INTRO TO OPTICS AND PHOTONICS

Ana Doblas
University of Massachusetts Dartmouth

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University of Massachusetts Dartmouth

ECE 513: Introduction to Optics and Photonics

Ana Doblas

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Course Topics:

This course exposes students to a broad array of topics in optics, with an emphasis on understanding over-arching concepts, general physical principles that dictate the behavior of light, as well as practical knowledge about common optical components and devices, and analytical tools for engineering simple optical systems. Topics covered in the course: ray optics, wave optics, Gaussian optics, Fourier optics, interferometry, and polarization optics.

Course Learning Objectives:

By successfully completing this course, students will be able to:

- Demonstrate a correct solution to problems of Ray Optics with and without ABCD matrix.
- Analyze the interaction between elementary wave functions and simple optical elements.
- Understand optical interferometers and analyze their total intensity distribution.
- Analyze Gaussian beams and their physical properties through arbitrary optical systems using ABCD matrix.
- Apply Fourier description to the light propagation.
- Demonstrate light as an electromagnetic field that satisfies the Maxwell equations.
- Analyze absorption and dispersion in a medium, and their relationship with the refractive index.
- Analyze the polarization state of the light through several elements by applying the Jones matrix.

Course Outcomes:

Students successfully completing this course will be able to:

1. Calculate the reflection and refraction angle in a planar interface between two different media.
2. Determine the image height and position of an optical system composed of one or two lenses.
3. Calculate the ABCD matrix of an arbitrary optical system.
4. Determine the condition in which a spherical wave can be described by a paraboloidal wave.
5. Determine the Fourier plane of an optical imaging system.
6. Determine a *4f* imaging system.
7. Determine the polarization state of the light through several polarization-sensitive elements.

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Licensing

A detailed breakdown of this resource's licensing can be found in ***Back Matter/Detailed Licensing***.

CHAPTER OVERVIEW

Module 1 - Geometrical Optics

The *Module 1: Geometrical Optics* module explores the principles governing light propagation and interaction with various optical systems. Students will study foundational postulates and apply them to understand light behavior through planar and spherical boundaries, including refraction, reflection, and image formation. The module delves into the design and functionality of optical components, including lenses and mirrors. We finish this module by introducing *Matrix Optics*, enabling the systematic analysis of complex optical systems.

This Module contains 5 classes:

Class 1 - Postulates and Rules in Ray Optics

Learning Objectives in this Class:

- Understand that light travels in the form of rays in Geometrical/Ray Optics
- Understand the physical meaning of the refractive index
- Understand that light travels in the form of straight rays if the medium is homogeneous (i.e., refractive index = constant).
- Estimate the reflected and refracted angles in a planar interface
- Apply Snell’s law in one and two planar interfaces
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Class 2 - Mirrors

Learning Objectives in this class:

- Identify the different types of mirrors based on their properties and focal points
- Identify the imaging equation for spherical mirrors under paraxial approximation
- Apply ray tracing in convex and concave mirrors to estimate the image’s key features

Class 3 - Planar Boundaries, External and Internal Refraction, Total Internal Reflection

Learning Objectives in this class:

- Understand the external and internal refraction
- Estimate the critical angle
- Estimate the incident angle so total internal reflection occurs in a planar interface

Class 4 - Spherical Boundaries and Lenses

Learning Objectives in this class:

- Define a spherical boundary and a spherical lens
- Illustrate the image of an object through a spherical boundary
- Estimate the focal length of a lens based on its refractive index, its radii, and the refractive index of the surrounding medium.
- Illustrate the image of an object through a spherical lens
- Differentiate the optical tracing between converging and diverging lenses

Class 5 - Matrix Optics and 4f Imaging Systems

Learning Objectives in this class:

- Understand the matrix formalism
- Identify the ray matrix for elemental optical components
- Apply the matrix formalism in cascade to estimate the transfer function of an arbitrary optical system
- Identify the lateral and angular magnification from the transfer function matrix of a 4f system

Module 1 - Summary

Summary Notes Module 1

Postulates of Ray Optics

- #1. Light travels in the form of rays
- #2. An optical medium is characterized by a refractive index, $n \geq 1$. The refractive index provides a measure of the speed of the light in that medium (v): $n = \frac{c}{v}$ where $c = 3 \times 10^8 \mathrm{~m/s}$.
- #3. The optical path length is the optical distance between two points:

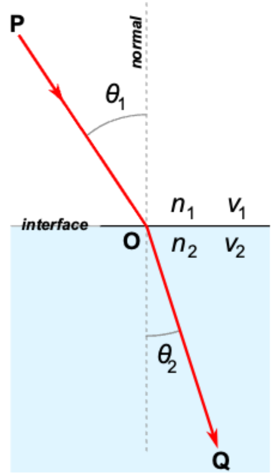
$$\mathrm{OPL} = \int_A^B n(x, y, z)\, ds, \tag{1}$$

  where ds is the differential distance between both points. If the medium is homogeneous, $n(x, y, z) = \text{constant}$, the optical path length is equal to $\mathrm{OPL} = n\Delta$, where Δ is the distance between points A and B.
- #4. Fermat's Principle: light traveling between two points follows a path in which the derivative of the OPL is zero. Therefore, the OPL is at a maximum, minimum, or a point of inflection.

Rules of propagation

- #1. In a homogeneous medium (i.e., $n(x, y, z) = \text{constant}$), the path of minimum distance between two points is a straight line. In other words, light travels in straight lines.
- #2. Law of reflection: the angle of reflection is the same as the angle of incidence.
- #3. The angle of refraction depends on the angle of incidence by Snell's law:

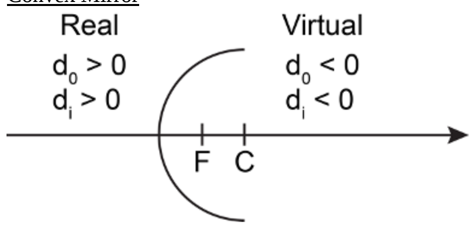
$$n_1 \sin\theta_1 = n_2 \sin\theta_2. \tag{2}$$



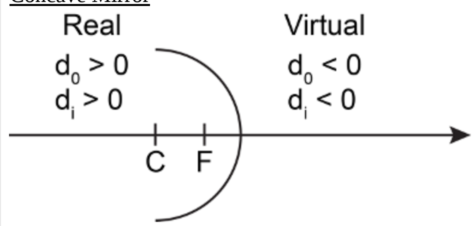
Mirrors

- Planar mirror
- Paraboloidal mirror (i.e., focusing mirror) – all parallel rays to its optical axis are focused in the same point F (i.e., there is a focal spot). The distance defined by F and the mirror's vertex P is the focal length.
- Elliptical mirror (i.e., image mirror) – all rays emitted from one focus are imaged onto the other focus. The distance traveled by the light between both focal spots is always the same regardless of the path. The elliptical mirror is defined by 2 focal spots.
- Spherical mirror – parallel rays are reflected to its optical axis at different positions. However, rays traveling close to its axis are approximately focused onto the same point F, with the distance equal to half the distance to its center. In other words, the focal length $f = -\frac{R}{2}$, where R is the radius of curvature. Note that convex mirrors have negative focal length $f = -\frac{R}{2}$, and concave mirrors have positive focal length, $f = \frac{R}{2}$.

Convex Mirror



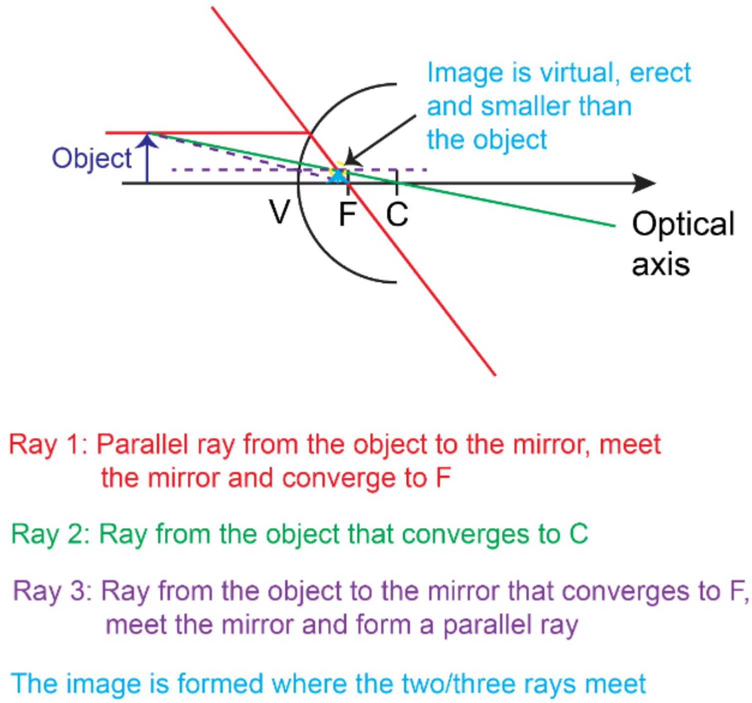
Concave Mirror



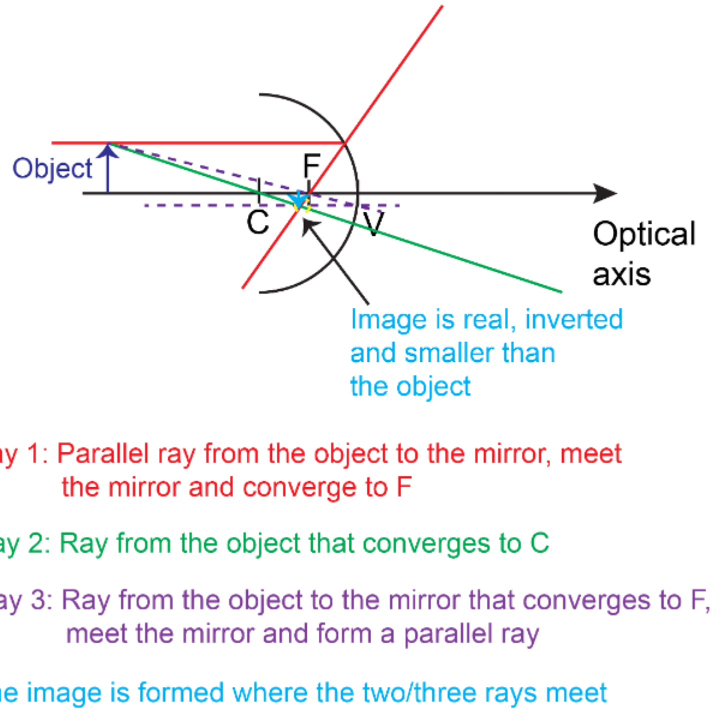
Note the following sign convention: $d_0, d_i < 0$ means points to the right side of the mirror (i.e., virtual) and $d_0, d_i > 0$ are related to points to the left side of the mirror (i.e., real).

Ray Tracing in Mirrors

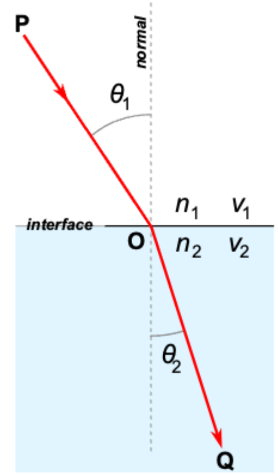
Convex Mirror



Concave Mirror



Planar Boundaries – a planar interface that separates two media of constant refractive index.



$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{3}$$

$$n_1\theta_1 \approx n_2\theta_2 \quad \text{if} \quad \theta_1, \theta_2 \ll 1. \tag{4}$$

- #1. $n_1 > n_2$ then $\theta_1 < \theta_2$ – refracted rays away from the boundary.
- #2. $n_1 < n_2$ then $\theta_1 > \theta_2$ – refracted rays towards the boundary.
- #3. $n_1 < n_2$ and $\theta_2 = 90^\circ$ – Phenomenon of Total Internal Reflection.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_2 \sin(90^\circ) = n_2 \quad \Rightarrow \quad \sin\theta_1 = \frac{n_2}{n_1} \quad \Rightarrow \quad \theta_{1,c} = \sin^{-1}\left(\frac{n_2}{n_1}\right). \tag{5}$$

If the incidence angle is higher than the critical angle $\theta_1 > \theta_{1,c}$, Snell's law cannot be satisfied (i.e., $\sin\theta_2 > 1$), so refraction does not occur and the incident ray is reflected (no refraction!). The phenomenon of total internal refraction occurs in fiber optics.

Lenses – two spherical surfaces of radii R_1 and R_2

- Lens' maker equation:

$$\frac{1}{f}=\frac{n-n_s}{n_s}\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \tag{6}$$

where f is the lens' focal length, n is the lens' refractive index, n_s is the refractive index of the surrounding media, and R_1 and R_2 are the radii of curvature of the first and second surfaces, respectively.

