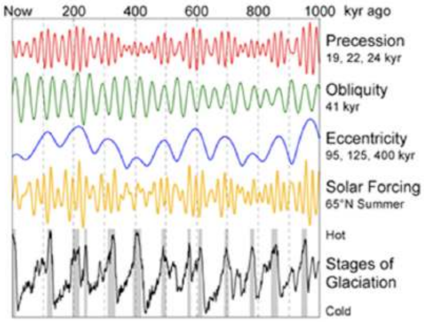

Figure 6.3.4: Variation of solar irradiance (i.e., forcing) observed as a result of the changes in eccentricity, obliquity, and precession. Links to glaciation are complex but the distribution of solar irradiance on Earth is important for climate. Credit: The Azimuth Project

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6.4: How is energy related to the wavelength of radiation?

We can think of radiation either as waves or as individual particles called photons. The energy associated with a single photon is given by

$$E = h\nu \tag{6.4.1}$$

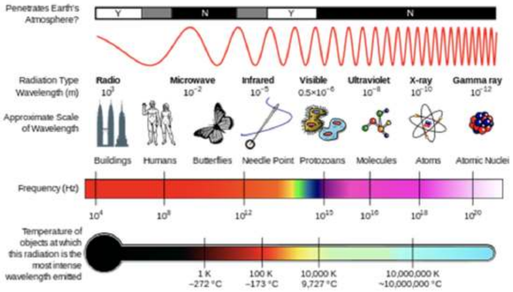
where E is the energy (SI units of J), h is Planck's constant (h = 6.626 x 10^{-34} J s), and ν is the frequency of the radiation (SI units of s^{-1} or Hertz, Hz) (see figure below). Frequency is related to wavelength by $\lambda = c/\nu$, where c, the speed of light, is 2.998 x 10^8 m s^{-1}. Another quantity that you will often see is wavenumber, $\sigma = 1/\lambda$, which is commonly reported in units of cm^{-1}.

The energy of a single photon that has the wavelength λ is given by:

$$E = \frac{hc}{\lambda} = \frac{1.986 \times 10^{-16} \mathrm{J\,nm\,photon}^{-1}}{\lambda} \tag{6.4.2}$$

Note that as the wavelength of light gets shorter, the energy of the photon gets greater. The energy of a mole of photons that have the wavelength λ is found by multiplying the above equation by Avogadro's number:

$$E_m = \frac{hcN_A}{\lambda} = \frac{1.196 \times 10^8 \mathrm{Jnmmol}^{-1}}{\lambda} \tag{6.4.3}$$



Energy scales: penetration through Earth’s atmosphere; radiation name by wavelength; physical object the size of that wavelength; frequency compared to wavelength; and temperature of an object that has its peak radiation at each wavelength. Credit: Wikimedia Commons

In the lesson on atmospheric composition, you saw how solar UV radiation was able to break apart molecules to initiate atmospheric chemistry. These molecules are absorbing the energy of a photon of radiation, and if that photon energy is greater than the strength of the chemical bond, the molecule may break apart.

? Exercise

Consider the reaction

$$\mathrm{O_3 + UV \longrightarrow O_2 + O\cdot}.$$

If the bond strength between O_2 and O* (i.e., excited state oxygen atom) is 386 kJ mol^{-1}, what is the longest wavelength that a photon can have and still break this bond?

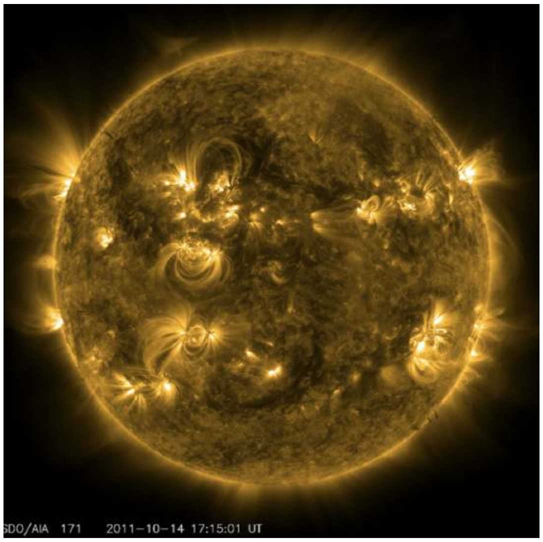
Click for answer.

Answer: Solve for wavelength in equation [6.2b]

$$\lambda = \frac{1.196 \times 10^8 \mathrm{Jnmmol}^{-1}}{E_m} = \frac{1.196 \times 10^8 \mathrm{Jnmmol}^{-1}}{386 \times 10^3 \mathrm{Jmol}^{-1}} = 309 \mathrm{nm}$$

6.5: The Solar Spectrum

The Sun emits radiation from X-rays to radio waves, but the irradiance of solar radiation peaks in the visible wavelengths (see figure below). Common units of irradiance are Joules per second per m^2 of surface that is illuminated per nm of wavelength (e.g., between 300 nm and 301 nm), or W m^{-2} nm^{-1} for the plot below. These units are the units of spectral irradiance, which is also simply called irradiance, but as a function of wavelength.



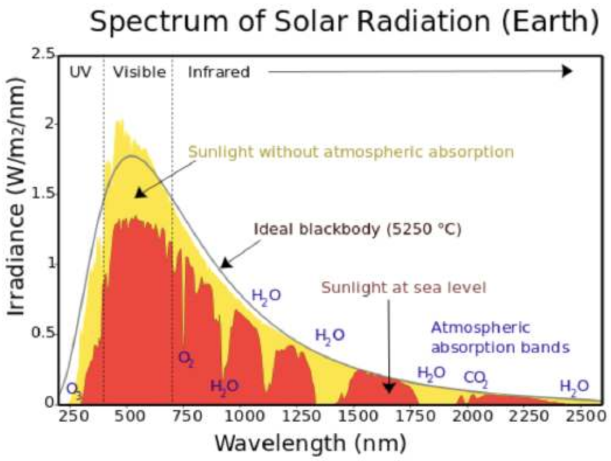
The Sun's emission in the extreme ultraviolet part of the solar emission spectrum. Credit: NASA Goddard Space Flight Center via flickr

To get the total irradiance in units of W m^{-2}, the spectral irradiance should be integrated over all the wavelengths.

Note the following for the solar spectrum:

- About half of the energy is in the visible wavelengths below 0.7 μm. We can tell this by doing a quick integration.
- O_3 and O_2 absorb much of the UV irradiance below 300 nm high in the atmosphere.
- About 70% of the visible irradiance makes it all the way to sea level.
- O_3 absorbs a little of the visible irradiance.
- A significant fraction of the visible irradiance is scattered by clouds and aerosol. Some is reflected back out into space so that this portion never deposits energy in the Earth system.
- There are large wavelength bands in which water vapor, CO_2, and O_3absorb infrared irradiance.

For solar wavelengths at which the absorptivity is high, the solar irradiance at sea level is small. Note that the big absorbers of infrared irradiance are water vapor, carbon dioxide, and ozone.



Solar spectrum and atmospheric absorbing gases from 240 nm to 2.5 µm wavelengths. Credit: Nick84 [CC BY-SA 3.0], via Wikimedia Commons

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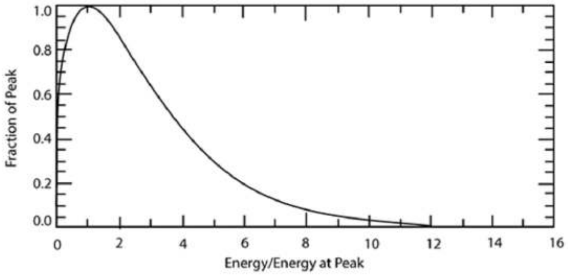
6.6: What is the origin of the Planck Function?

Recall that molecules have a wide range of speeds and thus a wide range of energies. The **Maxwell–Boltzmann Distribution**, which gives the distribution of molecules as a function of energy, is given approximately by the equation:

$$f(E) = \frac{2\sqrt{E}}{\sqrt{\pi}(kT)^{3/2}} \exp(-E/kT) \tag{6.6.1}$$

where f is the probability that a molecule has an energy within a small window around E, T is the absolute temperature, and k is the Boltzmann constant. The above equation, when integrated over all energies, gives the value of 1.

The functional form of this distribution is shown below:



Maxwell–Boltzmann distribution of molecules as a function of the energy of the molecules normalized to the number of molecules with the peak energy.

All objects—gas, liquid, or solid—emit radiation. If we think of radiation as photons, we would say that these photons have a distribution of energies, just like molecules do. However, photons cannot have continuous values of photon energy; instead, the photon energy is quantized, which means that it can have only discrete energy values that are different by a very very small amount of energy. When this quantized distribution is assumed, then the distribution of spectral irradiance leaving a unit area of the object's surface per unit time per unit wavelength interval into a hemisphere is called the **Planck Distribution Function of Spectral Irradiance**:

$$P_e(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp(hc/\lambda kT) - 1} \tag{6.6.2}$$

where h is Planck's constant, c is the speed of light, k is the Boltzmann constant (1.381 x 10^{-23} J K^{-1}), T is the absolute temperature, and λ is the wavelength. The integral of this function over all wavelengths leads to the **Stefan–Boltzmann Law Irradiance**, which gives the total radiant energy per unit time per unit area of the object's surface emitted into a hemisphere.

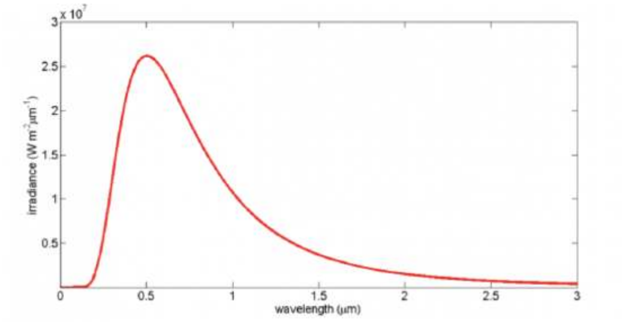
$P_e(\lambda)/\pi$ is called the **Planck Distribution Function of Spectral Radiance** and commonly has units of W $\text{steradian}^{-1}\text{m}^{-2}$ nm^{-1} and is often denoted by the letter I. A steradian is just a unit solid angle. Just as a radian for a circle is a length of the circle's arc that is equal to the circle's radius, a steradian is the area on a sphere's surface that is equal to the sphere's radius squared. There are 2π radians along a circle and 4π steradians (abbreviated sr) over a sphere. For some of this discussion we will use the spectral irradiance $P_e(\lambda)$ and consider the radiation from any area on the sphere to be emitted not in a single direction but into the full hemisphere of directions. Later on, when we start talking about absorption, we will need to think about the irradiance in very specific directions, in which case we will use radiance. Be mindful of the difference between irradiance and radiance.

The spectral irradiance is the amount of energy that is emitted from or falling on a unit area in space per unit time per unit wavelength (W m^{-2} μm^{-1}). So the m^2 in this case indicates a surface area on which energy is leaving or falling. To get the Sun's spectral irradiance at the top of Earth's atmosphere, we must multiply the spectral irradiance emitted by the Sun's surface by the Sun's surface area to get the total energy emitted per unit time and per unit wavelength by the Sun and then divide by the surface area of the sphere that has a radius equal to the Earth–Sun distance to get the energy per time per unit wavelength per unit area of the surface located at the top of Earth's atmosphere.

Physical Interpretation

All the radiation that is emitted from the Sun's surface continues to move outward at the speed of light until it hits objects, but there are very few objects between the Sun and Earth, except occasionally the Moon. Thus, in the absence of any objects, the total amount of solar irradiance striking the planets decreases as the square of the distance between the center of the Sun and the surface of the planet.

The Planck distribution function spectral irradiance for an object at a temperature of 5777 K (the Sun's surface temperature) is shown in the figure below. Note the rapid rise at the shorter wavelengths, the peak value, and then the slower decline at longer wavelengths. Look at the peak value of the irradiance, which is about 25 million W m^{-2} nm^{-1}! That's a lot of energy being radiated from the Sun's surface, but of course the Earth is 150 million km away from the Sun and thus intercepts less than half a billionth of the solar irradiance.



The Planck distribution function spectral irradiance, P_e (equation 6.4) emitted into a hemisphere for the Sun's surface temperature, T_{sun} = 5777 K. Credit: W. Brune

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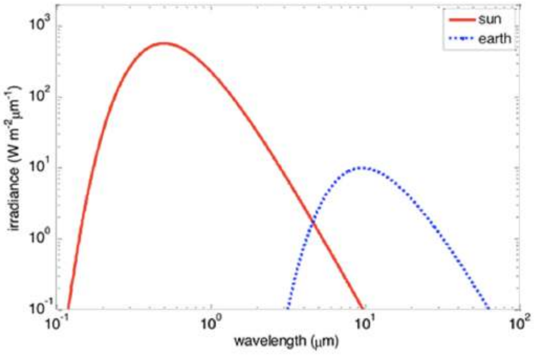
6.7: Which wavelength has the greatest spectral irradiance?

The peak of this distribution as a function of wavelength can be found by taking the derivative of $P_e(\lambda)$ with respect to wavelength, setting the value equal to 0, and solving for the wavelength. The result is the **Wien Displacement Law**:

$$\lambda_{\max} = \frac{2898 \mu \mathrm{mK}}{T} \tag{6.7.1}$$

For the sun with a photospheric temperature of about 5780 K, λ_{max} ~ 0.500 μm or 500 nm, which is the color green. However, for Earth with a mid-tropospheric temperature of about 260 K, the peak wavelength is closer to 11 μm, well into the infrared (see below).



The spectral irradiance of solar radiation for a surface located at the top of Earth's atmosphere (red solid curve) and the spectral irradiance emitted by Earth into the upward hemisphere above its surface (blue dashed curve) To get these curves, we assumed that the Sun and the Earth both emit radiation according to the Planck distribution function spectral irradiance, which they do not quite do.

Credit: W. Brune

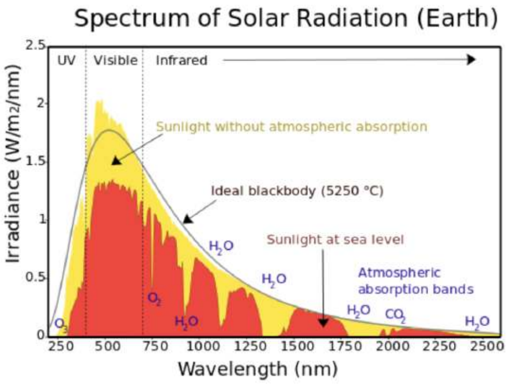
6.8: What is the total irradiance of any object?

If the Planck distribution function spectral irradiance is integrated over all wavelengths, then the total irradiance emitted into a hemisphere is given by the **Stefan–Boltzmann Law**:

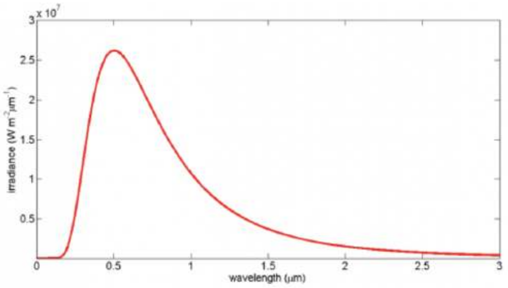
$$F_s = \sigma T^4 \tag{6.8.1}$$

where σ is called the Stefan-Boltzmann constant (5.67 x 10^{-8} W m^{-2} K^{-4}). F_s has SI units of W m^{-2}, where the m^2 refers to the surface area of the object that is radiating.

The Stefan–Boltzmann law (total) irradiance applies to an object that radiates according to the Planck distribution function spectral irradiance. If we look at the figure below, we see that the solar spectrum at the top of the atmosphere is similar to the Planck distribution function but does not follow it perfectly. However, the Planck distribution function with the same total irradiance as the sun has a temperature of 5777 K, as in the second figure.



Solar spectrum and atmospheric absorbing gases from 240 nm to 2.5 µm wavelengths. Credit: Nick84 [CC BY-SA 3.0], via Wikimedia Commons



The Planck distribution function spectral irradiance, P_e (equation 6.4), emitted into a hemisphere for the sun, T_{sun} = 5777 K. Credit: W. Brune

Exercise

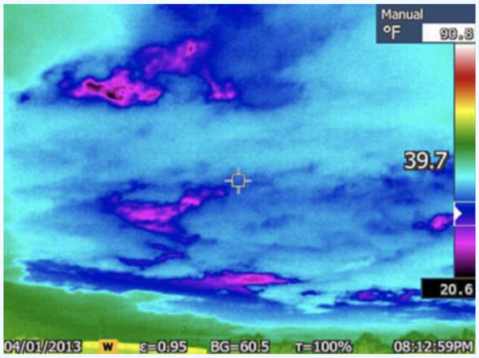
Clouds radiate. Assume two spherical clouds, one with a radius of 100 m and a temperature of 275 K and a second with a radius of 100 m and a temperature of 230 K. Assuming that they both radiate according to the Planck distribution function, calculate the emission for each cloud in W m^{-2} and in W. Which cloud is radiating more total energy and by how much?

Click for answer.

Answer:

Cloud T (K)	**Cloud radius (m)**	**F_s (W m^{-2})**	**F_s x $4\pi R_c^2$ (W)**
275	100	324	4.1 x 10^7
230	100	100	2.0 x 10^7

The warmer cloud is radiating about twice as much energy as the cooler cloud. These little clouds are radiating quite a lot of energy in all directions, but some of it is down toward Earth's surface. If we make the simple assumption that half the radiation goes up and the other half goes down, the amount of energy radiated toward Earth's surface per second is approximately 10 million W. If the clouds are not too far from the surface, this downward radiation could contribute a few hundred W m^{-2} of heating at Earth's surface. Thus clouds can act like additional heat sources for Earth's surface, keeping its temperature higher than it would be on a clear night. The image below is an infrared photograph of the sky above Ogden, Utah. Infrared radiation detected by the camera has been converted to temperature, with higher temperatures indicating more infrared emission.



Credit: *activerain*

6.9: Kirchhoff’s Law explains why nobody is perfect

Remember that when radiation encounters matter it may be absorbed or transmitted or scattered (including reflected). For an object acting as a perfect Planck distribution function, it must absorb all radiation completely with no scattering and no transmission. Some objects absorb very well at some wavelengths but not at others. For instance, water vapor absorbs little visible radiation but absorbs infrared radiation at some wavelengths very well.

At the same time, the sun, like other objects, does not radiate perfectly according to the Planck distribution function spectral irradiance, but instead radiates at a fraction of it at some wavelengths. This fraction, which goes from 0 to 1, is called the emissivity and is denoted by ε. How is an object’s emissivity related to its absorptivity?

Kirchhoff’s Law states that at any given wavelength, an object’s emissivity ε is equal to its absorptivity, that is:

$$\varepsilon(\lambda)=\alpha(\lambda) \tag{6.9.1}$$

Thus, if an object has some wavelengths at which radiation is scattered or reflected, then the object will have an emissivity less than 1 at the wavelength, and the fraction that is absorbed will be equal to the emissivity at each wavelength.

Thus, when we integrate the Planck distribution function spectral irradiance over wavelength to obtain the irradiance emitted by the object, it first has to be multiplied by the wavelength-dependent emissivity, thus leading to the modified form of the Stefan-Boltzmann law:

$$F=\varepsilon\sigma T^4 \tag{6.9.2}$$

where we understand that ε is some form of averaged emissivity.

Watch the following video (1:07), where the Stefan–Boltzmann Law is described in greater detail:

Stefan-Boltzmann Law

Click here for transcript of the Stefan-Boltzmann law.

> This formula, Stefan-Boltzmann law, is the one that we will use the most. Note that for a perfect emitter, epsilon equals to one, the total irradiance submitted into a hemisphere equals the product of the Stefan-Boltzmann constant, sigma, and the temperature to the fourth power. However, the irradiance is modified by the emissivity, which equals the [INAUDIBLE]. Note that this emissivity here is some sort of average over all emissivities for different wavelengths, and we've seen that emissivity can vary a lot with wavelengths. Water vapor, for example, has a very low emissivity invisible, but a very strong one in the infrared. The [INAUDIBLE] depends on the composition of matter, but it also depends on the number concentration of gaseous matter. And the pass length through that matter. Go back and look at Beer's Law of Absorption to see this dependence. With this form of the stuff on Boltzmann law, we can compare the irradiances of two different bodies of matter at different temperatures or different emissivities.

Some typical average emissivities are listed in the table below. These are emissivities averaged over all wavelengths. At any particular wavelength, the emissivity may be greater or less than the average.

Wavelength-Averaged Emissivity of Some Common Materials

Material	Emissivity, ε
ice	0.97
pure water	0.96
snow	snow
trees (oak, beech, maple, pine)	0.97-0.98
grass	0.98
soil	0.93
aluminum foil	0.03
asphalt	0.88-0.94

What about gases? Gases absorb and thus emit like all other matter. To know more about the emissivity of all objects, we need to know more about the absorption of objects.

6.10: Why do objects absorb the way that they do?

We see that there is a significant amount of absorption of radiation in the infrared but rather little in the visible. Also, we see that gases absorb strongly at some wavelengths and not at others. Why is this?

To answer this question, we need to look at the configurations of the electrons that are zooming around atoms and molecules. More than 100 year ago, scientists began using prisms to disperse the light from the sun and from flames containing different elements. While the sun gave the colors of the rainbow, the flames had light in very distinct lines or bands. This puzzle was finally resolved a little more than 100 years ago with the invention of quantum mechanics, which basically says that the electrons zooming around atoms and molecules and the vibrations and rotations of molecules can have only discrete energies that are governed by rules of conservation of angular momentum.

Perito Moreno Glacier Opal Iceberg in Argentina. The blue color results from selective absorption of radiation with wavelengths at the red end of the visible spectrum. In the figure below that shows the wavelength dependence of absorption by different molecules, you can see that water vapor has absorption in the red part of the visible spectrum, starting at about 600 nm (0.6 µm). Credit: Dominic Alves via flickr

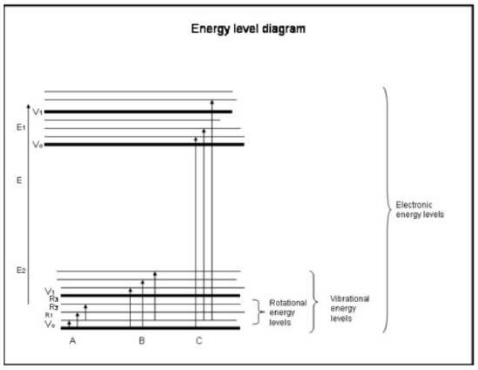
The following bulleted list is a crash course in absorption by the electrons in atoms and molecules. Refer to the figure below the box.

Crash Course: Absorption by the Electrons in Atoms and Molecules

- Chemical bonds and quantum mechanics together determine the energy levels that any electron, atom, or molecule can be in.
- The molecular energy is a sum of the energy related to the position of the electron relative to the stable ground electronic state, the molecular vibration, and the molecular rotation.
- Absorption occurs when the energy of the photon matches the difference between two energy levels in a molecule, $\Delta E = E_{final} - E_{initial} = hc/\lambda$.
- Rules set by conservation of angular momentum and electron spin determine which transitions are allowed.
- The amount of absorption, called simply the absorption cross section, σ, comes from many factors, but varies significantly from molecule to molecule and from transition to transition. The cross section has dimensions of area and commonly has units of cm^2.
- Electronic transitions occur when the electrons actually jump into other orbits around the nuclei. They have energies that are equivalent to radiation (i.e., photons) in the ultraviolet to visible wavelengths.
- Vibrational transitions occur when the molecule vibrates at a different frequency or in a different way. Diatomic molecules (e.g., O_2 and N_2) have only one way to vibrate—back and forth along the chemical bond that binds them. But more complicated molecules (e.g., H_2O and CO_2) can vibrate not only with the nuclei going toward and away from each other, but also by bending in three directions. These vibrational transitions, accompanied with motions that combine vibration and rotation, have energies equivalent to the near and middle infrared radiation (i.e., photons).
- Rotational transitions occur when a molecule changes its rate of rotation. These transitions have energies equivalent to radiation (i.e., photons) in the far infrared to radio wave wavelengths.
- Translational energies of molecules in Earth's atmosphere, $\sim kT$, are generally a little larger than the energy required to move from one rotational level to another, 10–100 times less than the energy required to go from one vibrational level to another, and hundreds to thousands of times less than what is required to go from one electronic level to another.

- We have already seen that some molecules and atoms have more kinetic energy than others. All molecules with kinetic energy larger than the energy difference between rotational levels can collide with the molecule and give it enough rotational energy to change to a higher rotational level. Thus, we see that rotational energy is distributed over many rotational levels, but that vibrational and electronic levels are usually the ground (lowest energy) levels.
- Atoms do not have molecule-like vibrations and rotations because they have only one nucleus, so their spectra consist only of electronic transitions.
- A sharp transition from one discrete level to another, which appears as a line in a spectrum, occurs in a narrow range of energies about the transition energy difference. The resulting radiation occurs in a narrow band of wavelengths about the line's central wavelength. The width of this line (measured at half the line's maximum height) is called the linewidth.
- This natural linewidth can be broadened by the molecule's motion, called **Doppler broadening**, or by collisions, called **pressure broadening**.
- High in the atmosphere, Doppler broadening is dominant because the pressure is low, but lower in the atmosphere, pressure broadening becomes dominant even though Doppler broadening also increases. So the absorption lines are broader near Earth's surface than they are higher up in the atmosphere.

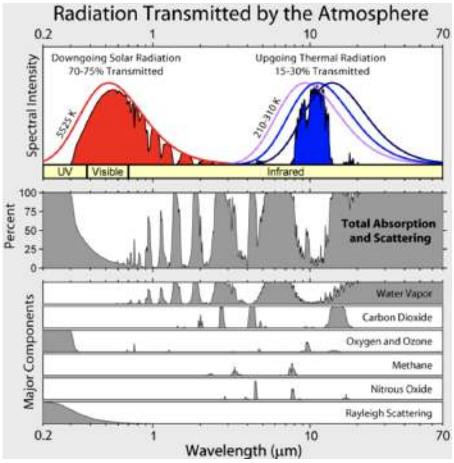
The absorption cross section, σ, varies significantly over the width of the absorption line. So it is possible for all the radiation to be absorbed in the middle of the line but very little absorbed in the "wings."



An energy level diagram for a molecule. The greater the distance between lines, the greater the energy of the absorbed or emitted photon and thus the shorter the photon's wavelength. Not all transitions are allowed between all levels because of conservation of angular momentum. Credit: UC Davis chemwiki

Physical Interpretation

Atoms and molecules can absorb radiation (a photon) only if their structure has an energy difference between levels that matches the photon's energy (hc/λ). Otherwise, the atom or molecule will not absorb the light. Once the molecule has absorbed the photon, it can either lose a photon and go back to its original lower energy level; or it can break apart if the photon energy is greater than the chemical bond holding the molecule together; or it can collide with other molecules, such as N_2 or O_2, and transfer energy to them while it goes back to its lower energy level. Collisions happen often, so the energy of the absorbed photon is often transferred to thermal energy.



Solar and terrestrial irradiance and absorption by molecules in the ultraviolet, visible, and infrared. Credit: Robert A Rohde, Global Warming Art, via Wikimedia Commons

Note that Earth's upgoing infrared irradiance is limited to a few atmospheric "windows" and the irradiance at all other wavelengths is strongly absorbed, mostly by water vapor, but also by carbon dioxide, ozone, nitrous oxide, methane, and other more trace gases that aren't shown in the figure above.

CHAPTER OVERVIEW

7: Applications of Atmospheric Radiation Principles

Learning Objectives

By the end of this chapter, you should be able to:

- demonstrate the effects of infrared absorbers on Earth’s temperature using a simple model
- explain the concept of radiative–convective equilibrium
- determine what a satellite is seeing by interpreting the observed spectrum of upwelling infrared radiation

Now that you are familiar with the principles of atmospheric radiation, we can apply them to help us better understand weather and climate. Climate is related to weather, but the concepts used in predicting climate are very different from those used to predict weather.

7.1: Prelude to Applications of Atmospheric Radiation Principles

For climate, we need to understand the global energy budget comprised of solar radiation coming into the Earth’s atmosphere and infrared radiation leaving the atmosphere to go into space. We will see that, when averaged over the Earth and over sufficient time, the energy associated with infrared radiation emitted to space by the Earth’s surface and atmosphere essentially always balances the energy associated with solar radiation absorbed by the Earth’s surface and atmosphere. By increasing atmospheric concentrations of CO_2 and other greenhouse gases during the industrial era we have slightly perturbed this balance such that less infrared radiation is currently leaving the Earth system as compared to solar radiation being absorbed by it. This leads to additional energy being deposited into the Earth system that has been exhibited, in part, as a rise in surface air temperatures. At Earth’s surface the energy budgets of both downwelling solar and downwelling longwave radiation at short (second to minute to hour) timescales depends strongly on the presence of gases that absorb, emit, and scatter radiation in the atmosphere. Thus, Earth’s local surface temperature is exquisitely sensitive to the amounts and radiative properties of those gases and particles. We will do some simplified radiation calculations to show you how the Earth’s atmosphere affects the surface temperature.

For weather, we make predictions using models that consist of the equations of thermodynamics, motion, and microphysics. We initialize the models with observations and then let the model calculate the air motions going into the model future, thus giving weather forecasts. The models are good, but not so good that they can run for many days and continue to make accurate forecasts. So periodically, the models are adjusted by adding more observations, a process called **data assimilation**, in order to correct them and keep the forecasts accurate. Increasingly, satellite observations are being assimilated into the models to improve weather forecasts.

Satellite instruments observe atmospheric radiation: both visible sunlight scattered by Earth’s surface, clouds, and aerosols; and infrared radiation emitted by Earth’s surface and many of its atmospheric constituents. What the satellites measure depends on the wavelengths at which they collect radiation coming up to them. Typically, satellites observe in different wavelength bands, some of which cover wavelengths at which water vapor absorption is much stronger than for others. Taken together, the radiation in these different bands tells us much about the atmosphere’s temperature and moisture structure, which is just the kind of information that the models need to assimilate. You will learn how to interpret satellite observations of atmospheric radiation in support of applications such as vertically resolved temperature and moisture retrievals.

7.2: Applications of Atmospheric Radiation

Let’s use what you learned in Lesson 6 to examine two applications of atmospheric radiation. The first application involves the role of atmospheric radiation and greenhouse gases in Earth’s climate. The second application is the interpretation of upwelling infrared radiation spectra measured by satellite instruments in space with an eye on improving weather forecasting. These two applications use the principles of atmospheric radiation in quite different ways, but understanding both is critical to your becoming a competent meteorologist or atmospheric scientist.

Earth’s atmosphere is essentially always in radiative energy balance, which is also called **radiative equilibrium**. By this, I mean that, when averaged over the whole Earth, the total amount of solar radiation energy per second that is absorbed by the Earth’s surface and atmosphere is about equal to the total amount of infrared radiation energy per second that leaves the Earth’s surface and atmosphere to go into space. There can be periods when this balance is not exact because changes in atmospheric or surface composition can alter the absorption or scattering of radiation in the Earth system. It can take a little while for all of the temperatures of all of the Earth system's parts to adjust, but if the changes stop, the Earth system will adjust its temperatures to come back into balance. Right now we are in a period where atmospheric CO_2concentrations are increasing due to industrialization, the outgoing infrared radiation is slightly less than the incoming absorbed solar radiation, and the Earth system's temperatures are adjusting (by increasing) to try to bring the outgoing infrared radiation into balance with the incoming absorbed solar radiation. For most of the following discussion, we will use this concept of radiative equilibrium even though the current balance is not exact.

Always keep in mind that atmospheric radiation moves at the speed of light and that all objects are always radiating. Moreover, as soon as an object absorbs radiation and increases its temperature, its emitted radiation will increase. Thus energy is not “trapped” in the atmosphere and greenhouse gases do not “trap heat.” We will see instead that greenhouse gases act like another radiation energy source for Earth’s surface.

Before we do any calculations, let's summarize how different parts of the Earth system affect visible and infrared radiation (Table 1). Earth's surface either absorbs or scatters both visible and infrared radiation, while the atmosphere mostly transmits the visible radiation, with a little scattering; and the atmosphere mostly absorbs infrared radiation, with a little transmission. Clouds, an important part of the Earth system, strongly absorb infrared radiation and both scatter and absorb visible radiation.

Table 1: Absorptivity, Emissivity, Scattering, and Transmissivity of the Earth System

	Earth’s surface		**atmosphere**		**clouds**	
	visible	**visible**	**visible**	**IR**	**visible**	**IR**
absorptivity	large	opaque	tiny	large	large	opaque
emissivity	large	large	tiny	large	large	large
scattering (reflectivity)	large	large	moderate	none	large	small
transmissivity	none	none	large	small	none	none

Watch this video (52 seconds) to learn more:

[redacted]

Click here for transcript of the Absorption Scattering Transmitting video.

> Table 7-1 gives the absorptivity and thus, emissivity, as well as the scattering and transmissivity for the visible and infrared. Remember that the fractions of absorptivity, scattering, and transmissivity of radiants must add up to one when radiants encounters matter. I want to point out two features in the table. First, the atmosphere has little absorptivity and moderate scattering in the visible wavelengths while the atmosphere has large absorptivity, small transmissivity, and essentially no scattering in the infrared. Second, note that clouds behave a lot like Earth's surface in all aspects, except that scattering in the infrared can be large at Earth's surface while it is small for clouds.

Image of Earth in the visible. Clouds are scattering radiation at all visible wavelengths back out to space, and thus appear to be white, while Earth's surface selectively scatters visible radiation at only some wavelengths and absorbs the rest. Credit: NASA

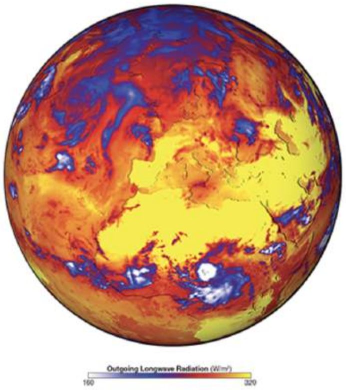


Image showing outgoing longwave (infrared) radiation emitted by the Earth and atmosphere during the European heatwave of 2003. The blue and white colors are clouds, which are radiating at the lower temperatures of the upper troposphere, while the yellows are the Earth's surface and the lower troposphere, which are radiating at higher temperatures. Credit: NASA

Extra Credit Reminder!

Here is another chance to earn **0.2 points of extra credit: Picture of the Week**!

1. You take a picture of some atmospheric phenomenon—a cloud, wind-blown dust, precipitation, haze, winds blowing different directions—anything that strikes you as interesting.
2. Add a short description of the processes that you think are causing your observation.
3. Upload it to the **Picture of the Week Discussion and add your description in the text box.**
4. The TA and I will be the sole judges of the weekly winners. A student can win up to five times.
5. Entries not chosen one week will be considered in subsequent weeks.

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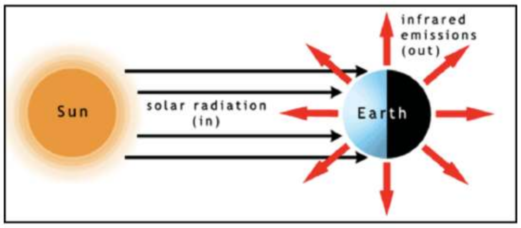
platform.

7.3: Atmospheric Radiation and Earth’s Climate

Let’s first look at the general energy balance—the radiative equilibrium—of the Earth system (see figure below). The solar irradiance is essentially composed of parallel radiation beams (or radiances) that strike half the globe. At the same time, outgoing infrared radiation is emitted to space in all directions from both the sunlit and dark sides of the globe. At the top of the atmosphere, the difference of the incoming solar radiation energy minus the amount of solar radiation energy that is scattered back to space (this difference being the amount of solar radiation energy absorbed by the Earth system) must balance the emitted infrared radiation energy for radiative equilibrium to hold. The total amount of solar radiation energy striking Earth per second is equal to the solar irradiance, $F\left(\mathrm{Wm}^{-2}\right)$, times the Earth's cross sectional area, $\pi R_{Earth}^{2}\left(\mathrm{m}^{2}\right)$. Some of the solar radiation energy is reflected by clouds, aerosols, snow, ice, and the land surface back to space and is not absorbed, hence does not contribute energy to raise Earth’s temperature. The fraction that is reflected is called the **albedo**, and we can account for it by subtracting the albedo from *1 and multiplying* $F\pi R_{Earth}^{2}$ *times the difference:* $F\pi R_{Earth}^{2}(1-a)$. *The* albedo has been estimated to be 0.294 (Stephens et al., 2012, Nature Geoscience 5, p. 691). On the other hand, Earth and its atmosphere radiate in all directions and the radiation can be described by the Stefan–Boltzmann Law, which, recall, is the integral of the Planck function over all wavelengths. Thus the emitted infrared energy per unit area (or emitted infrared irradiance) out the top of the atmosphere is $\sigma T_{\text{top}}^{4}\left(Wm^{-2}\right)$, where we have assumed an emissivity of *1* for the atmosphere at all emitted infrared radiation wavelengths. To get the total energy we must multiply this irradiance by the Earth’s total surface area, $4\pi R_{\mathrm{Earth}}^{2}$. The top of the atmosphere is at an altitude of $\sim 50-100$ km above the surface, compared to Earth’s radius of 6400 km, so we will ignore this small difference.



Distribution of solar radiation into the Earth system and Earth infrared radiation out of the Earth system. The Sun’s rays are roughly parallel when they reach Earth and deposit more energy per unit area on Earth’s surface in the tropics than near the poles. Earth is a little warmer in the tropics than at the poles, so it radiates in all directions, though a little stronger in the tropics than at the poles. *Credit: Help Save the Climate*

See the video (1:37) below for a more detailed explanation:

Earth Energy Balance

Click here for transcript of the Earth Energy Balance video

To calculate the average temperature at the top of Earth's atmosphere, we need to look at the balance between the solar radiation energy coming into the Earth's system against the infrared radiation going out of the Earth's system. The solar irradiance is essentially parallel by the time it gets to Earth, so it is intercepted by Earth's cross-section, which is just pi r Earth squared. Since a fraction of the solar radiation is immediately reflected and scattered back out into space-- this is what we call the albedo-- we have to correct the amount of radiation energy that Earth's system absorbed by subtracting off the albedo. On the other hand, Earth radiates in all directions. So assuming Earth's emissivity, than Earth's irradiance is in watts per meter squared. And if we multiply by Earth's surface area, really the surface area at the top of the atmosphere, then we get the energy leaving the Earth's system every second. That is in watts. We can use the laws of exponents to rearrange this equation to get an equation for temperature. When we put in typical values for the earth's system and solar irradiance, we calculate that the radiating temperature at the top of the atmosphere is 255 Kelvin or minus 18 degrees C or 0 Fahrenheit.

Equating the solar radiation energy absorbed by the Earth system to the infrared radiation energy emitted by the Earth system to space gives the equation:

$$\pi R_{Earth}^2 F(1-a) = 4\pi R_{Earth}^2 \sigma T_{top}^4 \tag{7.3.1}$$

$$T_{top} = \left(\frac{(F/4)(1-a)}{\sigma}\right)^{1/4} \tag{7.3.2}$$

But what is the temperature at the top of the atmosphere, T_{top}? Put in the values F = 1361 W m^{-2}, a = 0.294, and σ = 5.67 x 10^{-8} W m^{-2} K^{-4}. Therefore,

$$T_{top} = \left(\frac{\left(1361\mathrm{Wm}^{-2}/4\right)(1-0.294)}{5.67\times 10^{-8}\mathrm{Wm}^{-2}\mathrm{K}^{-4}}\right)^{1/4} \tag{7.3.3}$$

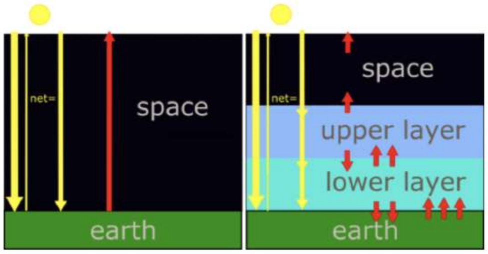
$$= 255\mathrm{K} \tag{7.3.4}$$

The temperature at the top of the atmosphere is 255 K, which equals –18 ^{o}C or 0 ^{o}F. It is substantially less than Earth’s average surface temperature of 288 K, which equals 15 ^{o}C or 59 ^{o}F. This top-of-the-atmosphere temperature is the same as what the Earth’s surface temperature would be if Earth had no atmosphere but had the same albedo. It is clear from these calculations that the atmosphere, modeled with an emissivity, and hence absorptivity, of 1 over all emitted infrared radiation wavelengths, is creating a difference between the temperature at the top of the atmosphere and the temperature at Earth’s surface.

In particular, let’s look at only the vertical energy balance averaged over the entire globe. We will think of everything in terms of the SI units of irradiance (or energy per second per unit area), which is W m^{-2}. Consider two idealized cases first before examining the actual atmosphere.

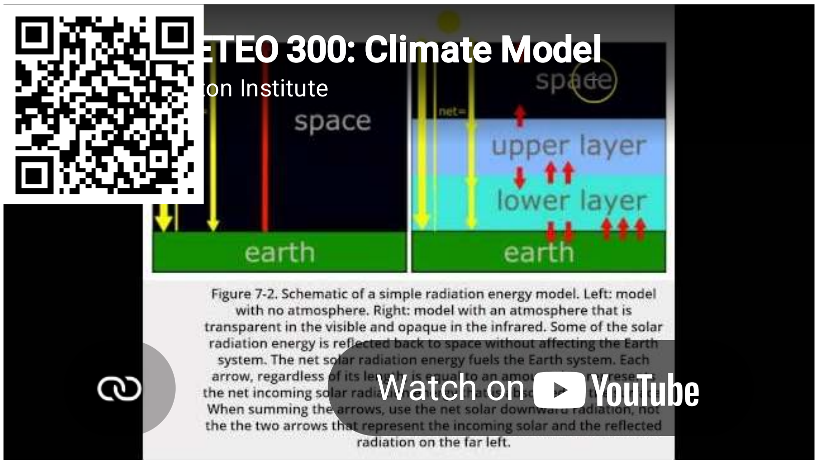
Let’s build a simple, flat atmosphere with all solar and infrared radiation energy moving only vertically (see figure below). We will make the following physical assumptions:

- solar irradiance is mostly in the visible (even though in reality half the solar irradiance is in the infrared with wavelengths greater than 0.7 μm);
- Earth's surface and atmosphere emitted radiation is in the infrared;
- the atmosphere is transparent to visible radiation;
- the atmosphere is opaque (that is, has an absorptivity of 1) to infrared radiation;
- radiation energy flowing up must equal radiation energy flowing down at every level in the atmosphere.



Schematic of a simple radiation energy model. Left panel: model with no atmosphere. Right panel: model with an atmosphere that is transparent in the visible and opaque in the infrared. Some of the solar radiation energy is reflected back to space without affecting the Earth system (second yellow arrow in each panel). The net solar radiation energy that is absorbed by the surface (third yellow arrow in each panel) fuels the Earth system. Each of the red arrows, regardless of its length, is equal to an amount that represents the net solar radiation energy that is absorbed by the surface. When summing the arrows, use the net solar radiation energy that is absorbed by the surface (third yellow arrow in each panel), not the incoming solar and the reflected radiation (first and second yellow arrows, respectively). Credit: W. Brune

Please watch the following video (2:18)



Climate Model

Click here for transcript of the Climate Model video.

> In the simplest climate model there is no atmosphere. Therefore, radiation is absorbed only by Earth's surface. And the atmosphere's emissivity is zero. That solar radiation energy, which is just the difference between the incoming solar radiation energy and the reflected solar radiation energy, equals Earth's infrared radiation energy outgoing to space. Let's represent that amount of energy with a single arrow. At the earth's surface, and at all levels above, there is one arrow coming down and one arrow going up to maintain radiative equilibrium. Consider next a more realistic climate model, one that has two atmospheric layers that do not absorb the incoming solar radiation, but do strongly absorb infrared radiation. Since they are good absorbers of the infrared, they are also good emitters of the infrared. The radiative equilibrium at each level, the number of arrows, which represent units of radiation energy, must be equal. Starting at the top of the atmosphere, the upper layer must emit one arrow of infrared radiation up to balance the solar visible radiation energy coming down. At the interface between the upper and lower layers there is one arrow of solar radiation energy going down. And the upper layer is emitting one arrow of infrared radiation down because if it is emitting one up, then it must also emit one down, since we are assuming that the layer has a uniform temperature. That puts two down arrows at the interface between the upper layer and the lower layer. To balance these two, the lower layer must be emitting to infrared arrows up. And since the lower layer also has a uniform temperature, it must also be emitting two arrows down to Earth's surface. With one solar and two infrared arrows down to earth's surface, Earth's surface must emit three arrows of infrared radiation up. To emit that much infrared, Earth's surface must be at a higher temperature, since it's irradiance is proportional to its temperature to the fourth power.

In the no-atmosphere model, the only radiating bodies are the Sun and the Earth. (By the way, if Earth had a pure nitrogen atmosphere, the results would be very similar to the no atmosphere scenario.) The solar radiation passes through the altitude levels where a stratosphere and troposphere would be and the fraction $1 - a$ of it is absorbed by the Earth’s surface. We assume that Earth’s albedo is still 0.294 so that 0.706, or 70.6%, of the solar radiation is absorbed at the surface with the rest reflected back to space. The Earth’s surface radiates infrared radiation energy back out to space with no absorption at the levels where the stratosphere and troposphere would be. The surface temperature in this model is such that the infrared radiation energy leaving the surface balances the incoming solar radiation energy absorbed by the surface. In terms of the arrows in the figure, there is one down arrow and one up arrow at every level.

