KINEMATICS WITH VIDEO EXAMPLES

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Kinematics

with Videos of Typical Problems

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CHAPTER OVERVIEW

1: One-Dimensional Kinematics

Objects are in motion everywhere we look. Everything from a tennis game to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. And even in inanimate objects, there is continuous motion in the vibrations of atoms and molecules. Questions about motion are interesting in and of themselves: *How long will it take for a space probe to get to Mars? Where will a football land if it is thrown at a certain angle?* But an understanding of motion is also key to understanding other concepts in physics. An understanding of acceleration, for example, is crucial to the study of force. Kinematics is the branch of classical mechanics which describes the motion of points, bodies, and systems of bodies without consideration of the masses of those objects, nor the forces that may have caused the motion.

1.1: Prelude to One-Dimensional Kinematics

Objects are in motion everywhere we look. Everything from a tennis game to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. And even in inanimate objects, there is continuous motion in the vibrations of atoms and molecules. Questions about motion are interesting in and of themselves: *How long will it take for a space probe to get to Mars? Where will a football land if it is thrown at a certain angle?* But an understanding of motion is also key to understanding other concepts in physics. An understanding of acceleration, for example, is crucial to the study of force.

Figure 1.1.1:The motion of an American kestrel through the air can be described by the bird's displacement, speed, velocity, and acceleration. When it flies in a straight line without any change in direction, its motion is said to be one dimensional. (credit: Vince Maidens, Wikimedia Commons)

Our formal study of physics begins with kinematics which is defined as the *study of motion without considering its causes*. The word "kinematics" comes from a Greek term meaning motion and is related to other English words such as "cinema" (movies) and "kinesiology" (the study of human motion). In one-dimensional kinematics and Two-Dimensional Kinematics we will study only the *motion* of a football, for example, without worrying about what forces cause or change its motion. Such considerations come in other chapters. In this chapter, we examine the simplest type of motion—namely, motion along a straight line, or one-dimensional motion. In Two-Dimensional Kinematics, we apply concepts developed here to study motion along curved paths (two- and three-dimensional motion); for example, that of a car rounding a curve.

1.2: Displacement

Learning Objectives

- Define position, displacement, distance, and distance traveled.
- Explain the relationship between position and displacement.
- Distinguish between displacement and distance traveled.
- Calculate displacement and distance given initial position, final position, and the path between the two.

These cyclists in Vietnam can be described by their position relative to buildings and a canal. Their motion can be described by their change in position, or displacement, in the frame of reference.

Figure 1.2.1: These cyclists in Vietnam can be described by their position relative to buildings and a canal. Their motion can be described by their change in position, or displacement, in the frame of reference. (credit: Suzan Black, Fotopedia)

Position

In order to describe the motion of an object, you must first be able to describe its position—where it is at any particular time. More precisely, you need to specify its position relative to a convenient reference frame. Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame. For example, a rocket launch would be described in terms of the position of the rocket with respect to the Earth as a whole, while a professor's position could be described in terms of where she is in relation to the nearby white board. In other cases, we use reference frames that are not stationary but are in motion relative to the Earth. To describe the position of a person in an airplane, for example, we use the airplane, not the Earth, as the reference frame.

Displacement

If an object moves relative to a reference frame (for example, if a professor moves to the right relative to a white board or a passenger moves toward the rear of an airplane), then the object's position changes. This change in position is known as displacement. The word "displacement" implies that an object has moved, or has been displaced.

Definition: Displacement

Displacement is the change in position of an object:

$$\Delta x = x_f - x_0,$$

where Δx is displacement, x_f is the final position, and x_0 is the initial position.

In this text the upper case Greek letter size Δ(delta) always means "change in" whatever quantity follows it; thus, size Δx means change in position. Always solve for displacement by subtracting initial position size x_0 from final position x_f.

Note that the SI unit for displacement is the meter (m) (see Section on Physical Quantities and Units), but sometimes kilometers, miles, feet, and other units of length are used. Keep in mind that when units other than the meter are used in a problem, you may need to convert them into meters to complete the calculation.

