8.2: The Biot-Savart Law

Learning Objectives

By the end of this section, you will be able to:

- Explain how to derive a magnetic field from an arbitrary current in a line segment
- Calculate magnetic field from the Biot-Savart law in specific geometries, such as a current in a line and a current in a circular arc

We have seen that mass produces a gravitational field and also interacts with that field. Charge produces an electric field and also interacts with that field. Since moving charge (that is, current) interacts with a magnetic field, we might expect that it also creates that field—and it does.

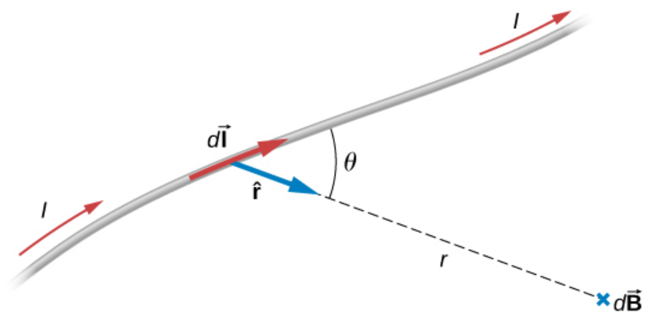

Figure [Math Processing Error]: A current element [Math Processing Error] produces a magnetic field at point [Math Processing Error] given by the Biot-Savart law (Equation [Math Processing Error]).

The equation used to calculate the magnetic field produced by a current is known as the Biot-Savart law. It is an empirical law named in honor of two scientists who investigated the interaction between a straight, current-carrying wire and a permanent magnet. This law enables us to calculate the magnitude and direction of the magnetic field produced by a current in a wire. The **Biot-Savart law** states that at any point [Math Processing Error] (Figure [Math Processing Error]), the magnetic field [Math Processing Error] due to an element [Math Processing Error] of a current-carrying wire is given by

[Math Processing Error]

The constant [Math Processing Error] is known as the **permeability of free space** and is exactly

[Math Processing Error]

in the SI system. The infinitesimal wire segment [Math Processing Error] is in the same direction as the current [Math Processing Error] (assumed positive), [Math Processing Error] is the distance from [Math Processing Error] to [Math Processing Error] and [Math Processing Error] is a unit vector that points from [Math Processing Error] to [Math Processing Error], as shown in Figure [Math Processing Error]. The direction of [Math Processing Error] is determined by applying the right-hand rule to the vector product [Math Processing Error]. The magnitude of [Math Processing Error] is

[Math Processing Error]

where [Math Processing Error] is the angle between [Math Processing Error] and [Math Processing Error]. Notice that if [Math Processing Error], then [Math Processing Error]. The field produced by a current element [Math Processing Error] has no component parallel to [Math Processing Error].

The magnetic field due to a finite length of current-carrying wire is found by integrating Equation [Math Processing Error] along the wire, giving us the usual form of the Biot-Savart law.

Biot-Savart law

The magnetic field [Math Processing Error] due to an element [Math Processing Error] of a current-carrying wire is given by

[Math Processing Error]

Since this is a vector integral, contributions from different current elements may not point in the same direction. Consequently, the integral is often difficult to evaluate, even for fairly simple geometries. The following strategy may be helpful.

Problem-Solving Strategy: Solving Biot-Savart Problems

To solve Biot-Savart law problems, the following steps are helpful:

1. Identify that the Biot-Savart law is the chosen method to solve the given problem. If there is symmetry in the problem comparing *[Math Processing Error]* and *[Math Processing Error]*, Ampère's law may be the preferred method to solve the question.
2. Draw the current element length *[Math Processing Error]* and the unit vector *[Math Processing Error]* noting that *[Math Processing Error]* points in the direction of the current and *[Math Processing Error]* points from the current element toward the point where the field is desired.
3. Calculate the cross product *[Math Processing Error]*.The resultant vector gives the direction of the magnetic field according to the Biot-Savart law.
4. Use Equation *[Math Processing Error]* and substitute all given quantities into the expression to solve for the magnetic field. Note all variables that remain constant over the entire length of the wire may be factored out of the integration.
5. Use the right-hand rule to verify the direction of the magnetic field produced from the current or to write down the direction of the magnetic field if only the magnitude was solved for in the previous part.

Calculating Magnetic Fields of Short Current Segments

A short wire of length 1.0 cm carries a current of 2.0 A in the vertical direction (Figure *[Math Processing Error]*). The rest of the wire is shielded so it does not add to the magnetic field produced by the wire. Calculate the magnetic field at point **P**, which is 1 meter from the wire in the **x**-direction.