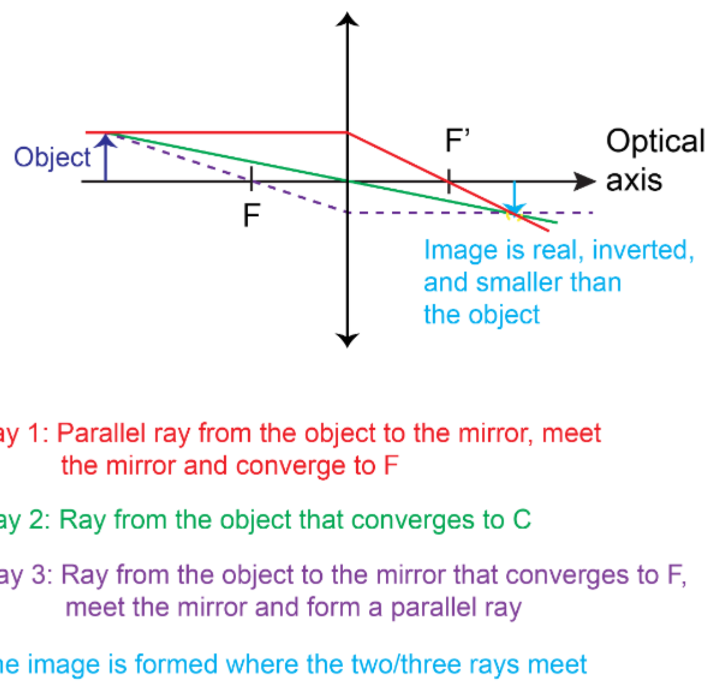
- $f>0$ is a converging lens and $f<0$ is a diverging lens.

- Type of lenses (TIP: the name is associated with the outside shape):
  - Biconvex: $R_1>0$ and $R_2<0$
  - Biconcave: $R_1<0$ and $R_2>0$
  - Plano-convex: $R_1>0$ and $R_2=\infty$
  - Plano-concave: $R_1=\infty$ and $R_2>0$
- Imaging equations:

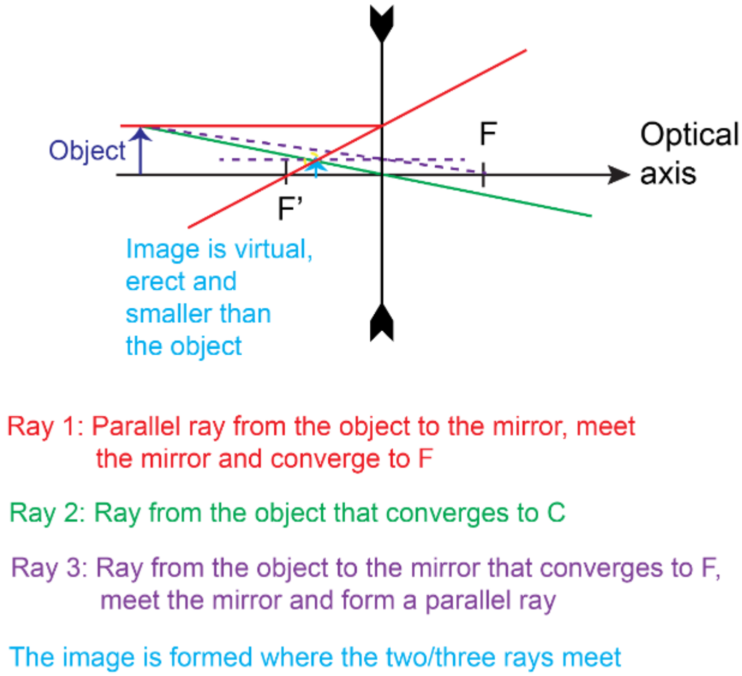
$$\frac{1}{d_0}+\frac{1}{d_i}=\frac{1}{f}, \quad y_i=-\frac{d_i}{d_0}y_0 \tag{7}$$

- Ray tracing

Converging Lens



Diverging Lens



Matrix Optics

$$\text{Input}\begin{bmatrix} A & B \\ C & D \end{bmatrix}\text{Output}$$

$$\begin{pmatrix} y_2 \\ \theta_2 \end{pmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix} \tag{8}$$

$$y_2 = Ay_1 + B\theta_1, \quad \theta_2 = Cy_1 + D\theta_1. \tag{9}$$

Matrix Element Value	Output Height or Angle	Consequence of the resultant system
$A = 0$	$y_2 = B\theta_1$	Focusing system
$B = 0$	$y_2 = Ay_1$	Imaging system where A is the lateral magnification
$C = 0$	$\theta_2 = D\theta_1$	Parallel rays remain parallel
$D = 0$	$\theta_2 = Cy_1$	Rays emerging from a point source are parallel after the system

Examples of ABCD Matrices

- Free propagation of a distance d: $\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$
- Lens of focal length f: $\begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$ where $f > 0$ for converging lenses and $f < 0$ for diverging lenses.

Cascade of Matrices – an optical system composed of N matrices:

$$\text{Input}\,[\,M_1 \quad M_2 \quad \ldots \quad M_N\,]\,\text{Output}$$

$$\begin{pmatrix} y_2 \\ \theta_2 \end{pmatrix} = [\,M_T\,]\begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix} \tag{10}$$

where $M_T = M_N M_{N-1} \ldots M_2 M_1$.

Multi-choice questions

Multi-choice Quiz Module 1

1. An optical fiber is illuminated by light from an external source. The numerical aperture NA = $\sin(\theta_a)$ of the fiber is 0.3. Knowing that the refractive index of the cladding is 1.46. What is the value of the refractive index of the core?

a. $n_{core} = 1.55$

b. $n_{core} < 1$ so this optical fiber cannot exist

c. $n_{core} = 1.43$

2. Yes/No. Light with an incidence angle of θ_1 = 45 degrees passes through a medium with sunflower oil (n_1 = 1.46) to an air medium. Considering that the interface is a planar boundary. Does the phenomenon of total internal reflection occur?

a. Yes

b. No

3. A 2-cm toy object is located 30 cm in front of a concave mirror of a focal length of 20 cm. How is the image of the toy?

a. The image is virtual and smaller

b. The image is virtual and larger

c. The image is real and smaller

d. The image is real and taller

4. Could a converging lens form virtual images?

a. No, never.

b. Yes, if the object to be imaged is placed at an axial position smaller than the (object) focal length, $z_{obj}<f$.

c. Yes, if the object to be imaged is placed at an axial position z_{obj} is $f < z_{obj} < 2f$.

5. The same double-convex thin lens with a refractive index of 1.52 is embedded in two different media: air (n=1) and water (n = 1.33). Which of the following sentences is true?

a. The focal length of the lens is independent of the medium in which it is embedded.

b. The focal length of the lens embedded in water is higher than the one embedded in air.

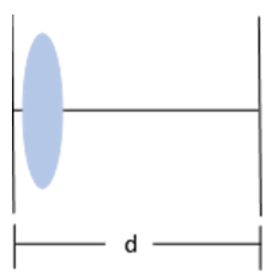
c. The focal length of the lens embedded in water is smaller than the one embedded in air.

6. Yes/No. There is an optical system made of a thin convex lens of focal length *f* and a thin concave lens of focal length -*f* separated by a distance *f*. Do parallel rays remain parallel after emerging from this system?

a. Yes

b. No

7. Which is the ray-transfer matrix of an optical system composed of an ideal lens of focal length *f* followed by a free-space propagator of distance *d*?



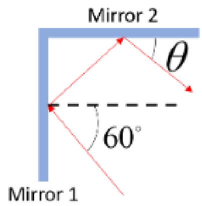
a. $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$

b. $\begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix}$

c. $\begin{pmatrix} \frac{f-d}{f} & d \\ -1/f & 1 \end{pmatrix}$

d. $\begin{pmatrix} 1 & d \\ 1/f & \frac{f-d}{f} \end{pmatrix}$

8. Two mirrors are located on a table. Between the two mirrors, there is an angle of 90deg, as the figure shows. Consider that on the Mirror 1 impinges a beam with an angle of 60-deg respect to the normal. What is the angle θ in which the beam reflected on the Mirror 2?



a. 15 deg

b. 30 deg

c. 45 deg

d. 60 deg

9. Light travels through a homogenous medium with a refractive index n = 1.56. What is its speed after transitioning to another homogenous medium with a refractive index of n=1.21?

a. 2.48×10^8 m/s

b. 3.00×10^8 m/s

c. 1.92×10^8 m/s

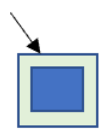
10. A 0.5-m tall object is placed 5 m away from a concave mirror with a focal length of 2 m. How far should a 0.75-m tall object be placed from a second concave mirror with a focal length of 2 m so that its image is the same height as the first's?

a. 47.6 mm

b. 6.5 m

c. 5 m

11. Light (initially in air) travels into a glass box (n=1.46) filled with water(n=1.33), with an initial incidence of 30°. At what angle does the light leave the box?



a. 20.03

b. 22.08

c. 30.00

12. A 10-cm toy boxer (B1) is placed 100 cm from a spherical mirror with a curvature radius of 10 cm. A second toy boxer (B2) of the same size is placed at 90 cm from the mirror. Which boxer's image is bigger/larger through the image and, therefore, wins the heavyweight title? How is the image: real or virtual?

a. B1 is bigger, and the image is real.

b. B1 is bigger, and the image is virtual.

c. B2 is bigger, and the image is real.

d. B2 is bigger, and the image is virtual.

13. Light is measured to travel through a new material at a speed of $1.97x10^8$ m/s. This material creates a spherical lens with radii of 15 cm and 25 cm. The spherical lens is submerged in olive oil (refractive index of 1.47). About how far from the lens should we observe a clear image of an object located at infinity?

a. 10.4 m

b. 8.2 m

c. The image will be placed at the infinity.

CHAPTER OVERVIEW

Module 2 - Wave Optics

The *Module 2: Wave Optics* module explores the foundational principles of wave optics, emphasizing the wave nature of light. It covers the postulates of wave optics and explores monochromatic waves through the Helmholtz equation. Students will analyze elementary waveforms such as plane, spherical, and paraboloidal waves, understanding their mathematical representations and physical significance. The module bridges ray and wave optics, providing insights into their interconnected nature. Key topics include the interference of two waves, principles of interferometry, and experimental setups like the Young's double-slit experiment.

This Module contains 4 classes:

Class 6 - Postulates of Wave Optics, Monochromatic Waves, Helmholtz equation

Learning Objectives of this class:

- Define an optical wave as a wavefunction that satisfies the wave equation.
- Identify the wave equation for waves traveling in any media
- Understand the superposition principle
- Define intensity and power
- Define a monochromatic wave
- Define the complex amplitude and understand that it satisfies the Helmholtz equation

Class 7 - Elementary Waves- plane, spherical and paraboloidal waves

Learning Objectives of this class:

- Identify the characteristics of elementary waves: plane, spherical, and paraboloidal waves.
- Determine if a spherical wave can be approximated to a paraboloidal wave.

Class 8 - Relation ray-wave optics, interference of two waves, interferometers

Learning Objectives of this class:

- Identify the relationship between Ray and Wave Optics
- Identify the procedure to estimate the wavefunction of an arbitrary wave using the Eikonal equation
- Understand optical interference phenomenon as the summation of the complex amplitude distributions of the interfering waves.
- Express the total intensity distribution of two interfering waves based on their phase difference
- Distinguish constructive and destructive interference
- Deduce the condition between two interference waves to form constructive and destructive interference

Class 9 - Young experiment

Learning Objectives of this class:

- Describe the Young double-slit experiment
- Derive the total intensity distribution in the Young double-slit experiment
- Deduce the condition for constructive and destructive interference in terms of the separation between the slit and screen planes

Module 2 - Summary

Summary Notes Module 2

• Light propagates in the form of waves with a speed $v = \frac{c}{n}$ where $c = 3 \times 10^8\ \mathrm{m/s}$ and n is the refractive index. Since $n \geq 1$ ($n = 1$ for air), the speed of light in any medium is $v \leq c$.

• The speed of light relates the wavelength λ and frequency f by $v = f\lambda$.

• An optical wave is described by its wavefunction $u(\mathbf{r}, t)$ where $\mathbf{r} = (x, y, z)$ represents the 3D position.

• Any optical wave satisfies the wave equation:

$$\nabla^2 u(\mathbf{r}, t) - \frac{1}{v^2} \frac{\partial^2 u(\mathbf{r}, t)}{\partial t^2} = 0 \tag{1}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplacian operator in Cartesian coordinates.

• The intensity of a wave can be defined by the wavefunction as $I(\mathbf{r}) = \langle |u(\mathbf{r}, t)|^2 \rangle$, where $\langle \cdot \rangle$ means the average over a temporal interval.

• The power of a wave is calculated at the integrated intensity over an area normal to the light's propagation axis:

$$P = \int_{Area} I(\mathbf{r})\, dA \tag{2}$$

• The wavefunction for monochromatic waves is:

$$u(\mathbf{r}, t) = a(\mathbf{r}) \cos(2\pi f t + \varphi(\mathbf{r})) \tag{3}$$

where $a(\mathbf{r})$ is the amplitude and $\varphi(\mathbf{r})$ is the phase, f is frequency [Hz], and $\omega = 2\pi f$ is the angular frequency [rad/s], $T = \frac{1}{f}$ is the temporal period [s].

The intensity of monochromatic waves is $I(\mathbf{r}) = a^2(\mathbf{r})$, which is independent of time.

The complex wavefront of monochromatic waves is:

$$U(\mathbf{r}, t) = a(\mathbf{r}) e^{j\varphi(\mathbf{r})} e^{j2\pi f t} \tag{4}$$

where $U(\mathbf{r}) = a(\mathbf{r}) e^{j\varphi(\mathbf{r})}$ is the complex amplitude.