Comparison of Interface Fluxes in Two Radiation Energy Balance Models

model	no atmosphere		atmosphere transparent in visible opaque in infrared	
interface	down arrows	up arrows	down arrows	up arrows
space–stratosphere			1	1
stratosphere–troposphere			2	2
troposphere–surface			3	3
space–surface	1	1		

So what would the temperature at Earth's surface be if there was no atmosphere? Equation [7-2] applies to the no-atmosphere case and hence the Earth with no atmosphere has a surface temperature of 255 K. This temperature is the same as the radiating temperature at the top of our Earth with an atmosphere whose absorptivity, hence emissivity, is 1 at all emitted infrared radiation wavelengths. The surface would be so cold that any water on it would freeze and stay frozen.

Now consider Earth with an idealized atmosphere identical to that used to derive Equation [7-2] but now paying attention to the temperature of Earth's surface under such an atmosphere. As before, this atmosphere is transparent to all solar radiation energy coming down to Earth's surface and is opaque to all infrared radiation. "Opaque" means that the infrared radiation is completely absorbed over very short distances (i.e., the absorptivity, hence emissivity is 1, and the absorption optical depth is great, so by Beer's Law, very little infrared radiation is transmitted). The atmosphere itself is strongly emitting in all directions, both up and down, and the only infrared radiation that does not get absorbed is that emitted out the top of the stratosphere to space.

We know that the infrared radiation energy leaving the Earth system must come close to balancing the solar radiation energy absorbed by the Earth system. Otherwise, the temperatures of Earth's surface and atmosphere would adjust until this condition was true. So, we will assume radiative equilibrium. Our model is a two layer model—an upper layer and a lower layer—with a solid Earth beneath them. We are assuming that each layer is at a constant temperature and absorbs all infrared radiation energy impinging on it, and then emits infrared radiation out its top and its bottom in equal amounts (because the layer emits infrared radiation energy in both directions equally). The amount of infrared radiation energy emitted by the layer is determined by its temperature only because its emissivity is set to 1 at all wavelengths. Thus between the upper layer and space, we have one arrow going down and one arrow going up: the outgoing emitted infrared radiation energy exactly balances the incoming solar radiation energy that is absorbed.

The upper layer thus also emits one arrow of infrared radiation down. So, at the interface between the upper and lower layer, the solar radiation and the upper layer's infrared radiation are going down (two arrows), so to be in radiative equilibrium there must be enough upwelling infrared radiation from the lower layer to equal the incoming solar radiation energy that is absorbed and the downward infrared radiation emitted by the upper layer (two arrows). But that means that the lower layer must also be emitting the same amount of infrared radiation down to Earth's surface (two arrows).

At Earth's surface, there is the incoming solar radiation energy that is absorbed and the tropospheric downward emitted infrared radiation, equivalent to three times the incoming solar radiation energy that is absorbed. Thus Earth's surface must be radiating upwelling infrared radiation energy equivalent to this incoming energy to maintain radiative equilibrium. So, in this simple model Earth's surface is radiating three times the energy that the model without the atmosphere does. But to emit this larger amount of radiation the surface must be much warmer than without an atmosphere. We can calculate the surface temperature that would be required using Equation 7.3.2, but adding the downward emitted infrared radiation energy from the troposphere to the solar radiation energy. One way to look at this situation is that the lower layer is providing a source of radiation energy at the Earth's surface in addition to the solar radiation energy.

Mathematically, we can account for this extra energy near Earth's surface by simply multiplying the solar radiant energy by an IR multiplier, $multiplier_{IR} = 3$, in Equation 7.3.2:

$$T_{\text{Earth}} = \left(\frac{\text{multiplier }_{IR}(F/4)(1-a)}{\sigma}\right)^{1/4} \tag{7.3.5}$$

$$= \left(\frac{3\left(1361\text{Wm}^{-2}/4\right)(1-0.294)}{5.67\times10^{-8}\text{Wm}^{-2}\text{K}^{-4}}\right)^{1/4} = 336\text{K}$$

This temperature (336 K = 63 ºC = 145 ºF) is deadly and much higher than Earth's actual surface temperature, 288 K. So this model also fails to simulate the real Earth. The no-atmosphere model is too cold while the model with a two-layer, infrared-opaque atmosphere is too hot. So we can guess that something in between might be just right.

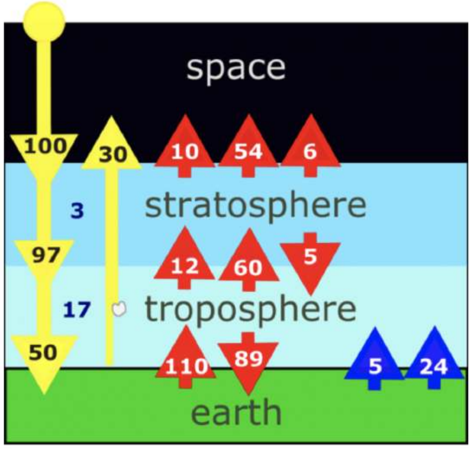
Indeed this is the case! If you look at the infrared absorption spectrum in Lesson 6, you will recall that there are some wavelengths at which all the infrared is absorbed and others, called windows, at which only a small fraction of the infrared radiation is absorbed. So, we find that a mix of total absorption, partial absorption, and no absorption at various wavelengths gives an atmosphere that allows Earth's surface to radiate much radiation directly to space at some wavelengths but not at other wavelengths, where troposphere absorption is strong. But a large absorptivity implies a large emissivity so that at those wavelengths for which there is strong absorption there is also emission; however, given that the troposphere is cooler than the surface, the troposphere emits less upwelling infrared radiation energy than it absorbs from the warmer surface underneath. But irrespective of wavelength, emission by the troposphere is downwards as well as upwards, and provides another radiation energy source to heat Earth's surface. This is called the greenhouse effect, which is poorly named because a greenhouse warms the Earth by suppressing heat loss by convection whereas the troposphere warms the Earth by emitting infrared radiation.

A study by Kiehl and Trenberth (1997, Bulletin of the American Meteorological Society 78, p. 197) determined the contributions to the greenhouse effect. It was shown that 81% of the greenhouse effect is due to greenhouse gases and 19% is due to clouds. Of the greenhouse effect resulting from gases, 60% is contributed by water vapor, 26% by carbon dioxide, and 14% by ozone, nitrous oxide, and methane.

In parts of the spectrum where water vapor, carbon dioxide and other gases absorb more weakly, the atmosphere is less opaque. However, if the amounts of these gases are increased, then they will absorb more strongly and thus start emitting more strongly, thus increasing the radiation emitted by the atmosphere to the surface and thus increasing the surface temperature in order for the surface to come into radiative equilibrium. Remember that the energy going out of the top of the atmosphere is still essentially the same as the solar radiation energy coming into the atmosphere that is absorbed. In a sense, by adding carbon dioxide and other greenhouse gases to the atmosphere, we are moving Earth's surface temperature from being closer to the no-atmosphere model to being closer to the infrared-opaque model.

7.4: What does the energy balance of the real atmosphere look like?

The real atmosphere's energy balance includes not only radiation energy but also energy associated with evaporation and convection (see figure below). However, the atmosphere is still very close to total energy balance at each level.



The average vertical energy balance of the actual atmosphere. All energies are represented as a percentage of the incoming solar irradiance at the top of the atmosphere (340.2 W m^{-2} = 100 units). Solar irradiance is on the left (yellow arrows), infrared radiation is in the middle (red arrows), and convection (5 units) and evaporation (24 units) are on the right (blue arrows). Credit: W. Brune (after D. Hartmann)

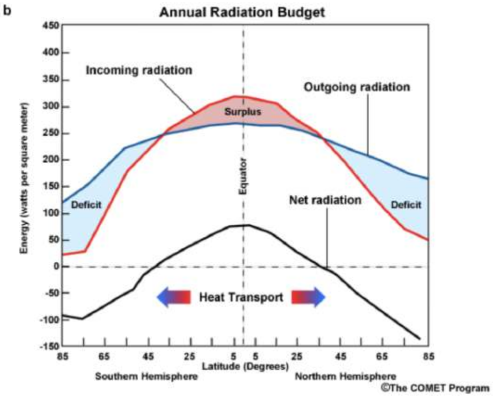
First, let's go through each set of arrows to see what is happening. The average solar irradiance at the top of the atmosphere is 340.2 W m^{-2}, which we will represent as being 100 units and then compare all other energy amounts to it.

- **Leftmost two columns of yellow arrows:** Of the solar irradiance coming into the atmosphere, most of the solar ultraviolet irradiance, about 3 units, is absorbed in the stratosphere and warms it, leaving 97 units to make it to the troposphere. 17 units, mostly at wavelengths just longer than solar visible wavelengths, are absorbed in the troposphere and another 30 units are scattered back out to space by bright objects, such as clouds, non-absorbing aerosols, snow, ice, and the land surface, leaving 50 units to be absorbed at Earth's surface.
- **First column of red upward arrows:** The Earth's surface emits upwelling infrared irradiance of 110 units, only 12 units of which are transmitted through the troposphere into the stratosphere, and 10 of these 12 units are subsequently transmitted through the stratosphere to space.
- **Second column of red upward arrows:** The troposphere radiates 89 units down and 60 units up; 54 of these 60 units escape to space. Unlike our simple two-layer model in which we assumed that the troposphere emitted equally up and down, the real troposphere is more complex and the downward radiation exceeds the upward radiation because of the vertical distribution of temperature (with temperature decreasing with height through the troposphere), water vapor, and carbon dioxide.
- **Third column of red upward arrows:** The stratosphere radiates 5 units downward and 6 units upward.
- **Rightmost blue columns:** There is significant non-radiation vertical energy transport at the surface. Of the net 29 units of irradiance absorbed at the Earth's surface, 24 units go into latent heat. Latent heat quantifies the amount of irradiance necessary to evaporate liquid water (mostly seawater) at Earth's surface to water vapor. This water vapor is transported upward by convection to form clouds, which releases this energy into the troposphere, warming it. The remaining 5 units of net irradiance absorbed by the surface goes into sensible heat. Sensible heat is the conduction of energy between the warmer Earth's surface and the cooler tropospheric air, thus warming the air and causing it to become less dense (higher virtual temperature) than its surrounding air, followed by convection, which moves warmer air upward.

At each level, the amount of energy going down must equal the amount of energy going up. Thus, at the top of the stratosphere, 100 units cross into the stratosphere from space, and to balance this downward energy are 30 units of reflected solar irradiance upward to space and 70 units upward emitted infrared radiation that makes it to space. At the top of the troposphere, the downelling of 97 units of solar irradiance and 5 units of infrared irradiance is balanced by the upwelling of 30 units of reflected solar irradiance and 72 units of infrared irradiance. At Earth's surface, the downward fluxes of solar irradiance (50 units) and infrared irradiance (89 units) balance the upward fluxes of 110 units infrared irradiance, the 24 units of latent heat, and the 5 units of sensible heat.

In reality, the Earth’s surface and atmosphere are not in simple radiative equilibrium, but are instead in radiative–convective equilibrium. Furthermore, the atmosphere is in radiative–convective equilibrium globally, but not locally (see figure below). The absorbed solar irradiance is much greater near the equator than the poles because that is where the surface is most perpendicular to the incoming solar irradiance. The radiative and convective net upward energy transport is greatest at the equator as well (because Earth’s surface is warmer there than at the poles). Overall, there is significant net incoming radiation energy between 30^{o}S and 30^{o}N latitude and a net outgoing radiation energy poleward of 30^o in both hemispheres.

This uneven distribution of incoming and outgoing radiation results in a flow of energy from the tropics to the poles (see figure below). It unleashes forces that cause warm air to move poleward and cold air to move equatorward. The poleward motion of warmer air, coupled with the Coriolis force that curves moving air to the right in the Northern Hemisphere and to the left in the Southern Hemisphere, causes the atmosphere’s basic wind structure, and thus its weather. We'll talk more about these forces and the resulting motion in the next few lessons when we discuss atmospheric motion (kinematics) and the forces (dynamics) that cause the motion that results in weather.



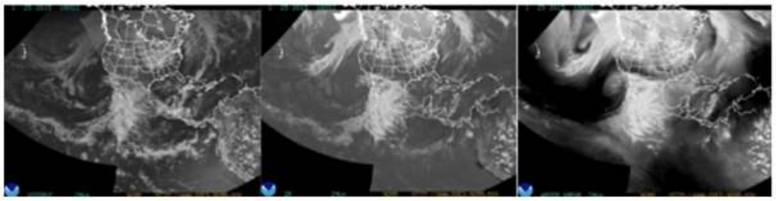
The uneven distribution of incoming solar radiant energy and outgoing radiant energy and the resulting net incoming energy near the equator and net outgoing radiant energy toward the poles. Credit: NOAA

A series of slides shows you the vertical energy balance in the Earth system and can be found at this site that depicts the energy balance.

7.5: Applications to Remote Sensing

Quick Refresher on the Major NOAA Geostationary Satellite (GOES) Data Products

The visible channel $(0.55-0.75 \mu m)$ records reflected sunlight radiances, where whiter shades are more reflected light and darker shades are less, just like in a black-and-white photograph. Land reflects more light than oceans and lakes; clouds and snow cover reflect more light than land. The visible channel goes dark at night.



NOAA composite images of the Northern Hemisphere on 25 January 2015. Left: visible (0.55–0.75 μmμm) radiances where white indicates bright (large) values and black dark (low) values; center: infrared window (10.2–11.2 μm) the WeatherTAP website for satellite tutorials. Credit: NOAA

The infrared window channel $(10.2-11.2 \mu m)$ is over a wavelength band where the cloud-free atmosphere is transparent. As a result, it primarily records infrared radiation emitted from Earth's surface and clouds, with emission and absorption by the gases in the atmosphere playing a secondary role. In the figure above, the greater the surface temperature (and hence the greater the radiance or radiation energy according to Equation [6-5]), the darker the shading. Thus clouds tops, which are at higher altitudes and thus colder, appear brighter.

NOAA thermal infrared (IR) channel temperature scale as a gray scale. Less radiant objects are colder and are given lighter shading. Credit: NOAA

The water vapor channel $(6.5-7.0 \mu m)$ covers a strong water vapor absorption band. Thus, radiation energy at this wavelength is strongly absorbed and the radiation energy recorded by the satellite for this channel must originate from the top of the highest moist layer. Within the moist layer, the absorptivity at this wavelength is effectively 1 and it is only near the top of the moist layer that the absorption optical thickness becomes small enough that the radiation energy can escape to space and be recorded by the satellite. Note that the higher the top of the moist layer, the lower the temperature and the less radiance recorded by the satellite. Lower radiances (and hence higher, colder moist layers) are given whiter shading; darker shading is given to higher radiances (and hence lower, warmer moist layers).

Please Note

A few remarks on the water vapor channel. Even the driest column of air will have enough water vapor to absorb all 6.5–7.0 μm infrared radiation emitted from Earth's surface and just above Earth's surface. Therefore, all the radiation energy at these wavelengths recorded by the satellite comes from atmospheric water vapor at least a kilometer or more above the surface.

Second, in a drier column, some of the radiation energy emitted by water vapor at lower altitudes will not be absorbed by the water vapor above, thereby making it to space. Because lower altitude water vapor has a higher temperature than the water vapor above, it emits a greater amount of infrared radiation than the overlying water vapor. Therefore, as a column dries and there is less high altitude water vapor, the water vapor channel radiance recorded by a satellite will go up in value (or become darker) in the water vapor image.

Thus brighter shades indicate emissions from higher altitudes and lower temperatures; darker shades indicate emissions from lower altitudes and thus higher temperatures. In no case, however, is the Earth's surface or the water vapor just above the Earth's surface observed. So whiter shades indicate more water vapor in a column at higher altitudes and can be used as a qualitative indicator of air moisture and as a tracer of atmospheric motion because the amount of moisture does not change significantly on daily time scales.

7.6: What is the math behind these physical descriptions of the GOES data products?

In Lesson 6, we derived an equation (Schwarzschild's equation) for the change in radiance as a function of path between an infrared source and an observer:

$$\frac{dI}{ds}=\kappa_a\left(P_e-I\right) \tag{7.6.1}$$

where I is the directed beam of radiation (or radiance) along the path between the object and the observer, P_e is the Planck function radiance at the temperatures of the air (really the greenhouse gases in the air) along the path, and κ_a is the absorptivity of the air along the path.

Let's apply this equation to the point-of-view of an Earth-observing satellite. Define ττ (tau) as the optical path between the satellite (τ=0)(τ=0) and some arbitrary point along the optical path given by ττ . We are not using Earth's surface as the zero point as we often do, but instead, we are using the satellite as the zero point and letting the distance, *s*, and thus the optical path, change from there. The change in the optical path equals:

$$d\tau=-\kappa_a ds \tag{7.6.2}$$

because *ds* is going down and becoming more negative while the optical path $d\tau$ grows. κ_a is just the absorptivity (m^{-1}).

Integrating both sides from the satellite to some distance *s* from the satellite:

$$\int_{\text{satellie}}^{s} d\tau=\tau(s)=-\int_{\text{satellie}}^{s} \kappa_a\left(s'\right) ds'=\int_{s}^{\text{satellite}} \kappa_a\left(s'\right) ds' \tag{7.6.3}$$

To make it easier to understand what is going on, we will switch the variable in [6.15] from the distance *ds* to the optical path *dt*, because it is the optical path, not the actual distance, that determines what the satellite detects.

$\frac{dI}{-ds}=\kappa_a\left(P_e-I\right.$ where s is from the satellite going down (negative)

or

$\frac{dI}{d\tau}=\left(P_e-I\right)$ from the point-of-view of the satellite.

This equation can be integrated to give the radiance observed by the satellite at an optical depth τiτi looking down at Earth:

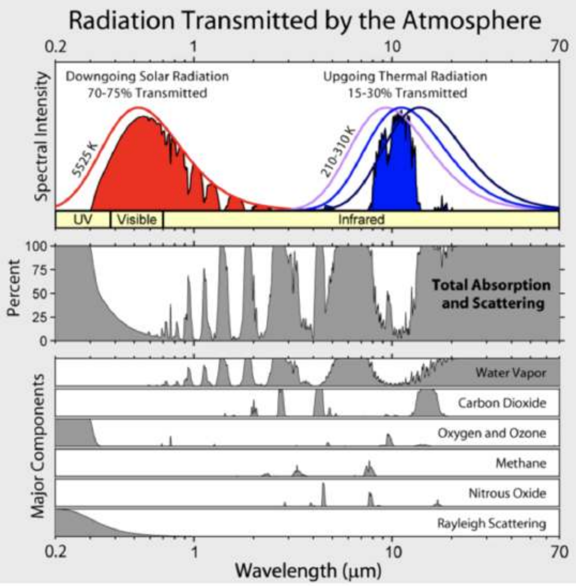
$$I\left(\tau=0, \text{ at the satellite}\right)=I\left(\tau_i\right)\exp\left(-\tau_i\right)+\int_0^{\tau_i} P_e \exp(-\tau) d\tau \tag{7.6.4}$$

So, what does this mean?

- The left-hand side is the radiance that the satellite observes.
- The first term on the right-hand side is a source's radiance that is absorbed along the path according to Beer's Law. I(τ)I(τ) could be the radiance emitted by Earth's surface and exp(−τi) exp(−τi) the transmittance from Earth's surface to the satellite.
- The second term on the right hand side is the emitted radiance of the atmosphere integrated over all points along the path, with transmission between each point of emission and the satellite accounted for by the exponential factor exp(−τ)exp(−τ) . For example, for the water vapor channel, P_e is the emission of radiance at the water vapor channel wavelength from some point along the path and exp(−τ)exp(−τ) represents the transmission of that radiance through all the other water vapor along the path between the emission point and the satellite.
- The satellite simply will not detect much radiance from an object, solid or gas, if the optical path, ττ , between it and the satellite is 3 or more because exp(-3) = 0.05.
- Remember that P_e depends on temperature (equation 6-7), so that P_e will be smaller at higher altitudes where the temperature is lower.

We have neglected scattering in these equations. Molecular scattering is insignificant at infrared and longer (for example, microwave) wavelengths. Cloud particle and aerosol scattering is important at visible and near-infrared $(1-4\mu m)$ wavelengths, but less so at thermal infrared $(4-50\mu m)$ wavelengths, where absorption dominates. In the thermal infrared, water clouds have an absorptivity, hence emissivity, close to 1 and emit according to the Planck function (Equation 6.3).

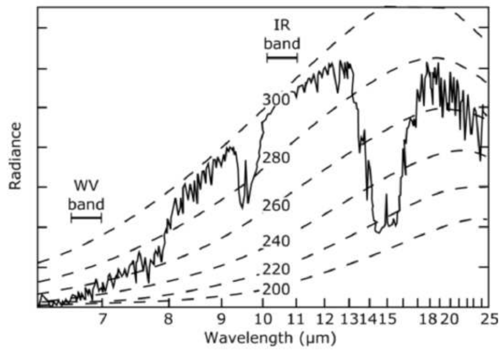
Looking back at a figure from Lesson 6, we can see at which wavelengths the greenhouse gases in the atmosphere, mostly water vapor and carbon dioxide, absorb and thus emit and at which wavelengths there are windows with low absorptivity that allow most infrared irradiance to leave Earth's surface and go out into space as indicated by the blue-filled spectral intensity (i.e., irradiance). Note that much of Earth's infrared irradiance is absorbed by the atmosphere. The radiation from the atmosphere is not included in the blue curve-filled curve called "Upgoing Thermal Radiation." This window extends from $\sim 8\mu m$ to $\sim 13\mu m$, with ozone absorption occurring in a fairly narrow band around $9.6\mu\mathrm{m}$.



Solar and terrestrial irradiance and absorption by molecules in the ultraviolet, visible, and infrared.

Credit: Robert A Rohde, Global Warming Art, via Wikimedia Commons

Satellites observe irradiance from both Earth's surface and from the atmosphere at different pressure levels (see figure below). The radiance observed in the $\sim 8\mu m$ to $\sim 13\mu m$ is coming from Earth's surface and has a temperature of about 295 K, or 22 ^{o}C. Note that the GOES weather satellite IR band $(10.2\mu m-11\mu m)$ is looking at the lowest opaque surface, which because the scene was clear, that surface was the ocean. At wavelengths lower than $8\mu m$, note that the radiance is coming from a source that is colder and, in fact, is coming from water vapor with an average temperature of 260 K when the absorptivity is a little weaker near $8\mu m$ and the radiance from the water vapor near $6\mu m$ has a temperature of 240 K. Because lower temperatures are related to higher altitudes, the satellite observed water vapor at lower altitudes near $8\mu m$ and water vapor at higher altitudes near $6\mu m$. Thus, the satellite can observe radiance from different depths in the atmosphere by using different wavelengths. Another example is the strong carbon dioxide and water vapor absorption near $15\mu m$. At wavelengths near $13\mu m$, the satellite is observing radiance mostly from CO_2 and H_2O from lower in the atmosphere because the emissivity of CO_2 is less at those wavelengths. At wavelengths nearer $15\mu m$, the CO_2 emissivity is much greater and the satellite is observing CO_2 and H_2O radiance from temperatures below 220 K and therefore much higher in the atmosphere, actually at the tropopause. Note the very narrow spike right in the middle of this strongly absorbing (and thus emitting) CO_2absorption band. Why does the temperature go up? Answer: In this most strongly absorbing part of the band the satellite is seeing the CO_2 radiance is coming from the stratosphere, which is warmer than the tropopause. Just to note - it's not that the CO_2 and H_2O at lower altitudes are not emitting in the 15 μmband - they are, but all of that radiance is being absorbed by the CO_2 and H_2O between the lower layers and the satellite and then these higher layers of CO_2 and H_2O are radiating, but only the layer that has no significant absorption above it can be observed by the satellite.



Infrared spectrum of Earth observed by a satellite. The spectrum extends from 6.0 μmμm to 25 μmμm for clear air over the tropical Western Pacific. Dashed lines are the Planck functions for objects at different temperatures from 300 K to 200 K. Where the measured radiance matches the dashed line the radiance came from water vapor and carbon dioxide at that temperature. Thus, if you know the atmospheric temperature profile, then you can guess at the altitude from which the radiance is coming (on average).

Credit: W. Brune (data from NOAA Star Center for Satellite Applications and Research)

Watch the following video (2:46) on infrared spectrum analysis:

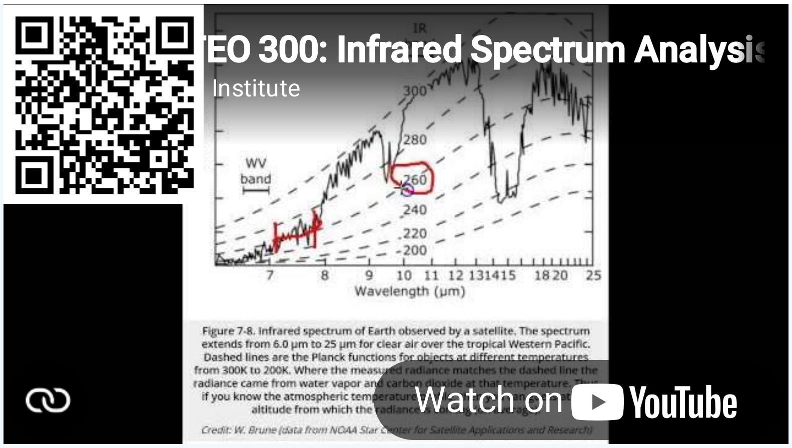


Figure 7-8. Infrared spectrum of Earth observed by a satellite. The spectrum extends from 6.0 µm to 25 µm for clear air over the tropical Western Pacific. Dashed lines are the Planck functions for objects at different temperatures from 300K to 200K. Where the measured radiance matches the dashed line the radiance came from water vapor and carbon dioxide at that temperature. Thus, if you know the atmospheric temperature [illegible] altitude from which the radiance [illegible]

Credit: W. Brune (data from NOAA Star Center for Satellite Applications and Research)

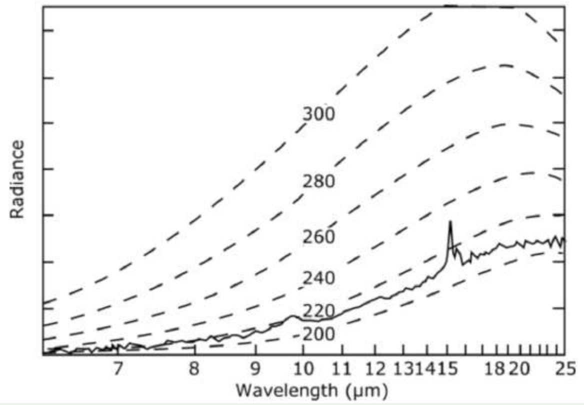
Infrared Spectrum Analysis

Click here for transcript of the Infrared Spectrum Analysis video.

> Let's examine the wavelength spectrum of radiance observed by satellite looking down at a location on Earth. Because the absorptivity [INAUDIBLE] of different gases changes dramatically from 6 to 25 microns, the satellite is observing radiance from different types of matter at different wavelengths. The radiance depends on temperature. So once we know the radiance, we know the temperature of the object that is radiating. The Planck distribution functions spectral radiance is plotted per curves of different temperatures from 200 kelvin to 300 kelvin. Thus the radiance gives us the object's temperature. And since we have a rough idea about the temperature profile of the atmosphere, we can make a pretty good guess at the height of the radiating object and what is actually radiating, whether it be Earth's surface or a gas, like water vapor, carbon dioxide, or ozone. Between 8 and 13 microns, no infrared gas absorbs very well in the atmosphere, except for ozone around 9.6 microns. Note that the radiance in this window came from matter at a temperature near 300 kelvin or 27 degrees C. From the satellite's position, this radiance is known to come from the ocean, the Pacific. At the edges of the strong water vapor absorption bend at 6 microns, say, at about 7 and 1/2 microns, note that the radiating temperature is about 260 kelvin. This radiance must be coming from water vapor at 10,000 to 20,000 feet altitude. At 6 microns, the temperature is quite a bit lower. And so therefore, this radiance comes from water vapor at a much greater altitude in the atmosphere. In the CO2 absorption band near 15 microns, the radiance is equivalent to a temperature of 220 kelvin, which is from CO2 near the tropopause since this is the lowest radiating temperature that we see. Note that little spike in the

> middle of this strongly-absorbing CO2 band. It is coming from CO2 that is warmer than the tropopause, but we know that it must be coming from above the tropopause because the center of the CO2 band absorbs the strongest and thus, this radiance must be becoming from the CO2 higher than above the tropopause. It must be coming from the stratosphere. This makes sense that the stratosphere is warmer than the tropopause. So we can actually learn a lot about what is being observed simply by looking at a satellite thermal infrared spectrum, like this one.

Look at another scene, which is the top of a thunderstorm in the tropical western Pacific. Remember that reasonably thick clouds are opaque in the infrared and therefore act as infrared irradiance sources that radiate at the temperature of their altitude. The cloud's radiance was equivalent to Planck distribution function irradiance with a temperature of 220–210 K. This temperature occurs at an altitude just below the tropical tropopause, which means that this storm cloud reached altitudes of 14–16 km. Note that in the middle of the 15μm CO_2 absorption band the satellite observed only the CO_2 in the stratosphere (there is essentially no water vapor in the stratosphere). We know this because the radiance temperature is higher and the absorption is so strong that the radiance must be coming from higher altitudes closer to the satellite.



Infrared spectrum of Earth observed by a satellite. The spectrum extends from 6.5 μmμm to 25 μmμm for a thunderstorm over the tropical Western Pacific. Dashed lines are the Plank functions for objects at different temperatures from 300 K to 200 K. Remember that clouds are opaque in the infrared and therefore radiate with the Planck distribution function spectral irradiance.

Credit: W. Brune (data from NOAA Star Center for Satellite Applications and Research)

Let’s put all of this together.

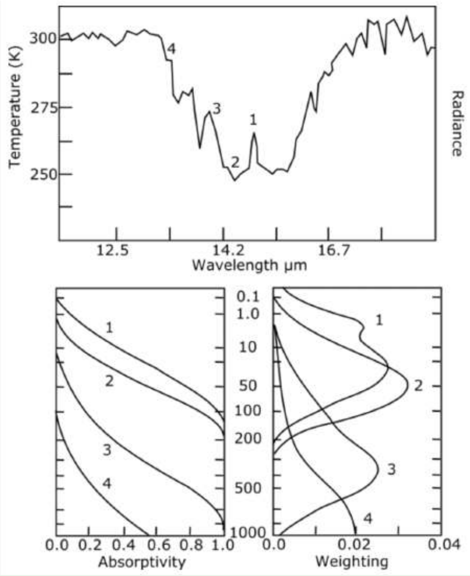
- Water vapor, carbon dioxide, and ozone have banded absorption in the thermal infrared due to vibrational–rotational transitions governed by quantum mechanical rules.
- As the absorption decreases, the emissivity decreases. Thus, weakly absorbing gases are also weakly emitting at the same wavelength.
- By looking at different wavelengths either inside, outside, or near absorption bands, a satellite can detect radiation emitted at different heights within the atmosphere.
- In the middle of an absorption band, where the absorption is greatest, the optical path is also the greatest; at these wavelengths a satellite detects only the emissions from the nearest (and highest) layers because the lower ones produce radiation that is absorbed before reaching the satellite.
- In wavelength “windows” between absorption bands, the absorption is small so that the satellite can detect radiation emitted all the way down to the Earth’s surface.
- On the edges of absorption bands, for which the absorption is weak but still significant, satellites detect radiation emitted from the middle troposphere but not from the surface .
- $$I_s = \sigma T^4 \tag{7.6.5}$$
- Thus, if a satellite detects only radiation emitted from the upper troposphere as a result of strong absorption below, its recorded radiance will correspond to temperatures of the upper troposphere (~ 200–220 K).

- If the satellite detects radiation emitted all the way down to Earth’s surface, then it will record a radiance with temperatures approaching that of the Earth’s surface.
- The water vapor ($6.7\mu m$) channel contains wavelengths at which water vapor absorption is fairly strong, and so it records radiation emitted from the middle troposphere but not below because there is always enough water vapor to absorb irradiance emitted by Earth's surface or the water vapor near Earth's surface. Because the distribution of water vapor is highly variable in time, horizontal position, and vertical position, satellites detect radiance originating from different depths in the atmosphere at different times and places. Basically, with a knowledge of temperature profiles and recorded radiances across the water vapor channel, the optical depths that result from water vapor can be retrieved and the relative humidities determined.

As I said earlier, by observing the CO_2 radiance at different wavelengths, the satellite can be sampling CO_2 radiance from different altitudes (see figure below . The top panel is the radiance from $12\mu m$ to $18\mu m$ centered on the strong $15\mu m\mathrm{CO_2}$ absorption band. Look at the wavelengths marked 1 through

4. The bottom left panel in the figure shows the absorptivity from the top of the atmosphere to a given pressure level as a function of pressure level at these four wavelengths. Note that for the most strongly absorbed wavelength, 1, the radiance of all the CO_2 and H_2O below a pressure level of about 150 hPa is completely absorbed. Thus, very little of the radiation received by the satellite comes from below this pressure level. On the other hand, very little of the radiance received from the satellite comes from above the 0.1 hPa pressure level because the absorptivity (and hence emissivity) there is zero. Thus, the radiance reaching space must primarily come from between the 150 and 0.1 hPa pressure levels. The panel on the lower right shows the relative contribution of each pressure level to the radiance that reaches space. For wavelength 1, we see that almost all radiance comes from the stratosphere.

Look at equation 7.6 to see that the absorption of lower layers is exponential so that there are no sharp layers that emit radiance at each wavelength, but instead, the radiance the satellite observes at any wavelength comes from a band that has soft edges. If we look at the wavelength at 2, 3, and 4, we see that the CO_2 and H_2O radiance comes from further down in the atmosphere. For wavelength 4, the satellite is observing radiance from Earth's surface as well as from the CO_2 and H_2O below about 500 hPa, whereas for the wavelength marked 3, the radiance is only slightly from Earth's surface—mostly from $CO_2$and H_2O in the middle troposphere.



Top panel: Infrared spectrum of Earth observed by a satellite between about 12 imand 18μn Numbers 1 through 4 indicate wavelengths of decreasing absorption (i.e., 1 indicates a strongly absorbing wavelength and 4 indicates a weakly absorbing wavelength). Bottom left panel: Absorptivity from the top of the atmosphere to a given pressure level as a function of pressure level (hPa) for the four wavelengths indicated in the top panel. Bottom right panel: The weighting function for each of the four wavelengths, which gives the relative contribution of each pressure level to the radiance that makes it to space.

Credit: W. Brune (data from NOAA Star Center for Satellite Applications and Research)

Discussion Activity: Greenhouse Gases and Climate Change

(3 discussion points)

This week's discussion topic asks you to reflect on the impact of this lesson's material on your own thinking. Please answer the following question:

How has studying this lesson altered your thoughts about greenhouse gases and climate change?

If it has not, say why not.

Your posts need not be long, but they should tie back to the material in Lesson 7 (and Lesson 6).

1. You can access the **Greenhouse Gases and Climate Change Discussion** in Canvas.
2. Post a response that answers the question above in a thoughtful manner that draws upon course material and outside sources.
3. Keep the conversation going! **Comment on at least one other person's post**. Your comment should include follow-up questions and/or analysis that might offer further evidence or reveal flaws.

This discussion will be worth 3 discussion points. I will use the following rubric to grade your participation:

Discussion Activity Grading Rubric

Evaluation	Explanation	Available Points
Not Completed	Student did not complete the assignment by the due date.	0
Student completed the activity with adequate thoroughness.	Posting answers the discussion question in a thoughtful manner, including some integration of course material.	1
Student completed the activity with additional attention to defending his/her position.	Posting thoroughly answers the discussion question and is backed up by references to course content as well as outside sources.	2
Student completed a well-defended presentation of his/her position, and provided thoughtful analysis of at least one other student’s post.	In addition to a well-crafted and defended post, the student has also engaged in thoughtful analysis/commentary on at least one other student’s post as well.	3

Quiz 7-2: Satellite remote sensing.

1. Please note: **there is no practice quiz** for Quiz 7-2 because the questions and answers follow directly from the text.
2. When you feel you are ready, take **Quiz 7-2 in Canvas**. You will be allowed to take this quiz only **once**. Good luck!

7.7: Summary and Final Tasks

Two applications of the theory of atmospheric radiation have been presented. The most important concepts used are:

- everything radiates
- solar visible irradiance strikes Earth on one side, but Earth radiates in the infrared in all directions
- the total energy for solar visible radiation *absorbed in the Earth system*closely balances the total energy for the infrared radiation *going out to space*
- the atmosphere is highly transparent in the visible and weakly transparent in the infrared.

For climate, these principles mean that water vapor, carbon dioxide, and other gases radiate energy to Earth's surface, keeping it warmer than it would be if the atmosphere did not have these gases. For satellite infrared observations, some wavelength bands are in windows, so that the satellites see radiation from Earth's surface. Other bands are completely absorbed by water vapor or carbon dioxide, so that the infrared getting to the satellite comes from the top of the water vapor column. Clouds are opaque in the infrared, so the satellite sees their tops, which are radiating at the temperature of that altitude.

Reminder - Complete all of the Lesson 7 tasks!

You have reached the end of Lesson 7! Double-check that you have completed all of the activities before you begin Lesson 8.

CHAPTER OVERVIEW

8: Math and Conceptual Preparation for Understanding Atmospheric Motion

Learning Objectives

By the end of this chapter, you should be able to:

- calculate partial derivatives
- implement vector notation, the dot product, the cross product, and the del operator
- explain the different coordinate systems and how they are used
- convert between math and meteorological wind directions
- calculate temperature advection at any point on a map of isotherms (lines of constant temperature) and wind vectors

In previous lessons, we were able to explain physical and chemical processes using only algebra and differential and integral calculus. Thermodynamics, moist processes, cloud physics, atmospheric composition, and atmospheric radiation and its applications can all be quantified (at this level of detail) with fairly simple mathematics. However, to understand and quantify the dynamics of the atmosphere requires more math skill.

8.1: Prelude to Math and Conceptual Preparation for Understanding Atmospheric Motion

This lesson introduces you to the math and mathematical concepts that will be required to understand and quantify **atmospheric kinematics**, which is the description of atmospheric motion; and **atmospheric dynamics**, which is an accounting of the forces causing the atmospheric motions that lead to weather. Weather is really just the motion of air in the horizontal and the vertical and the consequences of that motion. The motion is caused by wind and wind has both direction and speed, which are best described by vectors.

The Earth is a spinning, slightly squashed sphere. The atmosphere is a tenuous thin layer on this orb, so from a human's limited view, the Earth appears to be flat. For some applications, a simple Cartesian coordinate system, with three dimensions in the x, y, and z directions, seems like a good way to mathematically describe motion. For processes that occur on the larger scale, where the Earth's curvature is noticeable, we must resort to using coordinates that are natural for a sphere.

The way wind direction is described sprang out of wind observations, and is now firmly implanted in the psyche of every weather enthusiast: easterly, northerly, westerly, and southerly. This wind convention, however, is quite different than that used in the equations that govern atmospheric motion, which are the basis of weather forecast models. Here we will see that a conversion between the two conventions is straightforward but requires some care.

Finally, we will see that movement of air can either be described by fixed observers on the ground (called the **Eulerian framework**) or by someone riding along with a moving parcel of air (called the **Lagrangian framework**). These two points-of-view are very different, but we will see that they are related to each other by **advection**, which is just the movement of air with different properties (such as temperature, pressure, and relative humidity) from some place upwind of the place where you are.

With this math and these concepts you will be ready to take on atmospheric kinematics and dynamics.

8.2: This is why partial derivatives are so easy...

In your first calculus class you learned about derivatives. Suppose we have a function *f* that is a function of *x*, which we can write as *f*(*x*). What is the derivative of *f* with respect to *x*?

$$\frac{df(x)}{dx} \tag{8.2.1}$$

What about a new function that depends on two variables, *h*(*x*,*y*)? This function could, for example, give the height *h* of mountainous terrain for each horizontal point (*x*,*y*). So what is the derivative of *h* with respect to *x*? One way we determine this derivative is to fix the value of $y = y_1$, which is the same as assuming that *y* is a constant, and then take the ordinary derivative of *h* with respect to *x*. In a sense, we are taking a slice through the mountain in the *x*-direction at a fixed value of $y = y_1$. Thus,

$$\left(\frac{dh}{dx}\right)_{y=\text{ constant}} \equiv \frac{\partial h}{\partial x} \tag{8.2.2}$$

This is called the **partial derivative** of *h* with respect to *x*. It's pretty easy to determine because we do not need to worry about how *y* might depend on *x*.

? Exercise

Let $h = (x-3)^2 \cos(y)$. What is the partial derivative of *h* with respect to *x*?

Click for answer

$$\frac{\partial h}{\partial x} = \frac{\partial\left((x-3)^2 \cos(y)\right)}{\partial x} = 2(x-3)\cos(y)$$

We can also find the partial derivative of *h* with respect to *y*. Can you do this?

Click for answer

$$\frac{\partial h}{\partial y} = \frac{\partial\left((x-3)^2 \cos(y)\right)}{\partial y} = -(x-3)^2 \sin(y)$$

So you can see that the ∂h/∂x∂h/∂x may be different for each value of *y*and ∂h/∂y∂h/∂y may be different for each value of *x*. Thus, even if you are not entirely familiar with partial derivatives and their notation, you can see that they are no different from ordinary derivatives but you take the derivative for just of one variable at a time.

? Need more practice?

Partial Derivatives

Click here for transcript of the Partial Derivatives Video

Hey everyone! Welcome back! Today we're going to be doing some Partial Derivative problems. The first one that we're going to do is f of (x, y, z) equals x squared times y cubed times z to the four, and all that partial derivatives means is that we're going to be taking the derivative for every single variable in the problem. So, since there are three variables, we're actually going to have to take the derivative three times, once for each variable, and its' going to be a separate equation four each one. So we'll go in order. The first one we'll look at is x and the notation for partial derivative looks like this. This weird squiggly thing f, the weird squiggly thing x, so f, you get from here, so this is, you know, h of (x, y, z) then this becomes an h and then x is the first variable we're going to... we're going to do. So, we'll go ahead and say... now, when we're taking and... and when... when we take the partial derivative here with x, we say that we're taking the partial derivative with respect to x, that's how people call it, then we'll take the partial derivative with respect to y then with respect to z. So, looking at x, the way that we take the partial derivative with respect to x while still having these other variables in the equation is we treat the other variables like they're constants. And, what I like to do, and you get... you'll get faster and faster at it in your head, but the way that I like to do that just to make it really obvious because sometimes it's hard to understand how to hold those constant especially when... when you're first starting out, I like to actually put a constant in there for those numbers and then simplify the equation and then take the partial derivative so I can see it. So... so, what I would do here, for example, we're talking about holding y and z constant, as if they we're a constant number like two or three, so let's go ahead and put two in for... for... for y and z here. If we did, we would have... we would have x squared times two cubed times two to the fourth, right, because we... we plugged in two for y and for z. Okay? So, since we're keeping this constant, this is how the equation would simplify. So if... if you... if you multiply this out, let's see, this would actually be x squared times eight and this would be times sixteen, so this would be eighty and forty eight, so this would be x squared times a hundred and twenty eight, right, if you simplify that. So, what would be if we... if we were taking the derivative of this normally, we would be looking at a hundred and twenty eight x squared. We would take the derivative of this and if we would multiply two times the coefficient which is a hundred and twenty eight so that would be, what? Two hundred and fifty six, so the derivative of this would be two hundred and fifty six x, right? So, what I'm hoping that you can see from this is that it's... its... it's exactly the same thing. We're going to hold these two things constant and they are going to be like a coefficient and... and this two hundred and fifty six stays. So, this is actually going to be an... Let's go ahead and write out the answer and then we'll compare them. You're going to multiply these two out in front here so it's going to be two x and then y cubed z to the four. That's going to be the answer for the partial derivative and I that you can see the relationship here. We multiplied the two on the x squared out in front just like we did here, we... we brought this two out in front, and we ended up with a single x just like we ended up with a single x here and we left y cubed and z to the fourth because they were absorbed into the coefficient here. They are like... because they're constants and they're multiplied together, they are part of the coefficient, they're like part of this two which is why... which is why they get left in... in this equation. Let's go ahead and do y so that we see in another example and hopefully we'll start to understand. So, when we... when we take the partial derivative with respect to y as you might expect, it's going to be a partial derivative of f with respect to y, just like we did for x here. So now, with y, we're going to actually be holding... Oh, I hope you guys can't hear that fire truck. So with... with y, we're going to be keeping x and z constant so they're going to be like the coefficient as well, you could plug in numbers for them and... and go through the same exercise. But, they're like the coefficient so they're going to stay exactly the same because they're... they're multiplied here with the y so we're not even going to touch them. Remember, we didn't touched y cubed, we didn't touch z to the fourth so x and z, this time, are going to stay as well. So, all we're really looking at is... is the y and we're going to... we're going to do the same thing we did with x, take the derivative of y. So we're going to get that three out in front and then y squared, right, three y squared is the derivative of y cubed. So we took the derivative and then the x squared and the z to the fourth are just going to stay. So, that's the derivative with respect to y. I left the space because when you take the... the partial derivative, you always like to keep the variables in alphabetical order. So, I could have written three y squared x squared z to the fourth but we like to always keep them x, y, z in order. So, we'll go ahead and... and do the same thing here for z. So it's going to be the partial derivative of f with respect to z and we will go ahead and leave x squared and y cubed. We're not touching them because they're like part of the coefficient, they stay. So we'll go ahead and say x squared y cubed and then we... we take the derivative of z here. So the derivative... Of course, we subtract one from the exponent, so four minus one is three and the four gets multiply out in front so it comes out here. So our answer with respect to z is actually four x squared y cubed z cubed. And, your final answer is... is a three part answer if you're asked to take the... the... the derivative of this function or the partial derivative. Because

> there are three variables, you need each one of these equations and you would want to write all three of these down on the homework or on your test because... because your answer is actually all three of these. So, there you have it.