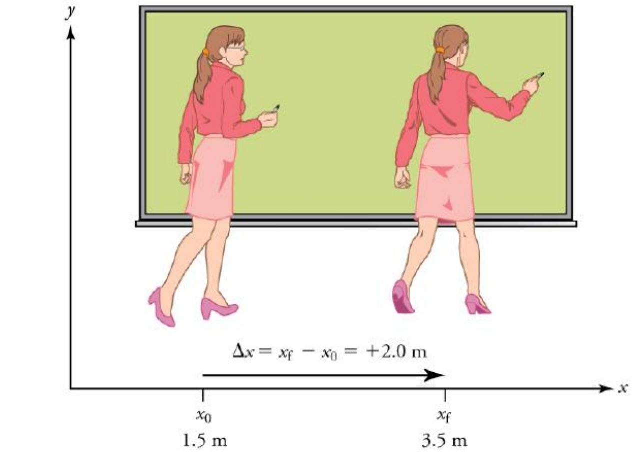


Figure 1.2.2: A professor paces left and right while lecturing. Her position relative to Earth is given by x. The +2 m displacement of the professor relative to Earth is represented by an arrow pointing to the right.

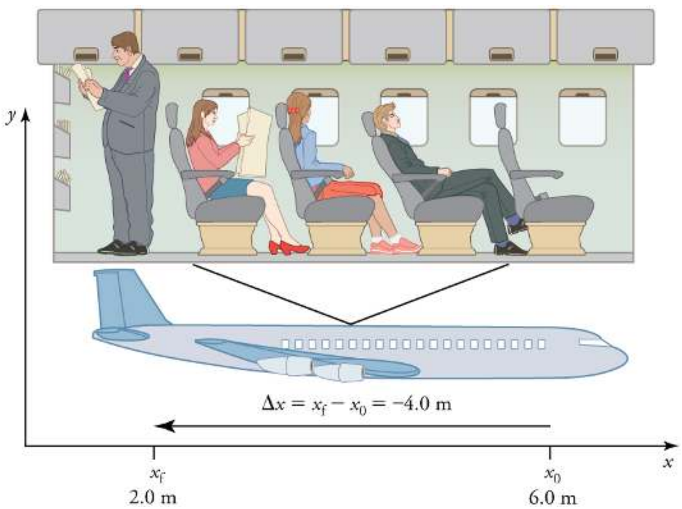


Figure 1.2.3: A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by x. The -4 m displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor (he moves twice as far)

Note that displacement has a direction as well as a magnitude. The professor's displacement in Figure 1.2.2 is 2.0 m to the right, and the airline passenger's displacement is 4.0 m toward the rear in Figure 1.2.3. In one-dimensional motion, direction can be specified with a plus or minus sign. When you begin a problem, you should select which direction is positive (usually that will be to the right or up, but you are free to select positive as being any direction). The professor's initial position is size x_0 = 1.5 m and her final position is x_f = 3.5 m. Thus her displacement is

$$\begin{aligned} \Delta x &= x_f - x_0 \\ &= 3.5\, m - 1.5\, m \\ &= +2.0\, m. \end{aligned}$$

In this coordinate system, motion to the right is positive, whereas motion to the left is negative. Similarly, the airplane passenger's initial position is $x_0 = 6.0m$ and his final position is $x_f = 2.0m$, so his displacement is

$$\begin{aligned} \Delta x &= x_f - x_0 \\ &= 2.0\, m - 6.0\, m \\ &= -4.0\, m. \end{aligned}$$

His displacement is negative because his motion is toward the rear of the plane, or in the negative size 12{x} {} direction in our coordinate system.

Distance

Although displacement is described in terms of direction, distance is not. **Distance** is defined to be the *magnitude or size of displacement between two positions*. Note that the distance between two positions is not the same as the distance traveled between them. **Distance traveled** is *the total length of the path traveled between two positions.* Distance has no direction and, thus, no sign. For example, the distance the professor walks is 2.0 m. The distance the airplane passenger walks is 4.0 m.