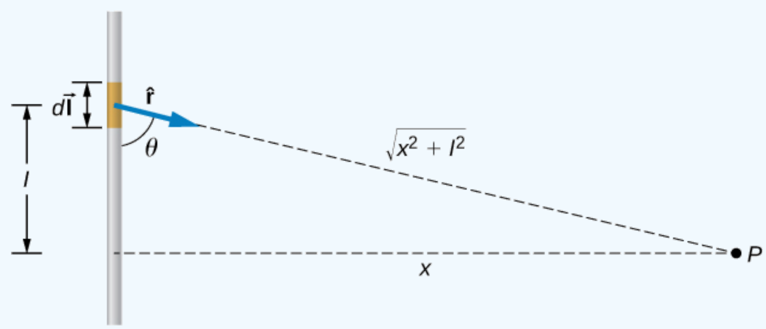

Figure *[Math Processing Error]*: A small line segment carries a current *[Math Processing Error]* in the vertical direction. What is the magnetic field at a distance **x** from the segment?

Strategy

We can determine the magnetic field at point *[Math Processing Error]* using the Biot-Savart law. Since the current segment is much smaller than the distance **x**, we can drop the integral from the expression. The integration is converted back into a summation, but only for small *[Math Processing Error]*, which we now write as *[Math Processing Error]*. Another way to think about it is that each of the radius values is nearly the same, no matter where the current element is on the line segment, if *[Math Processing Error]* is small compared to **x**. The angle *[Math Processing Error]* is calculated using a tangent function. Using the numbers given, we can calculate the magnetic field at *[Math Processing Error]*.

Solution

The angle between *[Math Processing Error]* and *[Math Processing Error]* is calculated from trigonometry, knowing the distances **l** and **x** from the problem:

[Math Processing Error]

The magnetic field at point *[Math Processing Error]* is calculated by the Biot-Savart law (Equation *[Math Processing Error]*):

[Math Processing Error]

From the right-hand rule and the Biot-Savart law, the field is directed into the page.

Significance

This approximation is only good if the length of the line segment is very small compared to the distance from the current element to the point. If not, the integral form of the Biot-Savart law must be used over the entire line segment to calculate the magnetic field.

Exercise [Math Processing Error]

Using Example [Math Processing Error], at what distance would **P** have to be to measure a magnetic field half of the given answer?

Solution

1.41 meters

Example [Math Processing Error]: Calculating Magnetic Field of a Circular Arc of Wire

A wire carries a current **I** in a circular arc with radius **R** swept through an arbitrary angle [Math Processing Error] (Figure [Math Processing Error]). Calculate the magnetic field at the center of this arc at point **P**.

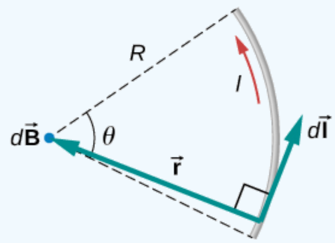

Figure [Math Processing Error]: A wire segment carrying a current **I**. The path [Math Processing Error] and radial direction [Math Processing Error] are indicated.

Strategy

We can determine the magnetic field at point **P** using the Biot-Savart law. The radial and path length directions are always at a right angle, so the cross product turns into multiplication. We also know that the distance along the path **dl** is related to the radius times the angle [Math Processing Error] (in radians). Then we can pull all constants out of the integration and solve for the magnetic field.

Solution

The Biot-Savart law starts with the following equation:

[Math Processing Error]

As we integrate along the arc, all the contributions to the magnetic field are in the same direction (out of the page), so we can work with the magnitude of the field. The cross product turns into multiplication because the path [Math Processing Error] and the radial direction are perpendicular. We can also substitute the arc length formula, [Math Processing Error]:

[Math Processing Error]

The current and radius can be pulled out of the integral because they are the same regardless of where we are on the path. This leaves only the integral over the angle,

[Math Processing Error]

The angle varies on the wire from 0 to [Math Processing Error]; hence, the result is

[Math Processing Error]

Significance

The direction of the magnetic field at point [Math Processing Error] is determined by the right-hand rule, as shown in the previous chapter. If there are other wires in the diagram along with the arc, and you are asked to find the net magnetic field, find each contribution from a wire or arc and add the results by superposition of vectors. Make sure to pay attention to the direction of each contribution. Also note that in a symmetric situation, like a straight or circular wire, contributions from opposite sides of point *[Math Processing Error]* cancel each other.

Exercise *[Math Processing Error]*

The wire loop forms a full circle of radius **R** and current **I**. What is the magnitude of the magnetic field at the center?

Solution

[Math Processing Error]