8.3: What you don’t know about vectors may surprise you!

Remember that a scalar has only a magnitude while a vector has both a magnitude and a direction. The following video (12:33) makes this difference clear.

Scalars and Vectors

Click here for transcript of the Scalars and Vectors video.

Hi it's Mr. Andersen and right now I'm actually playing Angry Birds. Angry Birds is a video game where you get to launch Angry Birds at these pig type characters. I like it for two reasons. Number one it's addictive, but number two it deals with physics and a lot of my favorite games deal with physics. So let's go to level two. And so, what I'm going to talk about today are vectors and scalars, and vectors and scalars are ways that we measure quantities in physics. Angry Birds would be a really boring game if I just use scalars because if I just use scalars I would input the speed of the bird and then I would just let it go, and it'd be boring because I wouldn't be able to vary the direction. And so Angry Birds I can vary the direction and let me try to skip this off of... nice. I can try to skip it off and and kill enough of these pigs at once. Now I could play this for the whole 10 minutes but that would probably be a waste of time. So, what I want to do is talk about scalars and vector quantities. Scalar and vector quantities I wanted to start with them at the beginning of physics because sometimes we get two vectors and people get confused and don't understand where did they come from. So, we have quantities that we measure in science especially in physics and we give numbers and units to those, but they come in two different types and those are scalar and vector. To kind of talk about the difference between the two, a scalar quantity is going to be a quantity where we just measure the magnitude, and so an example of a scalar quantity could be speed. So when you measure the speed of something and I say how fast does your car go, you might say that my car goes 109 miles per hour. Or, if you're a physics teacher you might say that my bike goes, I don't know like nine point six meters per second, and so this is going to be speed and the reason it's a scalar quantity is it simply gives me a magnitude. How fast, how far, how big, how quick. All those things are scalar quantities. What's missing from a scalar quantity is direction, and so vector quantities are going to tell you the not only the magnitude, but they're also going to tell you what direction that magnitude is in. So, let me use a different color maybe. Example of a vector quantity would be velocity, and so in science it's really important that we make this distinction between speed and velocity. Speed is just how fast something is going, but velocity is also going to contain the direction. In other words I could say that my bike is going 9.08 m/s West. Or, I could say this pen is being thrown with initial velocity of two point eight meters per second up or in the positive. And so, once we add direction to a quantity now we have a vector. Now you might think to yourself that's kind of nitpicky. Why do we care what direction that were flowing in and I have a demonstration that will kind of show you the importance of that, but a good example would be acceleration. So what is acceleration? Acceleration is simply change in velocity over time and so acceleration is going to be the change in velocity over time. and so I could ask you a question like this. let's say a car is driving down a road and it's going 23 meters per second and it stays at 23 meters per second. Is it accelerating? And you would say no of course it's not. Let's say it goes around a corner and during that movement around the corner it stays at 23 miles per hour. Well what would happen to the scalar quantity of speed around the corner? It would still be 23 meters per second, and so if you're using scalar quantities we'd have to say that it's not accelerating, but since the velocity is a vector if you're going 23 miles an hour and you go around a corner are you accelerating. Yeah, because you're not changing the magnitude of your speed, but you're clearly changing the direction and so a change in velocity is going to be acceleration. And so you are accelerating when you go around a corner. And so that be an example of why in physics, I'm not trying to be nitpicky I'm just saying that you have to understand the difference between a scalar quantity and then which is just magnitude, and a vector which is magnitude and direction. There's a review at the end of this minute video, and so I'll have you go through a bunch of these and so we'll identify a number of them, but for now I want to give you a little demonstration. To show you the importance of a scalar and vector quantities. So what I have here is a one thousand gram weight or one kilogram weight. It's suspended from a scale and I don't know if you can read that on there but the scale measures the number of grams. And so, if this is a thousand grams and this measures the number of grams and it's scaled right it should say and it does about a thousand grams is, is the weight of this. Now a question I could ask you is this, let's say I bring another scale and so I'm going to attach another scale to it. And so if we had one mass that had a mass of a thousand grams, and now I have two scales that are bearing the weight of that and I lift them directly up, what should what should each of the scales read. And if you're thinking well it's a thousand gram so each one should read 500 grams let me try it. The right answer is, yeah. Each of the scales ray right at about five hundred grands and so that should make sense to you. In other words 500 + 500 is a thousand so we have the force down of the weight force of tension that's holding these in position, and so we should be good to go. The problem becomes when I start to change the angle and so what I'm going to do and I'm sure this will go off screen, is I'm going to start to to hold these at a different angle. and so what if they look right here and now find that it's a six hundred and so this one is at 600 as well. and so as I increase the angle like this will find that that will increase as well and so when I get it to an angle like this I have a thousand gram weight and it's being supported by two scales now that are reading a thousand. and it's going to vary as I come back to here and if you do any weight lifting you understand kind of how that works. So the question becomes how do we do math? The problem with this then is the the numbers don't add up. And so, if I've got a 500 gram way excuse me a thousand gram weight being supported by two scales it made sense that it was weighing five hundred each. But now we all the sudden have a thousand gram weight being supported by two scales that are reading thousand and so this doesn't make sense or the math doesn't make sense. And the reason why is that you're trying to solve the problem from a scalar perspective, and you'll never be able to get the right answer because it's going to change its going to change depending on the angle that we lift them at. So, to understand this in a a vector method, and we'll get way into detail so I just kind of wanted to touch on it for just a second. What we had was a weight so we'll say there's a weight like this and will say that's a thousand gram weight and then we have two scales and each of those scales are pulling at 500 grams. So, if you add the vectors up, so this is 1 vector and this is another vector, so each of these are 500 grams so I make them 500 in length. Then we balance out in other words you have the balancing of this weight with these two weights that are on top of it. Now if we go to the vector problem the vector problem again we had a thousand gram weights a thousand grams in the middle, and then we had a force in this direction of a thousand and a force in that direction of the thousand. So we had the force down of a thousand, but we had a force of a thousand in this direction and a force of a thousand in that direction. And so, if you start to look of it at it like a vector quantity imagine this that we've gotta weight right here but you have to have two people pulling on it and so it's like this tug-of-war where it's not just in one direction but it's actually in two. And so you can start to see how these forces are going to balance out, but only if we look at it from the vector perspective. Let me show you what that would actually look like. So if we put these tails up this would be that force down of a thousand grams. This would be the force of the weight, but we also had a force in this direction so I'm doing the same rule where I'm lining up my vector from the tail to the tip and the tail to the tip. And so that diagram that I had in the last slide I'm actually moving this one force and you can see that they all sum up to 0. and so the reason I like to start talking about vectors and scalars with this problem is that you can never solve the problem if you're going to go at it from a scalar perspective. and we're going to do some really cool problems let's say I'm sliding a box across the floor, but how often do you slide a box across the floor and actually pull it straight across like that? if you're like me you're pulling a sled or something you normally pulling it at an angle and once we start playing at an angle becomes a totally different for us and we can't solve problems in the scalar way we have to go and solve it from the vector perspective and so that's the importance of vectors. on now it's a huge thing. So there are lots of things that we can measure in physics and so what I'm going to try to do hopefully can get this right is go through and circle all the scalar quantities and then go back and circle all the vector quantities. And so if you're watching this video a good thing to do

> would be pause right now and then you go through in and circle the ones that you think are scalar and vector, and then we'll see if we match up the end. Scalar quantities remember is simply going to be magnitude. And so the question I always ask myself when I'm doing this is, ok does it have a direction? Length is simply the length of a side of something, so I would put that in the scalar perspective. This is kind of philosophical, does time have a direction? I would say no. Acceleration we already talked about that. That's changing in velocity. What about density, the density of something. That definitely is a scalar quantity. If I say the density of that is 12.8 grams per cubic centimeter North that doesn't make sense at all. What are some other scalar quantities? Temperature would be a scalar quantity. It's just how fast the molecules are moving, but it's not in one certain direction. Pressure would be another one that is scalar. It's not directional. It's not in one direction, the pressure is remember, pressure air pressure is the one that I always think of is going to be in all direction, so we wouldn't say that. Let's see, mass. The mass of something is going to be a scalar quantity as well so it it doesn't change. Now wait and we'll talk more about that later and would actually be a a vector quantity. let's see if I'm missing any. now I think this would be good so let's change color for a second. So, displacement is how far you move from a location and that's in a direction. So we call that a vector quantity acceleration I mentioned before. force is going to be a vector and will do these force diagrams which are really fun later in the year. Drag is something slowing you down, so if your car it's what's slowing you down in the opposite direction of your movement, so the direction is important. Momentum is a product of velocity in the mass of an object, and lift we get from like an airplane wing. That would be a vector quantity because it's in a direction. So these are all vector quantities, the ones that I circled in red, but there are way more that we're going to find out there. And scalar quantities remember it's simply just magnitude or how big it is. And so as we go through physics be thinking to yourself is this a scalar quantity or vector? And if it's vector, my problem is a little bit harder, but like Angry Birds it's more fun when you go the vector route. And so, I hope that's helpful and have a great day!

Typically the vectors used in meteorology and atmospheric science have two or three dimensions. Let’s think of two three-dimensional vectors of some variable (e.g., wind, force, momentum):

$$\vec{A} = \vec{i}A_x + \vec{j}A_y + \vec{k}A_z \tag{8.3.1}$$

$$\vec{B} = \vec{i}B_x + \vec{j}B_y + \vec{k}B_z \tag{8.3.2}$$

Sometimes we designate vectors with **bold lettering**, especially if the word processor does not allow for arrows in the text. When Equations [8.3] are written with vectors in bold, they are:

$$\mathbf{A} = \mathbf{i}A_x + \mathbf{j}A_y + \mathbf{k}A_z \tag{8.3.3}$$

$$\mathbf{B} = \mathbf{i}B_x + \mathbf{j}B_y + \mathbf{k}B_z \tag{8.3.4}$$

> *Be comfortable with both notations for representing vectors.*

In the equations for vectors, A_x and B_x are the magnitudes of the two vectors in the *x* (east–west) direction, for which $\vec{i}$ or i is the unit vector; A_y and B_y are the magnitudes of the two vectors in the *y* (north–south) direction, for which $\vec{j}$ or ***j*** is the unit vector; and A_z and B_z are the magnitudes of the two vectors in the *z*(up–down) direction, for which $\vec{k}$ or ***k*** is the unit vector. Unit vectors are sometimes called direction vectors.

Sometimes we want to know the magnitude (length) of a vector. For example, we may want to know the wind speed but not the wind direction. The magnitude of $\vec{A}$, or *A*, is given by:

$$|\vec{A}| = \sqrt{(A_x^2 + A_y^2 + A_z^2)} \tag{8.3.5}$$

We often need to know how two vectors relate to each other in atmospheric kinematics and dynamics. The two most common vector operations that allow us to find relationships between vectors are the **dot product** (also called the scalar product or inner product) and the **cross product** (also called the vector product).

The dot product of two vectors ***A*** and ***B*** that have an angle β between them is given by:

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \tag{8.3.6}$$

$$= |\vec{A}||\vec{B}| \cos\beta \tag{8.3.7}$$

We can identify two extremes

$$\vec{A}\cdot\vec{B}=\begin{cases}|\vec{A}||\vec{B}| & \text{if } \vec{A}\parallel\vec{B}\\ 0 & \text{if } \vec{A}\perp\vec{B}\end{cases} \tag{8.3.8}$$

The dot product is simply the magnitude of one of the vectors, for example ***A***, multiplied by the projection of the other vector, ***B***, onto ***A***, which is just B cosβ ***A*** and ***B*** are parallel to each other, then their dot product is *AB*. If they are perpendicular to each other, then their dot product is 0. The dot product is a scalar and therefore has magnitude but no direction.

Also note that the unit vectors (a.k.a., direction vectors) have the following properties:

$$\vec{i}\cdot\vec{i}=\vec{j}\cdot\vec{j}=\vec{k}\cdot\vec{k}=1 \tag{8.3.9}$$

$$\vec{i}\cdot\vec{j}=\vec{i}\cdot\vec{k}=\vec{j}\cdot\vec{k}=\vec{j}\cdot\vec{i}=\vec{k}\cdot\vec{i}=\vec{k}\cdot\vec{j}=0 \tag{8.3.10}$$

$$\vec{i}\cdot\vec{A}=A_x \tag{8.3.11}$$

$$\vec{B}\cdot\vec{A}=\vec{A}\cdot\vec{B} \tag{8.3.12}$$

Note that the dot product of the unit vector with a vector simply selects the magnitude of the vector's component in that direction $\overrightarrow{(i}\cdot\vec{A}=A_x\Big)$ and that the dot product is commutative $(\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{A})$

Equation 8.3.7 can be rearranged to yield an expression for $\cos\beta$ in terms of the vector components and vector magnitudes:

$$\cos\beta=\frac{A_xB_x+A_yB_y+A_zB_z}{|\vec{A}||\vec{B}|} \tag{8.3.13}$$

The cross product of two vectors ***A*** and ***B*** that have an angle ββ between them is given by:

$$\vec{A}\times\vec{B}=\begin{pmatrix}\vec{i} & \vec{j} & \vec{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{pmatrix} \tag{8.3.14}$$

$$\vec{A}\times\vec{B}=(A_yB_z-A_zB_y)\,\vec{i}-(A_xB_z-A_zB_x)\,\vec{j}+(A_xB_y-A_yB_x)\,\vec{k} \tag{8.3.15}$$

The magnitude of the cross product is given by:

$$|\vec{A}\times\vec{B}|=|\vec{A}||\vec{B}|\sin\beta \tag{8.3.16}$$

We can identify two extremes

$$|\vec{A}\times\vec{B}|=\begin{cases}0 & \text{if } \vec{A}\parallel\vec{B}\\ |\vec{A}||\vec{B}| & \text{if } \vec{A}\perp\vec{B}\end{cases} \tag{8.3.17}$$

where β is the angle between ***A*** and ***B***, with β increasing from ***A*** to ***B***.

Note that the cross product is a vector. The direction of the cross product is at right angles to ***A*** and ***B***, in the right hand sense. That is, use the right hand rule (have your hand open, curl it from ***A*** to ***B***, and ***A*** x ***B*** will be in the direction of your right thumb). The magnitude of the cross product can be visualized as the area of the parallelogram formed from the two vectors. The direction is perpendicular to the plane formed by vectors ***A*** and ***B***. Thus, if ***A*** and ***B*** are parallel to each other, the magnitude of their cross product is 0. If *A* and *B* are perpendicular to each other, the magnitude of their cross product is *AB*.

The following video (2:06) reminds you about the right-hand rule for cross products.

Right-hand Rule for Vector Cross Product

Click here for transcript of the Right-hand Rule for Vector Cross product

> We're going to do a couple more examples of finding vector cross product. Suppose that I give you these two vectors a and B, which both lie in the plane of, look its my hands, which both lie in the plane of the page. Ok, so there are a and B. You want to find the direction of a cross B. To find the magnitude you do a times B times the sine of the angle between them, but we just want to find the direction right now, and to do this we're going to use the right hand rule, but first we can use a little bit of logic. So, first of all logic says this, whatever the direction of a cross B is which let's call that c, a cross b the we'll call that c. It has to be perpendicular to both a and B or perpendicular to the plane of the page. Well there are only two directions that that could be, right. What that means is that c either must point straight out of the page or it must point straight into the page. And, to figure out which one of those two directions it is, what we're going to have to do is we're gonna have to put our fingers along a. So there are two ways to do that. You can either put your fingers along a this way, or you could put your fingers along a this way, and you have to do it in the way that will let you swing a down into b like it was a little hinge. So, if you try that notice if you do it this way, yeah it's the wrong way right. You'd have to swing all the way the long way around. If you want to just simply fold a into b the way to do that is to put your fingers this way then you can curl them down this way. Notice when you do that your thumb is pointing into the page, so therefore, the answer is that c is into the page... and actually I got marker on my wall. Actually, the way we represent that is that's represented into the page is represented by a little X with a circle around it. You're supposed to think of it like the tail feathers of an arrow that's pointing into the page.

It follows that the cross products of the unit vectors are given by:

$$\vec{i} \times \vec{j} = \vec{k} \quad \vec{j} \times \vec{k} = \vec{i} \quad \vec{k} \times \vec{i} = \vec{j} \tag{8.3.18}$$

$$\vec{i} \times \vec{j} = -\vec{j} \times \vec{i} \tag{8.3.19}$$

Note finally that

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \tag{8.3.20}$$

We sometimes need to take derivatives of vectors in all directions. For that we can use a special vector derivative called the Del operator, $\vec{\nabla}$

Del is a vector differential operator that tells us the change in a variable in all three directions. Suppose that we set out temperature sensors on a mountain so that we get the temperature, T, as a function of x, y, and z. Then $\vec{\nabla}\, T$ would give us the change of T in the x, y, and z directions.

$$\vec{\nabla} = \vec{i}\,\frac{\partial}{\partial x} + \vec{j}\,\frac{\partial}{\partial y} + \vec{k}\,\frac{\partial}{\partial z} \tag{8.3.21}$$

The Del operator can be used like a vector in dot products and cross products but not in sums and differences. It does not commute with vectors and must be the partial derivative of some variable, either a scalar or a vector. For example, we can have the following with del and a vector ***A***:

$\vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$, which is a scalar

$\vec{\nabla} T = \vec{i}\,\frac{\partial T}{\partial x} + \vec{j}\,\frac{\partial T}{\partial y} + \vec{k}\,\frac{\partial T}{\partial z}$, which is a vector even though T is a scalar

$\vec{A} \cdot \vec{\nabla} T = A_x \frac{\partial T}{\partial x} + A_y \frac{\partial T}{\partial y} + A_z \frac{\partial T}{\partial z}$, which is a scalar

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8.4: Describing weather requires coordinate systems.

In meteorology and other atmospheric sciences, we mostly use the standard x, y, and z coordinate system, called the **Cartesian coordinate system**, and the **spherical coordinate system**. Let's review some of the main points of these two systems.

Cartesian Coordinate System

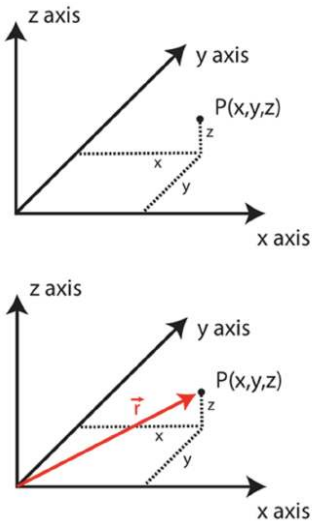
The Cartesian coordinate system applies to three dimensions (as seen in the figure below). The convention is simple:

- The zero point, $x = y = z = 0$ or (0,0,0), is arbitrary.
- x increases to the east; x decreases to the west.
- y increases to the north; y decreases to the south.
- z increases going up; z decreases going down.
- A distance vector extending from the origin to (x,y,z) as $\mathbf{L} = \mathbf{i}\,x + \mathbf{j}\,y + \mathbf{k}\,z$.

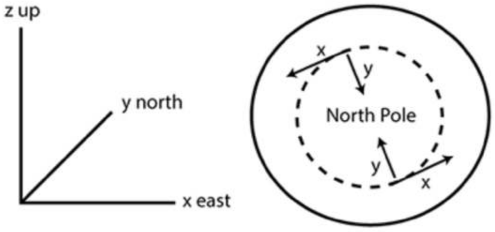
Unit vectors (length 1 along standard coordinates) are $\mathbf{i}$ (east); $\mathbf{j}$ (north); $\mathbf{k}$ (up).

Often we will consider motion in two dimensions as being separate from motions in the vertical. We usually denote the horizontal with a subscript H; for example, $\mathbf{L}_H = \mathbf{i}\,x + \mathbf{j}\,y$, where $\mathbf{L}_H$ is a horizontal distance vector.

We like this coordinate system because it works well over relatively small scales on Earth, perhaps the size of an individual state, where Earth's curvature is not important. However, it does not work so well for large-scale motion on Earth, which is spherical.



Cartesian coordinate system. Credit: W. Brune

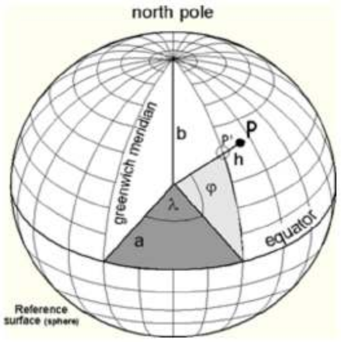


The Cartesian coordinate system when placed on the globe. Credit: W. Brune

When we place the Cartesian coordinate system on a sphere, note that x always points toward the east, y always points toward the North Pole, and z always points up along the direction of Earth's radius (as seen in the figure above).

Spherical Coordinate System

Life would be much easier if the Earth were flat. We could then use the Cartesian coordinate system with no worries. But the Earth is a sphere, which implies that to accurately describe motion, we must take the Earth's spherical shape into account.



Spherical coordinate system. Credit: Artima Developer

We use the following terms:

- r = distance from the center of the Earth
- $\Phi =$ latitude $(-90^\circ \text{ to } +90^\circ, \text{ or } -\pi/2 \text{to} + \pi/2)$
- $\lambda =$ longitude $(-180^\circ \text{ to } 180^\circ, \text{ or } -\pi \text{ to } +\pi)$
- State College PA is at $\Phi = 40.8^\circ$ and $\lambda = -77.9^\circ$

Note that 1^o of latitude is always 111 km or 60 nautical miles, but 1^o of longitude is 111 km only at the equator. It is smaller in general and equal to 111 km x cos(Φ). Note that 1 nm = 1.15 miles.

To find the horizontal distance between any two points on Earth's surface, we first need to find the angle of the arc between them and then we can multiply this angle by Earth's radius to get the distance. To find the angle of the arc, $\Delta\sigma$, we can use the **Spherical Law of Cosines**:

$$\Delta\sigma = \arccos\left(\sin\phi_1 \cdot \sin\phi_2 + \cos\phi_1 \cdot \cos\phi_2 \cdot \cos\Delta\lambda\right)$$

where the latitude and longitude of the two points are Φ_1, λ_1 and Φ_2, λ_2 respectively and $\Delta\lambda = |\lambda_1 - \lambda_2|$ is the absolute difference between the longitudes of the two points. Note that the angle of the arc must be in radians, where 2π radians $= 360^\circ$. To find the distance, simply multiply this arc angle by the radius of the Earth, 6371 km.

? Exercise

Show that 1^o of latitude = 111 km distance.

Click for answer

Distance = 6371 km * (1/360)*2π = 111.2 km

In summary, we will use a Cartesian coordinate system *when our scales of interest are not too large (synoptic scale or smaller)*, but will need to use spherical coordinates *when the scale of interest is larger than synoptic scale*.

For another explanation of these two systems, visit this Coordinate Systems website.

Vertical Coordinates

Three different vertical coordinates are used in meteorology and atmospheric science: height, pressure, and potential temperature.

We have already introduced the vertical coordinate ***z*,** which is a height, usually in m or km, above the Earth's surface in the Cartesian coordinate system; z is related to r in spherical coordinates through $r = a + z$, where a is the Earth's radius. The vertical coordinate z is the most commonly used in meteorology and in any process that involves getting off the ground, such as flight. Often pilots talk about flight levels, which are measured in hundreds of feet. So, flight level 330 is about 10 km altitude.

Another useful vertical coordinate is pressure, which decreases with height. Pressure is often a useful vertical coordinate for calculating dynamics. To a good approximation, pressure falls off exponentially with height,

$$p = p_o \exp(-z/H) \tag{8.4.1}$$

so that ln*(p)* is fairly linear with height. We’ll get into this in greater detail later. For now, consider the following table of typically used pressure heights:

Typically Used Pressure Heights

altitude (km)	altitude (kft)	pressure level (hPa or mb)
0	0	1000
1.5	4.4	850
3.0	9.9	700
5.5	18.3	500

A third important vertical coordinate is potential temperature, θ (Equation 2.58). This quantity is the temperature that an air parcel would have if it were brought to a pressure of 1000 hPa without any exchange of heat with its surroundings. This vertical coordinate has a nice property: air parcels tend to move on surfaces of constant potential temperature because moving on such a surface requires no energy. This coordinate is particularly useful in the stratosphere, where the rapid increase with altitude tends to keep air motion stratified.

8.5: Do you need a weathervane to see which way the wind blows?

Meteorologists talk of northeasterlies and southerlies when they describe winds. These terms designate directions that the winds *come from*. But when we think about the dynamic processes that cause the wind, we use the conventions for direction that are common in mathematics and in coordinate systems like the Cartesian coordinate system. The conversion between the two conventions—math and meteorology—is not simple. However, we will show you a simple way to do the conversion (see the second figure below).

Weathervane. Credit: Justin Otto via flickr

Math Wind Convention

The wind vector is given by $\boldsymbol{U} = \boldsymbol{i}\,u + \boldsymbol{j}\,v + \boldsymbol{k}\,w$. The wind vector points to the direction the wind *is going*.

The subscript "H" will be used to denote horizontal vectors, such as the horizontal velocity, $\boldsymbol{U_H} = \boldsymbol{i}\,u + \boldsymbol{j}\,v$ (though note that sometimes the symbols $\boldsymbol{V}$, $\boldsymbol{v}_H$, and $\boldsymbol{v}$ will be used to denote the horizontal velocity). The magnitude of $\boldsymbol{U_H}$ is $U_H = (u^2 + v^2)^{1/2}$. The ***math wind angle,*** $\alpha\alpha$ x-axis, so that $\tan(\alpha\alpha = v/u$ and the angle increases counterclockwise as the direction moves from the eastward x-axis ($\alpha\alpha = 0^o$) to the northward y-axis ($\alpha\alpha = 90^o$) .

Meteorology Wind Convention

In this station weather plot, the wind is blowing from the southwest. Credit: NOAA National Weather Service

The meteorology wind convention is often used in meteorology, including station weather plots. The wind vector points to the direction the wind *is coming from*. The angle is denoted by delta, δ, which has the following directions:

Wind Angles

direction wind is coming from	angle δ
north (northerlies or southward)	0^o
east (easterlies or westward)	90^o
south (southerlies or northward)	180^o
west (westerlies or eastward)	270^o

Relationship Between Math and Meteorology Wind Conventions

Meteorology angles, designated by δδy) axis. Math angles, designated by αα , increase counterclockwise from the east (*x*) axis.

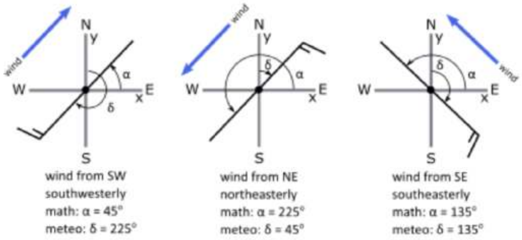


Diagram for converting between meteorology and math wind directions for winds blowing from three different directions. Note that the barbs on the wind line point toward the direction the wind is blowing from. In this diagram, we have extended the line past the center of the axis to indicate the direction that the wind is blowing to because this is the line needed for the math angle. Credit: W. Brune

In the diagram on the left, the wind is southwesterly, the meteorology angle (measured clockwise from the north or y-axis) $\delta = 225^\circ$, , and the math angle (measured counterclockwise from the east or x-axis) $\alpha = 45^\circ$. If the wind is northerly (southward), the wind vane points to the north, the wind blows to the south, $\delta = 0^\circ$, and $\alpha = 270^\circ$. If the wind is westerly (eastward), $\delta = 270^\circ$, and $\alpha = 0^\circ$.

Note that in all cases, we can describe the relationship between the math and the meteorology angles as:

$$\textbf{math angle} = 270^\circ - \textbf{meteorology angle}$$

When the meteorology angle is greater than 270^{o}, the math angle will be negative but correct. However, to make the math angle positive, simply add 360^{o}.

Drawing a figure like those shown in the figure above often helps when you are trying to do the conversion. The following video (2:17) explains the conversion between meteorology and math wind angles using the figure above.

Wind Meteo Math

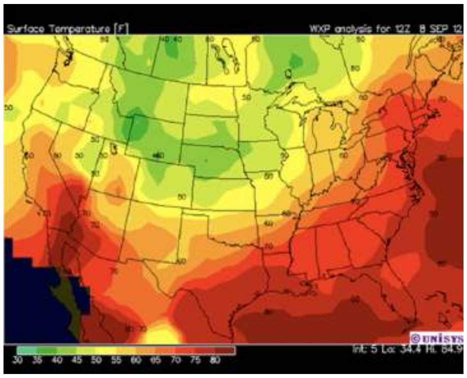
Click here for transcript of the Wind Meteo Math video.

> Meteorology description and wind direction originates from the compass and facing into the wind, where the wind comes from. The mathematical description of wind direction is based on the Cartesian xy grid and tracks the direction that the wind is going. We need to know both. Because the meteorology description, or medial angle, is used in the station weather plot. And we need the mathematical description, or math angle, for dynamics and a numerical weather prediction. Let's look at one example that relates the meteorology and math involves. First note that the grids are related, with positive x corresponding to east and positive y corresponding to north. Now let's add a wind, in this case a wind from the northeast, or northeasterly. From station weather plot the wind is from the northeast. Normally the wind bar would end in the center with a description of cloud cover. But we extend it past the center, toward the direction the wind is blowing, since that would be how we would draw the line and describe the wind direction in the mathematical xy coordinate system. The meteorology angle is measured clockwise from the north axis, just as it is for a compass-- 0, 90, 180, 270, 360, which is the same as 0. The math angle is measured

counterclockwise from the x, or east, axis-- 0, 90, 180, 270, 360 or 0. It turns out that the math angle equals 270 degrees minus the meteorology angle. And also therefore the meteorology angle equals 270 degrees minus the math angle. So for this case that we've drawn here the meteorology angle equals 45 degrees. So the math angle equals 270 minus 45, which is 225 degrees. The meteorology angle is drawn clockwise. And the math angle as drawn counterclockwise. If the resulting angle is negative, simply add 360 degrees to make it positive.

8.6: Gradients - How to Find Them

The **gradient** of a variable is just the *change in that variable as a function of distance*. For instance, the temperature gradient is just the temperature change divided by the distance over which it is changing: $\Delta T/\Delta$distance. The gradient is a vector and thus has a direction as well as magnitude.



Surface temperature map for North America on 8 September 2012. Temperatures are in ^{o}F. For a point in western Kentucky, the temperature gradient is to the southeast. Credit: Unisys

Consider the surface temperature contour plot from NOAA for 8 September 2012 in the figure above. A strong temperature variation is draped across the eastern and southern US from New York down to Texas. How do we quantify this temperature variation? First we have to specify where we want to measure the temperature gradient. Then we simply need to choose isotherms on either side of the point, take the difference between the isotherms, figure out how far apart they are in horizontal distance, and divide the temperature change between the isotherms by the distance between the isotherms. The direction for the gradient is on the normal (perpendicular to the isotherms) from the lower temperatures to the higher temperatures. It's pretty easy to figure out where the gradient vector points just by quick examination, but it is a little harder to figure out what the gradient magnitude and actual direction are.

Now watch this video (2:12) on finding distances:

Finding Distances

Click here for transcript of the Finding Distances video.

> We're often interested in finding distances on a map so that we can calculate quantities we are interested in, such as the temperature gradient. Note first that often the projection of the map that we have does not have east-west, parallel, and straight across. In fact, the east-west line is curved a little bit, so take that into account when you're doing your calculations. Also note the north-south lines run a little bit not parallel as well. So how do we find distances? Well, there are many different ways, but one good way is to take a known distance on the map, scale it with a ruler, and then use that ruler in other places to give us distances in other places. So for instance, we know that the height of Pennsylvania between the two

> parallel borders, the north and south borders is 135 nautical miles. So we can scale that with a ruler, and I have a ruler here. I put the ruler in, and I see, in this particular case, the distance between the two is just about exactly 1 centimeter or 10 millimeters. So what that means is each millimeter on my scale is equal to 13.5 nautical miles on the map. So then I can use this in other places to measure other distances. So for instance, if I want to know the height of Kansas between its parallel North and South borders, I can put my ruler on there. And if I look carefully, I get a number that's about 13 and 1/2 millimeters. So 13 and 1/2 times 13 and 1/2 is about 182. And that's what I would say this distance is. The actual distance is 180 nautical miles, so in fact, the scaling I have is actually pretty good.

Mathematically, if we know the algebraic expression for the temperature change, such that $T = T(x,y)$, we can find the gradient by using the **del operator**, which is also called the **gradient operator**.

Recall the del operator:

$$\vec{\nabla} = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z} \tag{8.6.1}$$

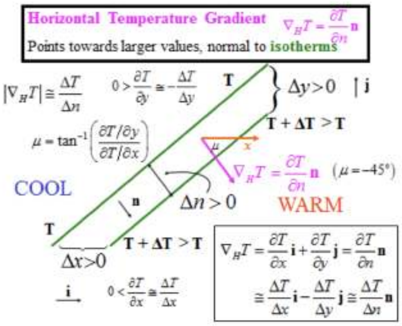
If we are looking only at changes in x and y, then we can define a horizontal del operator:

$$\vec{\nabla}_H = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} \tag{8.6.2}$$

At any point, we can determine the gradient of the temperature:

$$\vec{\nabla}_H T = \vec{i}\frac{\partial T}{\partial x} + \vec{j}\frac{\partial T}{\partial y} \tag{8.6.3}$$

Note that this quantity has dimensions of θ/Lθ/L and a magnitude and a direction. The gradient direction is always normal to the isolines and pointing in the direction of an increase. We can define the normal vector, which is just the unit vector in the direction of the increasing temperature. We will call this normal vector ***n***.



Example of a gradient and the math required to calculate the gradient magnitude and direction.

Credit: H.N. Shirer

We can calculate a gradient for every point on the map, but to do this we need to know the change in the temperature over a distance that is centered on our chosen point. One approach is to calculate the gradients in the x and $y$directions independently and then determine the magnitude by:

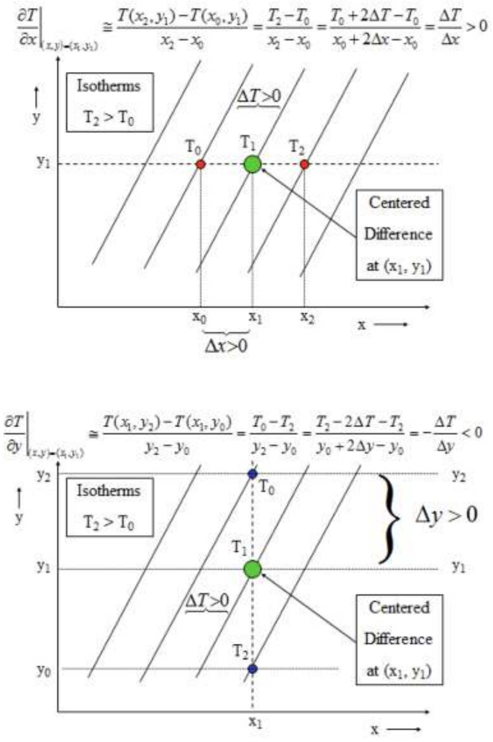
$$\left|\vec{\nabla}_H T\right| = \sqrt{\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2} = \left|\frac{\partial T}{\partial n}\right| \tag{8.6.4}$$

and the direction by:

$$\mu = \tan^{-1}\left(\frac{\partial T/\partial y}{\partial T/\partial x}\right) \tag{8.6.5}$$

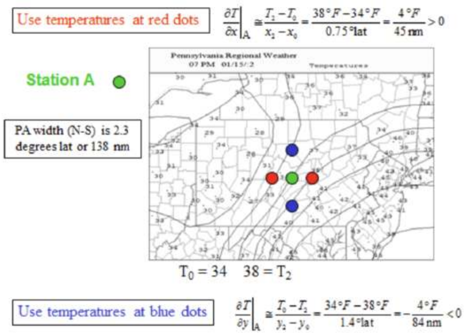
We can program a computer to do these calculations.

However, often we just want to *estimate* the gradient. The gradient can be determined by looking at the contours on either side of the point and computing the change in temperature over the distance. These partial derivatives can be approximated by small finite changes in temperatures and distances, so that ∂ is replaced by Δ in all places in these equations. We can calculate gradients by using "centered differences" as shown in the figures below.



Calculating the temperature gradient in the *x* (top) and *y* (bottom) directions using the centered difference method. Credit: H.N. Shirer

We then calculate the magnitude with Equation [8.12] and the direction with Equation [8.13], where we replace the partial derivatives with the small finite differences in all places in these equations.



Example of an estimation of the gradients in the *x* and *y* directions. To get an idea of the horizontal scale, you can estimate distances using the known size of a state or country, in this case, Pennsylvania, which is on average 470 km (254 nm, nautical miles, 290 miles) in the *x* (east–west) direction and 250 km (135 nm, 155 miles) in the *y* (north–south) direction for the parts where the north and south borders are parallel lines. Credit: H.N. Shirer

The magnitude and direction are:

$$|\nabla_H T| = \sqrt{\left(\frac{\Delta T}{\Delta x}\right)^2 + \left(\frac{\Delta T}{\Delta y}\right)^2} = \sqrt{\left(\frac{4°\mathrm{F}}{45\mathrm{nm}}\right)^2 + \left(\frac{-4°\mathrm{F}}{84\mathrm{nm}}\right)^2} = 0.1°\mathrm{F/nm} \tag{8.6.6}$$

$\mu = \tan^{-1}\left(\frac{\Delta T/\Delta y}{\Delta T_{/\Delta x}}\right) = \tan^{-1}\left(\frac{-4/84}{4/45}\right) = -28^{\circ}$, which points to the southeast

When you calculate the arctangent, keep in mind that the tangent function has the same values every 180°, or every π in radians. If you get an answer for the arctangent that is 45°, how do you know whether the angle is really 45° or 45°+ 180° = 225°? The gradient vector always points toward the higher temperature air, so always choose the angle so that the gradient vector points toward the warmer air.

Recap of the process for calculating the temperature gradient:

1. Determine the distance scale by any means that you can. Sometimes it is given to you; sometimes you can scale off a ruler; sometimes you just estimate it using the size of known boundaries.
2. Determine the spacing between the isotherms.
3. Find the temperature change in the *x* and *y* directions using the centered difference method. These two numbers, ΔT/Δx and ΔT/ΔyΔT/Δx and ΔT/Δy ΔT/Δx and ΔT/ΔyΔT/Δx and ΔT/Δy can be either positive or negative.
4. Calculate the magnitude by finding the square root of the squares of the gradients in the *x* and *y* directions (i.e., ΔT/Δx and ΔT/ΔyΔT/Δx and ΔT/Δy
5. Calculate the direction of the gradient vector by finding the arctangent of the *y*-gradient divided by the *x*-gradient. Pay attention to the direction—make sure that it points toward the warmer air.

Now watch this video (3:52) on finding gradients:

Finding Gradients

Click here for transcript of the Finding Gradients video.

> We can calculate the gradient by using the method that's described in the lesson. Let's pick a point in Pennsylvania about here, and then we'll calculate the gradient for that point. We will first look at the gradient in the x direction which goes along, and it's parallel with the North and South boundaries of Pennsylvania. We'll use the method of center differences that's described. We'll look at this contour here, this isotherm, and this one over here on the other side. And we note that this distance here is very, very similar to the distance of Pennsylvania between the parallel borders, which is 135 nautical miles. And so each one of these contours is four degrees Fahrenheit. So we have two of them, so 8 divided by 135 nautical miles gives us a gradient in the x direction of 0.059 degrees Fahrenheit per nautical miles. Now we can do the y direction, so we pick two points here. One here and one about here to be on the gradients. And we note that this is a little bit more than half the height of Pennsylvania. It's actually about 80 nautical miles. But also note that as y goes more positive, the temperature becomes more negative and therefore, we have to use minus 8 over 80. And we get, for the gradient in the y direction, minus 0.1 degrees Fahrenheit per nautical miles. When we put these in to get the magnitude-- it's the square root of the squares-- we see that we end up with 0.12 degrees Fahrenheit per nautical mile with a magnitude of the gradient. To find the direction of the gradient we see that mu, the angle with respect to the x-axis-- so this is a math angle. It's equal to the arctangent of the gradient in y divided by the gradient in x. And so that would be the arctangent of minus 0.1 over 0.059, which is minus 59 degrees. And that is, of course, measured from the x-axis here. So that is measured from this direction

> here like this, and so that's minus 59. And it's the same as if we went all the way around and we would get 301 for alpha if we were looking at the math angle. Now, we can look and get an idea about gradients and other places really quickly, so let's just take this point in Central Oregon. So now x is going like this over here, and so we see that the gradient in the East West direction, or x direction is-- to go to another country you have to go very, very far, and so that would be 8 degrees. It's so far the really the gradient is essentially 0. Whereas if we go in the North South direction-- that is in the y direction here-- we see that there's quite a substantial distance here. And so, since this is 8 degrees, just like this is 8 degrees over here, then what that means is the gradient is going to be quite a bit smaller in this direction than it is in Pennsylvania here where the isotherms are much, much closer together. So we would expect a gradient that's a fourth or a fifth of the gradient that we got for Pennsylvania, and so it'll be very weak. Nonetheless, it'll point toward the hotter air, and it'll look something like this.

A word about finding gradients in the real world. Sometimes the centered differences method is difficult to apply because the gradient is too much east–west or north–south. For instance, in the temperature map at the beginning of this section, the *x*-gradient is hard to determine by the centered difference method in the Oklahoma panhandle and the *y*-gradient is hard to determine in central Pennsylvania because in both cases, the temperature hardly changes. In these cases, you could say that the gradient in that direction equals 0, but then your computer program might have a hard time finding the arctangent. One way around this problem is to put in a very small number for the gradient in that direction, say 1 millionth of your typical gradient numbers, to do the calculation.

A second word about finding gradients in the real world. When you are finding temperature gradients from a temperature map, it is sometimes hard to determine the temperature gradient at some locations because the isotherms are not evenly spaced and can be curvy. Don't despair! Use your best judgment as to what the gradients are. Check your answers for the magnitude and direction of the temperature gradient vector by estimating the magnitude and direction by eyeballing the normal to the isotherms at that location and pointing the gradient vector to the warmer air. If your calculated direction is 160 degrees when your eyeball check says about 220 degrees, check your math again.

8.7: What You Experience Depends on Your Point of View - Eulerian vs. Lagrangian

Suppose you were driving underneath an eastward-moving thunderstorm at the same speed and in the same direction as the storm, so that you stayed under it for a few hours. From your point-of-view, it was raining the entire time you were driving. But from the point-of-view of people in the houses you passed, the thunderstorm approached, it rained really hard for twenty minutes, and then stopped raining.

A car travels at the same speed and direction as a thunderstorm. Credit: Nicholas A.Tonelli via flickr

The people in houses formed a network of observers, and if they talked to each other, they would find that the thunderstorm moved rapidly from west to east in a path that rained on some houses and missed others. This point-of-view is called the **Eulerian** description because it follows the thunderstorm through a fixed set of locations. You, on the other hand, followed along with the thunderstorm; any changes you saw were due to changes in the thunderstorm’s intensity only. Your point-of-view was a **Lagrangian** description because you stayed with the storm.

We can now generalize these ideas to any parcel of air, not just a thunderstorm. An air parcel is a blob of air that hangs more-or-less together as it moves through the atmosphere, meaning that its mass and composition are conserved (i.e., not changing) as it moves. It has a fairly uniform composition, temperature, and pressure, and has a defined velocity.

Air parcels do not live forever. They are just air moving in air, so they change shape as they bump into other air parcels and they mix until they disappear. Larger air parcels tend to live longer than smaller air parcels. Even though air parcels do not live forever, they are very useful concepts in explaining the differences between the Eulerian and Lagrangian frameworks.

To Summarize

Eulerian Framework:

- Observes atmospheric properties and their changes at *fixed points* in space
- Tracks motion by reporting those observations from time-to-time
- Is the way most weather observations are taken and the way the most weather prediction models do computations

Lagrangian Framework:

- Observes atmospheric properties and their changes in a *moving*air parcel
- Tracks changes within the air parcel as it moves
- Is a conceptual or numerical approach and is difficult to realize with observations

Here's an interesting fact. Radiosonde measurements are considered Eulerian because they measure properties like temperature and humidity at a fixed location. However, the horizontal velocity is measured by a radiosonde’s lateral movement with the horizontal wind and so is actually a Lagrangian measurement. A cup anemometer or sonic anemometer is an example of a Eulerian method of measuring wind velocity.

Discussion Activity: Eulerian and Lagrangian Points-of-View From Your Everyday Life

(3 discussion points)

In the lesson, I presented an example of a driver in a car that happened to be traveling at the same speed as a rainstorm so that to the driver it was raining the entire time, while to the observers in the houses that the driver passed, the rain showers were brief. We could have given a second case in which there was widespread rain so that both the car driver and the house occupants observed constant rainfall for several hours. In the first case, the advection exactly matched the local rainfall change wherever the driver was, so that the driver was in rain constantly, but the house occupants saw rain only for a short while. In the second case, there was no gradient in the rainfall rate, so even if the storm was moving, neither the driver nor the house occupants observed a change in rainfall.