Misconception Alert: Distance Traveled vs. Magnitude of Displacement

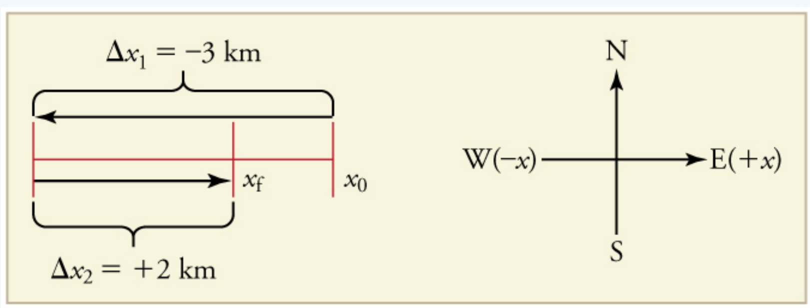
It is important to note that the distance traveled, however, can be greater than the magnitude of the displacement (by magnitude, we mean just the size of the displacement without regard to its direction; that is, just a number with a unit). For example, the professor could pace back and forth many times, perhaps walking a distance of 150 m during a lecture, yet still end up only 2.0 m to the right of her starting point. In this case her displacement would be +2.0 m, the magnitude of her displacement would be 2.0 m, but the distance she traveled would be 150 m. In kinematics we nearly always deal with displacement and magnitude of displacement, and almost never with distance traveled. One way to think about this is to assume you marked the start of the motion and the end of the motion. The displacement is simply the difference in the position of the two marks and is independent of the path taken in traveling between the two marks. The distance traveled, however, is the total length of the path taken between the two marks.

Exercise 1.2.1

A cyclist rides 3 km west and then turns around and rides 2 km east.

a. What is her displacement?
b. What distance does she ride?
c. What is the magnitude of her displacement?

Answer



Answer a

The rider's displacement is $\Delta_x = x_f - x_0 = -1 km$. (The displacement is negative because we take east to be positive and west to be negative.)

Answer b

The distance traveled is $3 km + 2 km = 5 km$.

Answer c

The magnitude of the displacement is $1 km$.

Summary

- Kinematics is the study of motion without considering its causes. In this chapter, it is limited to motion along a straight line, called one-dimensional motion.
- Displacement is the change in position of an object.
- In symbols, displacement Δx is defined to be

$$\Delta x = x_f - x_0 \ ,$$

where x_0 is the initial position and xf is the final position. In this text, the Greek letter Δ (delta) always means "change in" whatever quantity follows it. The SI unit for displacement is the meter (m). Displacement has a direction as well as a magnitude.

- When you start a problem, assign which direction will be positive.
- Distance is the magnitude of displacement between two positions.
- Distance traveled is the total length of the path traveled between two positions.

Glossary

kinematics
the study of motion without considering its causes

position
the location of an object at a particular time

displacement
the change in position of an object

distance
the magnitude of displacement between two positions

distance traveled
the total length of the path traveled between two positions

1.3: Vectors, Scalars, and Coordinate Systems

Learning Objectives

By the end of this section, you will be able to:

- Define and distinguish between scalar and vector quantities.
- Assign a coordinate system for a scenario involving one-dimensional motion.

What is the difference between distance and displacement? Whereas displacement is defined by both direction and magnitude, distance is defined only by magnitude. Displacement is an example of a vector quantity. Distance is an example of a scalar quantity. A vector is any quantity with both *magnitude and direction*. Other examples of vectors include a velocity of 90 km/h east and a force of 500 newtons straight down.

Figure 1.3.1: The motion of this Eclipse Concept jet can be described in terms of the distance it has traveled (a scalar quantity) or its displacement in a specific direction (a vector quantity). In order to specify the direction of motion, its displacement must be described based on a coordinate system. In this case, it may be convenient to choose motion toward the left as positive motion (it is the forward direction for the plane), although in many cases, the x -coordinate runs from left to right, with motion to the right as positive and motion to the left as negative. (credit: Armchair Aviator, Flickr)

The direction of a vector in one-dimensional motion is given simply by a plus (+) or minus (−) sign. Vectors are represented graphically by arrows. An arrow used to represent a vector has a length proportional to the vector's magnitude (e.g., the larger the magnitude, the longer the length of the vector) and points in the same direction as the vector.