Whereas the complex wavefront $U(\mathbf{r}, t)$ satisfies the wave equation, the complex amplitude satisfies the Helmholtz equation:

$$\nabla^2 U(\mathbf{r}) + k^2 U(\mathbf{r}) = 0 \tag{5}$$

• The complex amplitude of a plane wave is:

$$U(\mathbf{r}) = a(\mathbf{r}) e^{-j2\pi \mathbf{k}\cdot\mathbf{r}} = a(\mathbf{r}) \exp(-j2\pi[k_x x + k_y y + k_z z]) \tag{6}$$

where $\mathbf{k} = (k_x, k_y, k_z)$ is the wave vector and $k^2 = k_x^2 + k_y^2 + k_z^2$.

The intensity of a plane wave is constant:

$$I(\mathbf{r}) = |U(\mathbf{r})|^2 = |U(\mathbf{r})| = |a(\mathbf{r}) e^{-j2\pi \mathbf{k}\cdot\mathbf{r}}|^2 = |a(\mathbf{r})|^2 \tag{7}$$

The complex amplitude of a plane wave traveling along the z-direction is:

$$U(\mathbf{r}, z) = a(\mathbf{r}) e^{-j2\pi \mathbf{k}\cdot\mathbf{r}} e^{-j2\pi k_z z} \tag{8}$$

The wavefunction of a plane monochromatic wave traveling along the z-direction is periodic in terms of time and z:

$$u(\mathbf{r}, t) = |a(\mathbf{r})| \cos(2\pi f t - k_z z + \arg(a(\mathbf{r}))) \tag{9}$$

or equivalently,

$$u(\mathbf{r}, t) = |a(\mathbf{r})| \cos\left(2\pi f \left(t - \frac{z}{c}\right) + \arg(a(\mathbf{r}))\right) \tag{10}$$

since $k=\frac{f}{c}$.

• As monochromatic waves propagate through media of different refractive indexes, their frequency remains the same, but their velocity, wavelength, and wavenumber change:

$$v=\frac{c}{n} \tag{11}$$

$$v=\lambda f \rightarrow \lambda=\frac{v}{f}=\frac{c}{nf}=\frac{\lambda_0}{n}, \quad \text{with} \quad \lambda_0=\frac{c}{f} \tag{12}$$

$$k=\frac{2\pi}{\lambda}=\frac{2\pi}{\lambda_0/n}=nk_0, \quad \text{where} \quad k_0=\frac{2\pi}{\lambda_0} \tag{13}$$

• The complex amplitude of a spherical wave $U(\mathbf{r})=\frac{a_0}{r}e^{-j\mathbf{k}\cdot\mathbf{r}}$ where a_0 is a constant, r is the radius of the spherical wave from the origin, and $k=\frac{2\pi}{\lambda}=\frac{2\pi f}{v}=\frac{w}{v}$.

In general, the origin of the spherical wave can be shifted: $U(\mathbf{r})=\frac{a_0}{|\mathbf{r}-\mathbf{r_0}|}e^{-j\mathbf{k}\cdot|\mathbf{r}-\mathbf{r_0}|}$.

The intensity of a spherical wave is inversely proportional to the square of the distance, $I(\mathbf{r})=|U(\mathbf{r})|^2=\frac{a_0^2}{r^2}$.

• The complex amplitude of a paraboloidal wave, $U(\mathbf{r})=\frac{a_0}{z}e^{-j\mathbf{k}\cdot z}\exp\left(-j\frac{k}{2z}(x^2+y^2)\right)$, can be understood as a plane wave $a_0e^{-j\mathbf{k}\cdot z}$, modulated by a factor $\frac{1}{z}\exp\left(-j\frac{k}{2z}(x^2+y^2)\right)$.

For larger z, the phase factor $\exp\left(-j\frac{k}{2z}(x^2+y^2)\right)\approx 0$, so the paraboloidal wave can be approximated as a plane wave.

The paraboloidal waves satisfy the paraxial Helmholtz equation, $\nabla_T^2 A(x,y,z)-j2k\frac{\partial A(x,y,z)}{\partial z}=0$, where $A(x,y,z)=a_0\exp\left(-j\frac{k}{2z}(x^2+y^2)\right)$.

A spherical wave that satisfies the Fresnel approximation becomes a paraboloidal wave. The Fresnel approximation is met for points (x,y) whose radius $a=\sqrt{x^2+y^2}$ is at least one order of magnitude smaller than $(8\lambda z^3)^{1/4}$, i.e., $a^4 \ll 8\lambda z^3$. If one defines the Fresnel number: $N_F=\frac{a^2}{\lambda z}$, and the maximum angle $\theta_m=\frac{a}{z}$, the Fresnel approximation is given by:

$$\frac{N_F\theta_m^2}{4}\ll 1. \tag{14}$$

• Interference:
Coherent: superposition of two or more waves whose wavefunctions are related to each other. For example, waves that come from the same point source. The resultant wavefunction is the sum of the individual ones.
Incoherent: superposition of two or more waves whose wavefunctions are not related. For example, the waves emitted from each point of a lamp are completely unrelated, therefore they do not interfere. Nonetheless, there is a superposition of their individual intensities.

In this module, interference relates to the coherent superposition, i.e., the sum of the wavefunctions so that $U=U_1+U_2$. Therefore, the resultant intensity of the sum of two or more coherent wavefunctions is not equal to the sum of their individual intensities, i.e., $I=|U|^2\neq I_1+I_2$.

Example of the interference of two coherent waves whose complex amplitude distributions are, respectively, $U_1=\sqrt{I_1}e^{j\varphi_1}$ and $U_2=\sqrt{I_2}e^{j\varphi_2}$. The resultant intensity is:

$$I=|U_1+U_2|^2=|U_1|^2+|U_2|^2+U_1U_2^*+U_2U_1^*. \tag{15}$$

$$I=I_1+I_2+\sqrt{I_1}e^{j\varphi_1}\sqrt{I_2}e^{-j\varphi_2}+\sqrt{I_1}e^{-j\varphi_1}\sqrt{I_2}e^{j\varphi_2}. \tag{16}$$

$$I=I_1+I_2+2\sqrt{I_1}\sqrt{I_2}\cos(\varphi_1-\varphi_2). \tag{17}$$

The term $2\sqrt{I_1}\sqrt{I_2}\cos(\varphi_1-\varphi_2)$ depends on the phase difference between both waves. If both waves have the same intensity, $I_0=I_1=I_2$, then:

$$I=2I_0[1+\cos(\varphi_1-\varphi_2)]. \tag{18}$$

Because the resultant intensity is $I=2I_0[1+\cos(\varphi_1-\varphi_2)]$, there are different scenarios:

- $I = 2I_0$ when $\cos(\varphi_1 - \varphi_2) = 0$. In other words, the phase difference between both waves is a multiple of $\frac{\pi}{2}$, i.e., $\varphi_1 - \varphi_2 = (m + \frac{1}{2})\pi$, where $m = 0, 1, 2, \ldots$
- $I_{\max} = 4I_0$ when $\cos(\varphi_1 - \varphi_2) = 1$. This is the condition for constructive interference (i.e., maximum value of the resultant intensity). In other words, the phase difference between both waves is a multiple of 2π, i.e., $\varphi_1 - \varphi_2 = 2\pi m$, where $m = 0, 1, 2, \ldots$
- $I_{\min} = 0$ when $\cos(\varphi_1 - \varphi_2) = -1$. This is the condition for destructive interference (i.e., minimum value of the resultant intensity). In other words, the phase difference between both waves is an odd multiple of π, i.e., $\varphi_1 - \varphi_2 = (2m + 1)\pi$, where $m = 0, 1, 2, \ldots$

The interference creates a fringe-like pattern whose visibility is:

$$V = \frac{I_{\max} + I_{\min}}{I_{\max} + I_{\min}}. \tag{19}$$

In a coherent interference, the intensity of the superposition between two coherent waves depends on the phase of the individual waves. The phase difference between the waves can be expressed as:

$$I = |U_1 + U_2|^2 = |U_1|^2 + |U_2|^2 + U_1 U_2^* + U_2 U_1^*. \tag{20}$$

Example of the interference of two tilted coherent waves with the same intensity:

$$U_1 = \sqrt{I_0} e^{-jkz}, \quad U_2 = \sqrt{I_0} e^{-jk(\cos\theta z + \sin\theta x)}. \tag{21}$$

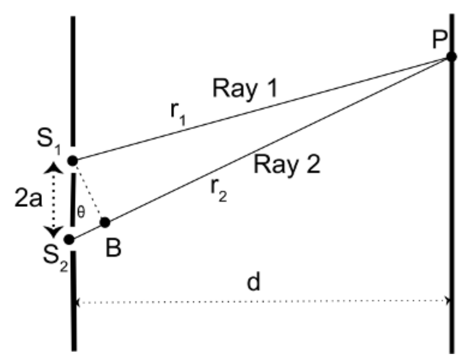
Thus, the intensity becomes:

$$I = 2I_0 + \sqrt{I_0}\sqrt{I_0} e^{-jkz} e^{jk(\cos\theta z + \sin\theta x)} + \sqrt{I_0}\sqrt{I_0} e^{jkz} e^{-jk(\cos\theta z + \sin\theta x)}. \tag{22}$$

$$I = 2I_0 \left[1 + \cos(k(1 - \cos\theta)z + k\sin\theta x)\right]. \tag{23}$$

The intensity changes axially and laterally. The axial and lateral periods are: $T_z = \frac{\lambda}{1-\cos\theta}, \quad T_x = \frac{\lambda}{\sin\theta}$.

• Young's Experiment



The optical path difference between the optical rays 1 and 2 is the distance S_2B.

$$r_1 = S_1P = \text{distance between the source } S_1 \text{ and the observation plane } P \tag{24}$$

$$r_2 = S_2P = S_2B + S_1P = \text{distance between the source } S_2 \text{ and the observation plane } P. \tag{25}$$

The interference pattern at the observation plane is:

$$I = 2I_0 \left[1 + \cos\left(\frac{2\pi}{\lambda}\frac{2a}{d}x\right)\right]. \tag{26}$$

Because the resultant intensity is: $I = 2I_0 \left[1 + \cos\left(\frac{2\pi}{\lambda}\frac{2a}{d}x\right)\right]$, we have the following conditions:

- $I_{\max} = 4I_0$ when $\cos\left(\frac{2\pi}{\lambda}\frac{2a}{d}x\right) = 1$.
- The maxima are located at $x_{\max} = m\frac{\lambda d}{2a}$.
- The separation between two consecutive maxima is $x_{\max,m+1} - x_{\max,m} = \frac{\lambda d}{2a}$.
- $I_{\min} = 0$ when $\cos\left(\frac{2\pi}{\lambda}\frac{2a}{d}x\right) = -1$.

- The minima are located at $x_{\min} = (m + \frac{1}{2})\frac{\lambda d}{2a}$.
- The separation between two consecutive minima is $x_{\min,m+1} - x_{\min,m} = \frac{\lambda d}{2a}$.

Multi-choice questions

Multi-choice Quiz Module 2

1. Select the correct statement. When two waves are mutually coherent, it means that

a. Their combined intensity is the linear sum of their individual intensities

b. The phase difference between the two waves does not change over time

c. The waves always interfere constructively

2. A light wave is described with the following equation $u\left(x,t\right)=10\sin\left(10\sin\left(1.5707\times10^{8}x-3.1416\times10^{15}t+\frac{\pi}{4}\right)\right)$. What is the medium's index of refraction of the propagating wave?

a. Light waves can only travel through air, n =1.

b. We do not have enough information to retrieve the refractive index.

c. The refractive index is n = 1.5.

3. When a monochromatic wave propagates through media of different refractive indexes, which one of the following parameters does not change?

a. The velocity of the light.

b. The wavelength of the light.

c. The frequency of the light.

4. Which of the following statements regarding the Fresnel approximation is correct?

a. The Fresnel approximation is invariant of the light wavelength

b. When the Fresnel approximation is satisfied, the spherical wave can be approximated by a plane wave.

c. For a fixed observation plane and wavelength, the smaller the radius of the circle, the more suitable the Fresnel approximation is.

5. Which is the maximum radius of a circle in which a spherical wave of wavelength \lambda = 488 nm, originating at a distance 2 m away, may be approximated by a paraboloidal wave?

a. The maximum radius is 55.038 mm

b. The maximum radius is 21.01 mm

c. The maximum radius is 66.49 mm

d. The maximum radius is 42.038 mm

6. Yes/No. Is the Fresnel approximation applicable for a radius of 10 cm at the observation plane located at 100 cm? The light wavelength is 550 nm.

a. Yes

b. No

7. If two waves are in phase and have the same amplitude, then the resultant wave has

a. half of the amplitude of the single wave

b. the same amplitude as the single wave
c. twice of the amplitude of the single wave

8. Imaging that there is a Young's double-slit experiment in which the separation of the slits is *2a*, and the distance between the slits and observation plane is *d*. For a particular reason, the observation plane is moved away so that the distance between the slits' and observation plane is doubled. Which is the new separation of the slits if one wants to keep constant the same fringes' spacing on the screen?

a. 2*a*

b. $2a/3$

c. a

d. $4a$

9. Assume that we have an optical system in which two beams, which come from the same point source (e.g., the beams are mutually coherent), interfere. Which of the following optical parameters is governed by the superposition principle?

a. Amplitude

b. Intensity

c. Power

10. Given that the intensity of two waves is equal, what would the value of the phase difference be to get destructive interference?

a. 0

b. π

c. $\pi/2$

11. You have built a demonstration of Young’s experiment with a distance of 0.25mm between the holes and a distance of 10 m to the screen. Unfortunately, you are colorblind, and you cannot remember the color of the laser beam that you used. However, you have measured that the distance between two maxima is 25.3mm. What must be the wavelength (and thus color) of the light used in the experiment?

a. 632.5 nm i.e. red

b. 532 nm i.e. green

c. 442 nm i.e. blue

12. Two monochromatic coherent waves of the same intensity (I_0) are interfering with a phase difference of 35deg. What is the resulting Intensity?

a. $3.6I_0$

b. $0.36\ I_0$

c. $3.1\ I_0$

d. $1.8\ I_0$

13. Your professor wants you to make an interferometer, but you have raided the storage cabinet and could only find a single beamsplitter and a pair of mirrors. Which is the only type of interferometer that you can make?

a. A Mach-Zehnder interferometer

b. A Michelson interferometer

c. A Sagnac interferometer

CHAPTER OVERVIEW

Module 3 - Beam Optics

The *Beam Optics* module focuses on the theory and applications of Gaussian beams, which form the foundation for modern laser optics. Students will explore the mathematical description and fundamental features of Gaussian beams, such as beam waist, divergence, and Rayleigh range. The module highlights key properties like phase fronts, intensity profiles, and beam quality factors. Additionally, students will analyze the propagation of Gaussian beams through various optical systems, including lenses and mirrors, using matrix optics. By mastering these concepts, students will gain a solid understanding of beam dynamics, preparing them for advanced studies in optical engineering and photonics.