Think of one event or phenomenon from the Eulerian and Lagrangian points-of-view, one in which the event or phenomenon looks very different from the two points-of-view. These events do not have to be weather related. Be creative.

1. You can access the **Eulerian and Lagrangian Points-of-View Discussion** in Canvas.
2. Post a response that answers the question above in a thoughtful manner that draws upon course material and outside sources.
3. Keep the conversation going! **Comment on at least one other person's post**. Your comment should include follow-up questions and/or analysis that might offer further evidence or reveal flaws.

This discussion will be worth 3 discussion points. I will use the following rubric to grade your participation:

Discussion Activity Grading Rubric

Evaluation	Explanation	Available Points
Not Completed	Student did not complete the assignment by the due date.	0
Student completed the activity with adequate thoroughness.	Posting answers the discussion question in a thoughtful manner, including some integration of course material.	1
Student completed the activity with additional attention to defending his/her position.	Posting thoroughly answers the discussion question and is backed up by references to course content as well as outside sources.	2
Student completed a well-defended presentation of his/her position, and provided thoughtful analysis of at least one other student's post.	In addition to a well-crafted and defended post, the student has also engaged in thoughtful analysis/commentary on at least one other student's post as well.	3

8.8: Can the Eulerian and Lagrangian frameworks be connected?

$$R=R(x,y,z,t) \tag{8.8.1}$$

To find the change in the rainfall rate *R* in an air parcel over space and time, we can take its differential, which is an infinitesimally small change in *R*:

$$dR=\frac{\partial R}{\partial t}dt+\frac{\partial R}{\partial x}dx+\frac{\partial R}{\partial y}dy+\frac{\partial R}{\partial z}dz \tag{8.8.2}$$

where *dt* is an infinitesimally small change in time and *dx*, *dy*, and *dz* are infinitesimally small changes in *x*, *y*, and *z* coordinates, respectively, of the parcel.

If we divide Equation [8.15] by *dt*, this equation becomes:

$$\frac{dR}{dt}=\frac{\partial R}{\partial t}+\frac{\partial R}{\partial x}\frac{dx}{dt}+\frac{\partial R}{\partial y}\frac{dy}{dt}+\frac{\partial R}{\partial z}\frac{dz}{dt} \tag{8.8.3}$$

where *dx/dt*, *dy/dt*, and *dz/dt* describe the velocity of the air parcel in the *x*, *y*, and z directions, respectively.

Let’s consider two possibilities:

Case 1: The air parcel is *not* moving. Then the change in *x*, *y* and *z* are all zero and:

$$\frac{dR}{dt}=\frac{\partial R}{\partial t} \tag{8.8.4}$$

So, the change in the rainfall rate depends only on time. $\frac{\partial R}{\partial t}$ is called the Eulerian or **local time derivative**. It is the time derivative that each of our weather observing stations record.

Case 2: The air parcel is moving. Then the changes in its position occur over time, and it moves with a velocity, $\vec{U}=\vec{i}u+\vec{j}v+\vec{k}w$, where:

$$\frac{dx}{dt}=u \quad \frac{dy}{dt}=v \quad \frac{dz}{dt}=w \tag{8.8.5}$$

$$\frac{dR}{dt}=\frac{\partial R}{\partial t}+u\frac{\partial R}{\partial x}+v\frac{\partial R}{\partial y}+w\frac{\partial R}{\partial z} \tag{8.8.6}$$

A special symbol is given for the derivative when you follow the air parcel around. It is called the **substantial, or total, derivative** and is denoted by:

$$\frac{DR}{Dt}=\frac{\partial R}{\partial t}+u\frac{\partial R}{\partial x}+v\frac{\partial R}{\partial y}+w\frac{\partial R}{\partial z} \tag{8.8.7}$$

Mathematically, we can express this equation in a more general way by thinking about the dot product of a vector with the gradient of a scalar as we did in an example of the del operator:

$$\frac{DR}{Dt}=\frac{\partial R}{\partial t}+\vec{U}\cdot\vec{\nabla}R \tag{8.8.8}$$

where the second term on the right hand side is called the advective derivative, which describes changes in rainfall that are solely due to the motion of the air parcel through a spatially variable rainfall distribution.You should be able to show that equation [8.19] is the same as equation [8.18].

We can rearrange this equation to put the local derivative on the left.

$$\frac{\partial R}{\partial t}=\frac{DR}{Dt}-\vec{U}\cdot\vec{\nabla}R \tag{8.8.9}$$

The term on the left is the local time derivative, which is the change in the variable *R* at a fixed observing station. The first term on the right is the total derivative, which is the change that is occurring in the air parcel as it moves. The last term on the right, $-\vec{U}\cdot\vec{\nabla}R$, is called the advection of *R*, Note that advection is simply the negative of the advective derivative.

To go back to the analogy of the thunderstorm, the change in rainfall that you observed driving in your car was the total time derivative and it depended only on the change in the intensity of the rain in the thunderstorm. However, for each observer in a house, the change in rainfall rate depended not only on the intensity of the rainfall as the thunderstorm was over the house but also on the movement of the thunderstorm across the landscape.

R can be any scalar. Rainfall rate is one example, but the most commonly used are pressure and temperature.

Equation [8.20] is called **Euler's relation** and it relates the Eulerian framework to the Lagrangian framework. The two are related by this new concept called advection.

Let's look at advection in more detail, focusing on temperature.

We generally think of advection being in the horizontal. So often we only consider the changes in the *x* and *y* directions and ignore the changes in the *z*direction:

$$\text{horizontal temperature advection}$$
$$= -\vec{U}_H \cdot \vec{\nabla}_H T = -\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) \tag{8.8.10}$$

So what's with the minus sign? Let's see what makes physical sense. Suppose *T*increases only in the *x*-direction so that:

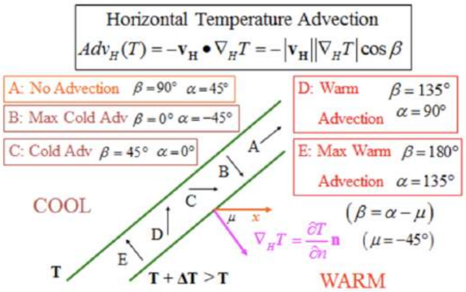
$$\frac{\partial T}{\partial y} = 0 \text{ and } \frac{\partial T}{\partial x} > 0$$

If $u > 0$ (westerlies, blowing eastward), then both u and $\frac{\partial T}{\partial x}$ are positive so that temperature advection is negative. What does this mean? It means that colder air blowing from the west is replacing the warmer air, and the temperature at our location is getting decreasing from this advected air. Thus $\frac{\partial T}{\partial t}$ should be

negative since time is increasing and temperature is decreasing due to advection.

If the temperature advection is *negative*, then it is called *cold-air advection*, or simply cold advection. If the temperature advection is *positive*, then it is called *warm-air advection*, or simply warm advection.

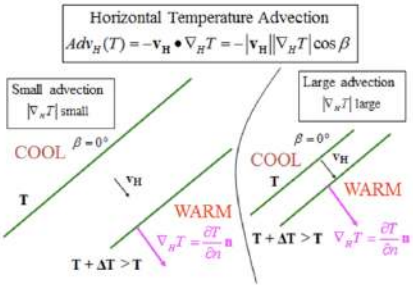
Some examples of simple cases of advection show these concepts (see figure below). When the wind blows along the isotherms, the temperature advection is zero (Case A). When the wind blows from the direction of a lower temperature to a higher temperature (Case B), we have cold-air advection. When the wind blows as some non-normal direction to the isotherms, then we need to multiply the magnitude of the wind and the temperature gradient by the cosine of the angle between them. We can estimate the temperature advection by doing what we did for the gradient, that is, replace all derivatives and partial derivatives with finite Δs.



Examples of horizontal temperature advection for cases with different angles between the normal to the isotherms and the wind direction (in math terms). When the temperature gradient and wind direction are parallel, then the advection is maximum; when they are perpendicular, then it is zero. Warm air moving toward cold air will be warm advection; cold air moving toward warm air will be cold advection. Note: $\mathbf{v}_H$ used in this figure is the same as $\mathbf{U}_H$ in the text.

Credit: H.N. Shirer

When the isotherms with the same temperature difference are further apart on the map (see figure below), then the horizontal temperature advection will be less than when the isotherms are closer together, if the wind velocity is the same in the two cases.



Examples of horizontal temperature advection for different distances between the isotherms for constant wind velocity. Note: $\mathbf{v}_\mathrm{H}$ used in this figure is the same as $\mathbf{U}_\mathrm{H}$ in the text.

Credit: H.N. Shirer

In summary, to calculate the temperature advection, first determine the magnitude and the direction of the temperature gradient. Second, determine the magnitude and direction of the wind. The advection is simply the negative of the dot product of the velocity and the temperature gradient.

Watch this video (2:20) on calculating advection:

Finding Advection

Click here for transcript of the Finding Advection video.

Temperature advection is just a dot product of the velocity vector and the temperature gradient vector at that point. Let's choose this point in Pennsylvania where we've already calculated the gradient of this point. Let's look at the wind vector. So the station weather plot has a wind barb that's northwesterly and five knots. And so we can estimate, since this is x direction, and since this is north, we can estimate that this is about 300 degrees in terms of meteorology angle. So to find the math angle, which is what we need for the calculation here, we need to take 270 degrees. And we subtract 300 degrees from that, and we get alpha equals minus 30, which is 330 degrees if we start from the x-axis and we go counterclockwise all the way around to this direction like this. We've already figured out that the gradient has an angle that's 301 degrees, and that's from the x-axis going all the way around. So that's something like this. And therefore, the difference between the two is 29 degrees. And that's beta. We know that the magnitude of the temperature gradient is 0.12 degrees Fahrenheit. So we multiply the magnitude of the velocity times the magnitude of the temperature gradient times cosine of 29 degrees. We end up getting a value of 0.52 and the minus sign degrees Fahrenheit per hour. So the minus sign is here, because this is positive, positive, and positive. And so the advection is minus 0.52 degrees Fahrenheit per hour. This is cold air advection, or cold advection.

Quiz 8-4: The advection connection.

1. Find **Practice Quiz 8-4** in Canvas. You may complete this practice quiz as many times as you want. It is not graded, but it allows you to check your level of preparedness before taking the graded quiz.

2. When you feel you are ready, take **Quiz 8-4**. You will be allowed to take this quiz only **once**. Good luck!

8.9: Summary and Final Tasks

Once you successfully complete the activities in this lesson, you will be ready to learn about atmospheric kinematics (the description of air movement) and atmospheric dynamics (the study of why air moves the way that it does).

Reminder - Complete all of the Lesson 8 tasks!

You have reached the end of Lesson 8! Double-check that you have completed all of the activities before you begin Lesson 9.

CHAPTER OVERVIEW

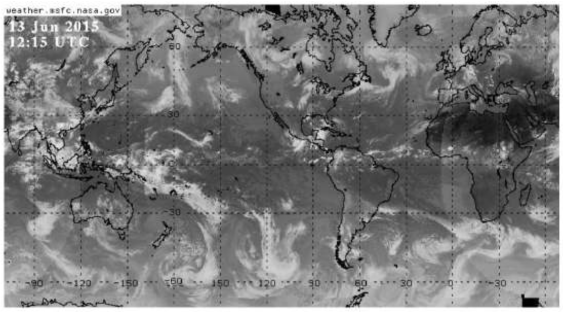
9: Kinematics

Learning Objectives

By the end of this chapter, you should be able to:

- identify regions of convergence, divergence, positive vorticity, and negative vorticity on a weather map
- calculate the strength of the different flow types from observations
- relate vertical motion to horizontal convergence and divergence

The study of kinematics provides a physical and quantitative *description* of our atmospheric motion, while the study of dynamics provides the physical and quantitative *cause-and-effect* for this motion. This lesson discusses kinematics. When we look at weather in motion from a satellite, we see very complicated swirls and stretching that evolve over time. We can see the same types of motions on a much smaller scale by observing swirling leaves. These complex motions can be ascribed to combinations of just five different types of atmospheric motion. Quantifying these motions with mathematics, without assigning a cause to the motion, is the focus of this lesson on kinematics.



Global composite infrared satellite image for 13 June 2015. The air motion is indicated by the infrared radiation from clouds, which were formed from vertical air motion resulting in cooling. Credit: NASA Global Hydrology and Climate Center

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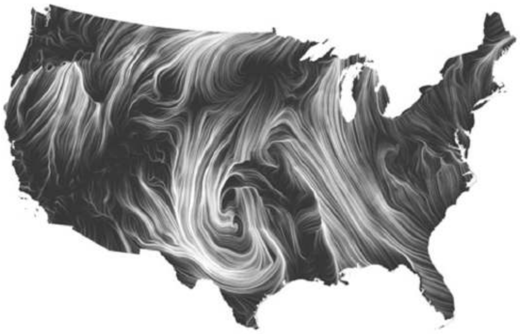
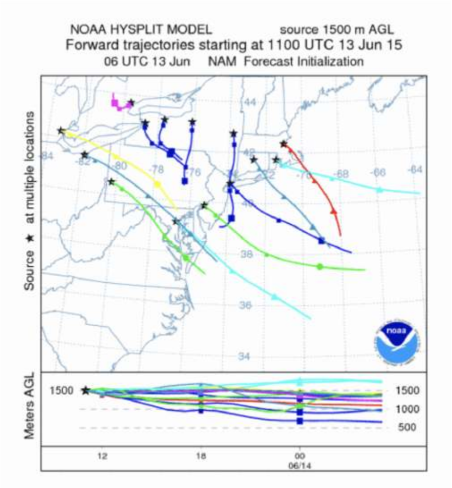
9.1: Streamlines and trajectories aren’t usually the same.

Streamlines are lines that are everywhere parallel to the velocity vectors at a fixed time. They consider the direction of the velocity but not the speed. Sometimes more streamlines are drawn to indicate greater speed, but this is not usually done. Streamlines generally change from one time to the next, giving us “snapshots” of the motion of air parcels. For maps of wind observations for a fixed time, we often look at streamlines. On a map of streamlines, you will see that the lines aren’t always straight and don’t always have the same spacing. **Confluence** is when streamlines come together. **Diffluence** is when they move apart.



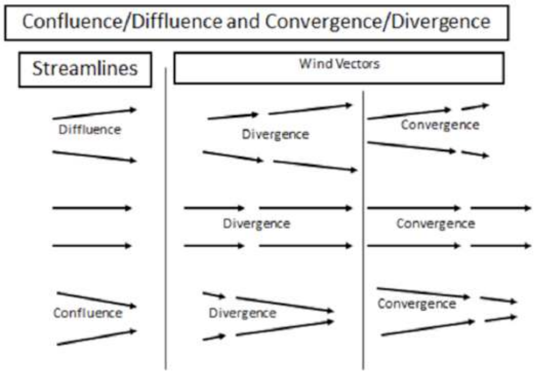
Wind streamlines over the continental United States. You can see many different examples of confluence (streamlines coming together) and diffluence (streamlines moving apart). See the wind map in motion as a series of streamline maps. Credit: Creative Commons via Fernanada Viegas and Marten Wattenberg

Trajectories are the actual paths of the moving air parcels, and indicate both the direction and velocity of air parcels over time. **Convergence** is when the velocity of the air slows down in the direction of the streamline. **Divergence** is when the velocity of the air speeds up in the direction of the streamline. We will talk more about convergence/divergence later, but for now, you should understand that convergence/divergence come from *changes in velocity* while confluence/diffluence come from *changes in spacing between streamlines*.



NOAA HYSPLIT model forecast of wind trajectories for June 13, 2015. The top figure is the horizontal view; the bottom figure shows the vertical motion of the trajectories. The distance between squares on individual trajectories indicates 6 hours of travel time. When the squares on a trajectory get closer together with time, there is convergence. When the squares get further apart with time, there is divergence. Credit: NOAA ARL

Confluence/diffluence and convergence/divergence are illustrated in the figure below:



Examples of confluence/diffluence and convergence/divergence. Credit: H.N. Shirer

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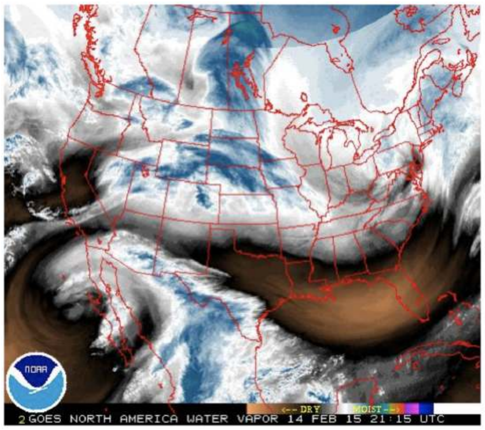
9.2: Watch these air parcels move and change.

The water vapor image from the GOES 13 satellite, above, indicates different air masses over the United States. As we know from Lesson 7, the water vapor image actually shows the top of a column of water vapor that strongly absorbs in the water vapor channel wavelengths, but it is not a bad assumption to think that there is a solid column of moister air underneath the water vapor layer that is emitting and is observed by the satellite. In a single snapshot, it is not possible to see what happens to the air parcels over time. But if we look at a loop, then we can see the air parcels moving and changing shape as they move.



Water vapor in the atmosphere over North America showing the behavior of different air parcels as they interact. Credit: NOAA

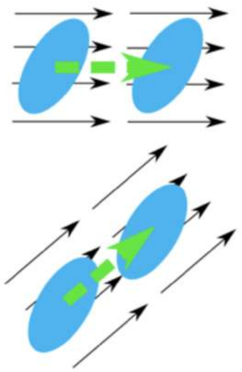
Watch a Loop

Visit this website to see a loop. Pick any air parcel with more water vapor in the first frame and then watch it evolve over time. What does it do? Maybe it moves; it spins; it stretches; it shears; it grows. Maybe it does only a few of these things; maybe it does them all.

We can break each air parcel’s complex behavior down into a few basic types of flows and then mathematically describe them. We will just describe these basic motions here and show how they lead to weather.

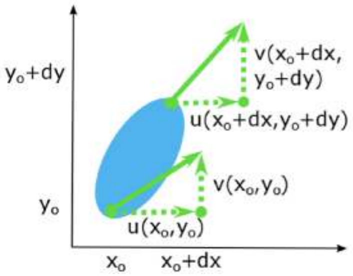
An air parcel Credit: W. Brune

Assume that we have an air parcel as in the figure above. We focus on motion in the two horizontal directions to aid in the visualization (and because most motion in the atmosphere is horizontal) but the concepts apply to the vertical direction as well. If the air parcel is moving and does not change its orientation, shape, or size, then it is only undergoing *translation* (see figure below).



Air parcel undergoing translation in *x*-direction (top) and at 45° (bottom). Black arrows are the wind field; green arrows are the motion of the air parcel. The orientation, shape, and size of the air parcel does not change as it is translated. Credit: W. Brune

The air parcel can do more than just translate. It can undergo changes relative to translation, and its total motion will then be a combination of translation and relative motion. Let's suppose that different parts of the air parcel have slightly different velocities. This situation is depicted in the figure below.



Air parcel with relative motion for two different points in the air parcel separated by *dx* in the *x* direction and *dy* in the *y* direction and with different velocities at each point. Credit: W. Brune, after R. Najjar

If we consider very small differences *dx* and *dy*, then we can write *u* and *v* at point $(x_o + dx, y_o + dy)$ as a **Taylor series expansion** in two dimensions:

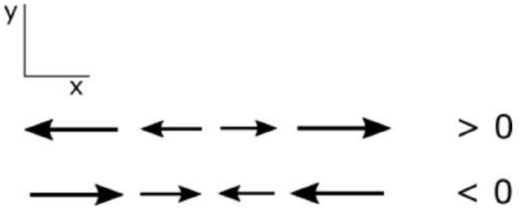
$$u\left(x_o+dx, y_o+dy\right) \approx u\left(x_o, y_o\right)+\frac{\partial u}{\partial x} dx+\frac{\partial u}{\partial y} dy \tag{9.2.1}$$

$$v\left(x_o+dx, y_o+dy\right) \approx v\left(x_o, y_o\right)+\frac{\partial v}{\partial x} dx+\frac{\partial v}{\partial y} dy \tag{9.2.2}$$

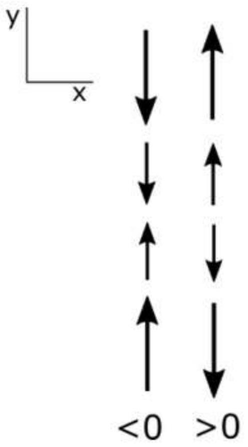
We see that $u(x_o,y_o)$ and $v(x_o,y_o)$ are the translation, and the relative motion is expressed as gradients of *u* in the *x* and *y* directions and gradients of *v* in the *x* and *y* directions.

There are four gradients represented by the four partial derivatives. Each can be either positive or negative for each partial derivative.

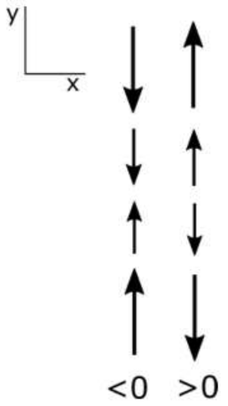
$\frac{\partial u}{\partial x}$ is the following change in velocity in the x direction:



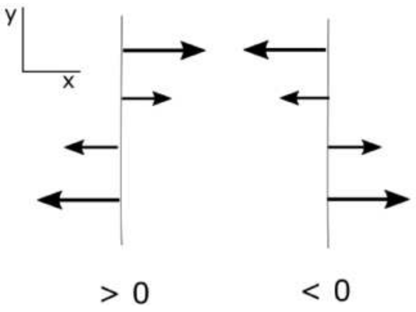
$\frac{\partial v}{\partial y}$ is the following change in velocity in the y direction:



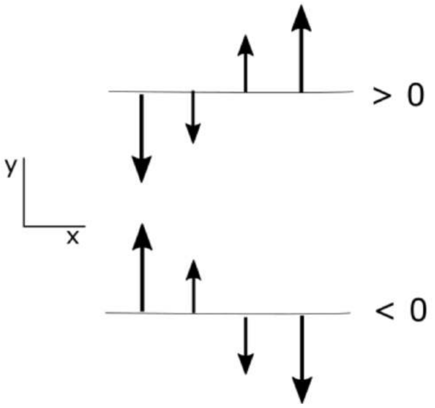
$\frac{\partial u}{\partial v}$ is the following change in velocity in the y direction:



\dfrac{\partial u}{\partial v} \text { is the following change in velocity in the } y \text { direction: }



\dfrac{\partial v}{\partial x} \text { is the following change in velocity in the } x \text { direction: }



Note that a partial derivative is positive if a positive value is becoming more positive or a negative value is becoming less negative. Similarly, a negative partial derivative occurs when a positive value is becoming less positive or a negative value is becoming more negative. Be sure that you have this figured out before you go on.

Watch this video (2:38) for further explanation:

Partials Velocity Distance

Click here for transcript of the Partials Velocity Distance video.

> I want to make sure that you understand the partial derivatives of the u and v velocity with respect to x and y because we will soon be using these terms a lot. Let's start with the partial derivative u with respect to x. Consider a constantly increasing x so that the change in x is positive. As x increases, u becomes initially less negative, hence a positive change; then becomes positive, another positive change; and then becomes more positive, another positive change. Since the change in u and the change in x are both always positive, the partial derivative is positive, greater than 0. Look at the case where a partial derivative is less than 0, or negative. As x increases, u becomes less positive hence, a negative change. Then becomes negative, another negative change, then becomes more negative, another negative change. Since the change in u is always negative with a positive change in x, the partial derivative is always negative. Same logic applies to the partial derivative of v with respect to y. Up is positive for y, so you should look at how v changes as y becomes more positive. Look at the case of the change in u with respect to y. It does not matter that u is in the x direction perpendicular to y because we are interested in how u changes as a function of y. Let's look at what happens as y becomes more positive. On the left, u becomes less negative, a positive change in u, then positive, and more positive. Thus the partial derivative is a positive change in u over a positive change in y and therefore is positive, or greater than 0. The change in u with respect to y is always positive in this case. Using the same logic on the right, we see that the change in u with respect to y is always negative. And because a change in y is positive, the partial derivative is negative, or less than 0. The same logic applies to the partial derivative of v with respect to x. To the right is positive for x. So you can determine how v changes as x becomes more positive to see whether the partial derivative is positive or negative.

9.3: Five Air Motion Types You Must Get to Know

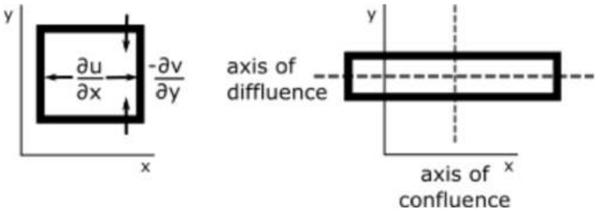
Generally, air velocities change with distance in such a way that more than one partial derivative is different from zero at any time. It turns out that any motion of an air parcel is a combination of five different motions, one being **translation**, which we have already discussed, and four of which can be represented by pairs of partial derivatives of velocity. Of these four, one is a deformation of the air parcel, called **stretching**, which flattens and lengthens the air parcel. A second is another deformation of the air parcel, called **shearing**, which twists the air parcel in both the x and y directions. A third is pure rotation, called **vorticity**. A fourth enlarges or shrinks the parcel without changing its shape, called **divergence**. Let's consider each of five types of air motion alone, even though more than one is often occurring for an air parcel.

Translation simply moves the air parcel without stretching it, shearing it, rotating, or changing its area. There are no partial derivatives of velocities involved with translation.

For the remaining four cases, ***we will provide examples in which the motion (stretching, shearing, vorticity, and divergence) has a positive value***. We could have provided examples in which the motion has negative value, but the conclusions would be the same.

Stretching deformation is represented by $\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}$,

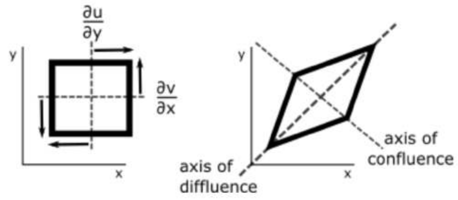
u gets more positive as $x$gets more positive and u gets more negative as x gets more negative (so that the derivative is always positive), making the parcel grow in the x direction. In the other direction, v gets more negative as y gets more positive and v gets more positive as y gets more negative (so that the derivative is always negative), making the parcel shrink in the y direction (see figure below). However, the total area of the air parcel will remain the same if $\partial u/\partial x=\partial v/\partial y$. Shown in the figure is positive stretching deformation; negative stretching deformation occurs when the parcel is stretched in the y direction.



Stretching deformation. The axis of confluence is in the direction that confluence is occurring; the axis of diffluence is in the direction that diffluence is occurring.

Credit: W. Brune

Shearing deformation is represented by $\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}$. In this case, v gets more positive as x gets more positive and v gets more negative as x gets more negative, resulting in the air parcel part at lower x getting pushed towards lower y, and the air parcel part at higher x getting pushed towards higher y. At the same time, u gets more positive as y gets more positive and u gets more negative as y gets more negative, resulting in the air parcel part at lower $y$getting pushed to lower x and the air parcel part at higher y getting pushed to higher x (see figure below). The total area of the air parcel remains the same after the shearing occurs. Shearing deformation is positive when the air parcel stretches in the southwest/northeast direction and contracts in the southeast/northwest direction (as in the figure below). Shearing deformation is negative when the parcel stretches in the southeast/northwest direction and contracts in the southwest/northeast direction.

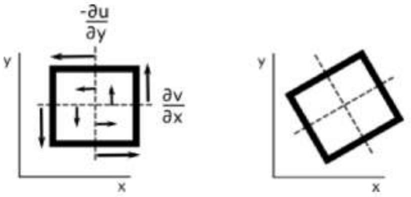


Shearing deformation. The axis of confluence is in the direction that confluence is occurring; the axis of diffluence is in the direction that diffluence is occurring.

Credit: W. Brune

As the two figures above show, both stretching and shearing deformation cause stretching along the axis of diffluence and contraction along the axis of confluence, with the two axes at right angles to each other. These deformations result in **weather fronts**. In both cases, these motions cause some parts of the air parcel to move away from each other and some parts of the air parcel to move towards each other. The air coming together is called **frontogenesis**.

Vorticity is represented by $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial v} \equiv \zeta$. Vorticity is special, and because it is special, it is represented by a Greek lower-case letter, zeta (ζ). In this case, the air parcel does not get distorted if $\partial v/\partial x = -\partial u/\partial y$ and does not change area. It simply rotates (see figure below).



Vorticity. The air parcel simply rotates. In this case, the vorticity is positive and the air parcel is rotating counter-clockwise.

Credit: W. Brune

This difference in partial derivatives may look familiar to you.

$$\vec{U}_H = \vec{i}\,u + \vec{j}\,v \quad \backslash)and\backslash(\quad \vec{\nabla}_H = \vec{i}\,\frac{\partial}{\partial x} + \vec{j}\,\frac{\partial}{\partial y} \tag{9.3.1}$$

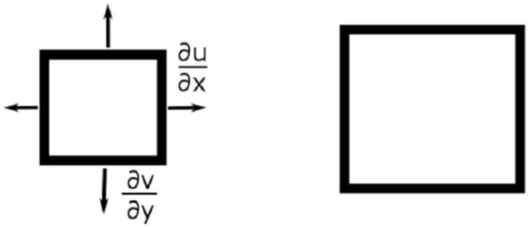
$$\vec{\nabla}_H \times \vec{U}_H = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k} \tag{9.3.2}$$

$$\zeta = \vec{k} \cdot \left(\vec{\nabla}_H \times \vec{U}_H\right) \tag{9.3.3}$$

Vorticity is actually a vector that follows the right-hand rule. Your fingers curve in the direction of the flow and your thumb is the vorticity vector. Here we are discussing only the vertical component of the vorticity. In a right-handed coordinate system, counter-clockwise flow in the *x*-*y* plane will result in your thumb pointing in the positive *z* direction. Hence, vorticity is positive if the rotation is counter-clockwise and is negative if the rotation is clockwise. In the Northern Hemisphere, low-pressure systems are typically characterized by counter-clockwise flow and thus have positive vorticity whereas high-pressure systems are typically characterized by clockwise flow and thus have negative vorticity. The vorticity definition is the same in the Southern Hemisphere (with counter-clockwise flow being positive and clockwise flow being negative), but low-pressure systems usually have clockwise flow and high-pressure systems usually have counterclockwise flow. Vorticity is an important quantity because low- and high-pressure systems are responsible for a lot of weather.

Divergence is represented by $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \equiv \delta$.

Divergence is also special, and because it is special, it is represented by a Greek lower-case letter, delta (δδ). When the divergence is positive, the air parcel grows (i.e., its area increases) (see figure below). If the divergence is negative, then the air parcel shrinks (i.e., its area decreases). Strictly speaking, δδ is the horizontal divergence because it describes a change in parcel area projected onto a horizontal plane. Adding ∂*w*/∂*z* to the horizontal divergence gives the 3-D divergence.



Divergence of an air parcel shown for the case when $\partial u/\partial x$ and $\partial v/\partial y$ are both positive.

Credit: W. Brune

The divergence can be written in vector notation:

$$\vec{U}_H = \vec{i}\,u + \vec{j}\,v \quad and \quad \vec{\nabla}_H = \vec{i}\,\frac{\partial}{\partial x} + \vec{j}\,\frac{\partial}{\partial y} \tag{9.3.4}$$

$$\vec{\nabla}_H \cdot \vec{U}_H = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \tag{9.3.5}$$

Watch this video (1:56) for further explanation:

Five Air Motion Types

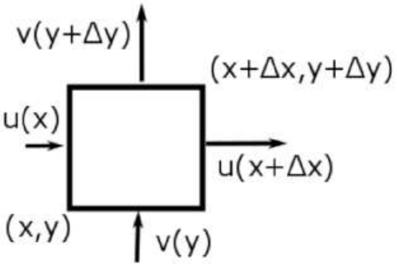
Click here for transcript of the Five Air Motion Types video.

> Airflow can be characterized by a combination of five basic flow types. Translation, which is just a motion of the air parcel, no change in area. Stretching deformation, which increases the parcel in one direction and decreases it in another. Shearing deformation, which shears the air parcel simultaneously in the x and y directions, creating a diamond shape out of a square. Vorticity, which thins the air parcel. And divergence, which grows the air parcel. The last four types can be represented by combinations of the partial derivatives of horizontal velocities, u and v, with respect to horizontal directions, x and y. Note that if we know the wind velocity vectors in the [INAUDIBLE] grid then we can calculate these five wind types for an airflow by determining the changes in the velocities as functions of x and y. And then combining these differentials that are shown here to find the actual values for stretching deformation, shearing deformation, vorticity, and divergence. The units for all of these motion types is per second, which is a frequency. In the figures, I have shown only those transformations that are positive. Negative translation goes to the left. Negative stretching deformation elongates the parcel in the y direction. Negative sharing deformation elongates the parcel in the northwest-southeast direction. Negative vorticity is clockwise. Negative divergence causes the air parcel to shrink, which is called convergence. Prove it to yourself that these transformations shown here are all possible. We will use the divergence heavily in the next section to the lesson.

9.4: How does divergence relate to the air parcel’s area change?

We see that divergence is positive when the parcel area grows and is negative when it shrinks. We call growth “divergence” and shrinking “convergence.” We wish to know whether air parcels come together (converge) or spread apart (diverge) or if the parcel area increases with time (divergence) or decreases with time (convergence).

Let’s see how divergence in the horizontal two dimensions is related to area change. We can do a similar analysis that relates divergence in three dimensions to a volume change, but we will stay with the two-dimensional case because it is easier to visualize and also has important applications. Consider a box with dimensions Δx and Δy. Different parts of the box are moving at different velocities (see figure below).



A box that is moving at greater velocity for parts that are at greater *x* and greater *y*.

Credit: W. Brune

The box's area, *A*, is given by:

$A = \Delta x \Delta y$

$$\frac{dA}{dt} = \frac{d(\Delta x \Delta y)}{dt} = \Delta x \frac{d(\Delta y)}{dt} + \Delta y \frac{d(\Delta x)}{dt} = \Delta x[v(y+\Delta y) - v(y)] + \Delta y[u(x+\Delta x) - u(x)] \tag{9.4.1}$$

divide by $A = \Delta x \Delta y$

$$\frac{1}{A}\frac{dA}{dt} = \frac{v(y+\Delta y) - v(y)}{\Delta y} + \frac{u(x+\Delta x) - u(x)}{\Delta x} \tag{9.4.2}$$

Let $\Delta y \to 0, \Delta x \to 0$

$$\frac{1}{A}\frac{dA}{dt} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial v} = \vec{\nabla}_H \bullet \vec{U}_H \tag{9.4.3}$$

So we see that the fractional change in the area is equal to the horizontal divergence. Note that the dimension of divergence is time^{-1} and the SI unit is s^{-1}.

We can do this same analysis for motion in three dimensions to get the equation:

$$\frac{1}{V}\frac{dV}{dt} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \vec{\nabla}_H \cdot \vec{U}_H + \frac{\partial w}{\partial z} = \vec{\nabla} \cdot \vec{U} \tag{9.4.4}$$

where *V* is the parcel volume. Thus, the 3-D divergence is just the fractional rate of change of an air parcel’s volume.

? Exercise

Suppose that an air parcel has an area of 10,000 km^2 and it is growing by 1 km^2 each second. What is its divergence?

Click for answer.

$\frac{\Delta A}{\Delta t} = 1 \text{km}^2 \text{s}^{-}$, so

$\left(\frac{1}{A}\right)\left(\frac{\Delta A}{\Delta t}\right) = \left(\frac{1}{10^4}\text{km}^2\right)\left(1\text{km}^2\text{s}^{-1}\right) = 10^{-4}\text{s}^{-1}$

Suppose that an air parcel has a area of 10,000 km^2 and has a divergence of -10^{-4} s^{-1}. Is the air parcel growing or shrinking?

Click for answer.

divergence $= \delta = \left(\frac{1}{A}\right)\left(\frac{\Delta A}{At}\right)$, or

$\frac{\Delta A}{A} = \delta \Delta t = \left(-10^{-4} \mathrm{s}^{-1}\right)(1 \mathrm{s}) = -10^{-4}$. The air parcel is shrinking.

Check out this video (1:33) for further explanation:

Divergence Area

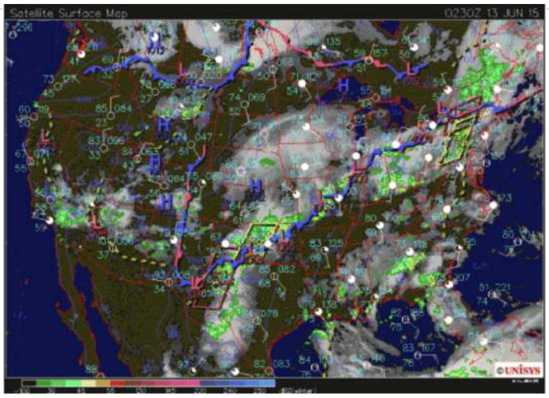
Click here for transcript of the Divergence Area video.

We can use a very simple demonstration to show how differences in velocity from one end of an air parcel to the other can cause changes in area. Let's let this be our air parcel here, outlined in the dark blue. Several things can happen to this air parcel. One, it can translate. So it can just move with a certain velocity across from left to right. The second thing it can do is it can have a zero velocity here and have a higher velocity here on this end. And it can then grow. And so you see that the area is increasing as time goes on. So we can combine these two motions and see what happens. And so we have some velocity at the parcel, but we have a greater velocity on the right hand side. And we see that as it moves, it grows. It's also possible that as it moves, it shrinks because the velocity on this side is less than the velocity on this side. Then as it moves along, you will see that actually the area's contracting. We can do the same sort of analysis in the y direction. And from this, we can show that in fact, the difference in velocity from here to here can result in the growth or the shrinking of the area of the parcel.

Quiz 9-1: The way the wind blows.

1. Find **Practice Quiz 9-1** in Canvas. You may complete this practice quiz as many times as you want. It is not graded, but it allows you to check your level of preparedness before taking the graded quiz.
2. When you feel you are ready, take **Quiz 9-1**. You will be allowed to take this quiz only **once**. Good luck!

9.5: How is the horizontal divergence/convergence related to vertical motion?



Composite of satellite infrared image overlaid with surface analysis and radar. Note that the precipitation is generally associated with the low-pressure regions and the high-pressure areas are generally free of significant clouds. As we will soon see, low-level air converges into low-pressure regions, causing uplift, which causes clouds and precipitation. Low-level air diverges from high-pressure regions, causing downwelling air and warming and drying by adiabatic descent. Take a look at the latest composite image.

Credit: Unisys

Our goal here is to relate horizontal convergence and divergence to vertical motion. If vertical motion is upward, then the uplifted air will cool, clouds will form, and it might rain or snow. If vertical motion is downward, then the downwelling air will warm by adiabatic descent, clouds will evaporate, and it will become clear.

To find out what will happen, we need to go back to a fundamental law of mass conservation, which we will derive in detail in Lesson 10. Here we simply quote the result:

$$\frac{1}{\rho}\frac{D\rho}{Dt} + \vec{\nabla} \bullet \vec{U} = 0 \tag{9.5.1}$$

where ρρ is the density and *D/Dt* is the total derivative.

For divergence, $\vec{\nabla} \bullet \vec{U} > 0$, volume decreases and density must increase to conserve mass.

However, to good approximation, density does not change with time for any given horizontal surface. Sure, density decreases exponentially with height, but for each height level, the density at that level is fairly constant.

So, to a good approximation:

$$\vec{\nabla} \bullet \vec{U} = 0 \tag{9.5.2}$$

and because we can separate out the horizontal and vertical components of divergence:

$$\vec{\nabla} \bullet \vec{U} = \vec{\nabla}_H \bullet \vec{U}_H + \frac{\partial w}{\partial z} \tag{9.5.3}$$

we see that:

$$\vec{\nabla}_H \bullet \vec{U}_H + \frac{\partial w}{\partial z} = 0, \quad or \quad \frac{\partial w}{\partial z} = -\vec{\nabla}_H \bullet \vec{U}_H \tag{9.5.4}$$

Thus, horizontal divergence is compensated by vertical convergence and horizontal convergence is compensated by vertical divergence.

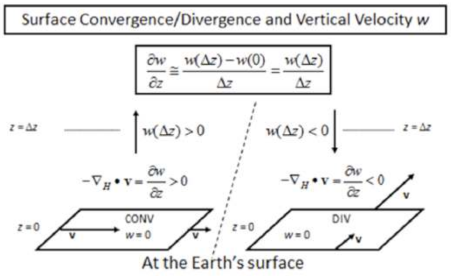
$$\vec{\nabla}_H \bullet \vec{U}_H > 0 \quad means \quad \frac{\partial w}{\partial z} < 0 \tag{9.5.5}$$

Horizontal divergence gives a *decrease* in vertical velocity with height.

$$\vec{\nabla}_H \bullet \vec{U}_H < 0 \quad means \quad \frac{\partial w}{\partial z} > 0 \tag{9.5.6}$$

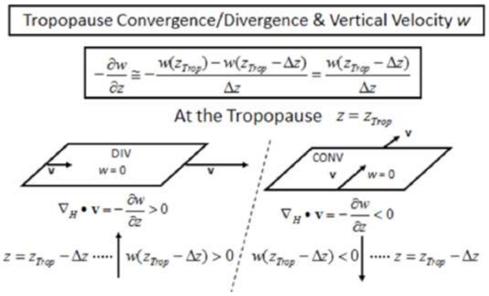
Horizontal convergence gives an *increase* in vertical velocity with height.

Now, in the troposphere, the vertical velocity is close to zero ($w \sim 0$) at two altitudes. The first is Earth's surface, which forms a solid boundary that stops the vertical wind. The second is the tropopause, above which the rapid increase in stratospheric potential temperature strongly inhibits vertical motion from the troposphere (see two figures below), so much so, that we can say that the vertical wind must be ~ 0 at the tropopause.



Surface convergence or divergence and the resulting vertical velocity (note: **v** used in this figure is the same as $\mathbf{U}_H$ in the text.).

Credit: H.N. Shirer



Tropopause convergence or divergence and the resulting vertical velocity (note: **v** used in this figure is the same as $\mathbf{U}_H$ in the text.).

Credit: H.N. Shirer

These processes can be summarized in the following table:

plane	**process**	**surface area change**	***∂w/∂z***	***w***
surface	convergence	decrease	+	up
surface	divergence	increase	–	down
aloft	convergence	decrease	+	down
aloft	divergence	increase	–	up

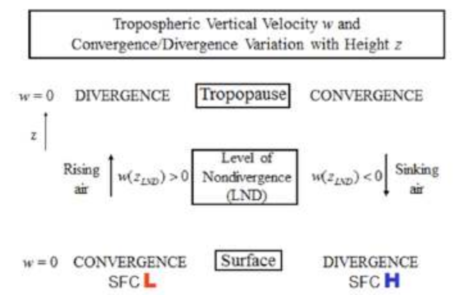
Let's now consider the effect that divergence/convergence aloft has on surface convergence/divergence (see figure below).

Divergence aloft is associated with rising air throughout the troposphere, which is associated with low pressure and convergence at the surface.

Convergence aloft is associated with sinking air throughout the troposphere, which is associated with high pressure at the surface and thus divergence at the surface.

So, starting at the surface, the vertical velocity becomes more positive with height when there is surface convergence, reaches some maximum vertical velocity, and then becomes less positive with height again toward the divergence aloft.

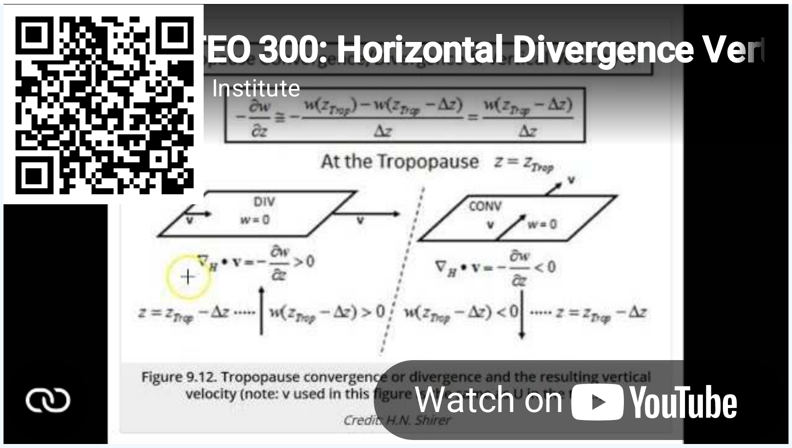
Similarly, starting again at the surface, the vertical velocity becomes more negative with height when there is surface divergence, reaches some maximum negative velocity, and then becomes less negative with height again near convergence aloft.



How divergence aloft connects to surface low pressure and convergence and how convergence aloft connects to surface high pressure and divergence.

