Module 5 - Summary

Summary Notes Module 5

• In Electromagnetic Optics, light is described as a vector.

• An electromagnetic field is described by 2 vector fields that are dependent on space, $\mathbf{r}=(x,y,z)$, and time, t.

- Electric field: $\mathbf{E}(\mathbf{r},t)=(E_x,E_y,E_z)$
- Magnetic field: $\mathbf{H}(\mathbf{r},t)=(H_x,H_y,H_z)$

• Maxwell's equation in free-space ($n=1$):

- $\nabla\times\mathbf{H}=\frac{\partial\mathbf{D}}{\partial t}=\epsilon_0\frac{\partial\mathbf{E}}{\partial t}$, where $\epsilon_0=\frac{1}{36\pi}\times10^{-9}\ \mathrm{F/m}$ is the electric permittivity.
- $\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}=-\mu_0\frac{\partial\mathbf{H}}{\partial t}$, where $\mu_0=4\pi\times10^{7}\ \mathrm{H/m}$ is the magnetic permeability.
- $\nabla\cdot\mathbf{E}=0$
- $\nabla\cdot\mathbf{H}=0$

Each component of $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{H}(\mathbf{r},t)$ should satisfy the wave equation (i.e., Module 2). For example, considering E_y, then:

$$\nabla^2E_y(\mathbf{r},t)-\frac{1}{v^2}\frac{\partial^2E_y(\mathbf{r},t)}{\partial t^2}=0$$

where $v=\frac{c}{n}$ and $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ is the Laplacian operator in Cartesian coordinates.

• In a medium with no free electric and magnetic charges, there are 4 vectors:

- Electric field: $\mathbf{E}(\mathbf{r},t)=(E_x,E_y,E_z)$
- Electric displacement field: $\mathbf{D}(\mathbf{r},t)=(D_x,D_y,D_z)$
- Magnetic field: $\mathbf{H}(\mathbf{r},t)=(H_x,H_y,H_z)$
- Magnetic flux density field: $\mathbf{B}(\mathbf{r},t)=(B_x,B_y,B_z)$

where $\mathbf{D}=\epsilon_0\mathbf{E}+\mathbf{P}$ and $\mathbf{B}=\mu_0\mathbf{H}+\mathbf{M}$. Note that $\mathbf{P}=\mathbf{M}=0$ in free space. Both $\mathbf{P}$ (i.e., polarization density field) and $\mathbf{M}$ (i.e., magnetization density field) are dependent on the medium.

• Maxwell's equation in a medium ($n>1$):

- $\nabla\times\mathbf{H}=\frac{\partial\mathbf{D}}{\partial t}$
- $\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$
- $\nabla\cdot\mathbf{D}=0$
- $\nabla\cdot\mathbf{B}=0$

• The Poynting vector, $\mathbf{S}=\mathbf{E}\times\mathbf{H}$, represents the flow of electromagnetic power. The Poynting vector is perpendicular to $\mathbf{E}$ and $\mathbf{H}$. For example, if $\mathbf{E}(\mathbf{r},t)=(E_x,0,0)$ and $\mathbf{H}(\mathbf{r},t)=(0,H_y,0)$, then the Poynting vector only has a unique component, $\mathbf{S}=\mathbf{E}\times\mathbf{H}=\begin{vmatrix}\hat{x}&\hat{y}&\hat{z}\\E_x&0&0\\0&H_y&1\end{vmatrix}=\hat{z}E_xH_y=(0,0,E_xH_y)$, so $S_z=E_xH_y$.

• Medium so $\mathbf{E}$ and $\mathbf{P}$ are related each other, and $\mathbf{H}$ and $\mathbf{M}$ are related each other.

Cases:

- $\mathbf{E}$ and $\mathbf{P}$ are linearly related → medium is dielectric
- $\mathbf{E}$ and $\mathbf{P}$ are invariant to space → homogeneous medium
- $\mathbf{E}$ and $\mathbf{P}$ are parallel → isotropic medium
- $\mathbf{E}(t_1)$ determines $\mathbf{P}(t_1)$ → nondispersive medium
- $\mathbf{E}(\mathbf{r}_1)$ determines $\mathbf{P}(\mathbf{r}_1)$ → spatially nondispersive medium

If a medium is linear, nondispersive, homogeneous, and isotropic, then $\mathbf{P}=\epsilon_0\chi\mathbf{E}$ and $\mathbf{D}=\epsilon\mathbf{E}$, where χ is the medium's electric susceptibility, and $\epsilon=\epsilon_0(1+\chi)$ is the medium's electric permittivity. Similarly, $\mathbf{M}=\mu_0\chi_m\mathbf{H}$ and $\mathbf{B}=\mu\mathbf{H}$, where χ_m is the medium's magnetic susceptibility, and $\mu=\mu_0(1+\chi_m)$ is the medium's magnetic permeability.

• Simplification of Maxwell's equations:

- $\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} = \epsilon \frac{\partial \mathbf{E}}{\partial t}$
- $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu \frac{\partial \mathbf{H}}{\partial t}$
- $\nabla \cdot \mathbf{E} = 0$
- $\nabla \cdot \mathbf{H} = 0$

where $v = \frac{1}{\sqrt{\epsilon\mu}}$, $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$, and $n = \sqrt{1+\chi}$ (i.e., $\mu = \mu_0$, medium without magnetic properties).

If $n = \sqrt{1+\chi}$, then $\chi = n^2 - 1$, the electric susceptibility can be estimated from the medium’s refractive index. In general, if the medium has electric and magnetic properties, $n = \sqrt{(1+\chi)(1+\chi_m)}$

• Monochromatic electromagnetic waves:

- $\mathbf{E}(\mathbf{r}, t) = \mathrm{Re}[\mathbf{E}(\mathbf{r})e^{j\omega t}]$
- $\mathbf{D}(\mathbf{r}, t) = \mathrm{Re}[\mathbf{D}(\mathbf{r})e^{j\omega t}]$
- $\mathbf{H}(\mathbf{r}, t) = \mathrm{Re}[\mathbf{H}(\mathbf{r})e^{j\omega t}]$
- $\mathbf{B}(\mathbf{r}, t) = \mathrm{Re}[\mathbf{B}(\mathbf{r})e^{j\omega t}]$

The Maxwell's equations for monochromatic electromagnetic waves:

- $\nabla \times \mathbf{H} = j\omega \mathbf{D}$
- $\nabla \times \mathbf{E} = -j\omega \mathbf{B}$
- $\nabla \cdot \mathbf{D} = 0$
- $\nabla \cdot \mathbf{B} = 0$

• Absorption – decrease of light intensity through propagation:

- Beer’s law: $I(x) = I_0 e^{-\alpha x}$, where $I(x)$ is the transmitted intensity after the light travels a distance x, I_0 is the incident light intensity, and α is the linear attenuation coefficient (units: m^{-1}). We can define the penetration depth as the inverse of the attenuation coefficient: $\delta = \frac{1}{\alpha}$, which is the distance in which the intensity of the transmitted light is reduced by a factor of $1/e$.
- Transmittance: $T = \frac{I(x)}{I_0} = e^{-\alpha x}$ and Absorptance: $A = \log_{10}(T) = 0.4343\alpha x$

• The complex refractive index when the susceptibility is complex ($\chi = \chi' + j\chi''$): $n - j\frac{\alpha}{2k_0} = \sqrt{1+\chi'+j\chi''}$, where $k_0 = \frac{2\pi}{\lambda_0}$ is the free-space wavenumber.

• Dispersion occurs when the refractive index changes with the light’s wavelength:

- Abbe number
- Cauchy’s equation
- Sellmeier’s equation