This module contains 3 classes.

Class 10 - Gaussian beam- features and mathematical description

Learning Objectives of this class:

- Describe the features of a Gaussian beam.
- Define the q parameter of a Gaussian beam
- Identify the observation plane and the Rayleigh range from the q parameter
- Define the radius of curvature and beam width of a Gaussian beam
- Determine the observation plane and the beam waist from the radius of curvature and beam width of a Gaussian beam.
- Describe the complex amplitude distribution and its irradiance distribution of a Gaussian beam.

Class 11 - Properties of Gaussian beams

The learning objectives of this class:

- Understand the behavior of the intensity distribution of a Gaussian beam.
- Recognize that the power of a Gaussian beam is constant.
- Understand the behavior of the beam width of a Gaussian beam.
- Calculate the beam divergence.
- Define the depth of focus.
- Identify the phase distribution of a Gaussian beam.
- Determine the minimum number of parameters to define a Gaussian beam.

Class 12 - Propagation of Gaussian beams through optical systems

The learning objectives of this class:

- Understand that a Gaussian beam becomes another Gaussian beam after propagating for an arbitrary optical system
- Apply the matrix formalism in cascade to estimate the parameters of the transmitted Gaussian beam
- Calculate the parameters of the newly transmitted Gaussian beam

Module 3 - Summary

Summary Notes Module 3

Amplitude distribution of a Gaussian beam

$$u(x,y,z) = A_0 \frac{w_0}{w(z)} \exp(-jkz) \exp\left(-\frac{x^2+y^2}{w^2(z)}\right) \exp\left(-j\frac{k(x^2+y^2)}{2R(z)}\right) \exp(j\varphi(z)) \tag{1}$$

$\exp(-jkz)$ represents a plane wave

$\exp\left(-\frac{x^2+y^2}{w^2(z)}\right)$ represents a Gaussian modulation

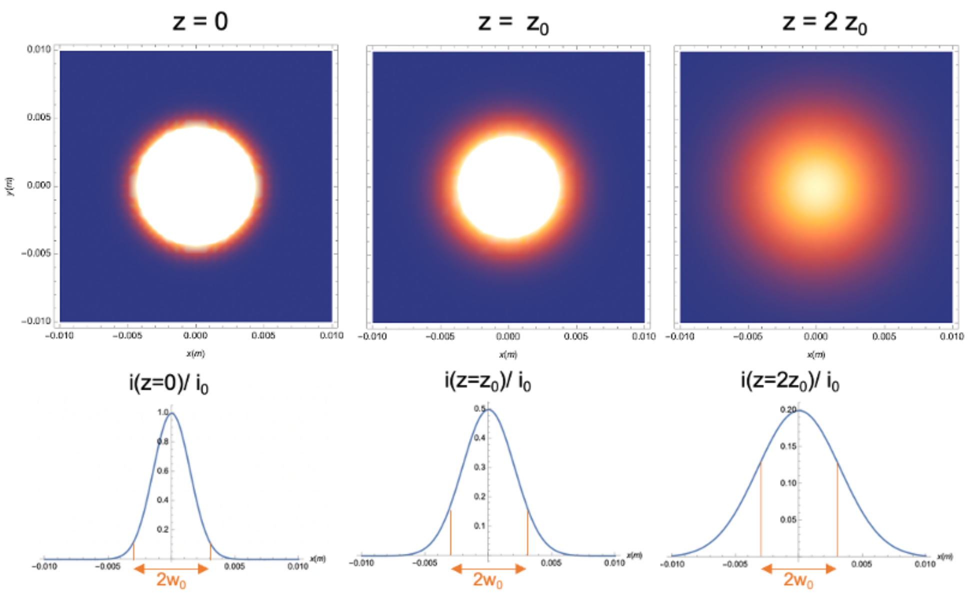
$\exp\left(-j\frac{k(x^2+y^2)}{2R(z)}\right)$ represents a paraboloidal wavefront

$\exp(j\varphi(z))$ represents a phase gained through propagation

Intensity distribution of a Gaussian beam

The irradiance/intensity distribution of a Gaussian beam follows a Gaussian function.

$$i(x,y,z) = |u(x,y,z)|^2 = \frac{A_0^2}{w_0^2}\frac{w^2(z)}{w_0^2} \exp\left(-2\frac{x^2+y^2}{w^2(z)}\right)$$



86% of the intensity is located within the beam waist w_0.

Beam Parameters

The beam width is given by:

$$w(z) = w_0\sqrt{1+\left(\frac{z}{z_0}\right)^2} \tag{2}$$

The beam waist is:

$$w_0 = \sqrt{\frac{\lambda z_0}{\pi}} \tag{3}$$

The Rayleigh range/distance is:

$$z_0 = \frac{\pi w_0^2}{\lambda} \tag{4}$$

The beam curvature is:

$$R(z)=z\left[1+\left(\frac{z_0}{z}\right)^2\right] \tag{5}$$

The phase retardance is:

$$\varphi(z)=\tan^{-1}\left(\frac{z}{z_0}\right) \tag{6}$$

Relation between intensity distributions at two axial planes

At $z=0$ (waist plane), the beam width coincides with the beam waist, $w(z=0)=w_0$. The intensity at $z=0$ is:

$$i_0(x,y)=i(x,y,z=0)=|u(x,y,z=0)|^2=\frac{A_0^2}{w_0^2}\exp\left(-2\frac{x^2+y^2}{w_0^2}\right) \tag{7}$$

The intensity at any axial plane z is:

$$i(x,y,z)=|u(x,y,z)|^2=\frac{A_0^2}{w_0^2}\frac{w^2(z)}{w_0^2}\exp\left(-2\frac{x^2+y^2}{w^2(z)}\right)=\frac{1}{M^2}i_0(x,y) \tag{8}$$

where $M=\frac{w(z)}{w_0}$

Power of Gaussian Beams. The power is given by:

$$P=\frac{1}{2}i_0\left(\pi w_0^2\right) \tag{9}$$

which is independent of z.

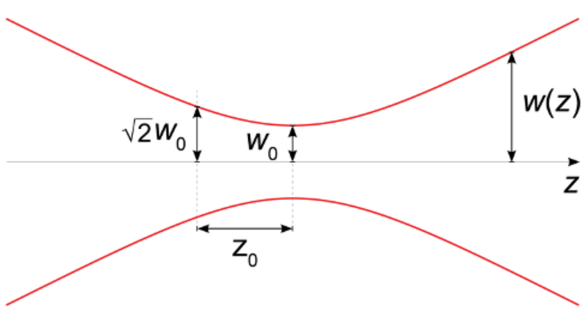
Energy Conservation Law for Gaussian Beams

$$\int_0^\infty i(\rho,z)2\pi\rho d\rho=\int_0^\infty i_0(\rho)2\pi\rho d\rho \tag{10}$$

$$\int_0^\infty i(\rho,z)2\pi\rho d\rho=\int_0^\infty\frac{1}{M^2}i_0(x,y)2\pi\rho d\rho=\int_0^\infty i_0(x',y')2\pi\rho' d\rho' \tag{11}$$

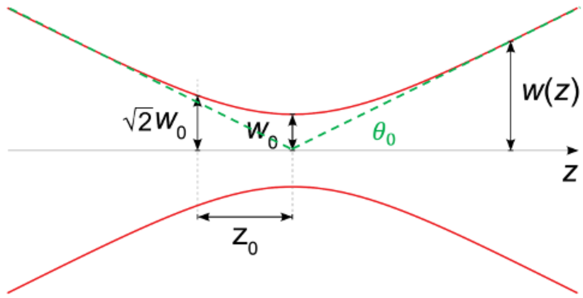
Where we have defined a new variable $x'=\frac{x}{M}$ and $y'=\frac{y}{M}$.

Features of Gaussian Beams



- At any transverse plane (z): $w(z)>w_0$
- If $z=\pm z_0$, $w(z=\pm z_0)=\sqrt{2}w_0$, which means that at the area of the spot beam $(\pi w^2(z))$ it is double $(2\pi w_0^2)$ at $z=\pm z_0$
- Depth of focus: $2z_0$, defined as the axial range in which the beam width is “almost” constant
- If $z=0$ (i.e., beam waist plane), $R(z=0)=\infty$, so $\exp\left(-j\frac{k(x^2+y^2)}{2R(z)}\right)=1$. In other words, there is no spherical wavefront term, i.e., the wavefront is a plane wave at the beam waist.
- If $z=\pm z_0$ (i.e., Rayleigh range), the radius of curvature is minimum: $R(z=\pm z_0)=\pm 2z_0$
- If $z\gg z_0$, the beam width is linearly proportional to the distance z:

$$w(z) = w_0 \frac{z}{z_0} \tag{12}$$



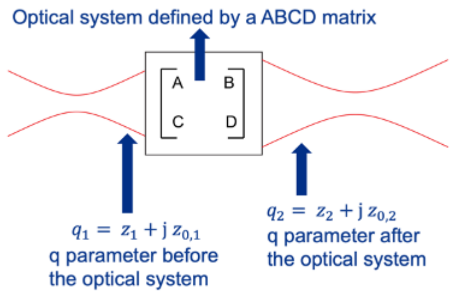
- Beam divergence \(\theta_0 = \frac{w_0}{z_0} = \frac{\lambda}{\pi w_0} \
- If $z \gg z_0$, $R(z) = z$, which means the Gaussian beam acts as a spherical wave confined within the divergence angle θ_0.

q-parameter: $q(z) = z + jz_0 = \frac{1}{R(z)} - j\frac{\lambda}{\pi w^2(z)}$ where z is the distance of the Gaussian beam to its beam waist, and z_0 is its Rayleigh range.

Propagation of Gaussian Beams through ABCD Matrix Optics: A Gaussian beam propagating through optical components remains a Gaussian beam. However, its features change on the ABCD matrix of the optical system.

$$q_2 = z_2 + jz_{02}, \quad q_2 = \frac{Cq_1 + D}{Aq_1 + B} \tag{13}$$

where q_2 defines a Gaussian beam at z_2 and Rayleigh range z_0^2.



Multi-choice questions

Multi-choice Quiz Module 3

1. Given a Gaussian beam with a wavelength of 500 nm and a beam waist of 2 mm. What is its Rayleigh range?

a. Approximately 50 meters

b. Approximately 25 meters

c. Approximately 12 meters

2. What is the radius of curvature $R(z)$ given a q parameter q = 5 + j2?

a. 14.5 m

b. 2.32 m

c. 5.8 m

d. 36.25 m

3. What is the beam waist of a He-Ne laser (wavelength 633 nm) if its beam divergence is 2 deg?

a. The beam waist is 0.17 µm

b. There is a need to know another parameter of the Gaussian beam in order to compute the beam waist.

c. The beam waist is 5.77 µm.

4. Which one of the following sentences is <u>not correct</u>?

a. The larger the beam waist, the larger the depth of focus.

b. The shorter the wavelength, the larger the depth of focus

c. The depth of focus depends proportionally on the beam waist, but it is invariant with the source’s wavelength.

5. Which one of the following sentences is <u>false</u>?

a. The beam width always increases with the axial distance *z*.

b. The power of a Gaussian beam is independent of the axial distance *z*.

c. The power of a Gaussian beam follows a Gaussian function.

6. Consider a green laser of wavelength λ = 532 nm that emits a Gaussian beam with a beam waist of 160 µm. At a particular axial plane, the beam waist has been increased to 1 mm. What is the distance of this plane with respect to the beam waist?

a. The axial distance *z* is 93.26 cm.

b. The axial distance is 6.17 m.

c. There is a need for some extra parameters to solve this problem.

7. A Gaussian beam, which is propagated through the positive z-axis, is reflected by a planar mirror located perpendicular to its axis. Which of the following sentences is <u>true</u>?

a. The position of the beam waist depends on the radius of curvature of the planar mirror.

b. Both the beam waist and the Rayleigh distance remain the same.

c. The beam waist after the reflection is double the incident beam waist due to the reflection phenomenon.

8. A Gaussian beam, which is propagated through the positive z-axis, is transmitted by a distance *d* through free space. Which of the following sentences is <u>false</u>?

a. The axial distance *d* changes the value of the beam waist inversely.

b. Both the beam waist and the Raleigh distance remain invariant to this free-propagation distance.

c. The free propagation affects the radius of curvature of the Gaussian beam.