Credit: H.N. Shirer

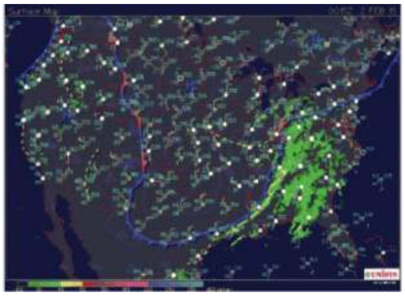
Now watch this video (3:52) on horizontal divergence:



Horizontal Divergence Vertical Motion

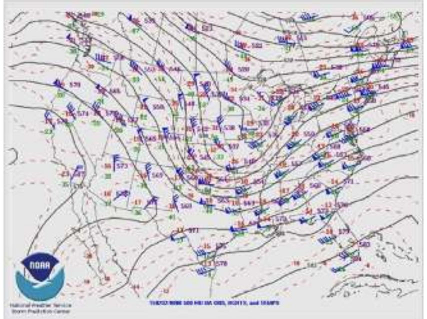
Click here for transcript of the Horizontal Divergence Vertical Motion video.

The key concepts that allow horizontal divergence to be converted into vertical motion are that mass is conserved, but the air density and density vertical structure are fairly constant with time, and that the vertical wind at Earth's surface and at the tropopause is effectively 0. This means that the total divergence must be approximately 0 so that the air parcel volume remains constant. Thus, changes in the horizontal area cause changes in the vertical height to maintain the air quality. A key to remember is that the vertical velocity w is partial derivative with respect to height z do not always have the same sign. The second key point to remember is that the partial derivative of w with respect to z is a negative divergence. We would look first at diverging mirror surface. But if there is horizontal convergence, then the air must go somewhere, and it cannot go down, so it goes up. The equation actually says that the partial derivative of w, the vertical velocity with respect to z, must be positive. But if w equals 0 at Earth's surface and w is increasingly with altitude, than w must be positive. For divergence near Earth's surface, we see that the partial derivative of w with respect to z is negative, which means that w must be negative above the surface since w equals 0 at earth's surface. So the air velocity w must be downward. But the tropopause, the rapid increase in stress for potential temperature acts like a lid on the troposphere. Effectively makes w go to 0 at the tropopause. There is a horizontal divergence aloft then w must be upward to maintain the air parcel volume as the air parcel spreads out horizontally near the tropopause. Mathematically, this means that w must be positive. But we know that it must go to 0 tropopause. Therefore, the partial of w with respect to z must be negative as it approaches the tropopause, i.e. w is decreasing with increasing height to 0 at the tropopause from a positive value in the troposphere. On the other hand, if there is convergence in the air near the tropopause then, the air must go down and the vertical velocity w must be negative. If we look at the changes in w with respect to height above the level of non-divergence, as z increases, w goes from more negative to less negative, which is a positive change in w, with a positive change in z. So the partial derivative is positive even though w is negative. Putting these pieces together, we see that if we have convergence at Earth's surface, which occurs in low pressure areas for reasons we will see in lesson 10, then at the tropopause, there's divergence. In between the two surfaces the velocity is upward-- that is, w is positive. If we have a divergence near Earth's surface, which occurs in high pressure areas, then there is convergence at the tropopause. In between, the vertical velocity is downward. That is, w is negative. The outward moving air above low pressure creates cooling, which leads to clouds and precipitation. The downward moving air above high pressure region causes warming and drying, resulting in clear conditions.



Surface map of weather for February 2, 2015. Fronts, lows, and highs are indicated. Radar shows where precipitation is falling.

Credit: Unisys



500 mb weather map for February 2, 2015. Black lines are surface heights and wind bars show the direction and magnitude of wind.

Credit: NOAA.

We have shown that convergence and divergence aloft near the tropopause is related to surface highs and lows. Now it's your turn to find some examples. Go to a source of information about surface pressure and upper-air winds and pick out some regions that show this relationship. One good source is the Penn State e-Wall, for which you can use the "U.S. Satellite Overlays."

9.6: How fast is the vertical wind and which way does it blow?

What are typical values of the vertical velocity caused by convergence or divergence and how do they vary with height? The vertical velocity, *w*, is typically too small to measure by a radiosonde. But we can estimate *w* from the convergence/divergence patterns:

$$\frac{\partial w}{\partial z}=-\vec{\nabla}_H \bullet \vec{U}_H=-\delta \tag{9.6.1}$$

Note that this equation just gives the derivative of the vertical velocity, not the vertical velocity itself. So to find the vertical velocity, we must integrate both sides of the equation over height, *z*.

Integrate this equation from the surface (*z* = 0) to some height *z*:

$$\int_0^z \frac{\partial w}{\partial z'} dz'=-\int_0^z \delta dz' \tag{9.6.2}$$

$$w(z)=-\int_0^z \delta dz' \tag{9.6.3}$$

where we have assumed that *w*(0) is equal to zero, which is true if the surface is horizontal. Equation [9.6] gives the **kinematic vertical velocity**.

To a good approximation, it has been determined that divergence/convergence for horizontal flow at large scales (e.g., synoptic scales, ~1000 km) varies linearly with altitude.

$$\delta=\delta_s+bz \tag{9.6.4}$$

where δ_s is the surface divergence and *b* is a constant. Substituting this expression for the horizontal divergence into Equation [9.6], we get:

$$w(z)=-\int_0^z \delta dz'=-\delta_s z-\frac{1}{2} b z^2 \tag{9.6.5}$$

The trick is to find *b* using some other information. To find *b*, note that $\frac{\partial w}{\partial z}$ must be 0 at both Earth's surface and the tropopause, so the derivative is positive near Earth's surface. It must become negative at c tropopause in order for *w* to go to zero, and so somewhere in between, being positive and negative, it must be zero. We call this level the **level of nondivergence**, $z_{LND,}$

$$0=-\delta_s-b z_{LND}, \quad or \quad b=-\frac{\delta_s}{z_{LND}} \tag{9.6.6}$$

$$w(z)=-\delta_s z+\frac{\delta_s}{2 z_{LND}} z^2 \tag{9.6.7}$$

The large-scale surface divergence typically has a value of 10^{-5} s^{-1}. The large-scale level of nondivergence is typically about 5000 m. So,

$$b=-\frac{\delta_s}{z_{LND}}=-\frac{10^{-5} \mathrm{s}^{-1}}{5000 \mathrm{m}}=-2 \times 10^{-9} \mathrm{m}^{-1} \mathrm{s}^{-1} \tag{9.6.8}$$

So, for typical large-scale surface divergence:

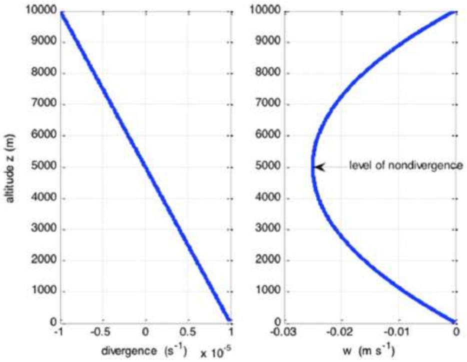
$$w(z)=\left(-10^{-5} \mathrm{s}^{-1}\right) z+\left(10^{-9} \mathrm{m}^{-1} \mathrm{s}^{-1}\right) z^2 \tag{9.6.9}$$

At

$$\begin{gathered} z=z_{LND}=5000 \mathrm{m}, \\ w\left(z_{LND}\right)=-2.5 \mathrm{cms}^{-1}=\left(-2.5 \times 10^{-5} \mathrm{kms}^{-1}\right)\left(86400 \mathrm{sday}^{-1}\right)=-2.2 \mathrm{km} \backslash) day \backslash(^{-1} \end{gathered} \tag{9.6.10}$$

(see figure below).

The result is that *w* is only a few cm s^{-1}. In a day, the air mass can rise or fall only a few kilometers. Compare this vertical motion dictated by large-scale convergence and divergence to the vertical motion in the core of a powerful thunderstorm (horizontal scale of a few km), where the vertical velocities can be many m s^{-1}. This simple model is called the ***bowstring model*** because the shape of the vertical velocity looks like a bowstring that is fixed at two points but can vary as a parabola in between.



Divergence (left) and vertical wind (right). For convergence aloft (negative divergence), the vertical wind is negative with a maximum value near 5000 m and remains negative as it decreases toward zero at the surface, where there is divergence (positive divergence) near the surface.

Credit: W. Brune

Quiz 9-2: Connecting the dots with vertical motion.

1. Find **Practice Quiz 9-2** in Canvas. You may complete this practice quiz as many times as you want. It is not graded, but it allows you to check your level of preparedness before taking the graded quiz.
2. When you feel you are ready, take **Quiz 9-2**. You will be allowed to take this quiz only **once**. Good luck!

9.7: Summary and Final Tasks

Kinematics describes the behavior of atmospheric motion but not the cause. Streamlines provide snapshots of that motion and trajectories show where individual air parcels actually go. All atmospheric motion in the horizontal is made up of one or more of five distinct types of motion: translation, stretching deformation, shearing deformation, vorticity, and divergence. Stretching and shearing deformation lead to the formation or the dissolution of surface weather fronts. Vorticity describes the counter-clockwise rotation around low pressure (in the Northern Hemisphere) and clockwise rotation around high pressure (in the Northern Hemisphere) and is thus associated with much of weather. Divergence/convergence aloft leads to vertical winds that connect to convergence/divergence at the surface, and through this mechanism, air motion aloft communicates with air motion at the surface.

This lesson showed the mathematics necessary to quantify all of these processes. So besides identifying streamline confluence/diffluence, you practiced quantifying the five flow types from weather maps of streamlines with wind vectors. Finally, you calculated the vertical wind and its direction (up or down) based on the divergence/convergence of the winds aloft.

CHAPTER OVERVIEW

10: Dynamics - Forces

Learning Objectives

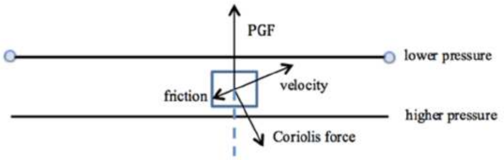
By the end of this chapter, you should be able to:

- explain mass conservation physically, recognize the mass conservation equation, and memorize its form when density is constant
- state the three main conservation laws in atmospheric science: the conservation of mass, the conservation of momentum, and the conservation of energy
- name and explain the three fundamental (real) forces in the atmosphere (gravity, pressure gradient, and friction)
- name and explain the two new (apparent) forces that emerge when momentum conservation is written in the rotating reference frame
- draw the balance of forces for geostrophic flow, gradient flow, geostrophic flow with friction, and cyclostrophic flow
- explain why midlatitude winds are westerly

10.1: What does turbulent drag do to horizontal boundary layer flow?

The turbulent drag force on the horizontal air flow within the atmospheric boundary layer can approach the size of the other terms in the horizontal equation of motion. Note that this turbulent drag force acts to reduce the air velocity and therefore is opposite the air flow velocity vector.

Let’s look at the force balance when the turbulent drag force is included. We will call this turbulent drag "friction" because it is commonly called that, but it is really quite different.



Horizontal map showing the geostrophic wind with friction in the Northern Hemisphere.

Credit: W. Brune

Note that the turbulent drag (friction) force is parallel to the velocity vector and is opposite in direction.

In the *x*-direction (along the isobars) the balance of forces is:

friction (*x*-component) = Coriolis (*x*-component).

In the *y*-direction (perpendicular to the isobars), the balance of opposing forces is:

friction (*y*-component) + Coriolis (*y*-component) = PGF

Because of the turbulent drag force and the velocity dependence of the Coriolis force, the parcel velocity points toward the lower pressure and the air parcel will tend to move across isobars toward the low pressure. Typical cross-isobaric flow in the boundary layer makes an angle of 30° with the isobars. Thus, surface air moves toward low pressure and away from high pressure.

Above the atmospheric boundary layer, however, turbulent drag is not generally important and geostrophic and gradient flows are good approximations.

The effects of turbulent drag are very important for weather. When divergence aloft causes upward-moving air below, it becomes associated with a low-pressure region near Earth's surface. A pressure gradient force is created, but the air moving toward the low pressure is turned to travel counterclockwise around the low by the Coriolis force. However, turbulent drag slows the wind and turns it to cross isobars toward the low pressure, creating convergence, which causes uplift, clouds, and perhaps precipitation. The opposite is true for surface high-pressure regions occurring under regions of convergence aloft, which causes descent. The anticyclonic winds around the surface high are slowed and turn outward, causing divergence near the surface, leading to descending air and clear skies. The following video (1:43) provides further discussion of friction:

Friction

Click here for transcript of the Friction video

Click here for transcript of the Friction video.

When the flow is in the upper part of the atmospheric boundary layer, which is a kilometer or two above the surface, the turbulence in the boundary layer acts to impede the horizontal wind. This resistance to flow is not really friction. But it does act to slow the wind down, no matter the wind's direction. As a result, we can assume that this turbulent drag is a force that opposes the wind velocity. Look at what adding a turbulent resistant turn does to the force balance for straight line flow in the upper boundary layer. The PGF force is perpendicular to the pressure gradient as usual. However, the friction opposes the velocity, and slows it down. At the same time, the Coriolis force is always perpendicular to the velocity and to the right. And because the velocity is slowed down, the Coriolis force is less. The velocity vector gets turned toward the PGF vector, and thus toward the low pressure. Note that the x direction of the frictional force must balance and be opposite to the x direction of the Coriolis force. And the y directions of the friction and Coriolis forces must be opposite to and balance the PGF in the y direction in the diagram. Boundary layer turbulent drag turns to velocity across isobars for low pressure, which causes convergence, while boundary layer turbulent drag turns to velocity across isobars away from high pressure, which causes divergence. These friction effects tend to amplify the convergence into surface low pressure and divergence from surface high pressure.

✓ Quiz 10-3: Balance of forces and motion.

1. Find **Practice Quiz 10-3** in Canvas. You may complete this practice quiz as many times as you want. It is not graded, but it allows you to check your level of preparedness before taking the graded quiz.
2. When you feel you are ready, take **Quiz 10-3**. You will be allowed to take this quiz only **once**. Good luck!

10.2: Why are midlatitude winds mostly westerly (i.e., eastward)?

The answer seems simple. More solar energy is deposited in the tropics than near the poles and as a result, the air is warmer in the tropics than the poles, where there is net radiative cooling. According to the hydrostatic equilibrium equation, the fall-off in pressure with altitude is less in the tropics than at higher latitudes, and as a result, for any height surface, the pressure is greater in the tropics than near the poles, setting up a pressure gradient force on each height surface that drives the wind poleward. As the air moves poleward, atmospheric processes such as the Coriolis force, which we saw occurs because of conservation of angular momentum, and synoptic-scale disturbances at higher latitudes cause the air to veer to the right in the Northern Hemisphere and to the left in the Southern Hemisphere.

Thermal Wind

With this broad concept in mind, we can consider the idea of the thermal wind, which is not really a wind, but instead is a difference in winds at two heights. We will see that the thermal wind is proportional to the horizontal temperature gradient. To show this relationship mathematically, we start with the geostrophic balance equation and apply the Ideal Gas Law and the hydrostatic equilibrium equation.

Look at the *x* and *y* components of Equation [10.33] for geostrophic winds:

$$v_g = \frac{1}{f}\frac{\partial \Phi}{\partial x} \tag{10.2.1}$$

$$u_g = -\frac{1}{f}\frac{\partial \Phi}{\partial y} \tag{10.2.2}$$

Use the hydrostatic equation and the Ideal Gas Law to relate T to Φ:

$$\frac{\partial p}{\partial z} = -\rho g \quad \rightarrow \quad \frac{\partial p}{g\partial z} = -\rho \quad \rightarrow \quad \frac{\partial p}{\partial \Phi} = -\rho \quad \rightarrow \quad \frac{\partial \Phi}{\partial p} = -\frac{1}{\rho} = -\frac{RT}{p}$$

Take $\frac{\partial}{\partial p}$ of equations in [10.41], starting with the equation for v_g:

$$\frac{\partial v_g}{\partial p} = \frac{1}{f}\frac{\partial}{\partial p}\left(\frac{\partial \Phi}{\partial x}\right) = \frac{1}{f}\frac{\partial}{\partial x}\left(\frac{\partial \Phi}{\partial p}\right) = \frac{1}{f}\frac{\partial}{\partial x}\left(-\frac{RT}{p}\right) \tag{10.2.3}$$

$$p\frac{\partial v_g}{\partial p} = -\frac{R}{f}\frac{\partial T}{\partial x} \quad and \quad p\frac{\partial v_g}{\partial p} = \frac{\partial v_g}{\partial \ln p} \tag{10.2.4}$$

Taking the same approach with the equation for ugug gives the **Thermal Wind Equations**:

$$\frac{\partial v_g}{\partial \ln p} = -\frac{R}{f}\left(\frac{\partial T}{\partial x}\right)_p \tag{10.2.5}$$

and

$$\frac{\partial u_g}{\partial \ln p} = \frac{R}{f}\left(\frac{\partial T}{\partial y}\right)_p \tag{10.2.6}$$

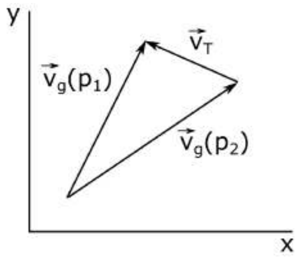
$$\frac{\partial \vec{V}_g}{\partial \ln p} = -\frac{R}{f}\vec{k} \times \vec{\nabla}_p T \tag{10.2.7}$$

in vector form

These equations are very powerful because they reveal that measurements of temperature only (which are relatively easy to make) make it possible to determine the changes in the horizontal wind with height, assuming that the geostrophic and hydrostatic approximations are valid (which is the case for large-scale flow in the free atmosphere). The thermal wind velocity is defined as the change in horizontal geostrophic velocity between two layers.

$$\vec{V}_T \equiv \vec{V}_g(p_1) - \vec{V}_g(p_2), \quad p_1 < p_2 \tag{10.2.8}$$

So the thermal wind velocity equals the horizontal velocity at the upper level minus the horizontal velocity at the lower level. Equation [10.43] can be drawn as a two-dimensional vector subtraction as if we were looking down on them from above (see figure below).



The thermal wind is the geostrophic wind vector at pressure level (p_1) higher in the atmosphere minus the geostrophic wind vector at a pressure level (p_2) lower in the atmosphere, where $p_1 < p_2$. Credit: W. Brune

We can integrate Equation [10.42] and if we let <*T*> be the average temperature between pressure surfaces p_1 and p_2 (where $p_1 < p_2$), yielding the expressions for the thermal wind vectors in the *x* and *y* directions:

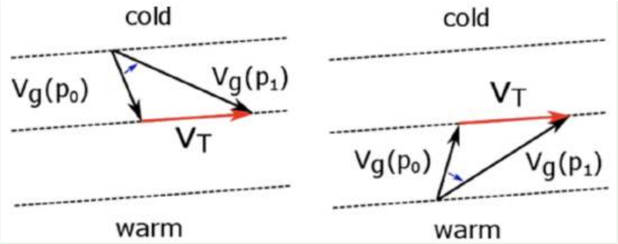
$$\vec{V}_T = u_T\vec{i} + v_T\vec{j} = \frac{R}{f}\left[\vec{j}\left(\frac{\partial\langle T\rangle}{\partial x}\right)_p - \vec{i}\left(\frac{\partial\langle T\rangle}{\partial y}\right)_p\right]\ln\left(\frac{p_2}{p_1}\right) \tag{10.2.9}$$

$$\vec{V}_T = \frac{R}{f}\ln\left(\frac{p_2}{p_1}\right)\vec{k}\times\vec{\nabla}_p\langle T\rangle \tag{10.2.10}$$

$$u_T = -\frac{R}{f}\left(\frac{\partial\langle T\rangle}{\partial y}\right)_p \ln\left(\frac{p_2}{p_1}\right) \tag{10.2.11}$$

$$v_T = \frac{R}{f}\left(\frac{\partial\langle T\rangle}{\partial x}\right)_p \ln\left(\frac{p_2}{p_1}\right) \tag{10.2.12}$$

The vertical change in geostrophic wind is called the **geostrophic vertical shear**. Since the geostrophic vertical shear is directly proportional to the horizontal temperature gradient, it is also called the **Thermal Wind.**



Thermal wind is the vector difference of the geostrophic wind at the upper-level pressure (p_1) minus the lower-level pressure (p_0, which should be written as p_2 to be consistent with the previous figure and the text). These figures show the view looking down on the *x*-*y* plane. Cold air is on the left of V_T. Left: Backing (flow turning counterclockwise with height) indicates cold advection. Right: Veering (flow turning clockwise with height) indicates warm air advection. Credit: W. Brune

We can learn a lot from the thermal wind (seen in the figure above).

- Thermal wind blows with cold air to the left (on average) and with the warm air on the right in the Northern Hemisphere (remember “air is light on the right”), and with warm air on the left in the Southern Hemisphere.
- If the geostrophic wind vector rotates counterclockwise with height (called *backing)*, then there is cold air advection. For the Northern Hemisphere, remember “CCC: counterclockwise cold.”
- If the geostrophic wind vector rotates clockwise with height (called *veering)*, then there is warm air advection.
- In the Northern Hemisphere, because low thickness means lower layer temperature and higher thickness means higher layer temperature, the thermal wind blows parallel to the lines of constant thickness with the low thickness on the left.
- This statement is an analogy to “The geostrophic wind blows parallel to the height contours on a pressure surface (pressure contours on a height surface) with the low height (pressure) to the left."

Quiz 10-4: Feeling the thermal wind.

1. Find **Practice Quiz 10-4** in Canvas. You may complete this practice quiz as many times as you want. It is not graded, but it allows you to check your level of preparedness before taking the graded quiz.
2. When you feel you are ready, take **Quiz 10-4**. You will be allowed to take this quiz only **once**. Good luck!

10.3: Why We Like Conservation

Scientists like things that are conserved. There are good reasons for this. First, if something is conserved, that means we can always count on it being the same no matter what happens. Second, when we write down the equation for the conserved quantity, we can use that equation to understand how the equation's variables will change with differing conditions. For example, in Lesson 2, we were able to use the First Law of Thermodynamics (a.k.a., conservation of energy) along with the Ideal Gas Law to derive the equation for potential temperature, which is very useful for understanding and calculating the vertical motion of air parcels.

In atmospheric dynamics, we like three conservation laws:

1. **conservation of energy** (The 1st Law of Thermodynamics)
2. **conservation of mass**
3. **conservation of momentum** (Newton's Second Law, but really three equations—one in each direction)

Conservation of Mass

So, let's step back and look at the mass of an air parcel, which equals the density times the volume of the parcel:

$$m=\rho V \tag{10.3.1}$$

In a parcel, the mass is conserved, and since $m = \rho V$,

$$\frac{D}{Dt}(m)=\frac{D}{Dt}(\rho V)=0 \tag{10.3.2}$$

Apply the product rule to Equation 10.3.1:

$$V\frac{D\rho}{Dt}+\rho\frac{DV}{Dt}=0 \tag{10.3.3}$$

Divide both sides by ρV:

$$\frac{1}{\rho}\frac{D\rho}{Dt}+\frac{1}{V}\frac{DV}{dt}=0 \tag{10.3.4}$$

Recall that the specific rate of change in parcel volume is equal to the divergence (Equation 9.4) and so we can write:

$$\frac{1}{V}\frac{DV}{dt}=\vec{\nabla}\bullet\vec{U} \tag{10.3.5}$$

Rearranging the equation gives us an expression for the conservation of mass:

$$\frac{1}{\rho}\frac{D\rho}{Dt}+\vec{\nabla}\cdot\vec{U}=0 \tag{10.3.6}$$

This equation is for the conservation of mass in a continuous fluid (i.e., the fluid particles are so small that the air parcel behaves like a fluid). It is also called the **Equation of Continuity**. Physically, this equation means that if the flow is converging ($\vec{\nabla}\bullet\vec{U}<0$), then the density must increase $\left(\frac{D\rho}{Dt}>0\right)$. Note that in Lesson 9.5 we said that density doesn't change much at any fixed pressure level, which is how we were able to relate horizontal divergence/convergence with vertical ascent/descent. What *did* change was the vertical size of the air parcel as the horizontal size increased or decreased. The total mass, however, remained the same.

"Isaac Newton woodcut, frontispiece to Mach" by an unknown artist after a portrait by Kneller, ca. 1689**.** (Public Domain via Wikimedia Commons).

Conservation of Momentum

Newton's 2nd Law, **F** = *m***a**, applies to a mass with respect to the inertial coordinate system of space. But we are interested in motion with respect to the rotating Earth. So, to apply Newton's 2nd Law to Earth's atmosphere, our mathematics will need to account for the forces of Earth's rotating coordinate system:

$$\vec{F} = m\vec{a} = \sum_i (\text{ real forces }) + \sum_{(\text{apparent forces })} = m\frac{d\vec{U}}{dt} \tag{10.3.7}$$

where the first set of forces are real forces and the second set are apparent (or effective) forces that will be used to correct for using a coordinate system attached to a rotating Earth.

When we use the word "specific" as an adjective describing a noun in science, we mean that noun divided by mass. So, specific force is **F**/m = **a**, acceleration. In what follows, we will use the terms "force" and "acceleration" interchangeably, assuming that if we say "force," we mean "force/mass," which is acceleration. At this point, you should be able to check the units—if there is no "kg," then obviously we are talking about accelerations.

$$\vec{a} = \frac{\vec{F}}{m} = \frac{\vec{d}\vec{U}}{dt} \tag{10.3.8}$$

10.4: What are the important real forces?

There are three real forces important for atmospheric motion:

1. Gravitational Force
2. Pressure Gradient Force (PGF)
3. Friction

Hence we can sum these real forces:

$$\sum \vec{F}_a = \vec{F}_g + \vec{F}_p + \vec{F}_f \tag{10.4.1}$$

We put the subscript "*a*" on these forces to indicate "absolute" because they are true in an inertial reference frame. Thus, in the absolute reference frame,

$$\frac{D_a \vec{U}_a}{Dt} = \frac{\sum \vec{F}_a}{m} \tag{10.4.2}$$

Let's examine each of these real forces in more detail.

Gravitational Force

Recall that the gravitational force on a mass *m* is simply the weight of the mass, which is given by:

\vec{F}_{g}=m \vec{g} *

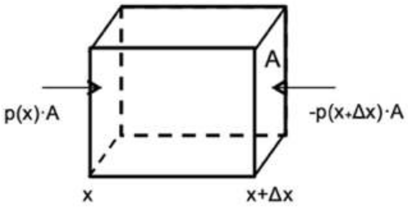
where

\vec{g} *=-\dfrac{G M}{r^{2}}\left(\dfrac{\vec{r}}{r}\right)

where *M* is Earth's mass (5.9722 x 10^{24} kg), \vec{r} is the distance vector originating from the Earth's center, and *G* is the gravitational constant (6.6741 × 10^{-11} m^3 kg^{-1} s^{-2}). Ignoring the minor effects of topography and the horizontal variation of Earth's density, the real gravitational force points directly towards Earth's center. The gravitational force per unit mass is simply \vec{e}^{*}.

Pressure Gradient Force (PGF)

The derivation of the pressure gradient force is similar to what we have already done in Lesson 2.2 to find hydrostatic equilibrium, except that we will look at only the pressure forces in this case, and will serve as a quick review. Consider the *x*-direction first:



Drawing of the forces involved in the pressure gradient force in the *x* direction. Credit: W. Brune

$$\frac{F_{px}}{m} = \frac{p(x)A - p(x+\Delta x)A}{m} = \frac{p(x)A - [p(x) + \Delta p]A}{m} \tag{10.4.3}$$

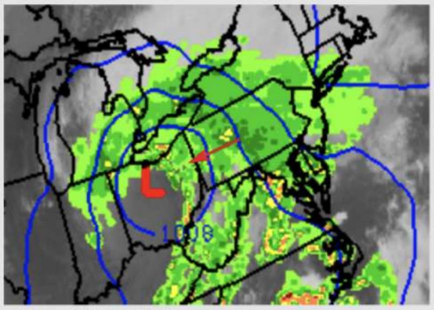
$$\frac{F_{px}}{m} = \frac{A\Delta x[p(x) - p(x) - \Delta p]}{m\Delta x} = -\frac{V}{m}\frac{\Delta p}{\Delta x} = -\frac{1}{\rho}\frac{\Delta p}{\Delta x} \approx -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{10.4.4}$$

Adding in the *y* and *z* directions, we get the 3-D vector form of the pressure gradient force per unit mass:

$$\frac{\vec{F}_p}{m} = -\frac{1}{\rho}\vec{\nabla} p \tag{10.4.5}$$

Example

Let's do a quick calculation of the pressure gradient force from a map of surface pressure on 26 June 2015. Note that Pennsylvania's northern border is about 250 km from its southern border.



PA is ~250 km high. Isobars are 4 mb apart.
Air density ~1.2 kg/m^3.
From 1008 mb isobar to 1016 mb isobar is ~300 km.

$|PGF| = (1/1.2)*(800\ Pa/(300*1000)=2.2x10^{-3}\ m\ s^{-2}$,
pointing due west (180°)

Credit: Penn State e-Wall and W. Brune

The following video (1:20) will explain the process:

PGF Example

Click here for transcript of the PGF Example video.

> Let's go through a quick calculation of the pressure gradient force, or a low pressure system that passed over Pennsylvania on June 26, 2015. Note that the pressure increases as x increases. But because the pressure gradient force is minus 1 over the density times the pressure gradient the pressure gradient force-- really the pressure gradient acceleration-- is negative. This makes sense since the pressure gradient force would move air from high pressure to low pressure which is to the west in this case. To find the pressure gradient we note that the height of Pennsylvania is about 250 kilometers, which is slightly smaller than the distance between the 1,008 millibar and the 1,016 millibar isobars, which is about 300 kilometers as a distance. So the air density is about 1.2 kilograms per meter cubed. When we put all these numbers together-- that is one over the density times the change in pressure over the change in distance-- we get that the pressure gradient force in this case is 2.2 times 10 to the minus 3 meters per second squared and is directed to 180 degrees, or due west.

Friction

We can think of friction as being processes that impede the air flow. There are two different types of friction that meteorologists are concerned with: **molecular friction** and **turbulent friction**. Molecular friction is a real force that appears in the conservation of momentum equation whereas turbulent friction is an additional term that arises out of averaging the conservation of momentum equation.

Molecular friction results from the random movement of molecules. Imagine two air parcels moving towards the east. One air parcel is just to the north of the other and is moving a bit faster than the other. Due to random molecular motion, the two parcels exchange air molecules that carry the momentum of their respective air parcels. When molecules collide, some of their momentum is transferred, resulting in the faster parcel (the one to the north) slowing down and the slower parcel (the one to the south) speeding up. There is thus a transfer of momentum from the faster parcel to the slower parcel. This transfer is proportional to the velocity difference between the air parcels and a quantity called the viscosity. The viscosity depends on the fluid in question (air in this case) and the temperature. Fluids with a relatively high resistance to motion, like honey, have relatively high viscosities. Think about air near the Earth's surface. The air right at the surface is stationary due to electromagnetic forces between the air and the surface. Due to molecular friction, the air near the surface will slow down the air just above it, just as that air slows the air a little bit higher. We show without derivation that the molecular friction force (sometimes called the viscous force) per unit mass is to a very good approximation given by:

$$\frac{\vec{F}_f}{m} = \nu\nabla^2\vec{U}_a \tag{10.4.6}$$

where v is the kinematic viscosity, $\nabla^2 = \vec{\nabla}\cdot\vec{\nabla} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is called the **Laplace operator** or the **Laplacian**, and \vec{U} is the air parcel velocity. The viscous force is important for resisting flow and dissipating air flow on small scales, such as for an individual raindrop, but it is not an important force on larger scales when compared to other forces such as gravity and the pressure gradient force (as will be demonstrated in Section 10.5).

Turbulent friction is important for larger-scale atmospheric motion, even synoptic-scale motion. The flow in the atmosphere's lowest kilometer or two, called the atmospheric boundary layer is often turbulent, with chaotic large and small swirls of air that, when taken together, have momentum in all directions. During the day, turbulence is generated by convection. During both day and night, turbulence is also generated by wind shear throughout the boundary layer. No matter how turbulence is generated, it provides a drag on the horizontal flow throughout the boundary layer because upward moving air with low horizontal momentum collides with air aloft with high horizontal momentum, slowing it down. This turbulent drag is often referred to as friction, even though the word "friction" really applies only to molecular-scale interactions.

Turbulent friction is not a fundamental force; it is represented in the conservation of momentum equation only after the equation has been averaged over time, space, or both. New terms representing turbulent friction arise from the averaging of the advective derivative, which we will discuss in more detail in Lesson 11. For now, we take the momentum conservation equation and average it so that all of the quantities that we are predicting—like velocity, pressure, and density—really reflect average quantities that vary gradually over space and time. For example, the wind velocity averaged over an hour and over the southeastern quarter of Pennsylvania would be a good example of a quantity one could predict from the averaged momentum conservation equation. On the other hand, a wind gust measured by an anemometer on top of a building would not be a good example of such a quantity.

For a turbulent boundary layer, the turbulent friction per unit mass is a function of four quantities: the dimensionless drag coefficient C_d the planetary boundary layer height h, the magnitude of the horizontal velocity $|\vec{v}_a|$, and the horizontal velocity itself:

$$-\frac{C_d}{h}\left|\vec{V}_a\right|\vec{V}_a \tag{10.4.7}$$

Even though this turbulent drag is not really friction, it is an important resistance to the average horizontal flow on large scales in the boundary layer and so we will keep it, and not molecular friction, as the friction term in the averaged momentum equation. Note that the turbulent drag is greatest within the boundary layer and becomes much smaller above the boundary layer, where it is assumed that the drag coefficient becomes very small.

Inertial (Real) Force Summary

The real forces can be summarized in the following two equations. The first equation represents how the instantaneous velocity of an individual air parcel varies with time. The second equation, which is an average of the first equation, represents how the average velocity of an air mass varies with time. Both equations include acceleration, gravity, and the pressure gradient force.

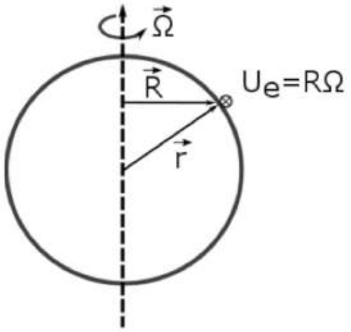
The first equation includes molecular friction and the second equation includes turbulent friction. The first equation is more accurate but the second equation is more practical for applications in weather and climate.

$$\frac{D_a\vec{U}_a}{Dt} = -\frac{1}{\rho}\vec{\nabla}p + \vec{g}* + \nu\nabla^2\vec{U}_a \tag{10.4.8}$$

$$\frac{D_a\vec{U}_a}{Dt} = -\frac{1}{\rho}\vec{\nabla}p + \vec{g}* - \frac{C_d}{h}\left|\vec{V}_a\right|\vec{V}_a \tag{10.4.9}$$

10.5: Effects of Earth’s Rotation- Apparent Forces

Newton’s Laws apply in an inertial reference frame, that is, one that is not accelerating. A point on the rotating Earth is not following a straight line through space, but instead is constantly accelerating by rotating away from a straight line. Therefore, Earth does not provide an inertial reference frame. From the point of view of an astronaut in distant space, the air motions she would observe obey Newton’s Law perfectly, but from the point of view of an Earth-bound observer, Newton’s Laws fail to capture the observed motion. To account for this crazy behavior, the Earth-bound observer needs to add some apparent forces to the real forces for the math to explain the observed motion from the point of view of someone standing on the rotating Earth.



Earth’s rotation is to the east. The symbol ⊗ indicates that the vector $\vec{U}_e$ s going into the page with a magnitude $R\Omega$. Credit: W. Brune

Suppose we have an air parcel moving through space with a velocity $\vec{U}_a$, which we will call the absolute velocity. We want to relate this absolute velocity to $\vec{U}$, the velocity observed with respect to the Earth reference frame. Let $\vec{U}_e$ be the velocity of the Earth. Here we only consider the velocity of the Earth due to rotation about its axis (the motion around the Sun is much less important), so $\vec{U}_e$ is always eastward, greatest at the equator and zero at the poles. The absolute velocity of an air parcel is simply the velocity of the air parcel with respect to the Earth plus the velocity of the Earth itself: $\vec{U}_a = \vec{U} + \vec{U}_e$

What is the velocity of the Earth? Consider a specific point on the Earth. Let $\vec{\Omega}$ be Earth’s angular velocity vector, $\vec{r}$ be the position vector from the Earth’s center to the point in question, and $\vec{R}$ be the shortest distance vector from the axis of rotation to the point in question (as in the figure above). The magnitude of $\vec{\Omega}$ is $|\vec{\Omega}| = \frac{2\pi}{23.934\text{hr}}\frac{1\text{hr}}{3600\text{s}} = 7.292 \times 10^{-5}\text{s}^{-1}$ and the direction of $\vec{\Omega}$ is determined by the right hand rule (the direction of your thumb when you curl the fingers of your right hand in the direction of rotation and point your thumb towards the North Star). To determine the angular velocity of the Earth, note that we have used the sidereal day length, 23.934 hr, which is the day length when Earth's rotation is measured with respect to the fixed stars (the inertial reference frame).

The following video (:51) will demonstrate the right hand rule:

UE Right Hand Rule

Click here for transcript of the UE Right Hand Rule video.

> u sub e is the eastward velocity of the Earth. It is pointed into the page. We know the u sub e equals r-- which is the shortest distance vector between Earth's rotation axis and the point on the surface-- times omega, which is Earth's rotation vector. The units of omega are seconds to the minus 1, which makes it a frequency. Note that u sub e equals r times omega, which is also equal to omega cross r. We can see this if you take your right hand with your fingers pointed in the omega direction and the palm in the r direction. And you fold your fingers into the palm. Your thumb will point into the page, which is the direction of u sub e and is in the positive x direction.

The magnitude of $\vec{U}_e$ is $R\Omega$, but we need to write $\vec{U}_e$ as a vector. Note that $\vec{U}_e$ is pointed into the page in the figure above, which is a direction perpendicular to both $\vec{\Omega}$ and $\vec{R}$. Hence we can use the cross product equation to write an expression for Earth's velocity: $\vec{U}_e = \vec{\Omega}\times\vec{R} = \vec{\Omega}\times\vec{r}$, since the component of $\vec{r}$ perpendicular to $\vec{\Omega}$ is $\vec{R}$. So:

$$\vec{U}_a = \vec{U} + \vec{\Omega}\times\vec{r} \tag{10.5.1}$$

We have thus related velocity in the absolute reference frame to velocity in the rotating reference frame.

Now we can consider acceleration. Since $\vec{U} = \frac{D\vec{r}}{Dt}$ and $\vec{U}_a = \frac{D_a\vec{r}}{Dt}$ we can write:

$$\frac{D_a\vec{r}}{Dt} = \frac{D\vec{r}}{Dt} + \vec{\Omega}\times\vec{r}$$

This equation describes the change in a position of an air parcel with time observed from an inertial reference frame (the derivative on the left) to the change with time observed from Earth’s reference frame (the derivative on the right). Equation [10.11] is general and applies not only to $\vec{r}$ but also to any other vector.

Let’s replace $\vec{r}$ on the left hand side with $\vec{U}_a$ and $\vec{r}$ on the right hand side with $\vec{U}+\vec{\Omega}\times\vec{r}$ since these two expressions equal each other in Equation [10.10]. By making these substitutions, we can relate acceleration in the absolute frame to acceleration in the rotating frame:

$$\frac{D_a\vec{U}_a}{Dt} = \frac{D\vec{U}}{Dt} + \frac{D\vec{\Omega}}{Dt}\times\vec{r} + \vec{\Omega}\times\frac{D\vec{r}}{Dt} + \vec{\Omega}\times\vec{U} + \vec{\Omega}\times(\vec{\Omega}\times\vec{r}) \tag{10.5.2}$$

We can simplify this equation and then we can make sense of it physically. First, $\vec{\Omega}$ is not changing significantly with time, so $\frac{D\vec{\Omega}}{Dt}$ can be set to zero. Second, $(\vec{\Omega}\times\vec{r})$ has the magnitude of ΩR and points to the east (by the right hand rule) and thus $\vec{\Omega}\times(\vec{\Omega}\times\vec{r})$ has the magnitude $\Omega^2 R$ and points toward $-\vec{R}$. Finally, noting that $\vec{U} = \frac{D\vec{r}}{Dt}$, we end up with the equation:

$$\frac{D_a\vec{U}_a}{Dt} = \frac{D\vec{U}}{Dt} + 2\vec{\Omega}\times\vec{U} - \Omega^2\vec{R} \tag{10.5.3}$$

The term on the left is the acceleration in the absolute inertial reference frame. The first term on the right is the acceleration in the Earth reference frame. The remaining terms are the apparent accelerations. The first one is the Coriolis acceleration and the second one is the centripetal acceleration.

We can now combine Equation [10.13] with the version of Equation [10.9] that is averaged to get:

$$-\frac{1}{\rho}\vec{\nabla}p + \vec{g}* - \frac{C_d}{h}|\vec{V}|\vec{V} = \frac{D\vec{U}}{Dt} + 2\vec{\Omega}\times\vec{U} - \Omega^2\vec{R} \tag{10.5.4}$$

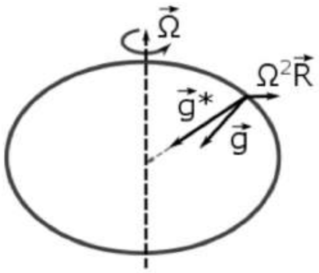
and then rearrange this equation to get:

$$\frac{D\vec{U}}{Dt} = -\frac{1}{\rho}\vec{\nabla}p + \vec{g}* - \frac{C_d}{h}|\vec{V}|\vec{V} - 2\vec{\Omega}\times\vec{U} + \Omega^2\vec{R} \tag{10.5.5}$$

The first three terms on the right hand side of Equation [10.14] are the real forces. The fourth and fifth terms on the right hand side are the apparent forces: the Coriolis force and the centrifugal force, respectively.

Centrifugal Force

The centrifugal force is directed away from the Earth's axis of rotation and is the same type of force that you feel when you are in a car going around a sharp curve. Over its long history, all the material that makes up the Earth has adjusted to the real gravitational force, \vec{g}^{*}, which is directed to Earth’s center, and the apparent centrifugal force that is directed away from Earth’s rotation axis (see figure below).



The gravity we feel is the sum of gravity pointed toward Earth’s center and the outward centrifugal force. The effect is greatly exaggerated to show the vectors. The gravity we feel, $\vec{g}$, is perpendicular to Earth's flat surfaces at rest (i.e., oceans). Credit: W. Brune

The resulting gravity that the Earth and everything on it feels is the vector sum of this real and this apparent force:

$$\vec{g} = \overrightarrow{g^*} + \Omega^2\vec{R} \tag{10.5.6}$$

Since the centrifugal force depends on $\vec{R}$, it is greatest at the equator and zero at the poles. As a result of the centrifugal force, the Earth has become slightly oblate, with an equatorial radius of 6378.1 km that is 0.34% greater than the polar radius of 6356.8 km. Note that $\overrightarrow{g}$ is always perpendicular to Earth’s surface, which is very useful because the vertical coordinate is always chosen to be perpendicular to Earth's surface, so that $\overrightarrow{g}$ is only in the *z* direction and, as the figure above indicates, does not point towards the center of the Earth (except at the poles and the equator). The value of g at the equator is $9.780\mathrm{ms}^{-2}$, which is $0.052ms^{-2}$ smaller than the value of g at the poles, which is 9.832 m s^{-2}. The centrifugal force at the equator is $\Omega^2 R = \left(7.27\times 10^{-5}\mathrm{s}^{-1}\right)^2\left(6.378\times 10^{6}\mathrm{m}\right)$ $= 0.033\mathrm{ms}^{-2}$, and hence accounts for more almost 2/3 of the difference in g between the equator and the poles. The rest of the difference is due to the difference in g*, which is overestimated by the difference in equatorial and polar radii--the problem is more complicated than it might appear because Newton's law of gravitation only applies to point masses. At any rate, the difference between g at the poles and equator is small enough for a constant value of g = 9.8 m s^{-2} to be suitable for most applications in atmospheric dynamics.

Combining Equations [10.14] and [10.15] yields a more useful form of the averaged momentum conservation in the rotating reference frame:

$$\frac{\overrightarrow{DU}}{Dt} = -\frac{1}{\rho}\vec{\nabla}_p + \vec{g} - \frac{C_d}{h}|\vec{V}|\vec{V} - 2\vec{\Omega}\times\vec{U} \tag{10.5.7}$$

We will now move on to a discussion of the Coriolis Force. The following video (3:05) gives a basic introduction.

Coriolis Effect

Click here for transcript of the Coriolis Effect video.

[MUSIC PLAYING]

What's going on here? Every time I throw the ball straight, it seems to bend to the side. No matter what I throw, no matter how straight I throw it, the ball seems to be curving.

Maybe it has something to do with this: That whole time I was spinning, which is why it looks like I'm about to puke.

That curve is created by the Coriolis effect. You may have heard that it makes water in the bathtub spiral down the drain a certain way or that it determines the way a toilet flushes. That's actually wrong. Those are not caused by the Coriolis effect. But the Coriolis effect does create hurricanes and is the reason why Jupiter’s Great Red Spot is spinning the way it is.

So, what is the Coriolis effect? Well, it's what happens when objects moving in a straight line appear to curve because you are rotating. And it affects all kinds of things. It bends the paths of missiles and sniper shots. But how does it work, and how does it create hurricanes?

