Module 3 - Summary

Summary Notes Module 3

Amplitude distribution of a Gaussian beam

$$u(x,y,z) = A_0 \frac{w_0}{w(z)} \exp(-jkz) \exp\left(-\frac{x^2+y^2}{w^2(z)}\right) \exp\left(-j\frac{k(x^2+y^2)}{2R(z)}\right) \exp(j\varphi(z)) \tag{1}$$

$\exp(-jkz)$ represents a plane wave

$\exp\left(-\frac{x^2+y^2}{w^2(z)}\right)$ represents a Gaussian modulation

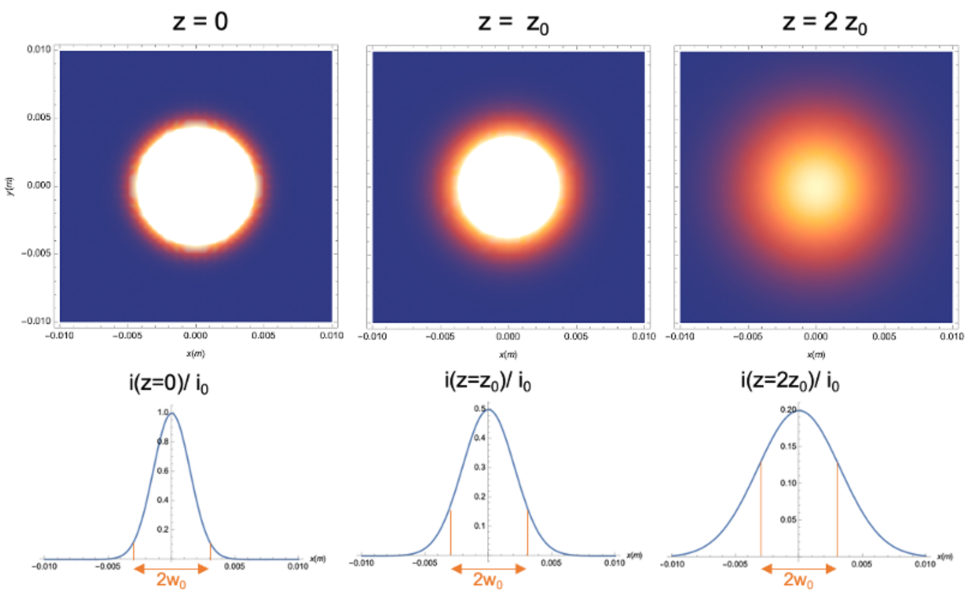
$\exp\left(-j\frac{k(x^2+y^2)}{2R(z)}\right)$ represents a paraboloidal wavefront

$\exp(j\varphi(z))$ represents a phase gained through propagation

Intensity distribution of a Gaussian beam

The irradiance/intensity distribution of a Gaussian beam follows a Gaussian function.

$$i(x,y,z) = |u(x,y,z)|^2 = \frac{A_0^2}{w_0^2}\frac{w^2(z)}{w_0^2}\exp\left(-2\frac{x^2+y^2}{w^2(z)}\right)$$



86% of the intensity is located within the beam waist w_0.

Beam Parameters

The beam width is given by:

$$w(z) = w_0\sqrt{1+\left(\frac{z}{z_0}\right)^2} \tag{2}$$

The beam waist is:

$$w_0 = \sqrt{\frac{\lambda z_0}{\pi}} \tag{3}$$

The Rayleigh range/distance is:

$$z_0 = \frac{\pi w_0^2}{\lambda} \tag{4}$$

The beam curvature is:

$$R(z)=z\left[1+\left(\frac{z_0}{z}\right)^2\right] \tag{5}$$

The phase retardance is:

$$\varphi(z)=\tan^{-1}\left(\frac{z}{z_0}\right) \tag{6}$$

Relation between intensity distributions at two axial planes

At $z=0$ (waist plane), the beam width coincides with the beam waist, $w(z=0)=w_0$. The intensity at $z=0$ is:

$$i_0(x,y)=i(x,y,z=0)=|u(x,y,z=0)|^2=\frac{A_0^2}{w_0^2}\exp\left(-2\frac{x^2+y^2}{w_0^2}\right) \tag{7}$$

The intensity at any axial plane z is:

$$i(x,y,z)=|u(x,y,z)|^2=\frac{A_0^2}{w_0^2}\frac{w^2(z)}{w_0^2}\exp\left(-2\frac{x^2+y^2}{w^2(z)}\right)=\frac{1}{M^2}i_0(x,y) \tag{8}$$

where $M=\frac{w(z)}{w_0}$

Power of Gaussian Beams. The power is given by:

$$P=\frac{1}{2}i_0\left(\pi w_0^2\right) \tag{9}$$

which is independent of z.