9. Select the correct answer that properly fills the gaps. The depth of focus of a Gaussian beam is proportional to the ___________ and inversely proportional to the ___________.

a. wavelength, area of the beam

b. area of the beam, wavelength

c. area of the beam, complex amplitude

d. wavelength, complex amplitude

10. Given a Gaussian beam whose beam waist is 797.88 nm and its radius of curvature at 4 µm is 10.25 µm, what is its wavelength λ?

a. 400 nm

b. 450 nm

c. 500 nm

11. Fill the gap**.** For distances $z \gg z_0$, a Gaussian beam can be approximated by a cone that contains _____ of the beam's power.

a. 66%

b. 86%

c. 96%

12. If a Gaussian beam propagates a distance *f* until it meets a lens of focal length f and then propagates a distance f again, what is the new q number of the Gaussian beam in relation to the original Gaussian beam?

a. -q

b. -2f/q

c. $-f^2/q$

13. Yes/No**.** A Gaussian beam propagates a distance of d_1= 10 cm along the positive z-axis until it gets refracted by a concave/diverging lens of focal length equal to 15 cm. Later, the resultant Gaussian beam propagates a distance of d_2 = 20 cm until it gets refracted again by a convex/converging lens of focal length 20 cm. Is the beam waist of the Gaussian beam affected?

a. Yes

b. No

CHAPTER OVERVIEW

Module 4 - Fourier Optics

The *Fourier Optics* module focuses on applying Fourier analysis to understand and design optical systems. Students will learn to describe optical signals as superpositions of tilted plane waves and define linear and shift-invariant (LSI) optical systems. The module emphasizes identifying criteria for determining if an optical system is LSI and exploring the mathematical implications. Key concepts include expressing the output of LSI systems as convolutions of input signals with impulse responses, facilitating system analysis. We will estimate the complex amplitude distribution in free propagation under Fresnel and Fraunhofer regimes. Additionally, students will explore image processing in the Fourier domain, gaining skills to analyze, filter, and manipulate images. This module bridges optics, mathematics, and image processing techniques.

This Module has 4 classes.

Class 13 - Space vs Fourier Domain, Principle of Fourier Optics, LSI systems

The learning objectives in this class:

- Describe any optical signal as a superposition of titled plane waves
- Define linear optical systems
- Define shift-invariant optical systems
- Identify the steps to determine if an optical system is LSI or not
- Express the output signal as a convolution of the input signal and an impulse response if the system is LSI
- Image process using Fourier domain

Class 14 - Impulse response and Transfer Function in free propagation

The learning objectives of this class:

- Understand the propagation of free-space optical wavefronts as an LSI system
- Define diffraction
- Recognize the three regions: Rayleigh-Sommerfield, Fresnel, and Fraunhofer regions.
- Calculate the impulse response and transfer function in free-space propagation

Class 15 - Fresnel and Fraunhofer diffraction patterns

The learning objectives of this class:

- Revisit the scalar diffraction theory
- Identify the conditions of the Fresnel Region
- Determine if the Fresnel condition is met
- Calculate the impulse response and transfer function in free-space propagation within the Fresnel Region
- Calculate the diffraction patterns in the Fresnel Region
- Identify the conditions of the Fraunhofer Region
- Determine if the Fraunhofer condition is met
- Calculate the diffraction patterns in the Fraunhofer Region

Class 16 - Lenses, Optical Fourier Transforms, 4F imaging systems and spatial filtering

The learning objectives of this class:

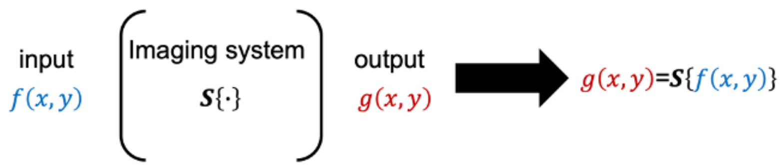
- Understand the operation principle of lenses
- Determine the transmittance of lenses
- Understand the use of the ABCD matrix to estimate the diffraction patterns in an arbitrary optical system
- Calculate the complex amplitude distribution at an arbitrary plane using the ABCD matrix
- Calculate the complex amplitude distributions at the Fourier and image planes in a coherent 4f imaging system using ABCD matrix
- Calculate the complex amplitude distribution at the image plane if there is a pupil in the Fourier plane of a coherent 4f imaging system
- Identify the coherent impulse response and its transfer function in a coherent 4f imaging system
- Understand spatial filtering

Module 4 - Summary

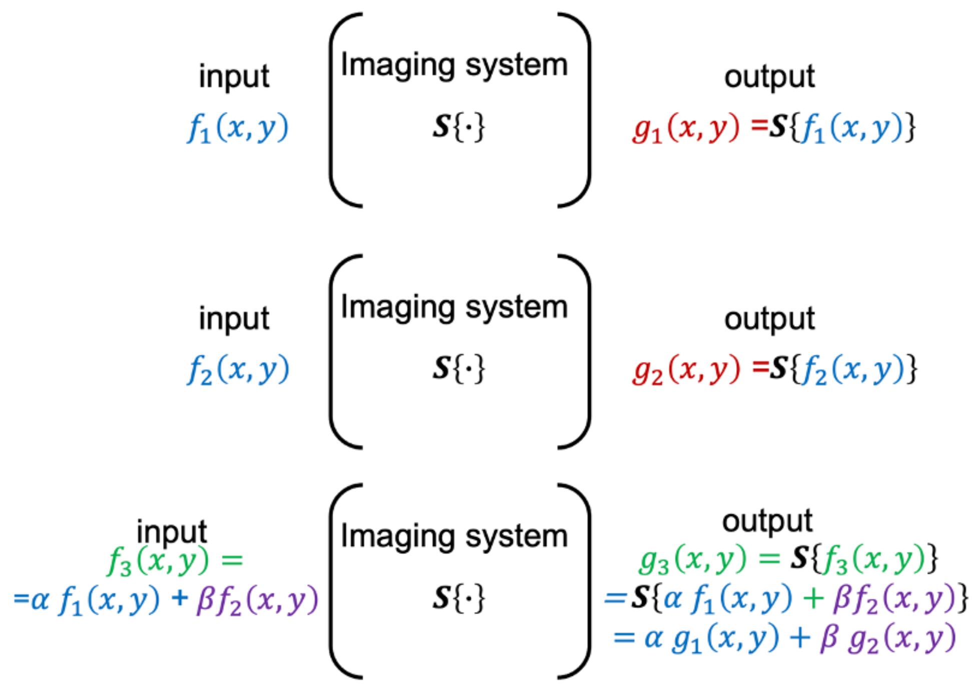
Summary Notes Module 4

• Any amplitude distribution can be expressed as a sum of plane waves, i.e., $\exp[j2\pi(ux+vy)]$. Compared to the mathematical expression of a plane wave in Module 2, $\exp[j(k_x x+k_y y)]$, then $k_x=2\pi u=\frac{2\pi}{\lambda}\sin\theta_x$ and $k_y=2\pi v=\frac{2\pi}{\lambda}\sin\theta_y$. This means that the angle of the plane waves along the x- and y-direction can be estimated by $\theta_x=\sin^{-1}(\lambda u)$ and $\theta_y=\sin^{-1}(\lambda v)$.

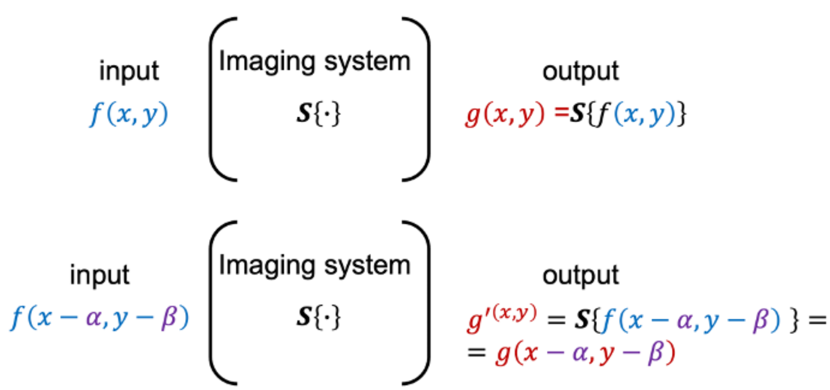
• An imaging system must be a linear shift-invariant system. Consider that an imaging system is represented by a $\mathbf{S}$ operator.



A system is linear if the output is equal to the sum of the individual outputs.



A system is shift-invariant if the output of a displaced input signal is the same function but displaced by the same amount.



Only when the optical system is linear space-invariant (LSI), the output signal, $g(x,y)$, can be expressed as the two-dimensional (2D) convolution ($\otimes_2$) between the input signal, $f(x,y)$, and the impulse response of the system, $h(x,y)$:

$$g(x,y)=f(x,y)\otimes_2 h(x,y). \tag{1}$$

In the Fourier domain, the spectrum of the output signal, $G(u,v)$, is the product between the spectrum of the input signal, $F(u,v)$, and the optical transfer function, $H(u,v)$:

$$G(u,v)=F(u,v)H(u,v). \tag{2}$$

This means that the transfer function $H(u,v)$ is the 2D Fourier transform of the impulse response, $h(x,y)$:

$$H(u,v)=\mathrm{FT}[h(x,y)]. \tag{3}$$

• Free-space propagator under the Fresnel approximation

$$h(x,y)=\frac{e^{jkz}}{j\lambda z}\exp\left[j\frac{k}{2z}(x^2+y^2)\right], \tag{4}$$

and

$$H(u,v)=\frac{e^{jkz}}{j\lambda z}\exp\left[-j\pi\lambda z(u^2+v^2)\right]. \tag{5}$$

Condition of the Fresnel region:

$$N_F\frac{\theta_m^2}{4}\ll 1, \tag{6}$$

where $N_F=\frac{a^2}{\lambda z}$ is the Fresnel number and $\theta_m=\frac{a}{z}$ is the maximum aperture angle.

• Free-space propagator under the Fraunhofer approximation ($z\gg$)

$$g(x,y)=f(x,y)\otimes_2 h(x,y)=F\left(\frac{x}{\lambda z},\frac{y}{\lambda z}\right)=F\left(\frac{x}{M},\frac{y}{M}\right), \tag{7}$$

where M is the magnification factor.

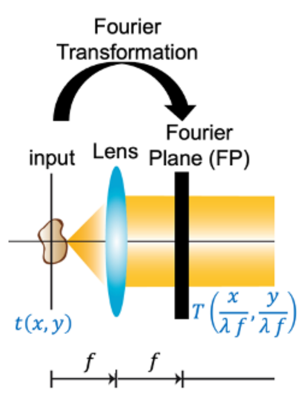
For $z\gg$, the amplitude distribution (also known as the diffraction pattern) of an object transmittance $f(x,y)$ is a scaled version of its Fourier Transform under Fraunhofer approximation.

Condition of the Fraunhofer region:

$$z\gg\frac{b_{\text{input}}^2}{\lambda}\quad\text{and}\quad z\gg\frac{a_{\text{output}}^2}{\lambda}, \tag{8}$$

where b_{input} is the maximum lateral extent in the input plane and a_{output} is the maximum lateral extent in the output plane.

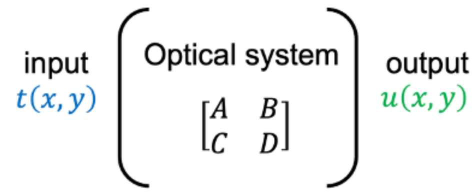
• Converging and diverging (i.e., optical) lenses make Fourier transforms.



Consider a converging lens. If an object with amplitude transmittance $t(x,y)$ is placed at the front focal plane of a lens (i.e., the distance between the object/input and lens planes is equal to the lens' focal length, f), a scaled replica of the Fourier transform of the object transmittance is found at its back focal plane:

$$T\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right). \tag{9}$$

• Calculation of the complex amplitude distribution of an arbitrary optical system with an input transmittance $t(x,y)$. The arbitrary optical system is represented by its ABCD matrix.



If $B=0$, the complex amplitude distribution at the output plane is the product of a quadratic phase wavefront and a scaled replica of the object transmittance:

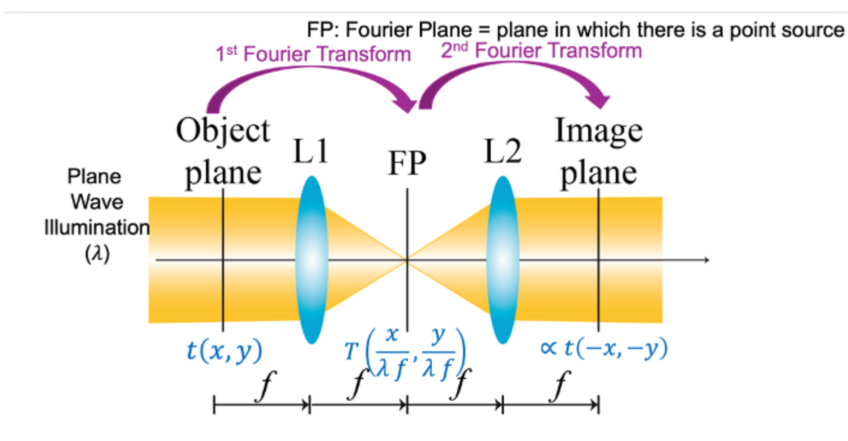
$$u(x,y) = e^{jkL_0/A} \exp\left[j\frac{kC}{2A}(x^2+y^2)\right] t\left(\frac{x}{A}, \frac{y}{A}\right), \tag{10}$$

where L_0 is the geometrical distance between the input and output planes and $k=\frac{2\pi}{\lambda}$ is the wave number.