Right now, it seems like the ball's curving. But let's take a look at what's really happening. My friends and I put together this little experiment using a sort of spinning seesaw. Here, the camera's standing still; it's just hanging from the ceiling. And watch what happens. From the moment the ball leaves my hand, it's just going straight. Let's watch that again, frame by frame. Keep your eye on the ball. You can see that it travels in a perfectly straight line. But what if you're spinning with the seesaw? Look at what happens when you rotate the footage so the seesaw stays sideways and everything moves around it? Now let's follow the path of the ball again but from this new perspective. The trajectory looks completely different. Now let's watch this one frame by frame. From this rotating perspective, it totally looks like the ball is curving. That's crazy because you already saw that exact same throw. Watch it again on the left. The ball is going straight, but on the right, when you're rotating with the seesaw, the ball really looks like it's curving.

You might be thinking, why should I care? I don't spend a lot of time on a spinning seesaw or on a merry-go-round. But you kind of are on a merry-go-round. The earth is constantly spinning. Have you ever seen someone in a movie making fine adjustments on their gun? Well, one effect they're correcting for is the Coriolis effect. The longest sniper shot, which was over 3,000 yards, would have had to correct for a one-foot deviation due to the Coriolis effect.

So, how does the Coriolis effect create hurricanes? Hurricanes form when air rushes from all directions into a low-pressure region. So, imagine there's a low-pressure region between the two of us. Air is going to rush toward the center. Let's see what that looks like from space, from our camera that's hanging from the ceiling. It's not spinning, so the air--or in this case, the balls--are clearly going in a straight line. But if you're rotating with the Earth, or with the seesaw, you'll see the air bend to the right. In the northern hemisphere, this creates hurricanes with counterclockwise spirals. In the southern hemisphere it does the opposite; it creates hurricanes with clockwise spirals. This storm on Jupiter, which is actually bigger than the Earth itself, has been spinning for around 200 years.

So, when you're in a rotating frame, the Coriolis effect seems to exert a very real force on objects. But there is no force; what you observe is just a result of your perspective. And you might be wondering, if this is simply a product of your frame of reference, then could other forces, like gravity or electromagnetism, also just be products of your frame of reference? Of where we live in the universe? Or in the multiverse? For more on that, check out our video "Do We Live in the Multiverse?" There are talking fish. And if you want to find out when new episodes come, click subscribe.

Credit: [NOVA PBS]

Coriolis Force

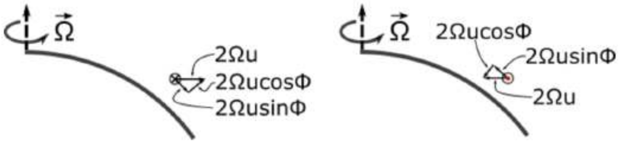
The Coriolis force, $-2\vec{\Omega} \times \vec{U}$, acts on an air parcel (or any other object) only when it is moving with respect to the Earth. It acts perpendicular to Earth's angular velocity vector and the air parcel's velocity vector. The explanation for the Coriolis force is usually broken into an explanation in the **zonal (constant latitude) direction** and the **meridional (constant longitude) direction**.

Zonal Flow (East–West Wind Velocity)

Consider an air parcel that is initially at rest and in hydrostatic balance but is impulsively accelerated to a velocity *u* to the east (see left side of figure below). Initially when it was at rest, it had the same acceleration as the Earth below it. However, after it accelerated to velocity *u*, it suddenly had more acceleration than it had before, throwing it out of hydrostatic balance. Look at the change in acceleration that comes from the air parcel suddenly acquiring a velocity to the east, which is just the acceleration after the velocity changes minus the acceleration before the velocity changes and equals $(\Omega + u/R)^2 R - \Omega^2 R$. To a very good approximation, this change equals the Coriolis force, $2\Omega u$.

There is a vertical component that points up, but there is also a horizontal component of force that points to the right of the motion in the Northern Hemisphere and to the left in the Southern Hemisphere.

Now consider an air parcel that is initially at rest but is impulsively accelerated to a velocity *u* to the west (see right side of figure below). The air parcel suddenly has less angular momentum than it had before and experiences a decreased centrifugal force. This decrease in angular momentum, to a very good approximation, equals the Coriolis force, $2\Omega u$, but is pointed toward Earth's axis of rotation. There is a vertical component that points down, but the horizontal component of force that points to the right of the motion in the Northern Hemisphere and to the left in the Southern Hemisphere.



Coriolis force on an air parcel traveling zonally (left) to the east, where $\otimes$ indicates flow into the page, and (right) to the west, where the red dot in the circle indicates flow out of the page, for the Northern Hemisphere. The air parcel speed is *u* and the latitude is *Φ*. *Credit: W. Brune*

We can write down the accelerations in the *y* and *z* directions due to the air moving to the east with speed *u*:

$$\text{Coriolis acceleration in the y direction}= \frac{Dv}{Dt} = -2\Omega u \sin\phi$$

$$\text{Coriolis acceleration in the z direction}= \frac{Dw}{Dt} = 2\Omega u \cos\phi$$

Meridional Flow (North–South Velocity)

What about an air parcel traveling to the north at a constant altitude? Note that the air parcel moving north starts at a greater distance from Earth’s axis and comes closer to Earth’s axis if it moves at the same height above the surface. Its angular momentum is conserved, so it is moving faster to the east than the Earth beneath it. As a result, it appears to move to the right or the east.

If the same air parcel moves to the south at the same height above Earth’s surface, then it moves to a greater distance from Earth’s rotation axis. Its angular momentum becomes less than that of Earth, it slows down relative to Earth, and it veers to the right of south or to the west.

In both the zonal and meridional flow cases, the air parcel's velocity with respect to the Earth causes the air parcel to have a different angular momentum from the Earth below it. Conservation of angular momentum during that motion requires that the apparent Coriolis force be added in order to describe the observed motion. See the video below (2:11) for further explanation:

Coriolis Explanation

Click here for transcript of the Coriolis Explanation video.

Coriolis force is an apparent force that accounts for motion on a rotating sphere, such as Earth. We can break the explanation of the Coriolis force in two cases-- zonal flow, which is east west, and Meridional flow, which is north south. The explanation for both cases relies on conservation of angular momentum. For zonal flow, imagine an air parcel moving to the east with velocity, u. Angular acceleration is just the angular velocity squared times the radius of rotation. If the parcel is moving at a velocity of u relative to Earth's surface, then it has some extra angular momentum, which is u divided by r. To find the total angular acceleration that the moving air parcel has, we need to square the angular momentum of the air parcel, which is omega plus u divided by r, and then multiply it by r. We then subtract the Earth's acceleration, which is just omega squared r. The difference, to good approximation, is 2 omega times u, which is just the Coriolis force, and, in the case of eastward motion, is pointed away from Earth's axis in the northern hemisphere. Thus, the Coriolis force turns the air parcel to the right for zonal flow. If the air parcel moves to the west, then by the same argument the Coriolis force points towards Earth's rotation axis in the northern hemisphere, which again turns the air parcel to the right. The explanation for the Meridional flow is simpler. An air parcel initially has the angular momentum of the Earth at its latitude. If it moves north at the same height, then it has more angular momentum than the Earth below it. And so it goes faster than the Earth and appears to move to the right. If it moves south at the height, then it has less angular momentum than Earth and appears to slow down relative to Earth and thus appears to move to the right.

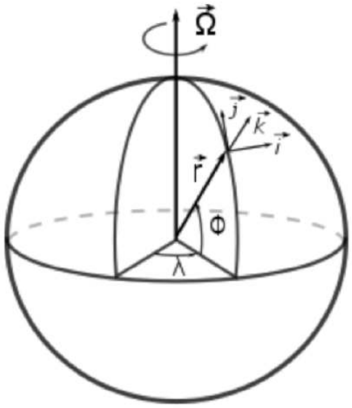
Finding the Magnitude and Direction of the Coriolis Force

The magnitude of the horizontal Coriolis force is simply $2\Omega|\vec{V}|\sin\varphi$, where φ is the latitude. This magnitude applies to both the Northern and Southern Hemispheres. The direction of the Coriolis force is 90 degrees to the right of the horizontal velocity vector in the Northern Hemisphere and 90 degrees to the left of the horizontal velocity vector in the Southern Hemisphere.

10.6: Equations of Motion in Spherical Coordinates

The three variables used in spherical coordinates are:

- longitude (denoted by λ)
- latitude (denoted by φ)
- vertical distance (denoted by r from Earth's center and by z from Earth's surface, where $z = r–a$ and a is Earth's radius)



Conversion between spherical and Cartesian coordinates. Credit: W. Brune

Note that the unit vectors in spherical coordinates change with position. For example, for an air parcel at the equator, the meridional unit vector, $\vec{j}$, is parallel to the Earth's rotation axis, whereas for an air parcel near one of the poles, $\vec{j}$ is nearly perpendicular to the Earth's rotation axis. In spherical coordinates, the velocity vector and its components are given by:

$$\vec{U} = u\vec{i} + v\vec{j} + w\vec{k} \tag{10.6.1}$$

$$u = r\cos\phi\frac{D\lambda}{Dt}, \quad v = r\frac{D\phi}{Dt}, \quad w = \frac{Dz}{Dt} \tag{10.6.2}$$

where *u*, *v*, and *w* are the eastward, northward, and upward components of the velocity, respectively. These velocities are derived from the changes in eastward, northward, and upward distances, which are given by:

$$dx = r\cos\phi d\lambda = \text{ change in eastward distance } \cong a\cos\phi d\lambda \tag{10.6.3}$$

$$dy = rd\phi = \text{ change in northward distance } \cong ad\phi \tag{10.6.4}$$

$$dz = dr = \text{ change in upward distance} \tag{10.6.5}$$

Let's now write the averaged momentum conservation equation [10.16] in component form in spherical coordinates. We will just show you how this conversion is done without actually taking you through all the steps. Note that we need to take the total derivatives of the unit vectors as well as the velocities:

$$\frac{D\vec{U}}{Dt} = \frac{D}{Dt}(\vec{i}u + \vec{j}v + \vec{k}w) = \vec{i}\frac{Du}{Dt} + u\frac{D\vec{i}}{Dt} + \vec{j}\frac{Dv}{Dt} + v\frac{D\vec{j}}{Dt} + \vec{k}\frac{Dw}{Dt} + w\frac{D\vec{k}}{Dt} \tag{10.6.6}$$

The terms containing derivatives of the unit vectors are called "metric terms." They depend on the Earth being a sphere. In Cartesian coordinates, they equal zero.

Consider just one of these metric terms:

$$\frac{D\vec{j}}{Dt} = \frac{\partial\vec{j}}{\partial t} + u\frac{\partial\vec{j}}{\partial x} + v\frac{\partial\vec{j}}{\partial y} + w\frac{\partial\vec{j}}{\partial z} = 0 + u\frac{\partial\vec{j}}{\partial x} + v\frac{\partial\vec{j}}{\partial y} + 0 \tag{10.6.7}$$

Since for any location, $\vec{j}$ is constant with time and $\vec{j}$ does not change as a function of altitude, that leaves $\vec{j}$ dependent only on latitude and longitude. Look at $\frac{\partial\vec{j}}{\partial y}$ first. Set *y* = 0 at the equator, and *y* = a (Earth's radius) near the pole. As noted above, at the equator, $\vec{j}$ is parallel to Earth's rotation axis, but near the pole, it is almost perpendicular to it. Thus the change in $\vec{j}$ going from south to north (increasing y) must be pointing down to Earth's center and so $\vec{j}$ changes by $-\vec{k}$ times a small angular change while y changes by a times the same small angular change. The net result is that:

$$\frac{\partial \vec{j}}{\partial y}=\frac{-\vec{k}}{a} \tag{10.6.8}$$

Using the same approach, we can show that:

$$\frac{\partial \vec{j}}{\partial x}=\frac{-\tan \phi}{a} \vec{i} \tag{10.6.9}$$

The total derivatives of all three unit vectors are:

$$\frac{D \vec{i}}{D t}=\frac{u}{a \cos \phi}(\vec{j} \sin \phi-\vec{k} \cos \phi) \tag{10.6.10}$$

$$\frac{D \vec{j}}{D t}=-\frac{u \tan \phi}{a} \vec{i}-\frac{v}{a} \vec{k} \tag{10.6.11}$$

$$\frac{D \vec{k}}{D t}=\frac{u}{a} \vec{i}+\frac{v}{a} \vec{j} \tag{10.6.12}$$

Putting this all together:

$$\begin{aligned} \frac{D \vec{U}}{D t}= & \left(\frac{D u}{D t}-\frac{u v \tan \phi}{a}+\frac{u w}{a}\right) \vec{i}+ \\ & \left(\frac{D v}{D t}+\frac{u^2 \tan \phi}{a}+\frac{u w}{a}\right) \vec{j}+ \\ & \left(\frac{D w}{D t}-\frac{u^2+v^2}{a}\right) \vec{k} \end{aligned} \tag{10.6.13}$$

Similar analysis can be done for the other terms in the averaged momentum equation.

Coriolis force:

$$-2 \vec{\Omega} \times \vec{U}=2 \Omega(v \sin \phi-w \cos \phi) \vec{i}-(2 \Omega u \sin \phi) \vec{j}+(2 \Omega u \cos \phi) \vec{k} \tag{10.6.14}$$

Gravity:

$$\vec{g}=-g \vec{k} \tag{10.6.15}$$

Pressure Gradient Force (PGF):

$$-\frac{1}{\rho} \vec{\nabla} p=-\frac{1}{\rho} \frac{\partial p}{\partial x} \vec{i}-\frac{1}{\rho} \frac{\partial p}{\partial y} \vec{j}-\frac{1}{\rho} \frac{\partial p}{\partial z} \vec{k} \tag{10.6.16}$$

Turbulent friction (in the boundary layer only):

$$-\frac{C_d}{h}|\vec{V}| \vec{V}=-\frac{C_d}{h}|\vec{V}| u \Big| u \vec{i}-\frac{C_d}{h} \Big| \vec{V}|v| \vec{v} \vec{j} \tag{10.6.17}$$

Adding together all of the forces, the averaged momentum equations in spherical coordinates in the zonal, meridional, and vertical directions are, respectively:

$$\frac{D u}{D t}-\frac{u v \tan \phi}{a}+\frac{u w}{a}=-\frac{1}{\rho} \frac{\partial p}{\partial x}+2 \Omega v \sin \phi-2 \Omega w \cos \phi-\frac{C_d}{h}|\vec{V}| u \tag{10.6.18}$$

$$\frac{D v}{D t}-\frac{u^2 \tan \phi}{a}+\frac{v w}{a}=-\frac{1}{\rho} \frac{\partial p}{\partial y}-2 \Omega u \sin \phi-\frac{C_d}{h}|\vec{V}| v \tag{10.6.19}$$

$$\frac{D w}{D t}-\frac{u^2+v^2}{a}=-\frac{1}{\rho} \frac{\partial p}{\partial z}-g+2 \Omega u \cos \phi \tag{10.6.20}$$

10.7: Are all the terms in these equations equally important? Let's use scale analysis.

Our equations of motion have several terms in them, and the question is, “Which ones are the largest and, thus, the most important?” The answer is, “It depends on the situation.” You can follow the following steps to determine which terms to keep and which you can safely ignore when you are trying to calculate the force balance for a specific atmospheric phenomenon. This process is called **scale analysis** and it can be applied to any conservation equation you would like to simplify.

1. Decide on the phenomenon of interest (e.g., cyclone, front, hurricane, tornado).
2. Determine the characteristic (i.e., typical) lengths and times over which the phenomenon occurs.
3. Determine the range of fluctuations of equation variables in space and time during the phenomenon.
4. Approximate derivatives (i.e., $\frac{\partial p}{\partial x} \sim \frac{\Delta p}{\Delta x}$)
5. Compare the magnitudes of the terms in the equation.
6. Keep only the relatively large terms (say, the top two orders of magnitude) and neglect the smaller terms.

The Coriolis Parameter

Define the Coriolis parameter as $f \equiv 2\Omega \sin\phi$. At 45°N

$f=(2)\left(7.27\times 10^{-5}\mathrm{s}^{-1}\right)\sin 45^{\circ} \sim 10^{-4}\mathrm{s}^{-1}$ We will use this parameter in the scale analysis in the next section and then throughout the rest of the lesson.

Example: Scale analysis of the averaged *x*-momentum equation for mid-latitude synoptic-scale flow in the free troposphere.

1. Phenomenon: mid-latitude synoptic-scale flow in the free troposphere.

2. $L\sim 1000\mathrm{km}=10^{6}\mathrm{m}; H\sim 10\mathrm{km}=10^{4}\mathrm{m}; \left(\text{ in boundary layer only, } C_d\sim 10^{-2} \text{ and } h\sim 1000\mathrm{m}\right); t\sim ?$

3.

$U\sim 10\mathrm{ms}^{-1}; \Delta p\sim 10\mathrm{hPa}=10^{3}\mathrm{Pa}$ (in the horizontal)

$t\sim L/u=\left(10^{6}\mathrm{m}\right)/\left(10\mathrm{ms}^{-1}\right)=10^{5}\mathrm{s}\sim 1$ day

$f_o\sim 2\Omega\sin\phi_o\sim 2\Omega\cos\phi_o\sim 10^{-4}\mathrm{s}^{-1}$

$W\sim H/t=10^{-1}\mathrm{ms}^{-1}; a\sim 10^{7}\mathrm{m}; \nu\sim 10^{-5}\mathrm{m}^{2}\mathrm{s}^{-1}; \rho\sim 1\mathrm{kgm}^{-3}$

4 and 5.

term	magnitude (variables)	magnitude (m s^{-2})
$\frac{Du}{Dt}=$	$\frac{U}{t}$	$\frac{10}{10^{5}}=10^{-4}$
$2\Omega v\sin\phi$	f_oU	$10^{-4}10=10^{-3}$
$-2\Omega w\cos\phi$	f_oW	$10^{-4}10^{-1}=10^{-5}$
$\frac{uw}{a}$	$\frac{UW}{a}$	$\frac{(10)\left(10^{-1}\right)}{10^{7}}=10^{-7}$
$\frac{uv\tan\phi}{a}$	$\frac{U^{2}}{a}$	$\frac{10^{2}}{10^{7}}=10^{-5}$
$-\frac{1}{\rho}\frac{\partial p}{\partial x}$	$\frac{\Delta p}{\rho L}$	$\frac{10^{3}}{(1)\left(10^{6}\right)}=10^{-3}$
$-\frac{C_d}{h}\lvert\vec{V}\rvert\vec{V}$	$\frac{C_dU^{2}}{h}$	$\frac{10^{-2}10^{2}}{10^{3}}=10^{-3}$
$\nu\nabla^{2}u$	$\frac{\nu U}{H^{2}}$	$\frac{\left(10^{-5}\right)(10)}{10^{8}}=10^{-12}$

6. Using just the most important (i.e., biggest) terms and knowing that the turbulent drag term goes to zero above the atmospheric boundary layer, we can write a simplified *x*-momentum balance as:

$$0=-\frac{1}{\rho}\frac{\partial p}{\partial x}+fv \tag{10.7.1}$$

Using the same scale analysis with the *y*-momentum equation, we can write a simplified *y*-momentum balance as:

$$0=-\frac{1}{\rho}\frac{\partial p}{\partial y}-fu \tag{10.7.2}$$

Only two terms remain in both equations. One term is the pressure gradient force and the other is the Coriolis force. Since $\frac{Du}{Dt}$ is much smaller than either the pressure gradient force or the Coriolis force, these two forces must be about in balance. We call this balance the **Geostrophic Balance**. It is very important for understanding atmospheric dynamics and we will talk about its consequences in more detail later.

The molecular friction term is the smallest of all the terms for the case of large-scale flow in the atmosphere. This term is almost always very small for most meteorological phenomena, which is why we had eliminated it from the averaged momentum conservation equation earlier.

We ignored the acceleration term, *Du/Dt*, because it is an order of magnitude smaller than the other two terms. We often must keep all terms that are within an order of magnitude of each other because our approximations may bias our results one way or the other. For instance, if we say that the velocity is 10 m s^{-1}, the spatial scale is 100 km, and the pressure change is 10 hPa when more accurate numbers are more like 20 m s^{-1}, 50 km, and 5 hPa, then we would be off almost an order of magnitude in our value for the centrifugal force, but get the same order of magnitude for the pressure gradient force. So, terms that are two orders of magnitude smaller than the rest you can easily neglect, but think carefully about terms that are only one order of magnitude different. For example, for very intense low-pressure systems, $\frac{Du}{Dt}$ must be considered because it can become about as large as the pressure gradient force and the Coriolis force.

When scale analysis is applied to the *z*-momentum equation for mid-latitude synoptic-scale flow, the result is the simplified *z*-momentum balance:

$$0=-\frac{1}{\rho}\frac{\partial p}{\partial z}-g \tag{10.7.3}$$

If you rearrange Equation [10.26], you will get the hydrostatic equilibrium equation, Equation [2.18]. Previously we derived it using a balance of forces on a slab of air, but here it comes naturally out of the *z*-momentum equation.

The following video (4:19) provides further explanation on how to complete the above example:

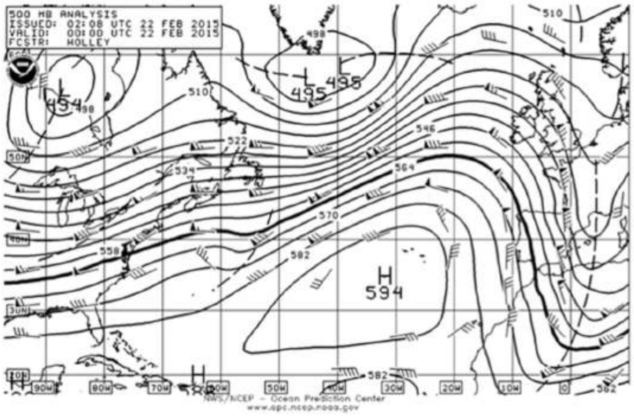
Scale Analysis video

Click here for transcript of the scale analysis video.

Scale analysis is very important. Because it tells us which terms in any equation are the most important and which terms we can ignore. In scale analysis you do not need to know the exact values for the variables. But instead you need to only know their order of magnitude. The process is straightforward. First, determine the phenomenon of interest whether it be cyclone, front, hurricane, tornado, synoptic-scale, winter weather. Determine the characteristic-- that is typical lengths and times-- over which phenomenon occurs. Determine the range of fluctuations of equation variables in space and time during the phenomenon. Approximate derivatives, that is the partial of p with respect to x would become delta p over delta x where they're roughly estimated. Compare the magnitudes of terms in the equation. And then keep only the relatively large terms-- say the top two orders of magnitude-- and neglect the much smaller terms. Let's look at this example of the x momentum equation for mid-latitude synoptic-scale flow. So in this case it's mid-latitude synoptic-scale flow. The lake is about 1,000 kilometers, which is 10 and 6 meters. The height is about 10 kilometers, which is 10 to the 4 meters. And if we were in the boundary layer only, we would find that the friction drag coefficients is 10 to the minus 2. And the height of the boundary layer is about 1,000 meters. Now we know that u is about 10 meters a second, roughly. It could be a lot less and a lot more. But it's that order of magnitude. Delta p is about 10 millibar over the length of interest. We see that the time then is equal to the scale of the synoptic-scale flow divided by the velocity, which is 10 to the 6 divided by 10, or 10 to the 5 seconds which is about a day. And we see that the Coriolis parameter is about 10 to the minus 4 per second. And we can estimate other factors, such as the w velocity which is height divided by time. So that's about 10 to the minus 1 meters per second. And so on. We continue on looking at derivatives and other terms. And so, for instance, the acceleration in the u direction is about 10 meters per second divided by 10 to the 5, which is about 10 to the minus 4. And so that's the size of that term. We see that the Coriolis term is about 10 to the minus 3. We see that other apparent terms are 10 to the minus 5 to 10 to the minus 7. They're quite a bit smaller. The pressure gradient force we see is 1 over the density, which is about 1 kilogram per meter cubed times the pressure difference which is about 10 to the 3 pascals divided by the distance, which is 10 to the 6 meters. So it's about 10 to minus 3. And we see that if we were in the boundary layer that the aerodynamic drag which causes friction is acting as friction. It's about 10 to the minus 3. So in the boundary layer we would need to consider this term because it's the same order of magnitude as the pressure gradient term and one of the larger terms. When we're not in the boundary layer then c sub d is actually very, very small. And this term is very small. We can ignore it. The last term is viscosity which is true friction. And we can see that for the case of viscosity is tiny. And therefore we can always ignore it for synoptic-scale flow. So when we look at the terms we have we see that we have away from the boundary layer we have two terms the count. That is we have the Coriolis term. And we have the pressure gradient. And those are the only two terms that we need to keep.

10.8: Why do weather maps use pressure surfaces instead of height surfaces?

You can’t find a weather map of winds at 5000 m, but you can find one for 500 mb, which is about the same altitude as 5000 m (see figure below).

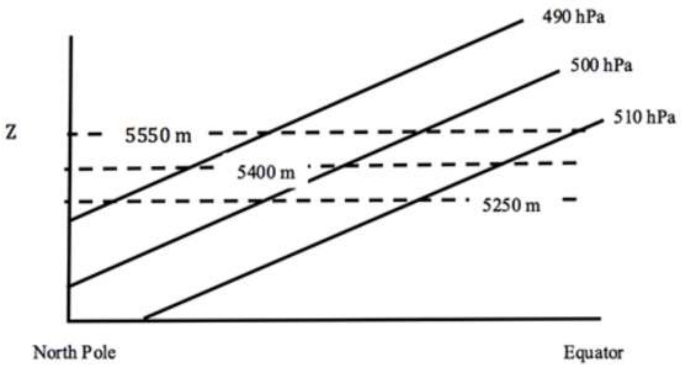


500 mb winds for 22 February 2015. Numbers are height above sea level in decameters (dam, tens of meters). For example, the height of the 500 mb surface in the high is 5940 m = 594 dam.

Credit: NOAA

We will learn why weather maps use pressure as the vertical coordinate, but for now, we will show that higher altitudes on a constant-pressure surface correspond to higher pressures on an constant-altitude surface.

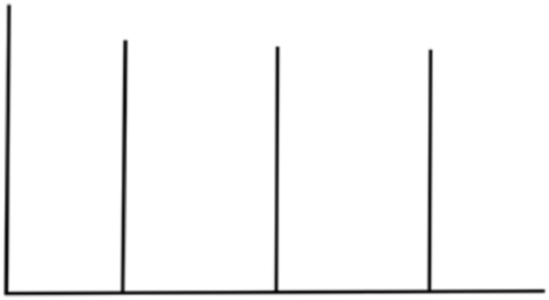
If we look down on the Earth, then we can plot the isobars as a function of latitude (*y*) and longitude (x). We can make a second plot of height surfaces on a constant-pressure surface (see figure below). Generally the pressure on average is greater at the equator on a given height surface than it is at the poles. This tilt makes sense if you think about the hydrostatic equilibrium equation because the temperature is greater at the equator than at the poles. Therefore, the scale height is greater, and so pressure decreases with height more gradually at the equator than it does at the poles.



A schematic of constant-pressure surfaces intersecting with constant-height surfaces from the pole to the equator.

Credit: W. Brune

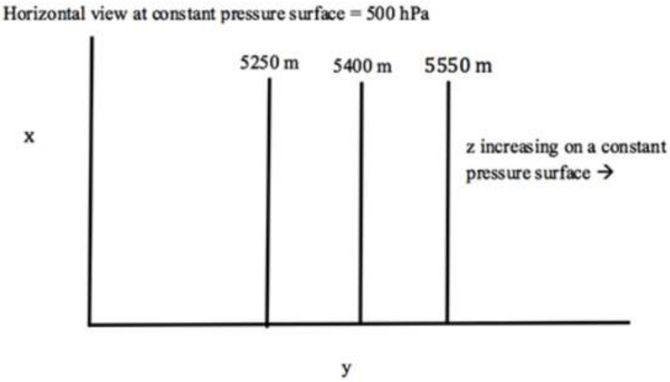
We can arbitrarily choose one height surface and see how the pressure changes as a function of latitude. We see that it increases from pole to equator (see figure below).



A schematic of isobars (constant pressure) on an *x*–*y* horizontal map at a constant height surface of 5400 m, as in the figure above.

Credit: W. Brune

If we now arbitrarily choose a constant-pressure surface of, say, 500 mb, then the change in the height on an *x*–*y* horizontal plot on the pressure surface also shows an increase from pole to equator (see figure below).



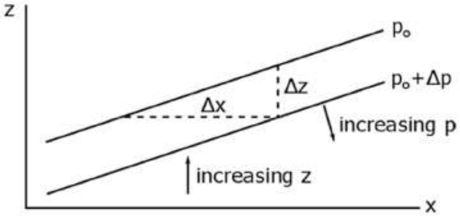
A schematic of isobars (constant pressure) on an *x*–*y* horizontal map at a constant height surface of 5400 m, as in the figure above.

Credit: W. Brune

Thus, low pressure on constant-height surfaces is related to low heights on constant-pressure surfaces. As a result of the hydrostatic approximation, for every height there is a unique pressure, so we can replace *z* with *p* as the vertical coordinate. We can then look at changes in variables as a function of *x*and *y*, but instead of doing this on a constant-height surface, we can do it on a constant-pressure surface.

We can now show how the equations of motion change when the vertical coordinate is switched from the height, *z*, to the pressure, *p*.

Consider first the pressure gradient force (PGF). The figure below provides a schematic of the math.



A cross section of pressure change on a *x*–*z* surface. Note that Δ*x*, Δ*z*, and Δ*p*are all positive and Δ*p* is the same in the horizontal and vertical.

Credit: W. Brune (after R. Najjar)

The slope of the isobar is just the change in *z* divided by the change in *x* on an isobar:

$$\frac{\Delta z}{\Delta x} \rightarrow \left(\frac{\partial z}{\partial x}\right)_p \tag{10.8.1}$$

where the subscript “*p*” means “constant pressure.” Since Δp is the same in the vertical and the horizontal:

$$\left(\frac{\partial p}{\partial x}\right)_z \Delta x = -\left(\frac{\partial p}{\partial z}\right)_x \Delta z \tag{10.8.2}$$

$$\left(\frac{\partial p}{\partial x}\right)_z = -\left(\frac{\partial p}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_p \tag{10.8.3}$$

where the subscripts "*x*" and "*z*" mean constant eastward distance and constant height, respectively.

Multiplying both sides by $1/\rho$ and using the hydrostatic equilibrium equation:

$$-\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_z = \frac{1}{\rho}\left(\frac{\partial p}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_p = -g\left(\frac{\partial z}{\partial x}\right)_p \tag{10.8.4}$$

or

$$-\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_z = -g\left(\frac{\partial z}{\partial x}\right)_p \tag{10.8.5}$$

Same for the *y* direction:

$$-\frac{1}{\rho}\left(\frac{\partial p}{\partial y}\right)_z = -g\left(\frac{\partial z}{\partial y}\right)_p \tag{10.8.6}$$

So, the geostrophic balance (Equation [10.24], [10.25]) in pressure coordinates becomes:

x-momentum equation: $0 = -g\frac{\partial z}{\partial x} + fv$

y -momentum equation: $0 = -g\frac{\partial z}{\partial y} - fu$

These equations can be rearranged to give the horizontal velocity on the pressure surface and can be rewritten as one in vector form:

$\overrightarrow{V_g} = \frac{g}{f}\vec{k} \times \vec{\nabla}_p z$

where $\overrightarrow{V_g}$ is designated geostrophic velocity and $\vec{\nabla}_p z = \vec{i}\frac{\partial z}{\partial x} + \vec{j}\frac{\partial z}{\partial y}$ is on a constant pressure surface.

Geopotential

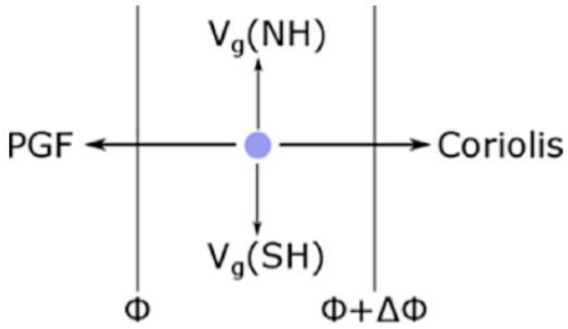
We can write these equations a little differently by using the concept of geopotential. Geopotential, *Φ*, is the potential energy per unit mass of an air parcel at a height *z*, where zero potential energy is defined at the surface (*z* = 0).

$$\Phi = \int_o^z g dz \tag{10.8.7}$$

$$d\Phi = g dz \tag{10.8.8}$$

In vector form, the velocities become:

$$\vec{V}_g = \frac{1}{f}\vec{k} \times \vec{\nabla}_p \Phi \tag{10.8.9}$$



Balance between Coriolis and pressure gradient forces and the resulting velocities of the wind in the Northern Hemisphere (NH) and Southern Hemisphere (SH).

Credit: W. Brune

A major advantage of using pressure coordinates is that the gradient of z or Φ is proportional to $\vec{V}_g$ for all pressure levels. This statement is not true for pressure gradients on height levels because you must know the density, ρ, as in Equations [10.24] and [10.25], which varies dramatically with height.

The following video (1:19) provides a good overview of pressure surfaces:

Pressure Surfaces

Click here for transcript of the Pressure Surfaces video.

> This schematic shows the relationship between height surfaces and pressure surfaces. Typically, pressure surfaces slope downward in height from the equator, where it is warmer, to the pole, where it is colder. You were able to show this in Activity 2.2, and you saw it again in Lesson 2.4 on thickness. Note that on the constant height surface, from the equator to the pole, the pressure surface is decreased with latitude. Now it also, on a constant pressure surface from the equator to the pole, the height surface is decreased on the constant pressure surface. Thus, changes in pressure are proportional to changes in height. After a little math, we can show that 1/rho-- that is, one over the density-- times the change in pressure with respect to x or y on a height surface is equal to g times the change in z with respect to x or y on a constant pressure surface. Finally, we note that gdz is just a differential of the geopotential phi, which has units of meters squared per second squared, which are the same units as energy divided by mass. So changes in height on a constant pressure surface are the same as changes in geopotential on a constant pressure surface.

10.9: Natural coordinates are better horizontal coordinates.

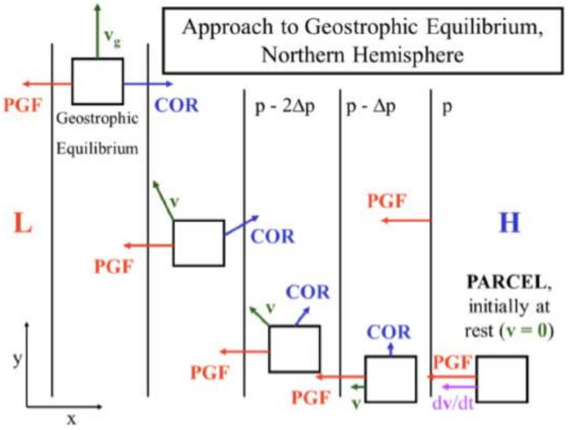
Understanding the results of a balance of forces can often be easier if we choose a horizontal coordinate system that is aligned naturally with the air flow, and not just set up in Cartesian coordinates x and y or spherical coordinates $\lambda$and φ. We can choose one direction—let's call it s—so that it is aligned with the streamline (and is thus always parallel with the flow) and increases in the direction of the flow (i.e, downwind). The second direction—let's call it n for normal—increases to the left of the flow. An interesting an noteworthy feature of natural coordinates is that the wind velocity is always positive because, by the definition of natural coordinates, the velocity vector is always pointed in the positive s direction.

For the horizontal momentum equation without friction:

10.10: A Closer Look at the Four Force Balances

Geostrophic Balance

This balance occurs often in atmospheric flow that is a straight line ($R = \pm\infty$) well above Earth's surface, so that friction does not matter. The Rossby number, R_O, is much less than 1. Let's think about how this balance might occur. Assume that an air parcel is placed in the midst of a fixed horizontal pressure gradient in the Northern Hemisphere and is initially at rest ($\vec{V} = 0$), as shown in the figure below. The Coriolis force is thus zero and the parcel begins to move from high pressure toward low pressure. However, as the parcel accelerates and attains a velocity perpendicular to the pressure gradient, the Coriolis force begins to increase perpendicular and to the right of the velocity vector and the PGF. The resulting acceleration is now the vector sum of the PGF and the Coriolis forces and turns the parcel to the right. As the velocity continues to increase, the Coriolis force increases but always stays perpendicular and to the right of the velocity vector while the PGF always stays perpendicular to the pressure gradient. Eventually, the PGF and Coriolis force become equal and opposite and the air parcel will move parallel to the horizontal pressure gradient. This condition is called **geostrophic balance**. We have simplified somewhat the approach to geostrophic equilibrium because, in reality, air parcels would overshoot and undergo inertial oscillations (discussed below) and because the pressure field would evolve in response to the motion.

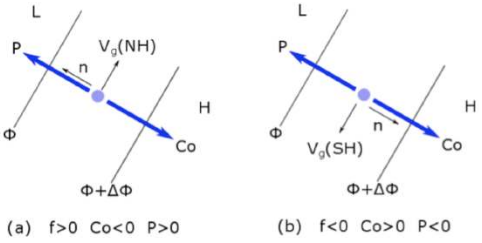


How geostrophic balance is achieved for an air parcel starting at rest. The PGF is always there, but the Coriolis force is zero until the air parcel acquires some velocity. In the figure, $\mathbf{v_g}$ is used to represent the geostrophic velocity. Credit: H.N. Shirer

$$-fV - \frac{\partial \Phi}{\partial n} = 0 \tag{10.10.1}$$

geostrophic wind balance

Let the Coriolis force per unit mass be designated as $Co = -fV$ and the pressure gradient force per unit mass as $P = -\frac{\partial \Phi}{\partial n}$. Then the force balances are shown in the figure below.



(a) $f>0$ $Co<0$ $P>0$ (b) $f<0$ $Co>0$ $P<0$

Geostrophic force balance in (a) the Northern Hemisphere and (b) the Southern Hemisphere shown in natural coordinates. Note that the *n*direction is always to the left of the velocity when looking downwind. In the figure, V_g is used to represent the geostrophic velocity. Credit: W. Brune (after R. Najjar and An Introduction to Dynamic Meteorology, Fifth Edition, J. R. Holton and G. J. Hakim, 2013)

Note that the Coriolis force is always to the right of the velocity vector in the Northern Hemisphere. It is always to the left of the velocity vector in the Southern Hemisphere. When the pressure gradient force and Coriolis force are in balance, the PGF is to the left of the velocity vector and the Coriolis force is to the right in the Northern Hemisphere. Watch the video below (1:10) for further explanation:

Into Geostrophic Balance

Click here for transcript of the into geostrophic balance video.

> Let's see how an air partial initially at rest achieves geostrophic balance. At rest, the air parcel velocity equals 0. And the only horizontal force acting on the parcel is the pressure gradient force, which has a constant magnitude and direction as long as the pressure gradient remains the same. As soon as the parcel has some velocity, the Coriolis force starts, perpendicular and to the right of velocity in the northern hemisphere. The Coriolis force begins to move the parcel to the right because the sum of forces on the parcel now has a y component. Note that the PGF is still always perpendicular to the pressure gradient, and the Coriolis force is always perpendicular to the velocity. Eventually, the PGF and the Coriolis force come into opposition with the velocity in between and Coriolis to the right of the velocity. In the end, the y component of the forces is 0 again so that the air parcel remains at the geostrophic velocity.

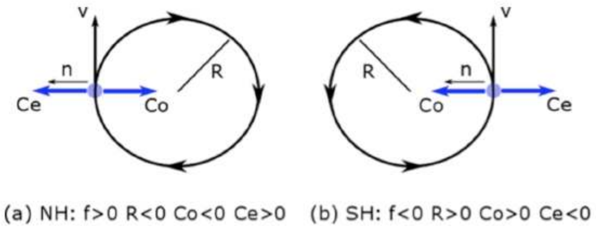
Inertial Balance

In this case, the pressure gradient force is minimal and the centrifugal and Coriolis forces are in balance.

$$-\frac{V^2}{R} - fV = 0 \tag{10.10.2}$$

inertial balance

Let the centrifugal force be designated by $Ce = -\frac{V^2}{R}$.



(a) NH: f>0 R<0 Co<0 Ce>0 (b) SH: f<0 R>0 Co>0 Ce<0

Inertial balance in (a) the Northern Hemisphere and (b) the Southern Hemisphere shown in natural coordinates. Note that the *n* direction is always to the left of the velocity when looking downwind. Credit: W. Brune (after R. Najjar)

We can manipulate Equation [10.37] to find the radius of the circle:

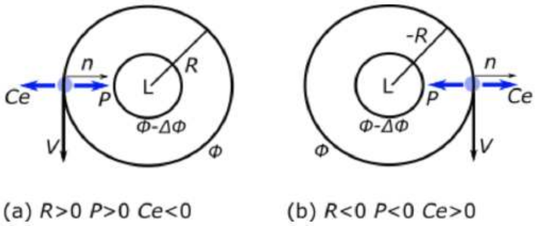
$$R = -\frac{V}{f} \tag{10.10.3}$$

For $f = 10^{-4}$ s^{-1} and $V = 10$ m s^{-1}, $R = -100$ km. Inertial balance is not a major balance in the atmosphere because there is almost always a significant pressure gradient, but it can be important in oceans.

Cyclostrophic Balance

The balance in this case is between the pressure gradient force and the centrifugal force.

$$-\frac{V^2}{R}-\frac{\partial\Phi}{\partial n}=0 \tag{10.10.4}$$



Cyclostrophic balance for (a) cyclonic flow and (b) anticyclonic flow in the Northern Hemisphere shown in natural coordinates. Note that the *n* direction is always to the left of the velocity when looking downwind. In the Southern Hemisphere, (b) is cyclonic and (a) is anticyclonic. Credit: W. Brune (after R. Najjar and An Introduction to Dynamic Meteorology, Fifth Edition, J. R. Holton and G. J. Hakim, 2013)

In this case, the scale of the motion is so small that Coriolis acceleration is not important. The Rossby number, R_o = centrifugal acceleration / Coriolis >> 1.

Examples of motion in cyclostrophic balance are tornadoes, dust devils, water spouts, and other small atmospheric circulations, such as the vortex you sometimes see when leaves get swept off the ground. These can be either cyclonic or anticyclonic and, in fact, a few percent of tornadoes in the Northern Hemisphere are anticyclonic. Another common example of cyclostrophic balance is the vortex seen when a bathtub or sink is draining.

NOAA Ship NANCY FOSTER dwarfed by waterspout. Gulf of Mexico. Summer, 2007. Credit: NOAA Photo Library via flickr

Gradient Balance

In gradient balance, the pressure gradient force, Coriolis force, and horizontal centrifugal force are all important. This balance occurs as wind in a pressure gradient field goes around a curve. There are many examples of this type of flow on any weather map —any synoptic-scale pressure gradient for which the isobars curve is an example of gradient flow.

$$-\frac{V^2}{R}-fV-\frac{\partial\Phi}{\partial n}=0 \tag{10.10.5}$$

To solve this equation for velocity, we can use the quadratic equation:

$$ax^2+bx+c=0 \quad\rightarrow\quad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \tag{10.10.6}$$

$$\frac{V^2}{R}+fV+\frac{\partial\Phi}{\partial n}=0 \quad\rightarrow\quad V=\frac{-f\pm\sqrt{f^2-4\left(\frac{1}{R}\right)\frac{\partial\Phi}{\partial n}}}{\frac{2}{R}} \tag{10.10.7}$$

$$V = -\frac{fR}{2} \pm \sqrt{\frac{f^2R^2}{4} - R\frac{\partial\Phi}{\partial n}} \tag{10.10.8}$$

$\frac{f^2R^2}{4}$ is always positive, so, for a given hemisphere (say, the Northern Hemisphere) there are eight possibilities because R can be either positive or negative, $\frac{\partial\Phi}{\partial n}$ can be positive or negative, and we have the ± sign in between the two terms on the right-hand side of Equation [10.40].

Gradient balance velocity solutions

Northern Hemisphere	$R > 0$	$R < 0$
$\frac{\partial\Phi}{\partial n} > 0$	no roots are physical	only positive root is physical
$\frac{\partial\Phi}{\partial n} < 0$	only positive root is physical	both roots are physical

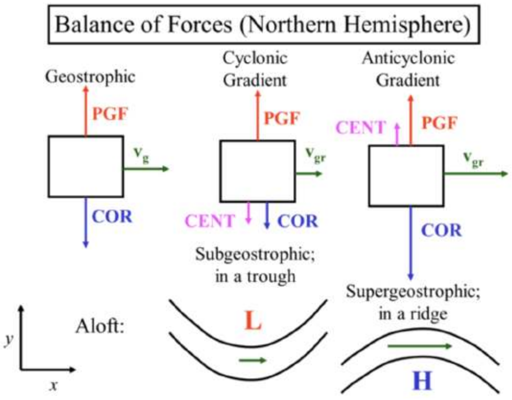
The table above gives the results for the Northern Hemisphere ($f > 0$). We are looking for whether positive or negative values of R and $\frac{\partial\Phi}{\partial n}$ give non-negative and real values for V because only non-negative and real values for V are physically possible. The reason why real negative values of V are not possible is because the gradient wind balance has been written down in natural coordinates.

- For $R > 0$ and $\frac{\partial\Phi}{\partial n} > 0$, is always negative, so there are no physical solutions.
- For $R > 0$ and $\frac{\partial\Phi}{\partial n} < 0$, only the plus sign gives a positive V and thus a physical solution.
- For $R < 0$ and $\frac{\partial\Phi}{\partial n} > 0$, only the plus sign gives a positive V and thus a physical solution.
- For $R < 0$ and $\frac{\partial\Phi}{\partial n} < 0$, both roots give positive V and thus physical solutions.

