Module 6 - Summary

Summary Notes Module 6

• In Transverse ElectroMagnetic (TEM) waves, the direction of the oscillation of the **E**- and **H**- waves is perpendicular to the propagation direction of the wave.

• Polarization is related to the orientation of the oscillation of the electric **E**-field

• Notation of the electric field: $\mathbf{E} = E_x\hat{\mathbf{x}} + E_y\hat{\mathbf{y}}$ where (E_x, E_y) are the components of the **E**-vector and are periodic functions of the time and propagation distance z

$$E_x = E_{0x}\cos(\omega t - kz + \phi_x) \tag{1}$$

$$E_y = E_{0y}\cos(\omega t - kz + \phi_y) \tag{2}$$

There is a relative phase difference between both transverse components, $\phi = \phi_y - \phi_x$.

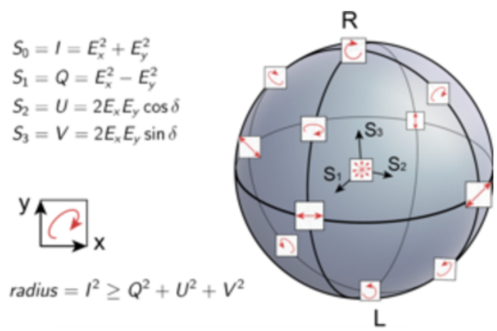
• The polarization state of the light can be determined from (E_{0x}, E_{0y}, ϕ)

- Linear polarization at 45 degrees if $E_{0x} = E_{0y}$ and $\phi = \pm 2\pi m$, where m is an integer number (i.e., $m = 0, 1, 2, \ldots$)
  - If $\phi = \tan^{-1}\left(\frac{E_{0x}}{E_{0y}}\right) > 0$, E_y leads E_x (i.e., left-hand or counterclockwise)
  - If $\phi = \tan^{-1}\left(\frac{E_{0x}}{E_{0y}}\right) < 0$, E_x leads E_y (i.e., right-hand or clockwise)
- Linear polarization at -45 degrees if $E_{0x} = E_{0y}$ and $\phi = \pm \pi m$, where m is an odd integer (i.e., $m = 1, 3, 5, \ldots$)
- Horizontal polarization (i.e., linear polarization that occurs along the horizontal axis) if $E_{0x} = 1$ and $E_{0y} = 0$.
- Vertical polarization (i.e., linear polarization that occurs along the vertical axis) if $E_{0x} = 0$ and $E_{0y} = 1$.
- Right-hand circular polarization if $E_{0x} = E_{0y}$ and $\phi = -\frac{\pi}{2} \pm 2\pi m$, where m is an integer number. In this polarization state, E_x leads E_y.
- Left-hand circular polarization if $E_{0x} = E_{0y}$ and $\phi = \frac{\pi}{2} \pm 2\pi m$, where m is an integer number. In this polarization state, E_y leads E_x.
- Elliptical polarization if $E_{0x} \neq E_{0y}$.

• A linear polarization state can be represented by the superposition of right- and left-hand circular polarization states.

• In linear polarization, the angle between both components of the E-field does not change (i.e., oscillation occurs in the same direction along the wave propagation).

• The Poincare Sphere allows us to represent any polarization state:



- Linear polarization (LP) lies on the equator of the Poincare Sphere
- Right-hand circular polarization (RCP) lies at the north pole of the sphere
- Left-hand circular polarization (LCP) lies at the south pole of the sphere

• Natural Light refers to randomly-polarized light, so it can be understood as the sum of two orthogonal polarization states.

  • When natural light is incident onto a linear polarizer with a transmission axis set at θ from the vertical, the polarization state of the transmitted light is parallel to the transmission axis of the linear polarizer (i.e., the polarization state of the transmitted light is linear with its transmission axis being θ from the vertical). In other words, only the E-component parallel to the transmission axis survives.

- When linearly polarized light with angle θ_i with respect to the vertical axis is incident onto a linear polarizer with a transmission axis θ from the vertical, the transmitted light has an intensity equal to:

$$I_t = I_i \cos^2(\theta - \theta_i) \tag{3}$$

where I_t is the transmitted intensity, and I_i is the incident intensity. Note that there is no transmitted light if $\theta - \theta_i = 90^\circ$.

- Birefringent materials are transparent optical materials that have two refractive indices (i.e., n_e for the extraordinary wave and n_o for the ordinary wave), causing light to split into two orthogonal linear polarizations. The birefringence parameter is defined by:

$$\beta = n_e - n_o \tag{4}$$

- Retarders are optical elements that introduce a phase shift between the components of the electric field. A retarder is defined by two axes: the fast and slow axis.
  - Full Wave Plate (FWP) is a retarder whose phase shift is $\pm 2\pi$ (i.e., invariant).
  - Half Wave Plate (HWP) is a retarder whose phase shift is $\pm\pi$ (i.e., polarizer rotator).
  - Quarter Wave Plate (QWP) is a retarder whose phase shift is $\pm\frac{\pi}{2}$.
- Stokes parameters $S = \{S_1, S_2, S_3, S_4\}$
  - Degree of polarization:

$$\text{Degree of polarization} = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} \tag{5}$$

which ranges from 0 (i.e., natural light) to 1 (linearly-polarized light or circularly-polarized light).

  - If $\mathbf{E} = E_x \hat{i} + E_y \hat{j}$, when $E_x = E_{0x}$ and $E_y = E_{0y} e^{j\phi}$, then:

$$S_0 = E_{0x}^2 + E_{0y}^2 \tag{6}$$

$$S_1 = E_{0x}^2 - E_{0y}^2 = S_0 \cos(2\chi)\cos(2\psi) \tag{7}$$

$$S_2 = 2E_{0x}E_{0y}\cos(\phi) = S_0 \cos(2\chi)\sin(2\psi) \tag{8}$$

$$S_3 = 2E_{0x}E_{0y}\sin(\phi) = S_0 \sin(2\chi) \tag{9}$$

  - If two beams of light interfere incoherently, then the resultant Stokes parameters is the sum of their individual ones:

$$\{S\} + \{S'\} = \{S_1 + S_1', S_2 + S_2', S_3 + S_3', S_4 + S_4'\} \tag{10}$$

- Jones vector:

$$\mathbf{E} = \frac{1}{\sqrt{E_x^2 + E_y^2}} \begin{pmatrix} E_x \\ E_y \end{pmatrix} \tag{11}$$

where $E_x = E_{0x} e^{j\phi_x}$ and $E_y = E_{0y} e^{j\phi_y}$.

- In coherent superposition, one should sum their Jones vectors instead of the Stokes parameters.
- Two polarization states are orthogonal if $\mathbf{E}_1 \cdot \mathbf{E}_2^* = 0$. Any polarization state can be represented by the coherent sum of two orthogonal Jones vectors.
- Polarizers and retarders are represented by 2×2 Jones matrices.

$$E_t = [T]E_i$$

$$E_t = [T_3T_2T_1]E_i$$