Some physical quantities, like distance, either have no direction or none is specified. A scalar is any quantity that has a magnitude, but no direction. For example, a 20ºC temperature, the 250 kilocalories (250 Calories) of energy in a candy bar, a 90 km/h speed limit, a person's 1.8 m height, and a distance of 2.0 m are all scalars—quantities with no specified direction. Note, however, that a scalar can be negative, such as a −20ºC temperature. In this case, the minus sign indicates a point on a scale rather than a direction. Scalars are never represented by arrows.

Coordinate Systems for One-Dimensional Motion

In order to describe the direction of a vector quantity, you must designate a coordinate system within the reference frame. For one-dimensional motion, this is a simple coordinate system consisting of a one-dimensional coordinate line. In general, when describing horizontal motion, motion to the right is usually considered positive, and motion to the left is considered negative. With vertical motion, motion up is usually positive and motion down is negative. In some cases, however, as with the jet in Figure \PageIndex{2}\), it can be more convenient to switch the positive and negative directions. For example, if you are analyzing the motion of falling objects, it can be useful to define downwards as the positive direction. If people in a race are running to the left, it is useful to define left as the positive direction. It does not matter as long as the system is clear and consistent. Once you assign a positive direction and start solving a problem, you cannot change it.

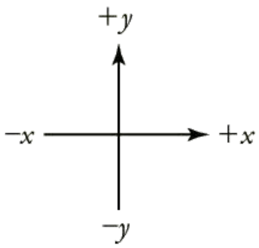


Figure 1.3.2: It is usually convenient to consider motion upward or to the right as positive (+) and motion downward or to the left as negative (−) .

Exercise 1.3.1

A person’s speed can stay the same as he or she rounds a corner and changes direction. Given this information, is speed a scalar or a vector quantity? Explain.

Answer

Speed is a scalar quantity. It does not change at all with direction changes; therefore, it has magnitude only. If it were a vector quantity, it would change as direction changes (even if its magnitude remained constant).

Summary

- A vector is any quantity that has magnitude and direction.
- A scalar is any quantity that has magnitude but no direction.
- Displacement and velocity are vectors, whereas distance and speed are scalars.
- In one-dimensional motion, direction is specified by a plus or minus sign to signify left or right, up or down, and the like.

Glossary

scalar
a quantity that is described by magnitude, but not direction

vector
a quantity that is described by both magnitude and direction

1.4: Time, Velocity, and Speed

Learning Objectives

By the end of this section, you will be able to:

- Explain the relationships between instantaneous velocity, average velocity, instantaneous speed, average speed, displacement, and time.
- Calculate velocity and speed given initial position, initial time, final position, and final time.
- Derive a graph of velocity vs. time given a graph of position vs. time.
- Interpret a graph of velocity vs. time.

There is more to motion than distance and displacement. Questions such as, “How long does a foot race take?” and “What was the runner’s speed?” cannot be answered without an understanding of other concepts. In this section we add definitions of time, velocity, and speed to expand our description of motion.

Figure 1.4.1: The motion of these racing snails can be described by their speeds and their velocities. (credit: tobitasflickr, Flickr)

Time

As discussed in Physical Quantities and Units, the most fundamental physical quantities are defined by how they are measured. This is the case with time. Every measurement of time involves measuring a change in some physical quantity. It may be a number on a digital clock, a heartbeat, or the position of the Sun in the sky. In physics, the definition of **time** is simple—time is *change*, or the interval over which change occurs. It is impossible to know that time has passed unless something changes.

The amount of time or change is calibrated by comparison with a standard. The SI unit for time is the second, abbreviated s. We might, for example, observe that a certain pendulum makes one full swing every 0.75 s. We could then use the pendulum to measure time by counting its swings or, of course, by connecting the pendulum to a clock mechanism that registers time on a dial. This allows us to not only measure the amount of time, but also to determine a sequence of events.

How does time relate to motion? We are usually interested in elapsed time for a particular motion, such as how long it takes an airplane passenger to get from his seat to the back of the plane. To find elapsed time, we note the time at the beginning and end of the motion and subtract the two. For example, a lecture may start at 11:00 A.M. and end at 11:50 A.M., so that the elapsed time would be 50 min. **Elapsed time** Δt is the difference between the ending time and beginning time,

$$\Delta t = t_f - t_0 \ ,$$

where Δt is the change in time or elapsed time, t_f is the time at the end of the motion, and t_0 is the time at the beginning of the motion. (As usual, the delta symbol, Δ, means the change in the quantity that follows it.)