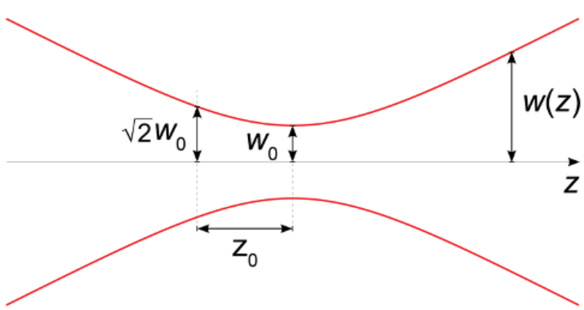
Energy Conservation Law for Gaussian Beams

$$\int_0^\infty i(\rho,z)2\pi\rho d\rho=\int_0^\infty i_0(\rho)2\pi\rho d\rho \tag{10}$$

$$\int_0^\infty i(\rho,z)2\pi\rho d\rho=\int_0^\infty \frac{1}{M^2}i_0(x,y)2\pi\rho d\rho=\int_0^\infty i_0(x',y')2\pi\rho' d\rho' \tag{11}$$

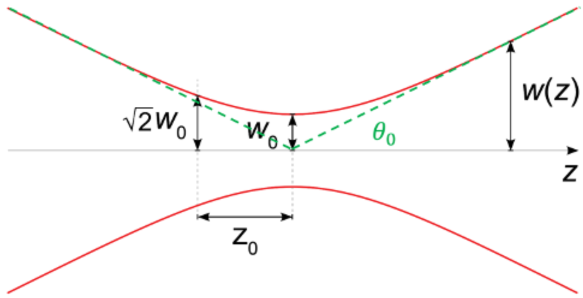
Where we have defined a new variable $x'=\frac{x}{M}$ and $y'=\frac{y}{M}$.

Features of Gaussian Beams



- At any transverse plane (z): $w(z)>w_0$
- If $z=\pm z_0$, $w(z=\pm z_0)=\sqrt{2}w_0$, which means that at the area of the spot beam $(\pi w^2(z))$ it is double $(2\pi w_0^2)$ at $z=\pm z_0$
- Depth of focus: $2z_0$, defined as the axial range in which the beam width is “almost” constant
- If $z=0$ (i.e., beam waist plane), $R(z=0)=\infty$, so $\exp\left(-j\frac{k(x^2+y^2)}{2R(z)}\right)=1$. In other words, there is no spherical wavefront term, i.e., the wavefront is a plane wave at the beam waist.
- If $z=\pm z_0$ (i.e., Rayleigh range), the radius of curvature is minimum: $R(z=\pm z_0)=\pm 2z_0$
- If $z\gg z_0$, the beam width is linearly proportional to the distance z:

$$w(z) = w_0 \frac{z}{z_0} \tag{12}$$



- Beam divergence \(\theta_0 = \frac{w_0}{z_0} = \frac{\lambda}{\pi w_0} \
- If $z \gg z_0$, $R(z) = z$, which means the Gaussian beam acts as a spherical wave confined within the divergence angle θ_0.

q-parameter: $q(z) = z + jz_0 = \frac{1}{R(z)} - j\frac{\lambda}{\pi w^2(z)}$ where z is the distance of the Gaussian beam to its beam waist, and z_0 is its Rayleigh range.

Propagation of Gaussian Beams through ABCD Matrix Optics: A Gaussian beam propagating through optical components remains a Gaussian beam. However, its features change on the ABCD matrix of the optical system.

$$q_2 = z_2 + jz_{02}, \quad q_2 = \frac{Cq_1 + D}{Aq_1 + B} \tag{13}$$

where q_2 defines a Gaussian beam at z_2 and Rayleigh range z_0^2.