Else, if $B \neq 0$, the complex amplitude distribution at the output plane becomes:

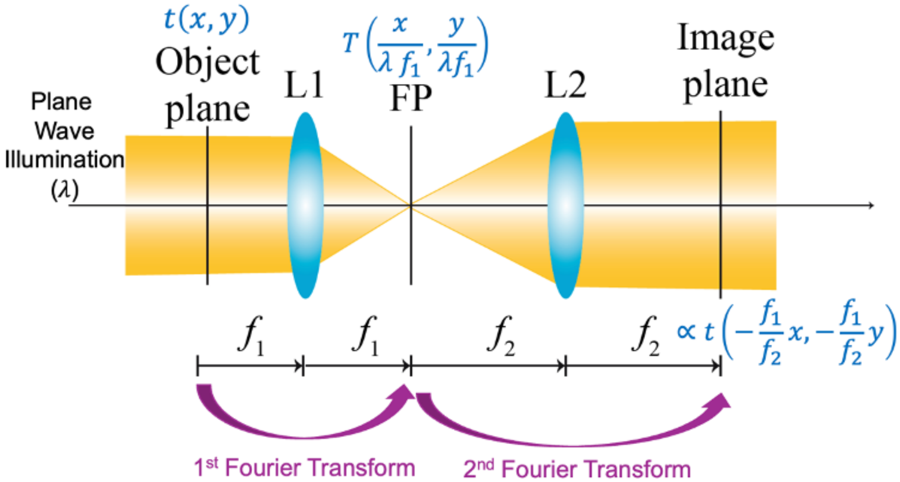
$$u(x,y) = \frac{e^{jkL_0}}{j\lambda B} \exp\left[j\frac{kD}{2B}(x^2+y^2)\right] \int_{-\infty}^{\infty} t(x_0,y_0) \exp\left[j\frac{kA}{2B}(x_0^2+y_0^2)\right] \exp\left[-j\frac{2\pi}{\lambda B}(xx_0+yy_0)\right] dx_0\, dy_0. \tag{11}$$

• 4f imaging system if both lenses have the same focal length

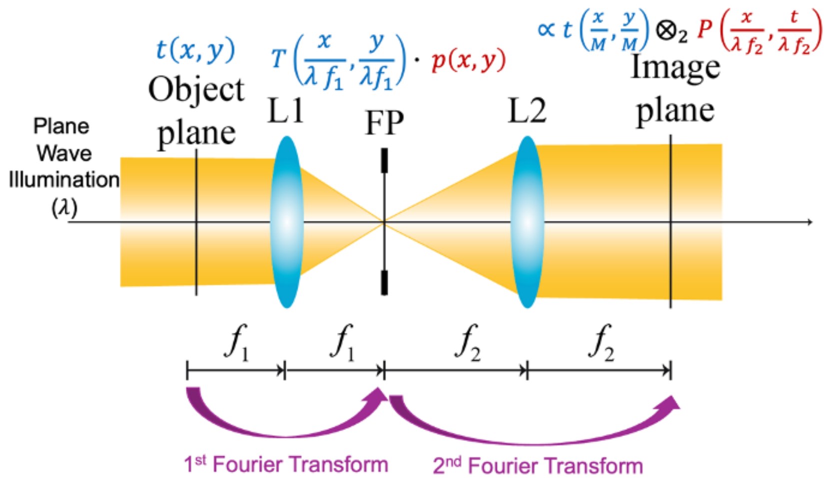


Hint – The Fourier Transform of a plane wave is a Delta function (i.e., point source)

• 4f imaging system if both lenses have different focal lengths



• Spatial filtering in a 4f imaging system by inserting a pupil with transmittance $p(x, y)$ at the Fourier plane of a 4f system. The pupil filters out some object frequencies.



Thus, the complex amplitude at the image plane (i.e., back focal plane of the L_2 lens) is:

$$u(x, y) = \frac{1}{M^2} t\left(\frac{x}{M}, \frac{y}{M}\right) \otimes_2 P\left(\frac{x}{\lambda f_2}, \frac{y}{\lambda f_2}\right) = \frac{1}{M^2} t\left(\frac{x}{M}, \frac{y}{M}\right) \otimes_2 h(x, y), \tag{12}$$

where $M = -\frac{f_2}{f_1}$ is the lateral magnification, $\otimes_2$ is the 2D convolution operator, and $P(u, v)$ is the 2D Fourier transform of the pupil transmittance. Based on the equation, the impulse response of the 4f imaging system is a scaled replica of the pupil's Fourier transform, $h(x, y) = P\left(\frac{x}{\lambda f_2}, \frac{y}{\lambda f_2}\right)$. Therefore, the transfer function is a scaled replica of the pupil's transmittance:

$$H(u, v) = \mathrm{FT}[h(x, y)] = p\left(-\frac{\lambda f_2 u}{f_1}, -\frac{\lambda f_2 v}{f_1}\right). \tag{13}$$

Multi-choice questions

Multi-choice Quiz Module 4

1. Any arbitrary function can be analyzed as the sum of harmonic functions in Fourier Optics. Which type of waves are these harmonic functions?

a. Any optical wave that satisfies the Helmonthz equation.

b. Paraboloidal waves

c. Plane waves

2. What are the conditions that an optical imaging system must satisfy?

a. It should be linear

b. It should be invariant under displacement, (i.e., shift-invariant)

c. Both conditions should be met.

d. An optical imaging system should not be linear and space invariant.

3. True/False. The output of a linear shift-invariant optical system is the 2D convolution between the input signal and the impulse response of the system.

a. True

b. False

4. True/False. The diffraction patterns of an object are a scaled copy of the 2D Fourier transform of the object amplitude distribution *t(x,y)* under Fraunhofer approximation.

a. True

b. False

5. Consider that light emerging from a source with wavelength λ = 0.5 μm propagates freely between two arbitrary planes separated by a distance of 1 m. Assume that the object points lie within a circular aperture of radius 1 cm and the observation points lie within a circular aperture of radius 2 cm. Which condition is satisfied?

a. Fraunhofer approximation.

b. Fresnel approximation

c. Both Fraunhofer and Fresnel approximation.

d. None of the above

6. Select the incorrect statement regarding lenses.

a. Lenses can provide images of objects if the distances between the object-lens (z_1) and the image-lens (z_2) satisfy the lens' imaging equation, $\frac{1}{z_1}+\frac{1}{z_2}=\frac{1}{f}$.

b. Lenses can perform real-time Fourier transforms of objects if objects are located at the front focal plane of the lens and the observation plane (i.e., Fourier plane) is located at the lens' back focal plane.

c. Only converging lenses can provide images and Fourier transform of an object.

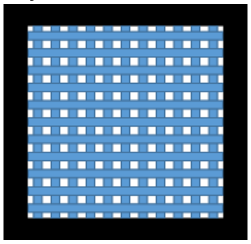
7. Select the incorrect statement regarding 4f imaging systems.

a. The impulse response of a 4f imaging system is the scaled copy of the Fourier transform of the mask transmittance.

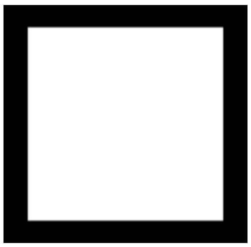
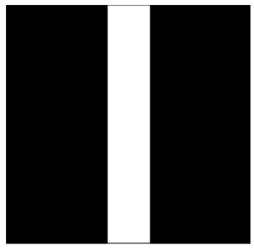
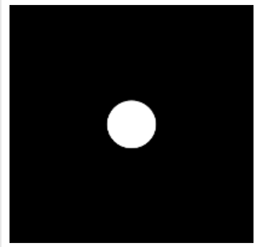
b. The transfer function of the system is a scaled copy of the mask transmittance.

c. If no mask is inserted at the Fourier plane, the system is not an imaging system.

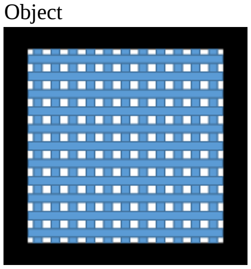
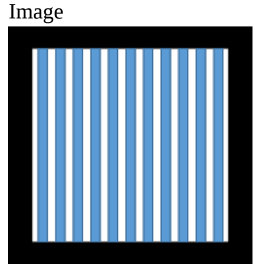
8. Consider a 4*f* imaging system whose object and image are shown below.

Object	Image
	

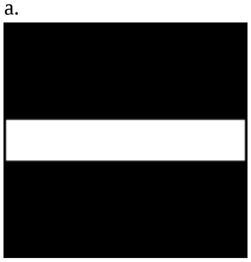
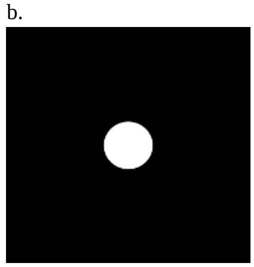
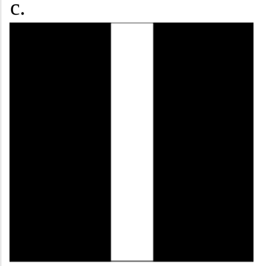
Select the mask of the 4f imaging system used to obtain the above image.

a.	b.	c.
		

9. Consider a 4*f* imaging system whose object and image are shown below.

Object	Image
	

Select the mask of the 4f imaging system used to obtain the above image.

a.	b.	c.
		

10. Select the <u>most restrictive</u> approximation in Fourier Optics.

a. Helmholtz

b. Fresnel

c. Fraunhofer

11. You are hired to build a system that operates under the Fraunhofer approximation using a 589-nm light source. Assuming that the maximum lateral extent of the input signal is 2 cm. How many football fields should be between the light source and observation planes? A football field is 91 meters long,

a. 2

b. 1

c. 10

d. 300

12. Consider that you have two identical lenses separated by the sum of their focal length. If an object is placed at the front focal plane of the first lens, what is the resulting image in the back focal plane of the second lens?

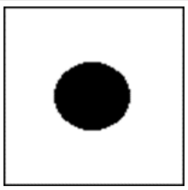
a. The original object

b. The inverted object

c. A shrunken image of the object

d. A magnified image of the object

13. Which one of the below masks would act as a high-pass filter when inserted into the middle of a 4f imaging system?

a. 	b. 	c. 

14. True/False. The output image is obtained when the input image passes through a 4f system with a low-pass filter.

Input image 	Output image 

a. True

b. False

CHAPTER OVERVIEW

Module 5 - Electromagnetic Optics

Module 5: Electromagnetic Optics provides a comprehensive understanding of light as an electromagnetic phenomenon. Students will begin with Maxwell's equations, exploring their role in describing the generation and propagation of electromagnetic waves. This module also covers boundary conditions at material interfaces, the Poynting theorem for energy flow, and electromagnetic wave behavior in dielectric media. It delves into monochromatic EM waves, emphasizing their mathematical representation and physical properties. Key topics include absorption, dispersion, and the interaction of light with matter.

This module has 2 classes.

Class 17 - Maxwell equations, Boundaries Conditions, Poynting theorem, EM waves in a dielectric medium

The learning objectives of this class:

- Review of electromagnetic theory for Optics
- Generalization of Maxwell equations for an arbitrary media
- Introduction of polarization and magnetization fields
- Describe optical electromagnetic waves in dielectric media

Class 18 - Monochromatic EM waves, absorption and dispersion

The learning objectives of this class:

- Simplify the Maxwell equation for monochromatic plane fields/waves
- Define absorption and dispersion as processes that occur when the light travels a medium
- Apply the Beer's law to determine the amount of transmitted light
- Quantify the dispersion using the Abbe value, the Cauchy equation, or the Sellmeier equation.

Module 5 - Summary

Summary Notes Module 5

• In Electromagnetic Optics, light is described as a vector.

• An electromagnetic field is described by 2 vector fields that are dependent on space, $\mathbf{r}=(x,y,z)$, and time, t.

- Electric field: $\mathbf{E}(\mathbf{r},t)=(E_x,E_y,E_z)$
- Magnetic field: $\mathbf{H}(\mathbf{r},t)=(H_x,H_y,H_z)$

• Maxwell's equation in free-space ($n=1$):

- $\nabla\times\mathbf{H}=\frac{\partial\mathbf{D}}{\partial t}=\epsilon_0\frac{\partial\mathbf{E}}{\partial t}$, where $\epsilon_0=\frac{1}{36\pi}\times10^{-9}\ \mathrm{F/m}$ is the electric permittivity.
- $\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}=-\mu_0\frac{\partial\mathbf{H}}{\partial t}$, where $\mu_0=4\pi\times10^{7}\ \mathrm{H/m}$ is the magnetic permeability.
- $\nabla\cdot\mathbf{E}=0$
- $\nabla\cdot\mathbf{H}=0$

Each component of $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{H}(\mathbf{r},t)$ should satisfy the wave equation (i.e., Module 2). For example, considering E_y, then:

$$\nabla^2E_y(\mathbf{r},t)-\frac{1}{v^2}\frac{\partial^2E_y(\mathbf{r},t)}{\partial t^2}=0$$

where $v=\frac{c}{n}$ and $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ is the Laplacian operator in Cartesian coordinates.