Life is simpler if the beginning time t_0 is taken to be zero, as when we use a stopwatch. If we were using a stopwatch, it would simply read zero at the start of the lecture and 50 min at the end. If $t_0 = 0$, then

$$\Delta t = t_f \equiv t. \tag{1.4.1}$$

In this text, for simplicity’s sake,

- motion starts at time equal to zero ($t_0 = 0$)

- the symbol t is used for elapsed time unless otherwise specified ($\Delta t = t_f \equiv t$)

Velocity

Your notion of velocity is probably the same as its scientific definition. You know that if you have a large displacement in a small amount of time you have a large velocity, and that velocity has units of distance divided by time, such as miles per hour or kilometers per hour.

Definition: AVERAGE VELOCITY

Average velocity is *displacement (change in position) divided by the time of travel,*

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0}. \tag{1.4.2}$$

where $\bar{v}$ is the average (indicated by the bar over the v) velocity, Δx is the change in position (or displacement), and x_f and x_0 are the final and beginning positions at times t_f and t_0, respectively. If the starting time t_0 is taken to be zero, then the average velocity is simply

$$\bar{v} = \frac{\Delta x}{t}. \tag{1.4.3}$$

Notice that this definition indicates that *velocity is a vector because displacement is a vector*. It has both magnitude and direction. The SI unit for velocity is meters per second or m/s, but many other units, such as km/h, mi/h (also written as mph), and cm/s, are in common use. Suppose, for example, an airplane passenger took 5 seconds to move −4 m (the negative sign indicates that displacement is toward the back of the plane). His average velocity would be

$$\bar{v} = \frac{\Delta x}{t} = \frac{-4\, m}{5\, s} = -0.8\, m/s. \tag{1.4.4}$$

The minus sign indicates the average velocity is also toward the rear of the plane.

The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point, however. For example, we cannot tell from average velocity whether the airplane passenger stops momentarily or backs up before he goes to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals.

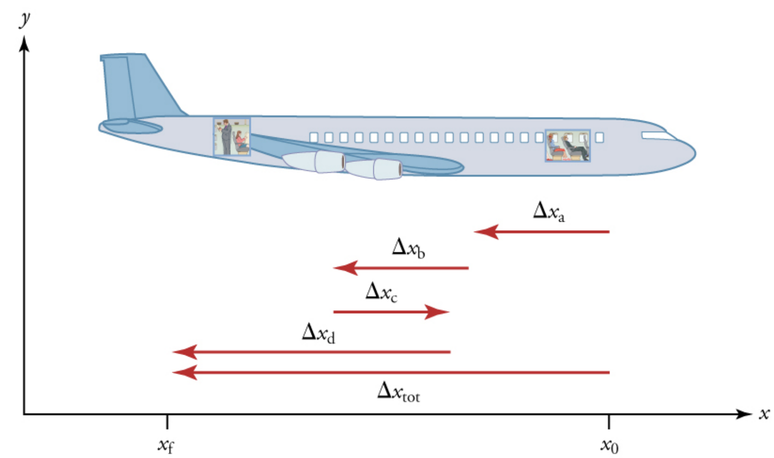


Figure 1.4.2: A more detailed record of an airplane passenger heading toward the back of the plane, showing smaller segments of his trip.

The smaller the time intervals considered in a motion, the more detailed the information. When we carry this process to its logical conclusion, we are left with an infinitesimally small interval. Over such an interval, the average velocity becomes the *instantaneous velocity* or the *velocity at a specific instant*. A car's speedometer, for example, shows the magnitude (but not the direction) of the instantaneous velocity of the car. (Police give tickets based on instantaneous velocity, but when calculating how long it will take to get from one place to another on a road trip, you need to use average velocity.) **Instantaneous velocity** v is the average velocity at a specific instant in time (or over an infinitesimally small time interval).