So there are four physical solutions. However, there is one more constraint. This additional constraint is that the absolute angular momentum about the axis of rotation at the latitude of the air parcel should be positive in the Northern Hemisphere (and negative in the Southern Hemisphere). Without proof, we state that only two of the four physically possible cases meet this criterion of positive absolute angular momentum in the Northern Hemisphere. They are:

1. Regular low: $R > 0$ and $\frac{\partial\Phi}{\partial n} < 0$ and $V = -\frac{fR}{2} + \sqrt{\frac{f^2R^2}{4} - R\frac{\partial\Phi}{\partial n}}$
2. Regular high: $R < 0$ and $\frac{\partial\Phi}{\partial n} < 0$ and $V = -\frac{fR}{2} - \sqrt{\frac{f^2R^2}{4} - R\frac{\partial\Phi}{\partial n}}$

These two cases are depicted in the second and third panels of the figure below.



Gradient balance in Northern Hemisphere. left: Geostrophic balance; center: regular low balance; right: regular high balance. Note that the PGF is independent of velocity but both the Coriolis force and the centrifugal force are dependent on velocity. In the figure, $\mathbf{v_g}$ is used to represent the geostrophic velocity (only the PGF and Coriolis force are important) and $\mathbf{v_{gr}}$ is used to represent the gradient wind velocity (the PGF, Coriolis force, and centrifugal force are all important). Credit: H.N. Shirer

The video below (3:22) explains these four force balances in more detail:

Four Force Balances

Click here for transcript of the Four Force Balances video.

Let's look at four force balances. Let's start which geostrophic balance, which occurs in straight line flow and [INAUDIBLE] troposphere. In geostrophic flow only the pressure gradient force and the Coriolis force are important. Pressure gradient points to low pressure on the height surface or low height and thus low geopotential on a constant pressure surface is opposed by the Coriolis force which is to the right of the velocity vector in the northern hemisphere and to the left of the velocity vector in the southern hemisphere. Note that we can find the geostrophic velocity if we know the pressure gradient on a constant height surface, or the geopotential or height gradient on a constant pressure surface. For inertial balance the Coriolis force is balanced by the horizontal centrifugal force with the Coriolis force to the right of the velocity vector in the northern hemisphere and to the left in the southern hemisphere. This balance is rarely seen in the atmosphere. Because there's almost always a pressure gradient force of the same magnitude as the centrifugal force and the Coriolis force. In cyclostrophic balance the pressure gradient force is balanced by the centrifugal force. In this case, the velocity vector can be either to the right or to the left of the centrifugal force in both hemispheres. And the Coriolis force is much smaller. This balance is seen in tornadoes and other small vortices. In the northern hemisphere, tornadoes are mostly cyclonic with only a few percent anticyclonic. While smaller vortices are about as often anticyclonic as they are cyclonic. For gradient wind balance the pressure gradient force, Coriolis force, and horizontal centrifugal force are all about equal. To two physical cases are shown for the northern hemisphere in the figure along with the geostrophic balance. With a cyclonic gradient-- that is curvature around the low pressure center-- the PGF points to the low and is constant as long as the pressure gradient is constant. In this case, the PGF is opposed by both the Coriolis force, which depends on the velocity, and the centrifugal force, which depends on the velocity squared. Since the PGF is constant then the sum of the centrifugal and Coriolis force must be equal to it. And since they both depend on velocity the velocity must be less than in the geostrophic case in order for them to be in force balance. This velocity is called subgeostrophic because it is less than the geostrophic velocity. For the anticyclonic gradient-- which is flow around a high-- the PGF points away from the high and is joined by the centrifugal force, which means that the Coriolis force must be stronger than in the geostrophic case because it must balance both the PGF-- which is the same in the geostrophic case-- and the centrifugal force. The Coriolis force can only be greater if the velocity is greater. Thus this velocity is called super geostrophic. Because it is greater than the geostrophic velocity.

10.11: See how the gradient wind has a role in weather.

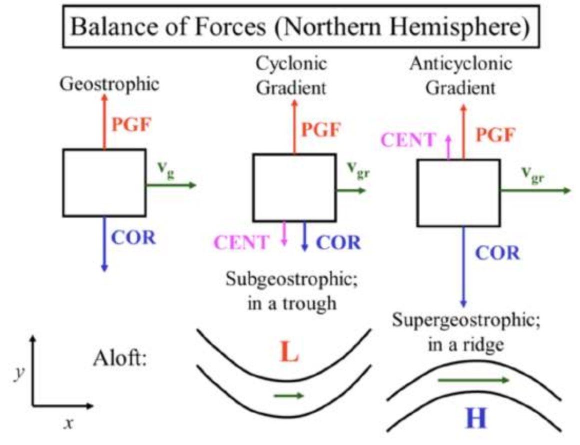
$$V_g = -\frac{1}{f}\frac{\partial \Phi}{\partial n} \tag{10.11.1}$$

Replacing the pressure gradient force $\left(\frac{\partial \Phi}{\partial n}\right)$ with $-fV_g$ in the gradient balance equation results in an equation that relates these gradient velocities to the geostrophic velocity:

$$\frac{V^2}{R} + fV - fV_g = 0 \quad \backslash)or\backslash(\quad \frac{V_g}{V} = 1 + \frac{V}{fR} \tag{10.11.2}$$

In a regular low (middle, figure below), $R > 0$ so that $V_g > V$. The velocity in a curve around a low-pressure area is *subgeostrophic*.

In a regular high (right, figure below), $R < 0$ so that $V_g < V$. The velocity in a curve around a high-pressure area is *supergeostrophic*.



Gradient balance in Northern Hemisphere. left: Geostrophic balance; center: regular low balance; right: regular high balance. Note that the PGF is independent of velocity but both the Coriolis force and the centrifugal acceleration are dependent on velocity. In the figure the geostrophic velocity is represented by $\mathbf{v_g}$ and the gradient wind velocity is represented by $\mathbf{v_{gr}}$.

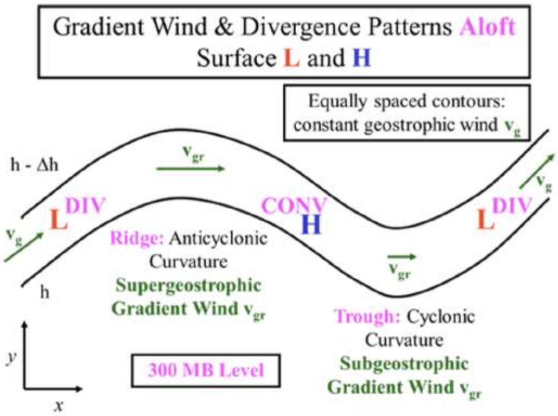
Credit: H.N. Shirer

Think of it this way. The pressure gradient force is independent of velocity and so is always there for a given geopotential gradient. In a regular low, the centrifugal and Coriolis forces, both dependent on velocity, sum together to equal the pressure gradient force, whereas for geostrophic flow, only the Coriolis force does. Thus, the velocity in the gradient balance case must be less than the geostrophic velocity for the same geopotential gradient.

So how do subgeostrophic and supergeostrophic flow affect weather?

Supergeostrophic flow around ridges and subgeostrophic flow around troughs helps to explain the convergence and divergence patterns aloft that are linked to vertical motions.

Look at the figure below, starting on the left. Going from geostrophic flow in the straight section to supergeostrophic flow at the ridge's peak causes divergence aloft. This divergence causes an upward vertical velocity, which causes a low pressure area and convergence at the surface. As the air rounds the ridge's peak, it slows down to become geostrophic, and then continues to slow down even more as the flow becomes subgeostrophic around the trough, thus causing convergence aloft. This convergence aloft causes a downward velocity, which causes high pressure and divergence at the surface.



Subgeostrophic and supergeostrophic velocities in flow aloft in the Northern Hemisphere.

Credit: H.N. Shirer

So downwind of a trough is the favored location for divergence aloft, upward motion, and a surface low. Downwind of a ridge is the favored location for convergence aloft, downward motion, and a surface high. Since ridges form around high pressure aloft and troughs form around low pressure aloft, we see that the high aloft is offset relative to the surface low and the low aloft is offset relative to the surface high.

Thus subgeostrophic flow and supergeostrophic flow aloft are directly related to the formation of weather at the surface. Other factors like vorticity are also very important. The video below (1:09) describes how the gradient wind flow aloft can affect surface weather.

Trough Aloft Surface Low Video

Click here for transcript of the Trough Aloft Surface Low video.

> Let's see how the gradient wind flow aloft can affect surface weather. Look at how the velocity changes as air flows around the ridge and then a trough aloft. Initially, the velocity is about geostrophic and straight-line flow. As it rounds the ridge, it speeds up. And then it slows down again to geostrophic in the straight section. As it goes through the trough, around the low pressure loft, it slows down to subgeostrophic and then speeds up to geostrophic in the next straight section. The speeding up causes divergence aloft. And the slowing down causes convergence aloft, just as you learned in lesson nine. You also saw how convergence aloft can lead to divergence at the surface. This contributes to a surface high. And how divergence aloft can lead to convergence at the surface, which contributes to a surface low. Thus, gradient flow contributes to surface weather. We often see a surface low forming on the downwind side of a trough.

10.12: Overview

To forecast the weather, we use numerical weather prediction models that are based on mathematical expressions for conservation of energy, mass, and momentum. Climate prediction models are based on the same conservation laws. Conservation simply means that the amount of a quantity such as total energy, mass, or momentum remains constant even though the forms of that quantity may change. Conservation of energy is described by the 1st Law of Thermodynamics, which was discussed in Lesson 2; conservation of mass and conservation of momentum are discussed in this lesson. The total mass of an air parcel is constant, but density and volume may change. The conservation of momentum is based on Newton's 2nd Law and involves forces that can change momentum. The conservation of momentum is relatively simple when cast in an inertial (non-accelerating) reference frame because there are only three real forces that really matter for atmospheric motion: gravity, the pressure gradient force, and friction. But when cast on a rotating Earth, we need to add apparent forces to this equation in order to compensate for the fact that an air parcel on Earth is always accelerating as the Earth rotates. We end up adding the apparent forces—Coriolis force and centrifugal force—to the real forces to get an equation of motion whose predictions we can readily match with our observations.

Some atmospheric motion occurs with air masses and waves that are thousands of kilometers across, while other motion, such as tornadoes, is at most a few kilometers across. Sometimes air flows in a straight line; sometimes it flows around ridges and troughs. In all of these different cases, the most important forces are different, allowing the momentum conservation equation to be simplified in different ways. We will discuss these different conditions and show how you can determine the wind velocity from knowledge of the balance of the most important forces and thus determine the impact of air motion at upper levels on the air motion near Earth's surface. Finally, we will describe why midlatitude winds are westerly.

Lesson Roadmap

Please see Canvas for a list of required assignments, due dates, and submission instructions.

Questions?

If you have any questions, please post them to the Course Questions discussion forum in Canvas. I will check that discussion forum daily to respond. While you are there, feel free to post your own responses if you, too, are able to help out a classmate.

10.13: Summary and Final Tasks

Summary

Understanding atmospheric dynamics is built upon three conservation laws: energy (the 1st Law of Thermodynamics), mass, and momentum. When we use conservation of momentum on the rotating Earth, we need to consider not only the real forces of gravity, pressure gradient force, and turbulent drag in the lower troposphere, but also apparent forces—centrifugal and Coriolis. With these terms, we can use conservation of momentum to write down the equations of motion in the Earth's reference frame and then show how they can be transformed into spherical coordinates or even pressure coordinates in the vertical and natural coordinates in the horizontal.

Using natural coordinates simplifies the equation of motion for geostrophic flow (the balance of Coriolis and pressure gradient forces), cyclostrophic flow (the balance of centrifugal and pressure gradient forces), inertial flow (the balance between centrifugal and Coriolis forces), and gradient flow (the balance among the pressure gradient force, Coriolis force, and horizontal centrifugal force). Upper air motion around high and low pressure causes upper-air convergence and divergence, which leads to high and low pressure at the surface.

Finally, the temperature decrease at each pressure level from tropics to the poles leads to a pressure gradient force that drives air toward the poles. The Coriolis force turns the air toward the east, creating westerlies observed in the midlatitudes of both hemispheres. This connection between the latitudinal temperature gradient and wind is expressed in the thermal wind equation. The thermal wind vector, which is the difference between the geostrophic winds at two different pressure levels, is parallel to isotherms, with cold air on the left in the Northern Hemisphere. If the geostrophic wind vector turns counterclockwise with height in the Northern Hemisphere, cold air advection is occurring in that layer of air. This relationship is a handy way to figure out if the advection is cold or warm.

Reminder - Complete all of the Lesson 10 tasks!

You have reached the end of Lesson 10! Double-check that you have completed all of the activities before you begin Lesson 11.

CHAPTER OVERVIEW

11: Atmospheric Boundary Layer

Learning Objectives

By the end of this chapter, you should be able to:

- draw the PBL and its diurnal variation
- perform Reynolds averaging on an equation and derive an equation for the turbulent parts
- explain kinematic fluxes
- show vertical motion using eddy fluxes
- explain turbulent kinetic energy (TKE) and its behavior
- sketch the surface energy budget for different conditions

11.1: How do these fluxes look?

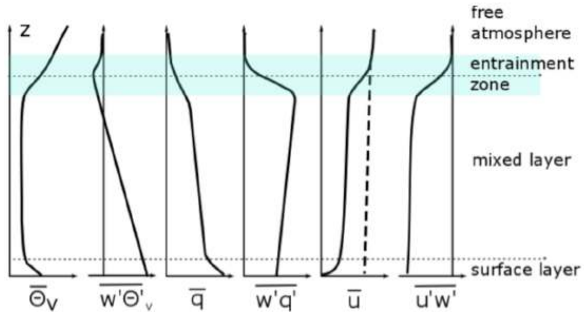


Figure: The mean values and turbulent vertical fluxes of virtual potential temperature, specific humidity, and horizontal momentum for daytime conditions. The top horizontal dashed line indicates the top of the boundary layer; the bottom horizontal dashed line indicates the top of the surface layer. The vertical dashed line is the calculated value of the geostrophic wind extending to the surface if there were no friction. The entrainment zone is an area of mixing between the mixed layer and the free atmosphere. Credit: W. Brune, after Deidonks and Tennekes, 1984

Check Yourself

First, start with some general observations:

- All these height variations scale with the PBL height, whether it be 1 km or 3 km.
- Eddy fluxes move quantities from higher to lower values. If a mean quantity increases with height, then the eddy flux will be downward (negative); if a mean quantity decreases with height, then the eddy flux will be upward (positive).
- The entrainment zone is a layer of mixing between the boundary layer and the free troposphere.

Before you read on, make sure that you believe these three concepts.

Virtual Potential Temperature, $\bar{\theta}_v$

The $\bar{\theta}_v$ profile is superadiabatic near the surface due to contact with the heated surface, neutral in the middle, and stable above. Eddies can rise from the surface to a height where $\boldsymbol{\bar{\theta}_v}$ equals its surface value (assuming no cloud formation and condensation). The mixed layer will grow if surface heating or increased humidity by evaporation causes the surface $\bar{\theta}_v$ to increase, which means that air parcels can rise and be neutrally buoyant at greater $\bar{\theta}_v$ aloft and thus greater heights.

We see that the eddy flux $\overline{w'\theta_v'}$ is greatest at the surface, decreases nearly linearly with height, becomes slightly negative above the PBL height *h* because eddies are bringing warmer air down from above. Remember, eddy fluxes carry a quantity like virtual potential temperature down the mean gradient. The eddy flux for virtual potential temperature (and all quantities) goes to very low values (essentially zero) above the entrainment zone even though a gradient is present because the eddy energy is much lower there.

Specific Humidity, $\bar{q}$

Specific humidity is greatest at the surface, where moisture sources, such as water bodies and vegetation, are present. The specific humidity falls off slowly with height until it reaches the PBL height, and then falls off rapidly into the free atmosphere. Because $\bar{q}$ falls off with height, the humidity flux $\overline{w'q'}$ increases with height until $\overline{w'q'}$ becomes small because $\bar{q}$ becomes small.

So what does the daytime convective boundary layer look like? There are isolated convective updrafts surrounded by slower descending air, giving rise to the large-scale eddy circulation, as seen in the following video of the convective boundary layer, viewed from the top. Associated with the large eddies are smaller eddies that come about as the upward and downward air parcels move past each other. If you look closely, you will see eddies of all sizes in the video (:24), some quite large and some quite small, but the smaller ones seem to originate in the larger ones.

Quiz 11-2: State of flux.

1. Find **Practice Quiz 11-2** in Canvas. You may complete this practice quiz as many times as you want. It is not graded, but it allows you to check your level of preparedness before taking the graded quiz.
2. When you feel you are ready, take **Quiz 11-2**. You will be allowed to take this quiz only **once**. Good luck!

11.2: Turbulent Eddies - A Cascade of Energy

If we start with the equation:

$$\frac{KE}{\rho} = \frac{1}{2}\left(u^2 + v^2 + w^2\right) \tag{11.2.1}$$

and we write each term as its mean and turbulent parts and then multiply all these terms out and take the Reynolds average, we can then apply the rules of averaging in Equation 11.2.1, and only two terms survive:

$$\frac{\overline{MKE}}{\rho} = \frac{1}{2}\left(\bar{u}^2 + \bar{v}^2 + \bar{w}^2\right) \tag{11.2.2}$$

$$\bar{e} = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right) \tag{11.2.3}$$

The first term is simply the kinetic energy associated with the mean wind. The second term is the kinetic energy associated with the turbulent wind and is called the **turbulent kinetic energy, or TKE**, for short.

Which size eddies have the most energy? We can look at the relative intensity of the different scales of wind by considering the energy associated with motions of different sizes. Remember that by the Taylor hypothesis, the size of the eddy and the period of the eddy are related so that large eddies have longer periods and smaller eddies have smaller periods.

So, relative spectral intensity is just the amount of kinetic energy associated with that size eddy and the eddy size is associated with a period required for the eddy to pass over a sensor (see figure below).

- The peak in energy at 100 hours is from fronts and weather systems as they pass over a location. The spatial scale of these phenomena is relatively large and is called the synoptic scale.
- The smaller peak at about 24 hours is the diurnal cycle of wind speed, which increases during the day and then decreases at night.
- There is often a mininum in energy (called a spectral gap) at a time scale of an hour, where the circulations or eddies are relatively weak.
- This smaller peak at about 0.1 to 0.01 hours is called the “turbulent scale." This peak is caused by production of turbulent kinetic energy by buoyancy production (i.e., convection) and shear production (i.e., viscous interaction of air masses with different velocities). These eddies have the time scales of minutes and the size of the PBL.
- As the period of the eddies decreases below about 0.01 hours (about a minute), the strength of the eddies decreases.
- Eventually, on the subsecond time scale, the eddies have very little energy indeed.

So what is happening? Energy is flowing from the larger-scale eddies to the smaller-scale eddies. Eventually, the energy is dissipated through viscosity, which is a molecular-scale process. So, the energy of the larger eddies is transferred into smaller eddies, and eventually that energy is lost to viscosity, which in turn generates heating.

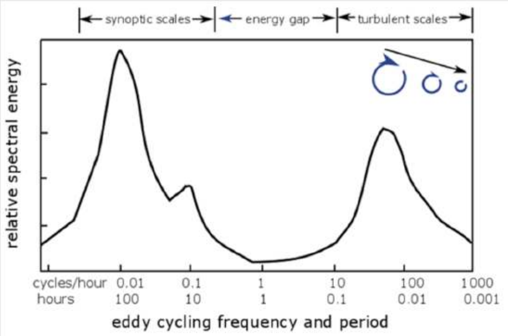


Figure 1: Relative spectral energy (i.e., energy per unit frequency) as a function of eddy cycling frequency or time period. As eddies generated at the PBL scale by convection and shear break down into smaller eddies, the energy is dissipated. Credit: W. Brune, after R. B. Stull's An Introduction to Boundary Layer Meteorology (1988), from a study of I. Van der Hoven (1957).

Lewis Richardson wrote a poem about this process for whorls (a.k.a. eddies) in 1922:

> *Big whorls have little whorls,*

> *Which feed on their velocity;*
>
> *And little whorls have lesser whorls,*
>
> *And so on to viscosity*
>
> *(in the molecular sense).*

11.3: The Surface Layer’s Energy Budget

In Lesson 7, we looked at the energy budget of the average atmosphere. Now let’s look at the energy budget in the surface layer. We can think of this budget in terms of energy fluxes and energy storage. As you learned in Lesson 2, energy is conserved—it must go somewhere.

There are several basic terms that we can put together in an energy equation that is appropriate for different environments or times of day. Upward fluxes will be positive; downward fluxes will be negative. The units are W m^{-2}.

The main energy terms are:

- Q_s: net radiation = Earth’s IR (up) – (solar + atmospheric IR) (down)
- Q_H: sensible heat flux (up or down)
- Q_E: latent heat flux (up or down)
- Q_G: surface heating (up or down)
- ΔQ: surface energy storage in trees, buildings, etc.

We will neglect the storage term, which is usually small compared to the fluxes.

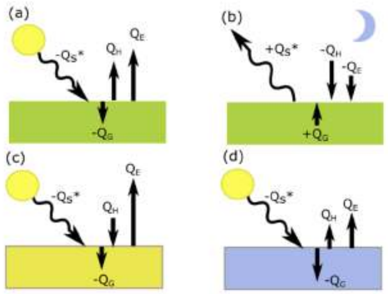
The ratio of sensible heat flux to latent heat flux is called the **Bowen ratio**:

$$B = \frac{Q_H}{Q_E}$$

It can be either positive or negative:

- > 10 over deserts,
- 2 – 6 over semi-arid lands,
- 0.4 – 0.8 over grasslands and forests,
- 0.1 – 0.3 in tropical rainforests,
- < 0.1 over oceans,
- negative over oases.

Looking at this list, it is easy to see that the water vapor comes not only from the surface moisture but also from trees and other plants by a process called **evapotranspiration**.



Energy balance for four different common cases: (a) daytime, land; (b) nighttime, land; (c) daytime, a desert oasis; and (d) daytime ocean. Arrow lengths indicate relative magnitude of the different fluxes.

Credit: W. Brune (after R. Stull)

Look at the figure above, which shows the relative values of energy fluxes for different environments and times of day:

- **(a) daytime:** solar heating dominates; heating is going into upward latent and sensible heating and warming the ground. The Bowen ratio (ratio of the sensible to the latent heat fluxes) depends on the amount of vegetation and surface moisture. The heating of the ground’s surface causes a downward energy flux into the ground.
- **(b) nighttime:** no solar shortwave; radiation is all infrared and is less than during the day, which has a net loss of energy to space. Both latent and sensible heat fluxes are negative because the ground is cooling and water vapor is condensing, leading to

a positive gradient in temperature and specific humidity. The subsurface ground is warmer than the surface, so there is an upward energy flux from the subsurface to the surface.

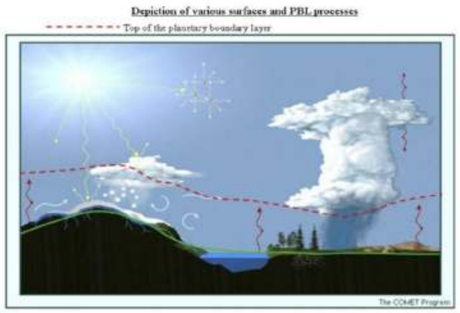
- **(c) daytime oasis with warm air advection over a moist surface:** solar heating, sensible heat flux downward from hot dry air, latent heat flux upward offsetting some of the heating from the solar flux and the downward sensible heat flux. The surface is still heating, so some energy flux goes into the ground. The oasis is cooler than the surrounding desert because of the large latent heat flux.
- **(d) daytime ocean:** sensible and latent heat fluxes are positive, but much of the downward surface flux carries significant energy down into the ocean away from the surface. The ocean has a lot of heat capacity, so that solar energy can be absorbed in the ocean with little temperature change. Most of the energy goes into the ocean, with fairly constant fluxes of latent heat and sensible heat.

✎ Quiz 11-3: Energy in the boundary layer.

1. Please note: **there is no practice quiz** for Quiz 11-3.
2. When you feel you are ready, take **Quiz 11-3** in Canvas. You will be allowed to take this quiz only **once**. Good luck!

11.4: The atmospheric boundary layer is your home.

The **atmospheric boundary layer (ABL)** is the tropospheric layer that is directly influenced by the presence of Earth’s surface and responds to surface forcings in an hour or less. It is also called the *planetary boundary layer* or just the *boundary layer*. The atmospheric boundary layer is typically 1 km deep during the day and ~100 m deep during the night. Above the boundary layer is the free atmosphere. We live in the atmospheric boundary layer.



Mid-day planetary boundary layer over different surfaces.

Credit: NOAA Earth System Research Laboratory

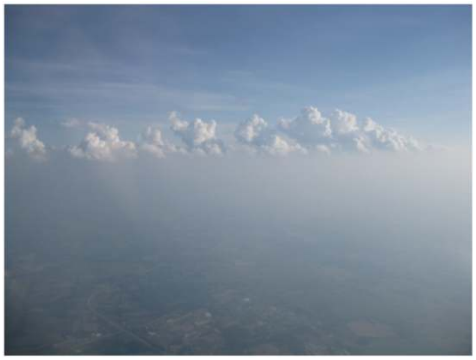
The surface forms a boundary for the atmosphere and is actually responsible for the existence of the planetary boundary layer.

The surface influences the atmosphere in three main ways:

- sensible heat flux
- moisture flux from the surface to the atmosphere
- radiation

The water vapor that is in the air came from the evaporation of liquid water or sublimation of ice in or on the surface. Once that water vapor is in the atmosphere, it can condense, thus providing energy that heats the air and creates buoyant energy, as you learned in Lesson 3.

The picture below was taken on a sunny summer day over Maryland. What do you see? Starting at the bottom, you can barely make out the buildings and the roads because the air is so uniformly hazy. The haze extends all the way up to the lower parts of the fair weather cumulus clouds, which seem to be bobbing on top of the haze, like Styrofoam peanuts bobbing on top of a pool of water. Above the clouds, it’s blue sky with some thin cirrostratus clouds. This haze layer is the atmospheric boundary layer and the fair weather cumulus clouds mark its top.



Fair weather cumulus riding on top of a hot hazy summer atmospheric boundary layer, somewhere over Maryland.

Credit: W. Brune

Where did the haze come from? How did the clouds form? How did the haze get to be so uniform? Why are the fair weather cumulus clouds bobbing on top with blue sky above? You already know answers to the first two questions. The haze is small particles that came from the surface or were made in the atmosphere by gas-to-particle conversion and then swelled in the hot humid air. The clouds come from moist air containing aerosol that rises and cools until its supersaturation is sufficient to form cloud drops. The condensation releases energy, creating buoyancy, and the clouds rise until they reach their **level of neutral buoyancy (LNB)**. You will learn in this lesson why the haze is so uniform (turbulent mixing) and why the clouds bob on top (a stable layer with increasing potential temperature above the boundary layer).

There are three basic ways that air can be moved: the mean wind, waves, and turbulence. You have already learned about the mean wind and its causes, but mostly we have dealt with the wind in the free troposphere. In the boundary layer, transport of moisture, heat, and momentum are dominated by the mean wind in the horizontal and by turbulence in the vertical. Turbulence is a much more persistent part of boundary layer flow than it is of flow in the free atmosphere.

For any variable over time or space, we can split that variable into two values—the *mean* and the *perturbation*. We will see more about this later.

Turbulent transport consists of swirling motions called eddies. These eddies occur in a range of sizes, and they can have different intensities, meaning wind velocities. We can plot the turbulent intensity as a function of size of the eddies to get a turbulence spectrum. To get an idea about the behavior of eddies and clouds, watch the following short video of cloud formation.

Extra Credit Reminder!

Here is your **last chance** to earn **one point of extra credit via Picture of the Week**!

1. You take a picture of some atmospheric phenomenon—a cloud, wind-blown dust, precipitation, haze, winds blowing different directions—anything that strikes you as interesting.
2. Add a short description of the processes that you think are causing your observation. A Word file is a good format for submission.
3. Use your name as the name of the file. Upload it to the **Picture of the Week Dropbox** in this week's lesson module. To be eligible for the week, your picture must be submitted by 23:59 UT on Sunday of this week.
4. I will be the sole judge of the weekly winners. A student can win up to three times.
5. This is your last chance to enter!

11.5: A Day in the Life of the Boundary Layer

The boundary layer is not frozen in time but instead changes dramatically during the course of the day. Let's start with the midday when the boundary looks like the hazy scene over Maryland (figure in 11.1). The boundary layer consists of a mixed layer that is stirred by solar heating of the surface and convection of warm moist air that pops up sporadically from place-to-place and time-to-time, and, as a result, mixes the air within the boundary layer. This convective stirring takes about ten to twenty minutes to go from bottom to top. As the air bubbles up, it mixes with the air surrounding it and with the air from the free troposphere at the top, thus creating an entrainment zone, which is where the clouds are.

? Exercise

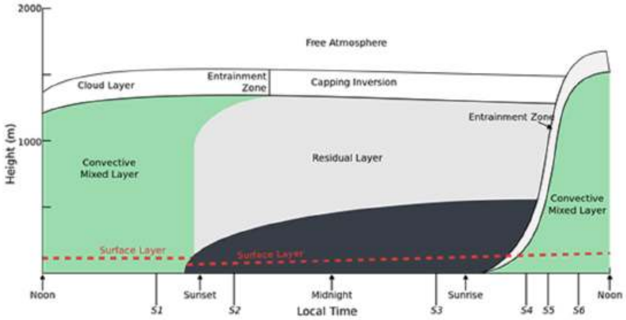
Does ten to twenty minutes for boundary layer vertical stirring make sense?

Click for answer.

You learned in Lesson 2 that buoyant acceleration equaled the gravity times the difference between the air parcel virtual temperature and the virtual temperature of its surroundings divided by the virtual temperature of its surroundings. Let's assume that temperature difference between an air parcel above a heated surface and its surroundings is 0.1 K, which seems pretty reasonable, and that the temperature is 300 K. The buoyant acceleration, B, is just 9.8 m s^{-2} times 0.1/300, or 0.0033 m s^{-2}. So, if the initial air parcel velocity is 0 m s^{-1} and the top of the PBL, z_0, = 1 km, then since $z_0 = 1/2\ Bt^2$, where t is time, then t is the square root of $2z_0/B \sim 13$ minutes. So now you can see that it takes a very small virtual temperature difference to stir the planetary boundary layer.

As the sun sets, the solar heating of the surface and the convection and associated turbulent eddies cease. Air from the surface no longer mixes with air throughout the convective boundary layer, and the air that was mixed during the day stays above the much lower nighttime stable boundary layer in a layer called the residual layer. Any gaseous or particle emissions from the surface are mixed within this nocturnal boundary layer. Because convection ceases at night, the winds in the residual layer are no longer affected by the friction caused by convection and they accelerate in the presence of a horizontal pressure gradient. So, the residual layer winds accelerate, blowing harder across the top of the more stagnant nocturnal boundary layer and a shear develops. This shearing is unstable and creates turbulence that mixes the boundary layer air and the residual layer air near the interface, so the nocturnal boundary layer grows a little during the night.

In the morning, the sun returns to heat the surface and to start driving convection and mixing again. This convection bubbles up, bumping into and entraining air from the residual layer. As the solar heating increases, the convection has more energy and can rise higher and entrain more air from the residual layer. Eventually, the air driven by convection reaches its maximum energy level and this maximum energy limits how high the boundary layer will grow into the stable free troposphere above it.



Diurnal evolution of the atmospheric boundary layer. The black region is the stable (nocturnal) boundary layer. Time markers S1–S6 are used in lesson 11.3. After R. B. Stull's An Introduction to Boundary Layer Meteorology (1988).

Credit: NikNaks (Own work, based on [1]) [CC BY-SA 3.0], via Wikimedia Commons

The following video explains the variation of the planetary boundary layer over the course of a typical day:

PBL Diurnal

Click here for transcrip

Let's look at the varia tart midday when the sun
is out and solar heatin ntial temperature matches
that of the overlying neutral buoyancy. In the
process, they entrain rcels collide with the air
above them and rub a s in mixing. These large,
buoyant eddies cycle i solar energy to power the
convection that stirs t al layer that contains the
mixed layer air that w er, but the boundary layer
height is much lower. With less turbulence in the residual layer, the air can speed up. The faster moving air above the slower moving air in the boundary layer causes a shear to develop between the two air masses. And sporadically, turbulence is generated when the shear breaks down, mixing air and increasing the boundary layer height. At sunrise, solar heating again begins to warm the surface, and the warm parcels rise up entraining residual layer air until eventually, the mixed layer reaches its maximum height again.

Summing Up

Let’s summarize the diurnal behavior of the boundary layer with a bulleted list of technical terms:

Mixed Layer (Convective Boundary Layer):

- turbulence driven by convection (large eddies or thermals)
- heat transfer from solar heating of the ground to the atmosphere
- mixed layer grows by entrainment of air from above it
- virtual temperature nearly adiabatic in middle; superadiabatic (i.e., potential temperature decreases with height) near surface; subadiabatic (i.e., potential temperature increases with height) at top, where exchange of air between the ABL and the free troposphere occurs
- wind speeds are sub-geostrophic in mixed layer, crossing isobars because of turbulent drag

Surface Layer

- directly in contact with Earth’s surface
- usually has vertical gradients in potential temperature, water vapor, and other quantities
- logarithmic wind speed profile with height, with low wind speed near ground
- typically is ~10% of the mixed layer

Residual Layer

- disconnected from boundary layer and Earth’s surface
- neutrally stratified, with small but near-equal turbulence in all directions
- contains moisture and trace atmospheric constituents from the day before

Stable Boundary Layer

- statically stable with weaker turbulence that occurs sporadically
- winds aloft may increase to supergeostrophic speeds (low-level jet or nocturnal jet)
- stability tends to suppress turbulence, except for occasional shear-generated turbulence caused by the low-level jet

---

11.6: The Story of Diurnal Boundary Layer Growth Told in Vertical Profiles of Virtual Potential Temperature

Recall the concept of virtual potential temperature, which was introduced in Lesson 2. The virtual potential temperature is found by replacing the temperature in the formula for virtual temperature with the potential temperature:

$$T_v = T(1 + 0.61q) \tag{11.6.1}$$

$$\theta = T\left(\frac{p_o}{p}\right)^{0.286} \tag{11.6.2}$$

$$\theta_v = \theta(1 + 0.61q), \tag{11.6.3}$$

for unsaturated air

θ_v is the virtual potential temperature. It is a useful quantity because it takes the moisture into account as well as temperature when considering buoyancy and stability. Thus, adiabatic ascent or descent in moist air follows the constant virtual potential temperature line all the way up to the **lifting condensation level** (LCL), where the potential temperature increases. So let's look at the evolution of the profile of virtual potential temperature in a cloud-free boundary layer.

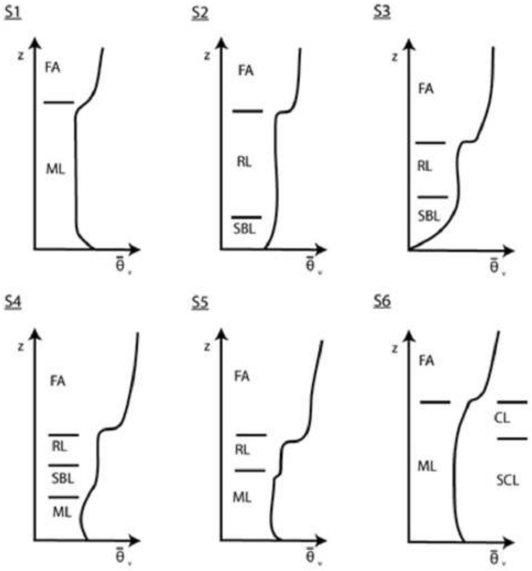


Figure: Boundary layer diurnal evolution as told by virtual potential temperature profiles. For context, see figure in lesson 11.2. ML = mixed layer, FA = free atmosphere, SBL = stable boundary layer, RL = residual layer. See S1 = late afternoon, S2 = just after sunset, S3 = just before sunrise, S4 = just after sunrise, S5 = mid-morning, S6 = late morning. From R. B. Stull's An Introduction to Boundary Layer Meteorology (1988).

Start with the late afternoon (S1 above). Surface heating causes the air near the surface to have a higher virtual potential temperature than the air just above it, so that the air is superadiabatic. Thus, air parcels of this warm moist air rise all the way up to the point in the free atmosphere where the virtual potential temperature is as great or greater than the air parcel's value. At this point the air parcel likely mixes with the surrounding air and therefore contributes to raising the mixed layer's temperature. As convection of air parcels continues, the slightly cooler air from the upper boundary layer sinks around the rising air parcels and continues the mixing process as the air parcels going up encounter air coming back down. The result is a well-mixed boundary layer.

Just after sunset (S2 above), the surface cools by infrared radiation and the air near the surface becomes stable with a virtual potential temperature (and thus temperature) inversion that prevents air from the surface to rise. As a result, the nocturnal boundary layer becomes quite stable, and as the surface continues to cool, the boundary layer becomes even more stable during the night (S3 above). Note also that the stable boundary layer grows, not by convective mixing, but by shear mixing.

Just after sunrise (S4 above), the surface is warmed by the sun, and as it continues to get warmed throughout the morning (S5 above), convection begins to mix warm air up first throughout the stable boundary layer and then into the mixed layer. Any trace atmospheric constituents left in the residual layer from the day before are now mixed back into the boundary layer as it grows higher. Finally, the residual layer is entirely mixed into the growing boundary layer (S6 above), and the boundary layer returns to its condition of the previous afternoon (S1 above).

Air pollution over Cairo, Egypt. Credit: Nina Hale via flickr

Think about morning and evening rush hours. During morning rush hour, which is near sunrise, the vehicle emissions mix into a shallow boundary layer and hence the mixing ratios of the pollutants can be quite substantial. This situation leads to the photochemistry that makes pollution, including ozone. In the evening, the rush hour traffic also emits the same amounts of pollutants into the boundary layer, but because the boundary layer height is so much greater in the early evening than in the morning, the pollutant mixing ratios are less because the same level of emissions are being mixed into a larger volume of air. Thus the effects of evening emissions are not as severe as the effects of morning emissions. In addition, modeling the planetary boundary layer height correctly is essential for accurate air quality modeling.

The diurnal variation of the planetary boundary layer height is more than just a curiosity - it has influence on our daily lives and health because we live, work, and breathe mostly in the atmospheric planetary boundary layer. Thus it is important that meteorologists and atmospheric scientists gain a better understanding of the atmospheric motions and energy budget of the planetary boundary layer. Gaining this understanding means learning something about atmospheric turbulence, which is the small-scale chaotic winds that are a significant factor in the planetary boundary layer.

11.7: Frozen - The Taylor Hypothesis

We would like to be able to take snapshots of the eddies in three dimensions and measure all their sizes each instant. Unfortunately, we do not have a good way to do this. Instead, we can simply measure the fluctuations of a variable such as wind speed, specific humidity, or temperature with a sensor at one location for a period of time. In this way, we watch the eddies drift by the sensor. But the eddies could be changing size and shape as they drift by the sensor. Let's put this physical concept in the context of the total derivative.

Take a variable like temperature, T. We know that the change in T with time at any location (such as where a sensor might be placed) is the sum of the total derivative and the **temperature advection**:

$$\frac{\partial T}{\partial t} = \frac{DT}{Dt} - \vec{U} \cdot \vec{\nabla} T \tag{11.7.1}$$

The temperature advection is the change in temperature at the sensor due to the advection of warmer or colder air past the sensor. The total derivative is the change in temperature of an air parcel moving past the sensor. Such a temperature change may be caused by any number of processes, such as the absorption or emission of radiation, condensation or evaporation (latent heating or cooling), or compression and expansion. **Taylor's hypothesis** says that we can assume that the turbulent eddies (which we can think of as large air parcels) are frozen as they advect past the sensor and thus the change in temperature within each eddy is negligible:

$$\frac{DT}{Dt} \sim 0 \tag{11.7.2}$$

so that:

$$\frac{\partial T}{\partial t} = -\vec{U} \cdot \vec{\nabla} T \tag{11.7.3}$$

Local temperature gradients, which might be present from one side of an eddy to another, are advected across the sensor by the mean wind without the eddy changing.

When is this condition valid? Experiments suggest that this hypothesis is valid when the variation of the wind speed due to turbulence is less than ½ of the mean wind speed.

We start this study with methods to separate wind motion driven by larger scale processes, such as gradient flow or geostrophic flow, from turbulence.

Anemometers. A cup anemometer and wind vane are on the left. A sonic anemometer, which uses sound to measure vertical as well as horizontal winds more than ten times a second, is on the right. You may have seen sonic anemometers at state weather stations along highways. Sonic anemometers are so fast that they are great for measuring turbulence and turbulent transport. Credit: Department of Energy Atmospheric Radiation Measurement Carbon Program

11.8: Here’s How Reynolds Did Averaging

For any variable, the observed value can be written as a sum of the mean value and a turbulent value:

$$u=\bar{u}+u^{\prime} \tag{11.8.1}$$

where $\bar{u}$ is the mean, or average value, and $u^{\prime}$ is the turbulent part.

The average can be obtained by averaging u over time or over space or even by doing a number of samples and averaging over the samples.

- **temporal averaging**:

$$\bar{u}=\frac{\int u(t) d t}{\int d t}=\frac{\sum_{i=0}^{N-1} u\left(t_i\right)}{N} \tag{11.8.2}$$

- **spatial averaging**:

$$\bar{u}=\frac{\int u(x) d x}{\int d x}=\frac{\sum_{i=0}^{N-1} u\left(x_i\right)}{N} \tag{11.8.3}$$

If the turbulence does not change with time and is homogeneous (i.e., the same in all directions and for all time), then these averages equal each other.

In Lesson 10, we developed the equation of motion without really considering the short-term and small-scale variations, except to say that they led to turbulent drag, which acts to resist the mean flow in the upper boundary layer. Now we want to think about how to correctly capture the dynamic effects of turbulent motion. What we want to do is to write down the equations of motion that you learned in Lesson 10; substitute mean and turbulent parts for the variables such as *u*, *v*, and *w*; average over all the terms; and then see if we can sort out the terms to create an equation for the mean wind and an equation for the turbulent wind. This type of averaging is called **Reynolds averaging**.

But first we need to learn the rules for averaging.

Rules of Averaging

c is constant; u and v are variables

$$\overline{u^{\prime}}=0 \tag{11.8.4}$$

$$\overline{c u}=c \bar{u} \tag{11.8.5}$$

$$\overline{u+v}=\bar{u}+\bar{v} \tag{11.8.6}$$

$$\overline{(\bar{u} v)}=\bar{u} \bar{v} \tag{11.8.7}$$

$$\overline{\left(\frac{\partial u}{\partial t}\right)}=\frac{\partial \bar{u}}{\partial t} \tag{11.8.8}$$

Now let’s apply these rules to a variable with a mean and a turbulent part. For example, consider the product *uv*, which is just the advection of horizontal wind in one direction by the horizontal wind in the other direction.

So, using the rules:

$$\begin{aligned} \overline{u v} &=\overline{\left(\bar{u}+u^{\prime}\right)\left(\bar{v}+v^{\prime}\right)} \\ &=\overline{\bar{u} \bar{v}}+\overline{\bar{u} v^{\prime}}+\overline{u^{\prime} \bar{v}}+\overline{u^{\prime} v^{\prime}} \end{aligned} \tag{11.8.9}$$

and

$$\overline{\bar{u} \bar{v}}=\bar{u} \bar{v} \tag{11.8.10}$$

$$\overline{\bar{u} v^{\prime}}=\bar{u} \overline{v^{\prime}}=\bar{u} \cdot 0=0 \tag{11.8.11}$$

$$\overline{u'\bar{\nu}} = \overline{u'\bar{\nu}} = 0 \cdot \bar{\nu} = 0 \tag{11.8.12}$$

so

$$\overline{uv} = \bar{u}\bar{v} + \overline{u'v'} \tag{11.8.13}$$

This second term, a product of two turbulent terms is *not necessarily zero*! Whereas the average of one turbulent term is zero by definition, the average of two turbulent terms is not necessarily zero.

The following video (3.11) further describes Reynolds averaging:

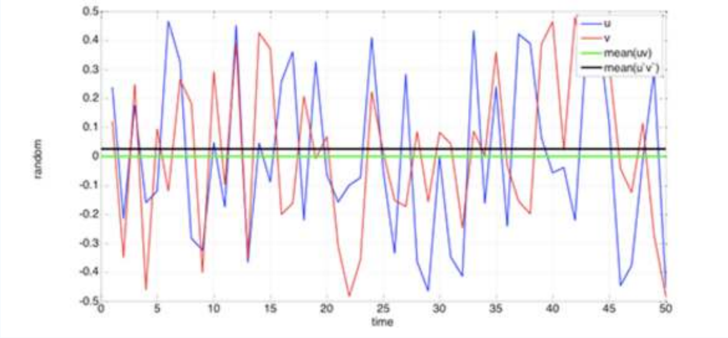
Reynold's Averaging

Click here for transcript of the Reynold's Averaging.