• In a medium with no free electric and magnetic charges, there are 4 vectors:

- Electric field: $\mathbf{E}(\mathbf{r},t)=(E_x,E_y,E_z)$
- Electric displacement field: $\mathbf{D}(\mathbf{r},t)=(D_x,D_y,D_z)$
- Magnetic field: $\mathbf{H}(\mathbf{r},t)=(H_x,H_y,H_z)$
- Magnetic flux density field: $\mathbf{B}(\mathbf{r},t)=(B_x,B_y,B_z)$

where $\mathbf{D}=\epsilon_0\mathbf{E}+\mathbf{P}$ and $\mathbf{B}=\mu_0\mathbf{H}+\mathbf{M}$. Note that $\mathbf{P}=\mathbf{M}=0$ in free space. Both $\mathbf{P}$ (i.e., polarization density field) and $\mathbf{M}$ (i.e., magnetization density field) are dependent on the medium.

• Maxwell's equation in a medium ($n>1$):

- $\nabla\times\mathbf{H}=\frac{\partial\mathbf{D}}{\partial t}$
- $\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$
- $\nabla\cdot\mathbf{D}=0$
- $\nabla\cdot\mathbf{B}=0$

• The Poynting vector, $\mathbf{S}=\mathbf{E}\times\mathbf{H}$, represents the flow of electromagnetic power. The Poynting vector is perpendicular to $\mathbf{E}$ and $\mathbf{H}$. For example, if $\mathbf{E}(\mathbf{r},t)=(E_x,0,0)$ and $\mathbf{H}(\mathbf{r},t)=(0,H_y,0)$, then the Poynting vector only has a unique component, $\mathbf{S}=\mathbf{E}\times\mathbf{H}=\begin{vmatrix}\hat{x}&\hat{y}&\hat{z}\\E_x&0&0\\0&H_y&1\end{vmatrix}=\hat{z}E_xH_y=(0,0,E_xH_y)$, so $S_z=E_xH_y$.

• Medium so $\mathbf{E}$ and $\mathbf{P}$ are related each other, and $\mathbf{H}$ and $\mathbf{M}$ are related each other.

Cases:

- $\mathbf{E}$ and $\mathbf{P}$ are linearly related → medium is dielectric
- $\mathbf{E}$ and $\mathbf{P}$ are invariant to space → homogeneous medium
- $\mathbf{E}$ and $\mathbf{P}$ are parallel → isotropic medium
- $\mathbf{E}(t_1)$ determines $\mathbf{P}(t_1)$ → nondispersive medium
- $\mathbf{E}(\mathbf{r}_1)$ determines $\mathbf{P}(\mathbf{r}_1)$ → spatially nondispersive medium

If a medium is linear, nondispersive, homogeneous, and isotropic, then $\mathbf{P}=\epsilon_0\chi\mathbf{E}$ and $\mathbf{D}=\epsilon\mathbf{E}$, where χ is the medium's electric susceptibility, and $\epsilon=\epsilon_0(1+\chi)$ is the medium's electric permittivity. Similarly, $\mathbf{M}=\mu_0\chi_m\mathbf{H}$ and $\mathbf{B}=\mu\mathbf{H}$, where χ_m is the medium's magnetic susceptibility, and $\mu=\mu_0(1+\chi_m)$ is the medium's magnetic permeability.

• Simplification of Maxwell's equations:

- $\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} = \epsilon \frac{\partial \mathbf{E}}{\partial t}$
- $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu \frac{\partial \mathbf{H}}{\partial t}$
- $\nabla \cdot \mathbf{E} = 0$
- $\nabla \cdot \mathbf{H} = 0$

where $v = \frac{1}{\sqrt{\epsilon\mu}}$, $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$, and $n = \sqrt{1+\chi}$ (i.e., $\mu = \mu_0$, medium without magnetic properties).

If $n = \sqrt{1+\chi}$, then $\chi = n^2 - 1$, the electric susceptibility can be estimated from the medium's refractive index. In general, if the medium has electric and magnetic properties, $n = \sqrt{(1+\chi)(1+\chi_m)}$

• Monochromatic electromagnetic waves:

- $\mathbf{E}(\mathbf{r}, t) = \operatorname{Re}[\mathbf{E}(\mathbf{r}) e^{j\omega t}]$
- $\mathbf{D}(\mathbf{r}, t) = \operatorname{Re}[\mathbf{D}(\mathbf{r}) e^{j\omega t}]$
- $\mathbf{H}(\mathbf{r}, t) = \operatorname{Re}[\mathbf{H}(\mathbf{r}) e^{j\omega t}]$
- $\mathbf{B}(\mathbf{r}, t) = \operatorname{Re}[\mathbf{B}(\mathbf{r}) e^{j\omega t}]$

The Maxwell's equations for monochromatic electromagnetic waves:

- $\nabla \times \mathbf{H} = j\omega \mathbf{D}$
- $\nabla \times \mathbf{E} = -j\omega \mathbf{B}$
- $\nabla \cdot \mathbf{D} = 0$
- $\nabla \cdot \mathbf{B} = 0$

• Absorption – decrease of light intensity through propagation:

- Beer's law: $I(x) = I_0 e^{-\alpha x}$, where $I(x)$ is the transmitted intensity after the light travels a distance x, I_0 is the incident light intensity, and α is the linear attenuation coefficient (units: m^{-1}). We can define the penetration depth as the inverse of the attenuation coefficient: $\delta = \frac{1}{\alpha}$, which is the distance in which the intensity of the transmitted light is reduced by a factor of $1/e$.
- Transmittance: $T = \frac{I(x)}{I_0} = e^{-\alpha x}$ and Absorptance: $A = \log_{10}(T) = 0.4343\alpha x$

• The complex refractive index when the susceptibility is complex ($\chi = \chi' + j\chi''$): $n - j\frac{\alpha}{2k_0} = \sqrt{1+\chi'+j\chi''}$, where $k_0 = \frac{2\pi}{\lambda_0}$ is the free-space wavenumber.

• Dispersion occurs when the refractive index changes with the light's wavelength:

- Abbe number
- Cauchy's equation
- Sellmeier's equation

Multi-choice questions

Multi-choice Quiz Module 5

1. Knowing that the Poynting vector is given by $\mathbf{S} = \mathbf{E} \times \mathbf{H}$, which is the value for an electric field **E** and a magnetic field **H**?

a. **S** = (0, 0, S_z)

b. **S** = (S_x, 0, S_z)

c. **S** = (S_x, Sy, S_z)

d. **S** = (S_x, S_y, 0)

2. Consider that the electric and magnetic fields **E** = (1, 2, 3) and **H =** (3, 2, 1), respectively. Select the correct Poynting vector knowing that $\mathbf{S} = \mathbf{E} \times \mathbf{H}$.

a. **S** = (3, 2, 3)

b. **S** = (− 4, 8, −4)

c. **S** = (− 3, 2, −3)

3. Consider that the medium is isotropic and the electric field is given by . Which of the following is a truthful representation of the polarization vector?

a. $\mathbf{P} = (0, 0, P_z)$

b. $\mathbf{P} = (P_x, 0, P_z)$

c. $\mathbf{P} = (P_x, P_y, P_z)$

d. $\mathbf{P} = (P_x, P_y, 0)$

e. $\mathbf{P} = (P_x, 0, 0)$

4. Consider that a medium is linear, nondispersive, homogeneous, and isotropic. Which one of the following sentences false?

a. The higher the value of the electric susceptibility, the higher the refractive index of the medium.

b. The refractive index does not depend on the electric and magnetic properties of the medium.

c. The electric and magnetic flux densities are related to the electric and magnetic fields, respectively.

5. What is the electric susceptibility of the water (n=1.33)? Consider that the water only has electric properties, and it can be considered a linear, nondispersive, homogeneous and isotropic medium.

a. We do not have enough information to estimate the electric susceptibility

b. The electric susceptibility is $\chi = 0.7689$

c. The electric susceptibility is $\chi = 0.33$

6. A certain medium has an absorption coefficient of $\alpha = 0.05\ m^{-1}$. What is its penetration depth?

a. Penetration depth is δ = 20 m

b. We need to know the incident intensity to provide this calculation

c. Penetration depth is δ = 3 m

7. Select the statement that is false to the Beer's law:

a. The lower the attenuation coefficient, the slower the penetration depth

b. In a no-amplification medium, there is always a reduction of the light intensity through the propagation.

c. Through propagation, the medium can only decrease the intensity via absorption or scattering.

8. Consider that Cauchy's equation gives the dispersion of a material with only two parameters: $n = A + \frac{B}{\lambda^2}$. How many resonances does the material have?

a. Two resonance wavelengths

b. A single resonance wavelength.

c. The resonance wavelengths are defined in Sellmeier's equation, so we cannot know them without enough information.

9. Select the statement that is true concerning dispersion:

a. The Abbe number can be estimated by knowing the refractive index of the material at any arbitrary wavelength.

b. The Cauchy's equation is a good approximation for materials that present normal dispersion in the visible regime.

c. The resonance wavelength is the wavelength in which the absorption curve is minimum.

10. True/False. The refractive index decreases with the increase of the wavelength in anomalous dispersion.

a. True

b. False

11. After overpassing a solution with an optical path length of x = 0.05 m, the intensity of light is reduced to half. The solution presents both absorption and scattering phenomena. If the linear attenuation absorption coefficient is $\alpha_A = 0.025\ cm^{-1}$. What is the value of the linear attenuation scattering coefficient α_S?

a. $\alpha_S = 0.025\ cm^{-1}$.

b. $\alpha_S = 1.412\ cm^{-1}$.

c. $\alpha_S = 0.11\ cm^{-1}$.

d. $\alpha_S = 2.361\ cm^{-1}$.

12. In which of these situations can the Cauchy equation be used?

a. Near resonant frequency

b. Within visible spectrum

c. In materials with anomalous dispersion

13. You want to get some tinted glass for your car that can reduce the incident sunlight to 10% of its original intensity over the 2-mm thickness of the window. What linear attenuation coefficient should you be shopping for?

a. $0.247\ m^{-1}$

b. $52.6\ m^{-1}$

c. $386\ m^{-1}$

d. $1150\ m^{-1}$

CHAPTER OVERVIEW

Module 6 - Polarization Optics

Module 6: Polarization Optics explores the fundamental concepts and applications of light polarization. Students will study the Poincaré sphere to understand linear, circular, and elliptical polarization states. The module introduces natural light and its transformation through polarizers, including Malus' law. Key topics include birefringent crystals, retarders, and polarization effects by reflection, such as the Brewster angle. Analytical frameworks like Stokes and Jones formalisms are used to describe and manipulate polarization states mathematically.

Class 19 - Poincare sphere – linear, circular and elliptical polarization; Natural light; Polarizers and Malus' law

The learning objectives of this class:

- Understand that the polarization is related to the orientation of the oscillations of the **E**-field
- Identify multiple applications of polarization
- Identify the different polarization states
- Determine the polarization state given an **E**-field
- Represent a polarization state using the Poincare Sphere
- Define natural light
- Understand the working principle of a linear polarizer
- Determine the transmitted light after linear polarizers (i.e., Malus' law)

Class 20 - Birefringent Crystals, Retarders, Stokes formalism

The learning objectives of this class:

- Define the terms birefringent materials, retarders, and Stokes parameters.
- Understand the principle of birefringence and how it leads to the splitting of light into two orthogonal polarization components.
- Describe how retarders change the polarization of the light.
- Predict the polarization state after a polarized light passes through full-wave, half-wave, and quarter-wave plates.
- Interpret the Stokes parameters.
- Measure the resultant Stokes parameters of an incoherent superposition.

Class 21 - Jones formalism, Polarization by reflection, Brewster angle

The learning objectives of this class:

- Define Jones vectors and matrix
- Derive the polarization state of the output beam of light after passing a cascade of polarizing elements
- Understand the reflected and refracted rays in terms of the polarization state

Module 6 - Summary

Summary Notes Module 6

• In Transverse ElectroMagnetic (TEM) waves, the direction of the oscillation of the **E**- and **H**- waves is perpendicular to the propagation direction of the wave.

• Polarization is related to the orientation of the oscillation of the electric **E**-field

• Notation of the electric field: $\mathbf{E}=E_x\hat{\mathbf{x}}+E_y\hat{\mathbf{y}}$ where (E_x,E_y) are the components of the **E**-vector and are periodic functions of the time and propagation distance z

$$E_x=E_{0x}\cos(\omega t-kz+\phi_x) \tag{1}$$

$$E_y=E_{0y}\cos(\omega t-kz+\phi_y) \tag{2}$$

There is a relative phase difference between both transverse components, $\phi=\phi_y-\phi_x$.