Mathematically, finding instantaneous velocity, v, at a precise instant t can involve taking a limit, a calculus operation beyond the scope of this text. However, under many circumstances, we can find precise values for instantaneous velocity without calculus.

Speed

In everyday language, most people use the terms "speed" and "velocity" interchangeably. In physics, however, they do not have the same meaning and they are distinct concepts. One major difference is that speed has no direction. Thus *speed is a scalar*. Just as we need to distinguish between instantaneous velocity and average velocity, we also need to distinguish between instantaneous speed and average speed.

Instantaneous speed is the magnitude of instantaneous velocity. For example, suppose the airplane passenger at one instant had an instantaneous velocity of −3.0 m/s (the minus meaning toward the rear of the plane). At that same time his instantaneous speed was 3.0 m/s. Or suppose that at one time during a shopping trip your instantaneous velocity is 40 km/h due north. Your instantaneous speed at that instant would be 40 km/h—the same magnitude but without a direction. Average speed, however, is very different from average velocity. **Average speed** is the distance traveled divided by elapsed time.

We have noted that distance traveled can be greater than displacement. So average speed can be greater than average velocity, which is displacement divided by time. For example, if you drive to a store and return home in half an hour, and your car's odometer shows the total distance traveled was 6 km, then your average speed was 12 km/h. Your average velocity, however, was zero, because your displacement for the round trip is zero. (Displacement is change in position and, thus, is zero for a round trip.) Thus average speed is *not* simply the magnitude of average velocity.

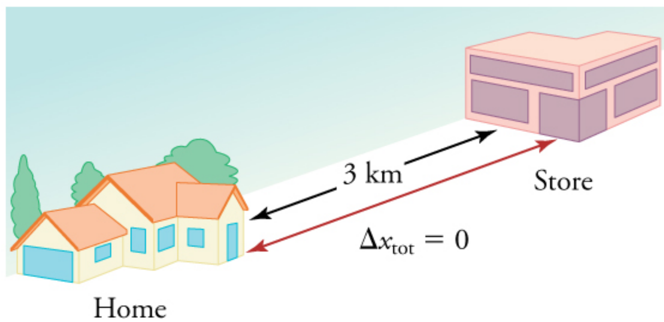


Figure 1.4.3: During a 30-minute round trip to the store, the total distance traveled is 6 km. The average speed is 12 km/h. The displacement for the round trip is zero, since there was no net change in position. Thus the average velocity is zero.

Another way of visualizing the motion of an object is to use a graph. A plot of position or of velocity as a function of time can be very useful. For example, for this trip to the store, the position, velocity, and speed-vs.-time graphs are displayed in Figure 1.4.4. (Note that these graphs depict a very simplified **model** of the trip. We are assuming that speed is constant during the trip, which is unrealistic given that we'll probably stop at the store. But for simplicity's sake, we will model it with no stops or changes in speed. We are also assuming that the route between the store and the house is a perfectly straight line.)