> Reynold's averaging is really pretty straightforward once you understand the rules. Each variable has an average and a perturbation, or turbulent, part. We need to determine the time over which we want to find the average. But after we do that, we can average all the values and then subtract the average from each individual value in order to find the perturbed or turbulent or fluctuation part of that value. The average the average value is, of course, the same for all the values in the average. I will use the words "mean" and "average" interchangeably for the noun meaning average. And we'll use the words like perturbation, fluctuation, and turbulent part to describe the variations of individual values about the average value. The rules are pretty simple. First, the average of a perturbed or turbulent term is 0, because if it were not, then the average value would be incorrect. Second, the average of the product of a constant times a variable is just a product of the average of the constant times the average of the variable. The average of the sum of two variables is just the sum of the average of the two variables. And the average of a product of the average value of one variable and another variable is just a product of the averages of the two variables. Note that the average of a variable is just a constant. Be careful. We will soon see that the average of the product of two variables is not just the product of the average of two variables. Finally, the average of the derivative of a variable is just a derivative of the average of the variable. Let's calculate the Reynold's average of one term of the equation for kinetic energy, which is just 1/2 mv squared. If we divide by the air density, then we have the kinematic kinetic energy. Each term can be written as its mean in turbulent parts. Let's look only at the u term. The v and w terms can be calculated in the same way. So we multiply all the terms out, then take the Reynold's average and apply the rules. Average values of average values are just average values. Because an average value is a constant, we get two terms of a constant times the average of the perturbed term, which is just 0. When we are done, we see that we have two terms left-- the average u squared, and the perturbation term squared. You can make a simple model with a random number generator to demonstrate the average of the product of two perturbation terms is not necessarily 0. This calculation was chosen so that the averages for u and v were 0. And so the average of the product of average u and average v is 0, but the average of the perturbations is not 0.

Example 11.8.1

Consider two random numbers varying between –0.5 and 0.5, called *u* and *v*. The figure below shows *u*, *v*, the mean of *uv* and the mean of *u'v'*. Of course, the mean of *u'v'* might be zero, but is it not necessarily zero, as shown in this figure.



Time series of two random variables whose means are zero (*u* in blue and *v*in red). The green line is the product of the mean of *u* and the mean of *v*; this product equals zero. The black line is the mean of the product *u'v'*; this product is not zero! Credit: W. Brune

The same thinking applies to u^2.

$$\overline{u^2}=\overline{(\bar{u}+u')(\bar{u}+u')}=\ldots=\bar{u}^2+\overline{u'^2} \tag{11.8.14}$$

Remember your statistics and the concept of variance:

$$\sigma_u^2=\frac{1}{N-1}\sum_{i=0}^{N-1}(u_i-\bar{u})^2\approx\frac{1}{N}\sum_{i=0}^{N-1}(u_i-\bar{u})^2=\frac{1}{N}\sum_{i=0}^{N-1}(u_i')^2=\overline{u^2} \tag{11.8.15}$$

So, the variance is the same as the mean value for the square of the turbulent part of the variable.

The covariance of *u* and *v* is given by the equation:

covariance

$$(u,v)=\overline{u'v'} \tag{11.8.16}$$

We can get a better sense of how large this variance is by dividing by the mean value:

$$\mathrm{I}=\frac{\sigma_u}{\bar{u}} \tag{11.8.17}$$

11.9: How Kinematic Fluxes Move Air Vertically

Now that we know how to separate out the mean and turbulent parts, we can look at the concept of **kinematic flux**. A flux is the transfer of some variable per unit area per unit time. Generally in meteorology, we care about variables like mass, heat (i.e., temperature), kinetic energy, moisture, momentum. Those who study the atmosphere's composition are also interested in the flux of chemicals emitted into the atmosphere from the surface and the flux of atmospheric pollutants, such as ozone, back to Earth's surface. For this discussion, let's consider only the fluxes directly relevant to meteorology.

So the Sl units of the following fluxes would be $\mathrm{kgm^{-2}s^{-1}}$ for mass (m) $\mathrm{kg_{water}\,m^{-2}s^{-1}}$ for moisture (m_y) ; $(\mathrm{kg\,ms^{-1}})\,\mathrm{m^{-2}s^{-1}} = \mathrm{kg}$ $\mathrm{m^{-1}s^{-2}}$ for momentum (mv), and $\mathrm{Jm^{-2}s^{-1}}$ for heat (mc_pT_v) or $\mathrm{Wm^{-2}}$ (just like radiation).

We can write these in the kinematic form by dividing by the density of air:

- kinematic mass flux $= (\mathrm{kg\,m^{-2}\,s^{-1}})\,/\,(\mathrm{kg\,m^{-3}}) = \mathrm{ms^{-1}}$
- kinematic moisture flux $= (\mathrm{kg_{water}\ m^{-2}s^{-1}})\,/\,(\mathrm{kg_{air}\ m^{-3}}) = \mathrm{kg_{water}\ kg_{air}^{-1}\ ms^{-1}}$
- kinematic momentum flux $= \mathrm{kgm^{-1}s^{-2}}/\,(\mathrm{kgm^{-3}}) = \mathrm{m^2s^{-2}}$

For **kinematic heat flux**, the heat flux is usually divided by the air density times the specific heat of air to give units of $(\mathbf{Jm^{-2}s^{-1}})\,/\,(\mathbf{kgm^{-3}Jkg^{-1}K^{-1}}) = \mathbf{Kms^{-1}}$. Note that this is just a temperature times a velocity.

The word "kinematic" is often dropped and assumed.

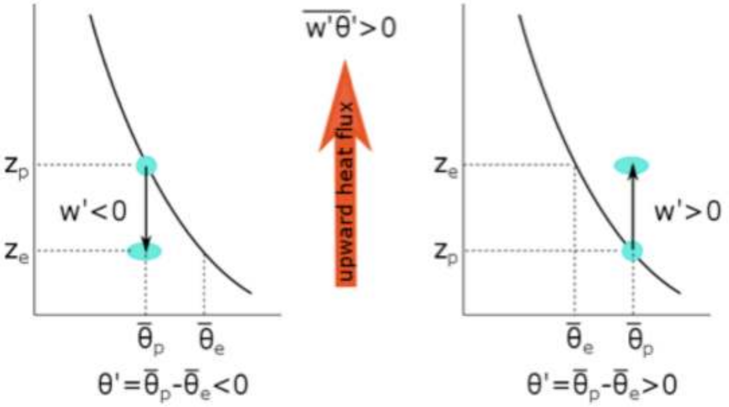
Let's look at the kinematic heat flux in the vertical direction. Also, we are interested in the vertical transport of thermal energy and so we do not want to think about the adiabatic variation in temperature due to vertical motion, since an air parcel's temperature decreases with height even in adiabatic ascent. We can accomplish this by using the potential temperature and not the virtual temperature, which falls off with height adiabatically while the potential temperature is constant with height. θ does not equal T except at 1000 hPa, so we need to keep that in mind.

Write down the expression for the vertical flux of potential temperature, which is just equal to the vertical wind times the potential temperature, and then take the Reynolds average of that flux:

$$\overline{w\theta} = \bar{w}\bar{\theta} + \overline{w'\theta'} \tag{11.9.1}$$

The first term on the right is the vertical flux of potential temperature due to the average vertical motion, but w ~ 0 on average in the boundary layer, so we can usually drop the first term on the right.

Let's consider how this flux works at moving thermal energy. A typical condition for the mixed boundary layer is a superadiabatic layer near the heated surface (i.e., θ decreases with height).



How turbulence is able to move heat vertically by turbulent heat flux for the daytime case with surface heating. Subscripts p and e refer to parcel and environment, respectively. θ' is always θ of the parcel before it moves minus θ of its new environment at its new height. Eddies move air both down (left panel) and up (right panel). In both cases, the net heat flux is up. Note that eddies cycle in the vertical and so they typically move air both up and down as shown in the panels at the same time. Credit: W. Brune

Let's look at the different cases.

Daytime case with surface heating such that$\partial\theta/\partial z < 0$(as in figure above). Eddies can make air trade places. They move some air parcels down,

$$\text{for}$$

$$w' < 0 \tag{11.9.2}$$

$$\text{and}$$

$$\theta' < 0 : \overline{w'\theta'} > 0 \tag{11.9.3}$$

and at the same time move some up,

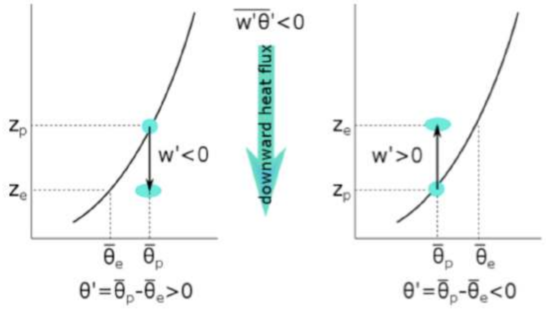
$$\text{for}$$

$$w' > 0 \tag{11.9.4}$$

$$\text{and}$$

$$\theta' > 0 : \quad \overline{w'\theta'} > 0 \tag{11.9.5}$$

Both of these processes are happening simultaneously. If the air parcels mix at their new heights, they move cooler air lower (left panel above) or warmer air higher (right panel above) and in both cases, they move heat up. Thus, when the potential temperature decreases with height (i.e., is superadiabatic), the turbulent eddies move warmer air up and the heat flux is positive.



How turbulence is able to move heat vertically by turbulent heat flux for the nighttime case with surface cooling. Subscripts p and e refer to parcel and environment, respectively. θ' is always θ of the parcel before it moves minus θ of its new environment at its new height. Eddies move air both down (left panel) and up (right panel). In both cases, the net heat flux is down. Note that eddies cycle in the vertical and so they typically move air both up and down as shown in the figures at the same time. Credit: W. Brune

Nighttime case with surface cooling such that $\partial\theta/\partial z > 0\partial\theta/\partial z > 0$ (see figure above). Eddies make air parcels trade places vertically. They move some air parcels down,

$$\text{for}$$

$$w' < 0 \tag{11.9.6}$$

$$\text{and}$$

$$\theta' > 0 : \overline{w'\theta'} < 0 \tag{11.9.7}$$

and some up,

$$\text{for}$$

$$w' > 0 \tag{11.9.8}$$

$$\text{and}$$

$$\theta' < 0 : \quad \overline{w'\theta'} < 0 \tag{11.9.9}$$

Both of these processes are happening simultaneously. If the air parcels mix at their new heights, they either move warmer air lower (left panel above) or cooler air higher (right panel above), and in both cases, they move heat down. Thus, when the potential temperature increases with height (i.e., is subadiabatic), the turbulent eddies move warmer air down and the heat flux is negative.

We see that by turbulent motions, the thermal energy tends to flow from higher mean temperature to lower mean temperature, or to put it another way, down the gradient of the mean value.

So, the heat flux $\overline{w'\theta'}$ tells us about the transport of thermal energy by eddies.

Note that the transport is in the direction from the higher values to the lower values for the mean potential temperature. The heat flux is the way that sensible heat gets transferred to the free troposphere in the atmosphere's energy budget. The heat flux combined with the moisture flux create conditions that can lead to thunderstorms and severe weather.

Watch the following video (1:49) for further discussion of the eddy heat flux:

Eddy Heat Flux

Click here for transcript of the Eddy Heat Flux.

> Eddy fluxes are one of the most important concepts in the planetary boundary layer because they are responsible for most of the vertical transport of everything in the planetary boundary layer, both those things leaving the surface, and those things coming back down to Earth. In the horizontal, the mean winds do most of the transporting. But in the vertical, the eddies do most of the transporting. Let's look at the transport of potential temperature, which is the same as the transport of thermal energy, or heat. Consider first a super eddy body potential temperature profile, that is one in which the potential temperature decreases with height. We get these kinds of profiles on hot, sunny days. Usually eddies cycle some air down and other air up. An air parcel cycled up is warmer than the surrounding air, so the perturbation to the potential temperature, which is just the difference between the potential temperature of the parcel minus the potential temperature of the surrounding air, is positive. The perturbation to the vertical wind is also positive, since the air parcel is being brought up. So that the product of w prime and theta prime is positive. The eddy also moves colder air down into warmer surroundings. So the negative perturbation to the vertical wind brings a negative perturbation to the potential temperature, and the product that these two perturbations-- two negatives-- is positive. Taking these two parts of the eddy together, we see that the cold air has replaced warm air at the lower height, and warmer air has replaced colder air at the greater height, and thus the eddy flux is upward. Convince yourself that the drawing in figure 11.6 all give downward heat flux.

11.10: Can we relate this turbulent flux to a molecular flux?

$$F_{molecules} = -D_v \frac{\partial n}{\partial r} \tag{11.10.1}$$

where $F_{\text{molecules}}$ is the molecular flux (S) units of molecules $m^{-2} s^{-1}$, $\partial n / \partial r$ is the change in concentration nn (SI units of molecules m^{-3}) as a function of radial distance rr from the drop (SI units of m), and Dv is the molecular diffusion coefficient $(S| \textbf{ units of } m^2 s^{-1})$. When n increases with r, then the flux is negative, which means that the flux is toward the drop, in the negative rr direction.

Molecular diffusion, by the way, is very slow at transferring molecules from one place to another in the troposphere. By solving the equations of motion for a simple case, we find that the characteristic time to travel a distance L by molecular diffusion is:

$$\tau = \frac{L^2}{D_v} \tag{11.10.2}$$

Exercise

By molecular diffusion, how long would it take water vapor molecules to move from Earth's surface to the top of the planetary boundary layer, 1 km away? A typical value for D_V is $2 \times 10^{-5} \mathrm{m}^2 \mathrm{s}^{-1}$

Molecular diffusion cannot transport anything fast enough for the atmosphere except on small scales of a centimeter or less. However, on the spatial and temporal scales of the planetary boundary layer, eddies are quite effective at moving heat, molecules, and momentum. In the last section, we saw that turbulence tends to move heat from heights where the air is warmer to heights where the air is cooler. Eddy "diffusion" shares this characteristic with molecular diffusion.

We can write the heat flux in the same way that we write the molecular flux:

$$F_{\text{heat}} = \overline{w'\theta'} = -K \frac{\partial \bar{\theta}}{\partial z} \tag{11.10.3}$$

where K is the eddy diffusion coefficient. Since K is always positive, this equation makes it clear that the flux of any quantity goes from where there is more of that quantity to where there is less of that quantity.

11.11: Let’s see how vertical turbulent transport can be quantified.

$$\frac{\partial\theta}{\partial t}=\frac{D\theta}{Dt}-\vec{U}\bullet\vec{\nabla}\theta \tag{11.11.1}$$

or only in the vertical

$$\frac{\partial\theta}{\partial t}=\frac{D\theta}{Dt}-w\frac{\partial\theta}{\partial z} \tag{11.11.2}$$

look at the mean and perturbed parts:

$$\frac{\partial\left(\bar{\theta}+\theta'\right)}{\partial t}=\frac{D\left(\bar{\theta}+\theta'\right)}{Dt}-\left(\bar{w}+w'\right)\frac{\partial\left(\bar{\theta}+\theta'\right)}{\partial z}$$

... take the Reynolds average

After taking the Reynolds average, we achieve the equation:

$$\frac{\partial\bar{\theta}}{\partial t}=\frac{D\bar{\theta}}{Dt}-\bar{w}\frac{\partial\bar{\theta}}{\partial z}-\overline{w'\frac{\partial\theta'}{\partial z}} \tag{11.11.3}$$

The term on the left is the rate of change of the mean potential temperature at a given height, although it applies to any height. The first term on the right is the local heating from the divergence of the radiant energy and from phase changes. This term is generally small, except in clouds, so we can ignore it in the typical convective boundary layer. The second term on the right is the mean advection, but can typically be ignored in the fair-weather boundary layer. Scale analysis shows that the mean horizontal advection (ignored here for the moment) is usually quite large, and must be kept in the heat conservation equation.

If we assume that the density does not change, then we can basically say the volume of air doesn’t change (i.e., incompressibility). We used this concept to show that horizontal convergence results in vertical divergence. For the typical convective boundary layer, turbulence is fairly homogeneous, meaning that the velocity perturbations do not vary much in space (i.e., are about equal in the *x*, *y*,and *z* directions). Thus, *w'* is independent of *z*, which means that *w'* can be taken inside of the derivative in the third term on the right of Equation [11.10]. With the assumptions described above for a fair-weather convective boundary layer, we now have:

$$\frac{\partial\bar{\theta}}{\partial t}=-\frac{\partial(\overline{w'\theta'})}{\partial z} \tag{11.11.4}$$

What does this mean? It means that the change in the boundary layer potential temperature in the daytime boundary layer is driven by the negative of the vertical gradient of eddy flux of thermal energy. During the day, the eddy heat flux is greatest at the surface and decreases with altitude. So $\frac{\partial(\overline{w'\theta'})}{\partial z}<0$, which means that the mean potential temperature increases with time ($\frac{\partial\bar{\theta}}{\partial t}>$). At night, the opposite is generally true.

Consider the sensible heat flux, F_{SH} (SI units of W m^{-2}). As we saw in the average atmospheric energy budget, the sensible heat flux plays an important role.

$$F_{SH}=\rho_{\text{air}}c_p\overline{w'\theta'} \tag{11.11.5}$$

The average θ is often approximately constant over the height of the boundary layer. So, when we integrate both sides of Equation [11.11], we get the following:

$$\frac{1}{h}\int_0^h\frac{\partial\bar{\theta}}{\partial t}\partial z=-\frac{1}{h}\int_0^h\frac{\partial(\overline{w'\theta'})}{\partial z}\partial z \tag{11.11.6}$$

h = boundary layer height; z=0 is the surface

$$\frac{\partial\bar{\theta}}{\partial t}=\frac{1}{h}\left[(\overline{w'\theta'})_0-(\overline{w'\theta'})_h\right] \tag{11.11.7}$$

$$\cong\frac{1}{h}(\overline{w'\theta'})_0 \tag{11.11.8}$$

Note that in the last line of the above, we have assumed $(\overline{w'\theta'})_h$ is small because it turns out that it is driven by $(\overline{w'\theta'})_0$ and is a negative but small fraction of $(\overline{w'\theta'})_0$.

Example

We can put some numbers to these values to show how temperature changes from turbulent eddy fluxes alone. Reasonable values for the boundary layer depth and daytime surface kinematic heat flux are 1000 m and 0.2 K m s^{-1}, respectively. Thus,

$$\frac{\partial\bar{\theta}}{\partial t} \approx \frac{0.2\mathrm{Kms}^{-1}}{1000\mathrm{m}} \times 3600\mathrm{shr}^{-1} = 0.7\mathrm{Khr}^{-1} \tag{11.11.9}$$

So, the boundary-layer potential temperature (and thus the temperature) is increasing during the day at a little less than 1 K hr^{-1}.

Thus, the heating of the boundary layer is driven by the eddy heat flux from the surface. In this way, the heating at the surface due to the absorption of solar energy at Earth's surface is spread throughout the boundary layer.

11.12: What other fluxes are important?

We have focused on the sensible heat flux up to now, but turbulence creates other vertical fluxes. There are many vertical turbulent fluxes, but two important ones are the **latent heat flux**, which involves the vertical transport of water vapor, and the **horizontal momentum flux**, which involves the vertical transport of horizontal wind.

Latent Heat Flux

For the purpose of this discussion, use the specific humidity, *q*. There is the mean value for *q* at different heights, and then there is the kinematic eddy flux. Using the same methods as before, we find that the kinematic water vapor flux (or specific humidity flux) is given by:

$$F_v = \overline{w'q'} \tag{11.12.1}$$

This flux has Sl units of kg_water, $\mathrm{kg_{air}^{-1}\,ms^{-1}}$. Usually the specific humidity is greatest near Earth's surface and decreases with height. Using the same logic as for the sensible heat flux, we expect a flux of water vapor from the surface, where the specific humidity is greater, to the free troposphere, where the specific humidity is less.

However, we usually want to compare energy fluxes caused by different processes as in Lesson 7.3, so we multiply the specific humidity flux by the terms necessary to convert it into an energy flux that would result from the condensation of that water vapor. We end up with a latent heat flux:

$$F_{LH} = \rho_{air} l_v \overline{w'q'} \tag{11.12.2}$$

with SI units of

$$\left(\mathrm{kg_{air}\ m^{-3}}\right)\left(\mathrm{Jkg_{water}}\ -1\right)\left(\mathrm{kg_{water}\ kg_{air}}\ -1\mathrm{ms^{-1}}\right) = \mathrm{Jm^{-2}s^{-1}} \tag{11.12.3}$$

Note that we have multiplied the specific humidity flux by the density of air and the latent heat of vaporization to put the specific humidity flux in terms of an energy flux, which we see is comparable to the sensible heat flux and is a significant fraction of the global energy balance at Earth’s surface. In fact, on a global scale, the latent heat flux is about five times larger than the sensible heat flux and is about half the total absorbed solar irradiance.

Latent heat flux is the primary way that water vapor gets into the atmosphere and is thus the primary source of water vapor for convection and clouds. Predicting convection and precipitation depends on knowing the latent heat flux.

Horizontal Momentum Flux

The mean horizontal wind velocity is the vector sum of the wind components in the *x*-direction and the *y*-direction. The magnitude of the mean horizontal wind velocity is given by:

$$\bar{V} = \sqrt{\bar{u}^2 + \bar{v}^2} \tag{11.12.4}$$

The horizontal momentum flux is basically vertical turbulent eddies bringing high-wind-velocity air down from above. You all have experienced this phenomenon if you have ever been out early in the morning, just as the solar heating of the surface has begun to create convection and mix calm near-surface air up and windier residual layer air down.

The equations for the (kinematic) vertical fluxes of *x*-momentum and *y*-momentum air are, respectively:

$$F_{mx} = \overline{u'w'} \tag{11.12.5}$$

and

$$F_{my} = \overline{v'w'} \tag{11.12.6}$$

where the SI units are $\mathrm{m^2\ s^{-2}}$ and where u' and v' are wind speed perturbations in the x and y directions, respectively.

Note that the horizontal wind speed, *V*, is zero at Earth's surface (because of molecular friction) and increases with height. Just as the turbulent heat flux moves air with a higher potential temperature to heights where the potential temperature is lower, the turbulent momentum flux moves air with higher horizontal momentum (i.e., horizontal velocity) to heights where the mean horizontal momentum is lower. That is, the horizontal momentum is moved downward through the boundary layer to the Earth's surface, where it is dissipated by molecular friction.

Just as the heat flux is equal to a constant times the vertical gradient of the mean potential temperature (Equation [11.9]), the *x*-momentum flux is equal to a constant times the vertical gradient of the mean *x*-wind:

$$\overline{u'w'} = -K\frac{\partial \bar{u}}{\partial z} \tag{11.12.7}$$

where K is the eddy diffusivity.

Just as the change with time of the mean potential temperature is related to the negative of the vertical gradient of the kinematic heat flux (Equation [11.11]), so is the change with time of the mean velocity related to the negative of the vertical gradient of the kinematic momentum flux. Thus, the *x*-component momentum equation in the boundary layer becomes (ignoring other terms for now, such as the pressure gradient force and the Coriolis force):

$$\frac{\partial \bar{u}}{\partial t} = -\frac{\partial(\overline{u'w'})}{\partial z} \tag{11.12.8}$$

Just as we assumed the mean potential temperature is constant with height in the boundary layer, we can assume that the mean *x*-momentum (i.e., zonal velocity, *u*) is constant with height in the boundary layer. We can then integrate the above equation from the surface (*z* = 0) to the top of the boundary layer (*z* = *h*) and make the same assumptions about the flux at the top being relatively small to get:

$$\frac{\partial \bar{u}}{\partial t} = -\frac{1}{h}(\overline{u'w'})_0 \tag{11.12.9}$$

where $(\overline{u'W'})_0$ is the vertical flux of *x*-momentum at the surface. As Equation [11.16b] indicates, this flux depends on the local eddy diffusivity and the local vertical gradient of the mean *u* velocity. The eddy diffusivity near the surface will increase as the mean wind speed in the boundary layer increases because vertical shear, which is responsible for the mechanical generation of turbulence, will be greater as the mean wind speed increases. Also, the vertical gradient of the mean *u* velocity near the surface will increase as the mean *u* of the boundary layer increases. Thus, we expect

$$(\overline{u'w'})_0 \propto \bar{V}\bar{u} \tag{11.12.10}$$

The coefficient of proportionality is called the drag coefficient, CdCd, which depends on the roughness of the surface and the thermal stability at the surface. Hence, combining Equations [11.16d] and [11.16e], we have for both horizontal components:

$$\frac{\partial \bar{u}}{\partial t} = -\frac{C_d\bar{V}}{h}\bar{u} \tag{11.12.11}$$

$$\frac{\partial \bar{v}}{\partial t} = -\frac{C_d\bar{V}}{h}\bar{v} \tag{11.12.12}$$

These are the turbulent resistance terms that we introduced as friction in Lesson 10. They come from the downward transfer of horizontal momentum to the surface, where it is dissipated by molecular friction at Earth's surface.

11.13: Summary and Final Tasks

Summary

The atmospheric boundary layer has a strong diurnal cycle, which consists of a well-mixed layer that is ~1 km high during the day but collapses down to 10% of that at night, leaving behind a residual layer. The mixing is caused by turbulence, a chaotic mix of swirling eddies ranging in size from the height of the PBL to the size of your fist. Understanding turbulence is the key to understanding the motion of air in the PBL. Taylor's Hypothesis shows that changes in eddy properties measured by a sensor in time can be used to describe the eddy properties in space.

Equations of motion by turbulent eddies arise from defining all quantities (e.g., wind speed, specific humidity, and potential temperature) in terms of their mean and turbulent parts, then substituting these two parts for each quantity into the appropriate conservation equations, expanding the equations, then taking the Reynolds averages. Terms with two turbulent components arise that are non-zero, and these give rise to turbulent fluxes of temperature, water vapor, and momentum. A key result is that eddies drive vertical transport of these quantities (and others) in the direction of where the mean quantities are the least from where they are the most. Furthermore, changes in the vertical profiles of mean quantities with time are caused by the vertical eddy fluxes of those quantities.

The energy in the PBL is a tale of the turbulent kinetic energy and the energy balance near the surface. Turbulence consists of eddies of all different sizes and lifetimes, ranging from sizes that are the height of the PBL to others as small as your fist. These eddies are related: the largest carrying the most energy, and the smaller ones fracturing off of the larger ones, thus cascading the energy from larger scales to smaller scales, ending eventually with molecular viscosity, which converts all that kinetic energy into heat. The driver of this turbulent kinetic energy is the exchange of energy at Earth's surface and consists of net radiation, sensible heat flux, latent heat flux, and subsurface ground flux. Depending on the environment and the time of day, the relative values of these fluxes can be quite different, but in every case, the total energy budget must balance.

Reminder - Complete all of the Lesson 11 tasks!

You have reached the end of Lesson 11! Double-check that you have completed all of the activities before you begin Lesson 12.

CHAPTER OVERVIEW

12: The Atmosphere - A Holistic View

Learning Objectives

By the end of this chapter, you should be able to:

- explain the physical and chemical phenomena that are responsible for an observation of the atmosphere
- demonstrate your mastery of the course learning objectives

This text has been compartmentalized into eleven chapters to aid your learning and to grow your analytical skills. But in the atmosphere, the fundamentals of atmospheric science work together to create the atmosphere that we observe. In this lesson, you will work to draw on your understanding of the atmosphere to explain an atmospheric observation that you have chosen. In addition, you will demonstrate your understanding of the lessons by taking a final exam that is made up of questions and problems from the eleven lessons. You will have worked some of the problems and answered some of the questions, but not all.

12.1: An Integrated View of the Atmosphere

The atmosphere is one of the Earth's most efficient integrators. The atmosphere connects to almost every part of the Earth system—the **lithosphere** (i.e., solid earth), the **hydrosphere** (i.e., oceans), the **cryosphere** (i.e., ice), and the **biosphere** (i.e., life from microbes to plants to animals). The atmosphere's constituents are essential for life. The atmosphere transports energy and atmospheric constituents—in days it mixes air through the troposphere; in weeks it circumnavigates the globe; in months it transports air from the equator to the poles; in a year it shifts air from one hemisphere to another. The atmosphere and the water it contains shape the land with wind and water erosion, move the ocean currents, and determine where and when life can thrive or die. The atmosphere has shaped human history. For all of these reasons and more, the atmosphere, its governing principles, and its behavior must be thoroughly understood in a way that makes it possible to accurately predict its future behavior.

METEO 300 is designed to give you a solid understanding of the atmosphere's physical and chemical principles and the skills to quantify its behavior and properties. In the following table, the accumulated learning objectives are laid out end-to-end in an impressive array. If you have worked hard and completed all the exercises, you can know and can do what is in this table.

METEO 300 Learning Objectives

Lesson	Learning Objectives
1	- correctly use significant figures, dimensions, and units - solve simple problems using integral and differential calculus - prepare and use a course Excel workbook for course calculations
2	- use the fundamental gas laws—Ideal Gas Law and Dalton’s Law—to determine the relative densities of different air masses - derive the hydrostatic equilibrium equation from force balance to show why atmospheric pressure decreases with height - use the 1st Law of Thermodynamics and conservation of energy (i.e., adiabatic processes) to explain air parcel temperature changes - determine stability for different dry environmental temperature profiles - calculate buoyancy and vertical velocity with time
3	- differentiate among the different ways that moisture can be expressed and choose the correct one for finding an answer to a given problem - explain the meaning of the lines and spaces on a water vapor phase diagram - calculate relative humidity using the Clausius–Clapeyron Equation - solve energy problems related to temperature and phase changes - demonstrate proficiency with using the skew-T diagram to find the lifting condensation level (LCL), potential temperature, relative humidity, wetbulb temperature, dry and moist adiabats, and equivalent potential temperature
4	- explain the role that each atmospheric constituent plays in atmospheric structure and weather - identify changes in minor and trace gas amounts and the impacts these changes have on the atmosphere - explain how the atmosphere cleanses itself, using methane as an example - use chemical equations to show how ozone is formed in the stratosphere and the troposphere and how they differ - diagram the lifecycle of aerosol particles with an emphasis on their role in weather
5	- identify cloud types - describe the essentials for cloud formation - on a Koehler curve, explain the behavior of a particle in different supersaturation environments - explain the lifecycle of cloud formation through precipitation
6	- identify the causes of changing solar radiation on Earth - calculate properties of the spectrum of solar and Earth radiation in terms of the Planck function - calculate the absorption between you and a light source - explain why the sky looks blue and hazy in the summer
7	- identify the causes of changing solar radiation on Earth

Lesson	Learning Objectives
	- calculate properties of the spectrum of solar and Earth radiation in terms of the Planck function - calculate the absorption between you and a light source - explain why the sky looks blue and hazy in the summer
8	- calculate partial derivatives - implement vector notation, the dot product, the cross product, and the del operator - explain the different coordinate systems and how they are used - convert between math and meteorological wind directions - calculate temperature advection at any point on a map of isotherms (lines of constant temperature) and wind vectors
9	- identify regions of convergence, divergence, positive vorticity, and negative vorticity on a weather map - calculate the strength of the different flow types from observations - relate vertical motion to horizontal convergence and divergence
10	- explain mass conservation physically, recognize the mass conservation equation, and memorize its form when density is constant - state the three main conservation laws in atmospheric science: the conservation of mass, the conservation of momentum, and the conservation of energy - name and explain the three fundamental (real) forces in the atmosphere (gravity, pressure gradient, and friction) - name and explain the two new (apparent) forces that emerge when momentum conservation is written in the rotating reference frame - draw the balance of forces for geostrophic flow, gradient flow, geostrophic flow with friction, and cyclostrophic flow - explain why midlatitude winds are westerly
11	- draw the PBL and its diurnal variation - perform Rayleigh averaging on an equation and derive an equation for the turbulent parts - explain kinematic fluxes - show vertical motion using eddy fluxes - explain turbulent kinetic energy (TKE) and its behavior - sketch the surface energy budget for different conditions
12	- explain the physical and chemical phenomena that are responsible for an observation of the atmosphere - demonstrate your mastery of the course learning objectives

There are fifty-one learning objectives listed here. Read through this list and think about how comfortable you are with your knowledge and your abilities in each area. If you don't remember some of them, review them now.

12.2: The Final Project

The final project will test your ability to make an observation of the atmosphere and to provide an integrated analysis of that observation using the knowledge and quantitative analysis skills that you have learned in this course.

Final Project 1

1. Choose an observation of the atmosphere. The observation could be a picture, video, radar image, satellite image, or figure. It can come from you, the course material, a site that allows copying, or a family member or friend. **Do not use an observation that has been previously analyzed** by anyone else. When you are choosing an observation, think about the story you want to tell and how well you think you can tell it. You will want to do a little research and maybe think about two or three different possibilities before you settle on one.
2. Check to make sure that your observation is at a time and place for which you can get weather information (e.g., temperature, relative humidity, pressure, weather maps, weather station model, upper air data) that you will need to analyze and explain your observation.
3. Before you start the analysis and writing, send me an email with your observation and a few words about it so that I can approve it. **I must approve your choice of an observation**.
4. Gather the evidence as you build your analysis of the observation. Be sure to document where your evidence came from.
5. First show and describe the observation. You should give as much information as you have about the observation – what, where, and when it was observed.
6. Write your analysis in formal language, such as you might see in a science news magazine and is in the example below. **You can submit this analysis as a Word or pdf file.** The length could be as short as a few pages, including figures, graphs, and equations, but it must be thorough and complete.
7. Be thorough but concise.
8. After you have completed your final project, go back through it and edit it. Make it as polished and appealing as possible.
9. Follow the instructions below for submitting it in Canvas. Late presentations will have points deducted.
    1. Put your final project in either Word or PDF format.
    2. Embed pictures and equations in the text.
    3. Name your file "Final_Project_YourLastName".
    4. Submit it to the **Final Project assignment** which is located in the **Lesson 12 Module** in Canvas.
10. Follow the instructions below for submitting it in Canvas.

Final Project 2

1. Choose an observation of the atmosphere. The observation could be a picture, video, radar image, satellite image, or figure. It can come from you, the course material, a site that allows copying, or a family member or friend. **Do not use an observation that has been previously analyzed** by anyone else. When you are choosing an observation, think about the story you want to tell and how well you think you can tell it. You will want to do a little research and maybe think about two or three different possibilities before you settle on one. Try to think of observations that are unique and interesting.
2. Check to make sure that your observation is at a time and place for which you can get weather information (e.g., temperature, relative humidity, pressure, weather maps, weather station model, upper air data) that you will need to analyze and explain your observation.
3. Before you start the analysis and writing, send me an email with your observation and a few words about it so that I can approve it. **I must approve your choice of an observation**.
4. Gather the evidence as you build your analysis of the observation. Be sure to document where your evidence came from.
5. First show and describe the observation. You should give as much information as you have about the observation—what, where, and when it was observed.
6. Then make a **5-minute PowerPoint presentation** of your observation and your analysis. The length could be as short as a few slides, including figures, graphs, and equations, but it must be thorough and complete. I suggest that you use anywhere from three to about six slides, depending on the number of figures you need to describe your analysis. Put in another slide with your

references to websites, books, and papers on it. You do not have to do a quantitative analysis, necessarily, but if you do, your analysis will be more impressive and convincing.

7. Be thorough but concise.
8. After you have completed your final project, go back through it and edit it. Make it as polished and appealing as possible.
9. Follow the instructions below for submitting it in Canvas. Late presentations will have points deducted.
   1. Put your final project in either PowewrPoint or PDF format.
   2. Embed pictures and equations in the text.
   3. Name your file "Final_Project_YourLastName".
   4. Submit it to the **Final Project assignment** which is located in the **Lesson 12 Module** in Canvas.
10. Presentations will run for two weeks. I will select the order of the presentations using random numbers, so you will need to attend all the classes. I will also take role during each class and will deduct participation points for students who do not have a valid excuse approved by me to miss class. It is only fair to the students who are presenting near the end that the audience is as big for the last presentation as it is for the first.

Example of a Final Project

Observation

I took this picture of southeastern Pennsylvania while flying to Atlanta, GA on 29 June 2015. The picture was taken at 14:30 EDT (18:30 UTC) from an altitude of about 20 kft. Note the fair weather cumulus clouds have little vertical development. Also, even though it was a fairly moist summer day, the boundary layer below the fair weather cumulus appears quite clear.

Explanation

The presence of clouds indicates that three conditions existed: moisture, aerosol, and cooling. The moisture came from the surface, which had seen heavy rain a day before. Surface heating by solar visible irradiance evaporated liquid water on the surface, which created pockets of moist, buoyant air. These air parcels rose relative to the nearby less buoyant environment, according to the Buoyancy Equation [2.66]:

$$B=\frac{(T_v'-T_v)}{T_v}g \tag{12.2.1}$$

until they reached the lifting condensation level (LCL). There, they became supersaturated, so that the aerosol that forms the cloud condensation nuclei nucleated and cloud drops were formed according to the Koehler Theory Equation [5.13]:

$$s_k=S_k-1=\frac{a_K}{r_d}-\frac{BiN_s}{r_d} \tag{12.2.2}$$

These clouds sat in the entrainment zone just above the convective boundary layer. The energy budget for such a recently wetted land surface would likely show significant downward net radiation, and significant upward latent heat flux.

The clouds showed little vertical development. This behavior would suggest that the air was quite stable. Indeed, the radiosonde recording (Figure 1) from Dulles Airport a few hours earlier indicated that the air was quite stable, with the ascent on a moist adiabat from the Lifting Condensation Level 5–10 K below the ambient temperature.

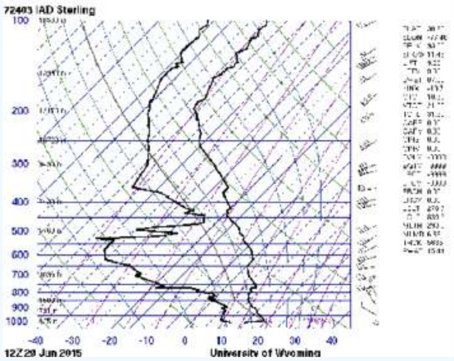


Figure 1. Skew-T from nearby Dulles International Airport on 29 June 2015.

Credit: NWS data, University of Wyoming skew-T website

This condition came about from the synoptic scale conditions, with high pressure over the region (Figure 2) suggesting downward vertical descent and divergence according to Equation [9.5]:

$$\frac{\partial w}{\partial z} = -\vec{\nabla}_H \bullet \vec{U}_H \tag{12.2.3}$$

which, by adiabatic compression, would lead to clearing skies.

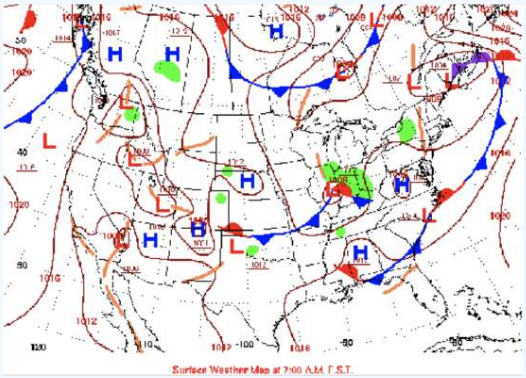


Figure 2. Surface weather map for 29 June 2015. Note the surface high pressure region over northern Virginia.

Credit: NOAA

It is most likely that these clouds in the observation were formed by adiabatic ascent by random localized buoyant air parcels. However, there was a fairly uniform stratus deck just to the northeast of this location and some evidence that this air mass was moving to the west or southwest toward the location (Figure 3).

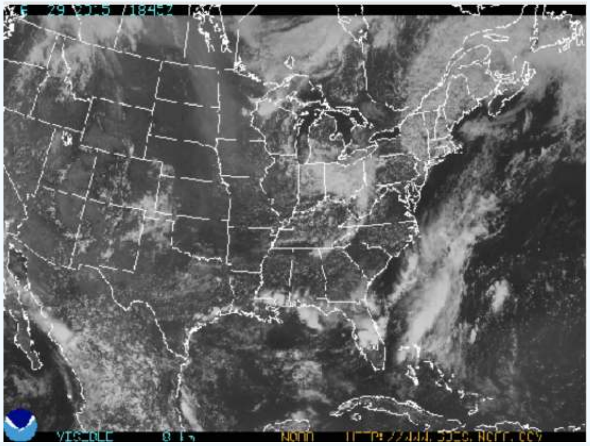


Figure 3. Visible satellite image for 29 June 2015. Note the breaking cloud cover in the Northeast and the fair weather cumulus over much of Pennsylvania and Maryland.

Credit: NOAA

As this stratus deck was mixed with drier air (Lesson 5.3, Equation [5.4]), the cloud deck could have broken up into evaporating individual clouds. Likely the clouds in the observation were from both adiabatic ascent and the evaporation of the stratus cloud deck.

Often with clear skies the pollution levels are high and the boundary layer is filled with haze. However, the visibility is quite good in the picture. There is generally enough PM2.5 (particle matter less than 2.5 microns in diameter) present in southeastern Pennsylvania. However, rain the previous day was able to remove some of the pollution from previous days, thus clearing the air. In addition, the particles that were there may not have been swollen to a size that efficiently scatters solar radiation, when in Equation [6.17]:

$$x \equiv \frac{2\pi r}{\lambda} \tag{12.2.4}$$

the size of the particles are approximately equal to the visible wavelength. Indeed, we have at least three pieces of evidence that the air was fairly dry (Figure 1, Figure 3, and Figure 4 (Lesson 7)), with dewpoints in the middle-to-high 50s (^{o}F) (see this image).

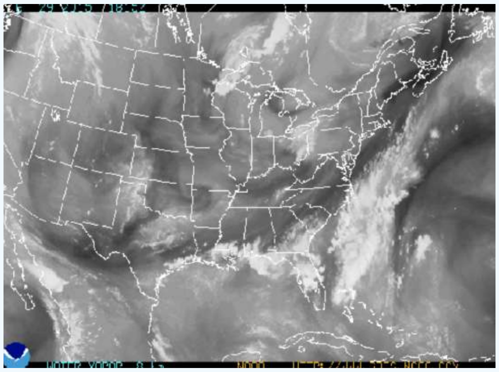


Figure 4. Satellite water vapor image for 29 June 2015. Note the tongue of fairly dry air going across western Virginia into Maryland and southeastern Pennsylvania.

Credit: NOAA

From the skew-T, the relative humidity was only about 50% (Lesson 3.5, $RH = w/w_s = 7$ g/kg / 14 g/kg). Thus, the recent scavenging of aerosol by heavy rain and the low relative humidity made for great visibility and clear boundary layer air even in the high pressure region, with light winds and clearing skies. This example is interesting because only after frontal passages with rain is the boundary layer air so clear under high pressure. If the high pressure were to persist, then the moisture levels would likely increase due to evaporation of surface water and the pollutant emissions and chemistry would make more particle pollution, both of which would lead to lower visibility in the boundary layer.

Self Evaluation of This Example. Note: This evaluation is given here to show you how the rubic will be used to evaluate your project. You should not include a self-evaluation with your project.

This example meets the overall standards for integration and explanation of the observation. The analysis addresses the presence of the fair weather cumulus and the reason for the clear air in conditions when the visibility is often not so good. On the other hand, my example would not receive a perfect score for a few reasons. First, the choice of observation is good but not very interesting. Second, all of the equations are appropriate, but some are not well integrated into the analysis. Third, some of the figures are fuzzy. And, fourth, the possible evaporation of the stratus deck is not particularly well explained.

For your reference, the grade for this example would likely be 11 to 12 on a scale of 15.

Final Project

(15% of final grade)

1. The final project is worth 15% of your final grade.
2. I will use the following rubric to grade your final project:

Final Project Grading Rubric

Evaluation	Explanation	Available % Points
Not Completed	Student did not complete the assignment by the due date.	0
Student completed the project with little attention to detail or effort.	Project is on a weak observation, has flawed analysis, and/or lacks a clear presentation. In addition, there is inadequate evidence of integration of course material, no references to equations that would help quantify the observation, or no/poor figures to support analysis.	3
Student completed the project, but it has many inadequacies.	Project is strong in one or two of the following areas but weak in the rest: good choice of observation, thorough analysis and evidence, conclusions supported by evidence, draws evidence from at least five different lessons, includes at least three different equations needed to do a quantitative analysis, contains figures/graphs taken from other sources to provide evidence and support conclusions, organization is logical, and presentation is clear and concise.	6
Student completed a pretty good project, but it had some inadequacies.	Project is strong in more than half of the following areas but weak in the rest: good choice of observation, thorough analysis and evidence, conclusions supported by evidence, draws evidence from at least five different lessons, includes at least three different equations needed to do a quantitative analysis, contains figures/graphs taken from other sources to provide evidence and support conclusions, organization is logical, and presentation is clear and concise.	9
Student completed a very good project, but it had a few inadequacies.	Project is strong in all but a few of the following areas: good choice of observation, thorough analysis and evidence, conclusions supported by evidence, draws evidence from at least five different lessons, includes at least three different equations needed to do a quantitative analysis, contains figures/graphs taken from other sources to provide evidence and support conclusions, organization is logical, and presentation is clear and concise.	12
Project is strong in all but a few of the following areas: good choice of observation, thorough analysis and	Project is strong in all of the following areas: good choice of observation, thorough analysis and evidence,	15

Evaluation	Explanation	Available % Points
evidence, conclusions supported by evidence, draws evidence from at least five different lessons, includes at least three different equations needed to do a quantitative analysis, contains figures/graphs taken from other sources to provide evidence and support conclusions, organization is logical, and presentation is clear and concise.	conclusions supported by evidence, draws evidence from at least five different lessons, includes at least three different equations needed to do a quantitative analysis, contains figures/graphs taken from other sources to provide evidence and support conclusions, organization is logical, and	

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