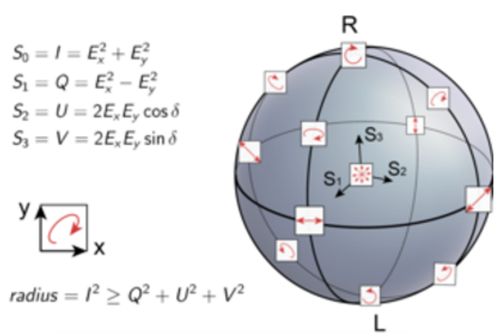
• The polarization state of the light can be determined from (E_{0x},E_{0y},ϕ)

- Linear polarization at 45 degrees if $E_{0x}=E_{0y}$ and $\phi=\pm 2\pi m$, where m is an integer number (i.e., $m=0,1,2,\ldots$)
  - If $\phi=\tan^{-1}\left(\frac{E_{0x}}{E_{0y}}\right)>0$, E_y leads E_x (i.e., left-hand or counterclockwise)
  - If $\phi=\tan^{-1}\left(\frac{E_{0x}}{E_{0y}}\right)<0$, E_x leads E_y (i.e., right-hand or clockwise)
- Linear polarization at -45 degrees if $E_{0x}=E_{0y}$ and $\phi=\pm\pi m$, where m is an odd integer (i.e., $m=1,3,5,\ldots$)
- Horizontal polarization (i.e., linear polarization that occurs along the horizontal axis) if $E_{0x}=1$ and $E_{0y}=0$.
- Vertical polarization (i.e., linear polarization that occurs along the vertical axis) if $E_{0x}=0$ and $E_{0y}=1$.
- Right-hand circular polarization if $E_{0x}=E_{0y}$ and $\phi=-\frac{\pi}{2}\pm 2\pi m$, where m is an integer number. In this polarization state, E_x leads E_y.
- Left-hand circular polarization if $E_{0x}=E_{0y}$ and $\phi=\frac{\pi}{2}\pm 2\pi m$, where m is an integer number. In this polarization state, E_y leads E_x.
- Elliptical polarization if $E_{0x}\neq E_{0y}$.

• A linear polarization state can be represented by the superposition of right- and left-hand circular polarization states.

• In linear polarization, the angle between both components of the E-field does not change (i.e., oscillation occurs in the same direction along the wave propagation).

• The Poincare Sphere allows us to represent any polarization state:



- Linear polarization (LP) lies on the equator of the Poincare Sphere
- Right-hand circular polarization (RCP) lies at the north pole of the sphere
- Left-hand circular polarization (LCP) lies at the south pole of the sphere

• Natural Light refers to randomly-polarized light, so it can be understood as the sum of two orthogonal polarization states.

• When natural light is incident onto a linear polarizer with a transmission axis set at θ from the vertical, the polarization state of the transmitted light is parallel to the transmission axis of the linear polarizer (i.e., the polarization state of the transmitted light is linear with its transmission axis being θ from the vertical). In other words, only the E-component parallel to the transmission axis survives.

- When linearly polarized light with angle θ_i with respect to the vertical axis is incident onto a linear polarizer with a transmission axis θ from the vertical, the transmitted light has an intensity equal to:

$$I_t = I_i \cos^2(\theta - \theta_i) \tag{3}$$

where I_t is the transmitted intensity, and I_i is the incident intensity. Note that there is no transmitted light if $\theta - \theta_i = 90^\circ$.

• Birefringent materials are transparent optical materials that have two refractive indices (i.e., n_e for the extraordinary wave and n_o for the ordinary wave), causing light to split into two orthogonal linear polarizations. The birefringence parameter is defined by:

$$\beta = n_e - n_o \tag{4}$$

• Retarders are optical elements that introduce a phase shift between the components of the electric field. A retarder is defined by two axes: the fast and slow axis.

- Full Wave Plate (FWP) is a retarder whose phase shift is $\pm 2\pi$ (i.e., invariant).
- Half Wave Plate (HWP) is a retarder whose phase shift is $\pm\pi$ (i.e., polarizer rotator).
- Quarter Wave Plate (QWP) is a retarder whose phase shift is $\pm\frac{\pi}{2}$.

• Stokes parameters $S = \{S_1, S_2, S_3, S_4\}$

- Degree of polarization:

$$\text{Degree of polarization} = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} \tag{5}$$

which ranges from 0 (i.e., natural light) to 1 (linearly-polarized light or circularly-polarized light).

- If $\mathbf{E} = E_x \hat{i} + E_y \hat{j}$, when $E_x = E_{0x}$ and $E_y = E_{0y} e^{j\phi}$, then:

$$S_0 = E_{0x}^2 + E_{0y}^2 \tag{6}$$

$$S_1 = E_{0x}^2 - E_{0y}^2 = S_0 \cos(2\chi)\cos(2\psi) \tag{7}$$

$$S_2 = 2E_{0x}E_{0y}\cos(\phi) = S_0 \cos(2\chi)\sin(2\psi) \tag{8}$$

$$S_3 = 2E_{0x}E_{0y}\sin(\phi) = S_0 \sin(2\chi) \tag{9}$$

- If two beams of light interfere incoherently, then the resultant Stokes parameters is the sum of their individual ones:

$$\{S\} + \{S'\} = \{S_1 + S_1', S_2 + S_2', S_3 + S_3', S_4 + S_4'\} \tag{10}$$

• Jones vector:

$$\mathbf{E} = \frac{1}{\sqrt{E_x^2 + E_y^2}} \begin{pmatrix} E_x \\ E_y \end{pmatrix} \tag{11}$$

where $E_x = E_{0x} e^{j\phi_x}$ and $E_y = E_{0y} e^{j\phi_y}$.

- In coherent superposition, one should sum their Jones vectors instead of the Stokes parameters.
- Two polarization states are orthogonal if $\mathbf{E}_1 \cdot \mathbf{E}_2^* = 0$. Any polarization state can be represented by the coherent sum of two orthogonal Jones vectors.
- Polarizers and retarders are represented by 2×2 Jones matrices.

$$\boldsymbol{E}_\mathrm{t} = [\mathrm{T}]\boldsymbol{E}_\mathrm{i}$$

$$\boldsymbol{E}_\mathrm{t} = [\mathrm{T}_3\mathrm{T}_2\mathrm{T}_1]\boldsymbol{E}_\mathrm{i}$$

Multi-choice questions

Multi-choice Quiz Module 6

1. Which of the following sentences is false in linear polarization?

a. In linear polarization, the component of the electric field rotates at a constant rate

b. In linear polarization, the electric field oscillates in a single direction

c. A linear polarization state can be represented as the superposition of right-hand circular polarization and left-hand circular polarization.

2. Which one of the following sentences is false about natural light?

a. Natural light refers to light with an orientation of the electric field that changes randomly in space and time.

b. Natural light can be represented as the superposition of two equal-amplitude, incoherent orthogonal linear polarization states.

c. If natural light is incident on a linear polarizer, the intensity of the transmitted light depends on the angle of the linear polarizer.

3. Fill the gap. Polarizers are polarization elements that reduce the incident intensity by half if the input light is ________________

a. Polarized light

b. Incoherent light

c. Coherent light

d. Natural light

4. A linearly-polarized beam along the horizontal direction (i.e., 90 deg from the vertical) impacts a linear polarizer whose transmission axis is vertical. What is the intensity of the transmitted beam if the incident beam has an intensity of 100 W/m^2?

a. 100 W/m^2

b. 0 W/m^2

c. 50 W/m^2

5. A beam of natural light with an intensity of 500W/m^2 impinges consecutively on two ideal linear polarizers. How apart are the transmission axes of the two polarizers if the intensity of the transmitted light is reduced to 250 W/m^2?

a. We do not have enough information to estimate the angle between both polarizers.

b. Their transmission axes are separated 0 deg.

c. Their transmission axes are separated 90 deg.

6. A beam of natural light with an intensity of 500W/m^2 impinges consecutively on two ideal linear polarizers. How apart are the transmission axes of the two polarizers if the intensity of the transmitted light is reduced to 0 W/m^2?

a. We do not have enough information to estimate the angle between both polarizers.

b. Their transmission axes are separated 0 deg.

c. Their transmission axes are separated 90 deg.

7. Considering that you have two optical polarizers. Both polarizers are localized in tandem and the angle between them is 45 degrees. If the incident light is natural light, which one is the ratio between the incident intensity I_0 at the first polarizer and the transmitted light I_2 after passing through both polarizers?

a. $I_2 = I_0$

b. $2I_2 = I_0$

c. There is not enough information

d. $4I_2 = I_0$

8. True/False. Retarders are polarization elements that advance or retard the phase of one of the two orthogonal components of **E**-field by some desired amount.

a. True

b. False

9. A light with a horizontal linear polarization state passes a half-wave retarder. Select the correct polarization state of the transmitted light.

a. Vertical linear polarization state

b. Horizontal linear polarization state

c. No light is transmitted.

d. It depends on the orientation of the retarder's fast axis

10. A beam of light is characterized by a Jones Vector, $\mathbf{E}=\frac{1}{\sqrt{13}}\begin{pmatrix}3\\2\end{pmatrix}$. Which of the following Jones vectors do not represent its orthogonal polarization state?

a. $\mathbf{E}=\frac{1}{\sqrt{13}}\begin{pmatrix}2\\-3\end{pmatrix}$

b. $\mathbf{E}=\frac{1}{\sqrt{13}}\begin{pmatrix}-4\\6\end{pmatrix}$

c. $\mathbf{E}=\frac{1}{\sqrt{13}}\begin{pmatrix}2\\3\end{pmatrix}$

11. You need left-hand circular polarized (LCP) light in your system. However, unfortunately, you just dropped your only LCP polarizer and it is now broken. Looking around in the lab, you find a quarter wave plate (QWP) and some linear polarizers. Select the correct angle of the transmission angle of the linear polarizer (LP) that you should insert before the QWP so that the system produces LHCP from natural light?

a. LP@45°

b. LP@-45°

c. LP@ 90 deg

d. LP @ 0 deg

12. Natural light impinges on a horizontal polarizer, passes through a half-wave plate whose fast axis is 0 degrees with respect to the horizontal axis, and then impinges on another polarizer set at 30° from the vertical. What percent of the initial light will remain?

a. 75%

b. 50%

c. 25%

d. Almost 13%

13. Two incoherent light beams with Stokes parameters {2, -1, 0, 0} and {2, 2, 1, 3}, respectively, superimposed upon each other. What are the resulting Stokes parameters and the degree of polarization?

a. {4, 1, 1, 3} with degree of polarization = 0.8292

b. {1, -2, 0, 0} with degree of polarization = 2

c. {2, 1, 1, 3} with degree of polarization = 1.6583

d. {1, -2, 0, 0} with degree of polarization = 4

External resources - Solutions of multi-choice questions

Module 1 Solutions

Module 2 Solutions

Module 3 Solutions

Module 4 Solutions

Module 5 Solutions

Module 6 Solutions

External Resources - Instructor Manual

This manual is designed to help instructors effectively use the open-educational book “ECE 5143: Intro to Optics and Photonics” in active learning environments. The goal of the course is to engage senior undergraduate and 1st-year graduate students in higher-order thinking activities during class, leveraging pre-class video content to maximize in-class learning outcomes.

Below is a list of guidelines to facilitate the adoption of this OER book:

Instructor Preparation

1. Review Pre-Class Material:
    - Ensure students have access to pre-class videos and any accompanying materials.
    - Identify key points and potential areas of confusion.
2. Prepare In-Class Activities:
    - Design 2-4 problems/activities that build on the video content.
        - Include a mix of individual, pair, and group activities.
        - Include problem-based learning, concept application activities, think-pair-share, peer teaching, and role play/simulation
3. Plan for Assessment:
    - Develop quick formative assessments (e.g., quizzes, polls) to check students’ understanding.
        - **Quizzes**: Use short quizzes at the start of each class to reinforce key concepts.
        - **Polls**: Conduct live polls using tools like Kahoot! or Mentimeter to gauge student understanding.
    - Develop summative assessments to dissect students' understanding
        - **Projects**: Assign 2-4 individual/group projects where students apply concepts to a comprehensive problem or application-based problem
        - **Exams**: Include both conceptual and application-based questions.
4. Providing Feedback
    - Offer immediate feedback during class activities.
    - Use peer feedback during group discussions and projects.
    - When it is possible, provide feedback within a week window.

Student Preparation

1. Students are required to watch instructional videos before coming to class. These videos introduce key concepts, allowing class time to be used for:

- Problem-solving exercises
- Group discussions
- Hands-on activities

2. Complete any required pre-class quizzes or reading assignments.

3. Submit a 1-page hand-written summary of the class notes taken while watching the class video.

4. Come prepared with questions.

Suggested Course Schedule

Class no.	Module	Lecture Topic
1	Geometrical/Ray Optics	Postulates and Rules in Ray Optics
2		Mirrors
3		Planar Boundaries, External and internal refraction, Total internal reflection
4		Spherical Boundaries and Lenses
5		Matrix Optics and 4f imaging system
6	Wave Optics	Postulates of Wave Optics, Monochromatic Waves, Helmholtz equation

7		Elementary Waves: plane, spherical and paraboloidal waves
8		Relation ray-wave optics, interference of two waves, interferometers
9		Young experiment
10	Beam Optics	Gaussian beam: features and mathematical description
11		Properties of Gaussian beams
12		Propagation of Gaussian beams through optical systems
13	Fourier Optics	Space vs Fourier Domains, Principle of Fourier Optics, Linear Systems and Shift-invariant Systems
14		Impulse response and Transfer function in free propagation
15		Fresnel and Fraunhofer diffraction patterns
16		Lenses, Optical Fourier Transforms, 4f imaging system and spatial filtering
17	Electromagnetics (EM) Optics	Maxwell equations, Boundaries Conditions, Poynting theorem, EM waves in a dielectric medium
18		Monochromatic EM waves, absorption and dispersion
19	Polarization Optics	Poincare sphere – linear, circular and elliptical polarization; Natural light; Polarizers and Malus' law
20		Birefringent Crystals, Retarders, Stokes formalism
21		Jones formalism, Polarization by reflection, Brewster angle

Note that other sequences are also possible. For example, some instructors may prefer the following sequence:

1. Module 1: Geometrical/Ray Optics
2. Module 2: Wave Optics
3. Module 3: Electromagnetics (EM) Optics
4. Module 4: Polarization Optics
5. Module 5: Gaussian Optics
6. Module 6: Fourier Optics

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