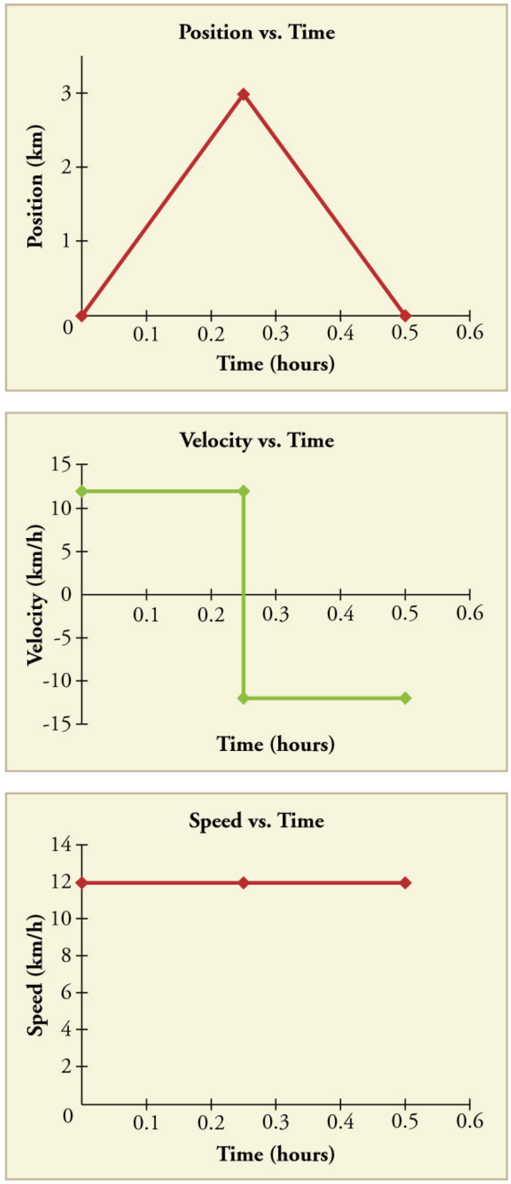


Figure 1.4.4: Position vs. time, velocity vs. time, and speed vs. time on a trip. Note that the velocity for the return trip is negative.

MAKING CONNECTIONS: TAKE-HOME INVESTIGATION - GETTING A SENSE OF SPEED

If you have spent much time driving, you probably have a good sense of speeds between about 10 and 70 miles per hour. But what are these in meters per second? What do we mean when we say that something is moving at 10 m/s? To get a better sense of what these values really mean, do some observations and calculations on your own:

- calculate typical car speeds in meters per second
- estimate jogging and walking speed by timing yourself; convert the measurements into both m/s and mi/h
- determine the speed of an ant, snail, or falling leaf

Exercise 1.4.1

A commuter train travels from Baltimore to Washington, DC, and back in 1 hour and 45 minutes. The distance between the two stations is approximately 40 miles. What is

a. the average velocity of the train, and
b. the average speed of the train in m/s?

Answer

(a) The average velocity of the train is zero because $x_f = x_0$; the train ends up at the same place it starts.

(b) The average speed of the train is calculated below. Note that the train travels 40 miles one way and 40 miles back, for a total distance of 80 miles.

$$\frac{distance}{time} = \frac{80 miles}{105 minutes}$$

$$\frac{80 miles}{105 minutes} \times \frac{5280 feet}{1 mile} \times \frac{1 meter}{3.28 feet} \times \frac{1 minute}{60 seconds} = 20 m/s$$

Summary

- Time is measured in terms of change, and its SI unit is the second (s). Elapsed time for an event is

$$\Delta t = t_f - t_0,$$

  where t_f is the final time and t_0 is the initial time. The initial time is often taken to be zero, as if measured with a stopwatch; the elapsed time is then just t.
- Average velocity $\bar{v}$ is defined as displacement divided by the travel time. In symbols, average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0}.$$

- The SI unit for velocity is m/s.
- Velocity is a vector and thus has a direction.
- Instantaneous velocity v is the velocity at a specific instant or the average velocity for an infinitesimal interval.
- Instantaneous speed is the magnitude of the instantaneous velocity.
- Instantaneous speed is a scalar quantity, as it has no direction specified.
- Average speed is the total distance traveled divided by the elapsed time. (Average speed is *not* the magnitude of the average velocity.) Speed is a scalar quantity; it has no direction associated with it.

Glossary

average speed
distance traveled divided by time during which motion occurs

average velocity
displacement divided by time over which displacement occurs

instantaneous velocity
velocity at a specific instant, or the average velocity over an infinitesimal time interval

instantaneous speed
magnitude of the instantaneous velocity

time
change, or the interval over which change occurs

model
simplified description that contains only those elements necessary to describe the physics of a physical situation

elapsed time
the difference between the ending time and beginning time

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1.5: How Fast Do I Need To Run to Catch the Bus (Video Solution)

A pedestrian is running at his maximum speed of 6.0 m/s to catch a bus stopped at a traffic light. When he is 15 m from the bus, the light changes and the bus accelerates uniformly at 1.00 m/s2 Does he make it to the bus? If so, how far does he have to run in order to catch it? If not, how close does he get?

Summary

This lecture demonstrates the solution to a physics problem involving a pedestrian trying to catch a bus. The instructor walks through a step-by-step approach to determine whether the pedestrian catches the bus and how far they travel.

00:00:30 Problem Introduction and Setup

00:01:41 Analysis of Motion Graphs

00:02:29 Deriving Position Equations

00:04:59 Setting Up and Solving the Quadratic Equation

00:09:05 Interpreting the Solution

 Transcript

1.6: Acceleration

Learning Objectives

By the end of this section, you will be able to:

- Define and distinguish between instantaneous acceleration, average acceleration, and deceleration.
- Calculate acceleration given initial time, initial velocity, final time, and final velocity.

In everyday conversation, to accelerate means to speed up. The accelerator in a car can in fact cause it to speed up. The greater the **acceleration**, the greater the change in velocity over a given time. The formal definition of acceleration is consistent with these notions, but more inclusive.

Figure 1.6.1: A plane decelerates, or slows down, as it comes in for landing in St. Maarten. Its acceleration is opposite in direction to its velocity. (credit: Steve Conry, Flickr)

Definition: Average Acceleration

The *average acceleration* is the rate at which velocity changes,

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0} \tag{1.6.1}$$

where $\bar{a}$ is average acceleration, v is velocity, and t is time. (The bar over the a means *average*.)

Because acceleration is velocity in m/s divided by time in s, the SI units for acceleration are m/s^2, meters per second squared or meters per second per second, which literally means by how many meters per second the velocity changes every second.

Recall that velocity is a vector—it has both magnitude and direction. This means that a change in velocity can be a change in magnitude (or speed), but it can also be a change in direction. For example, if a car turns a corner at constant speed, it is accelerating because its direction is changing. The quicker you turn, the greater the acceleration. So there is an acceleration when velocity changes either in magnitude (an increase or decrease in speed) or in direction, or both.

ACCELERATION AS A VECTOR

Acceleration is a vector in the same direction as the change in velocity, Δv. Since velocity is a vector, it can change either in magnitude or in direction. Acceleration is therefore a change in either speed or direction, or both.

Keep in mind that although acceleration is in the direction of the *change* in velocity, it is not always in the direction of *motion*. When an object slows down, its acceleration is opposite to the direction of its motion. This is known as **deceleration.**

Figure 1.6.2: A subway train in Sao Paulo, Brazil, decelerates as it comes into a station. It is accelerating in a direction opposite to its direction of motion. (credit: Yusuke Kawasaki, Flickr)

MISCONCEPTION ALERT: DECELERATION VS. NEGATIVE ACCELERATION

Deceleration always refers to acceleration in the direction opposite to the direction of the velocity. Deceleration always reduces speed. Negative acceleration, however, is acceleration in the negative direction in the chosen coordinate system. Negative acceleration may or may not be deceleration, and deceleration may or may not be considered negative acceleration. If acceleration has the same sign as the velocity, the object is speeding up. If acceleration has the opposite sign as the velocity, the object is slowing down. For example, consider Figure 1.6.2.

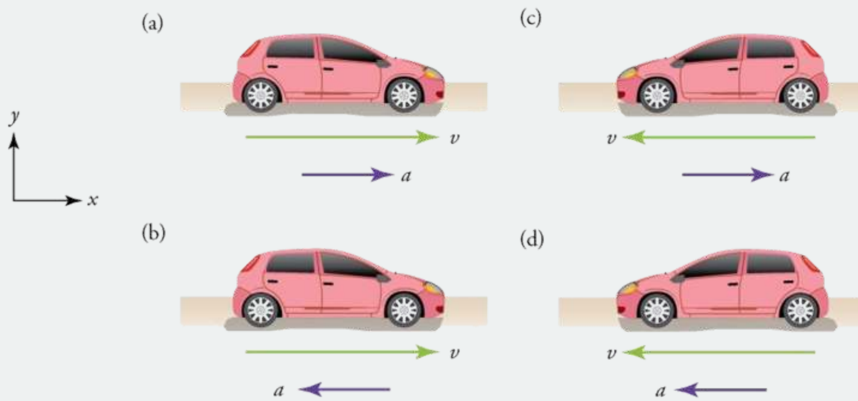


Figure 1.6.3: (a) This car is speeding up as it moves toward the right. It therefore has positive acceleration in our coordinate system. (b) This car is slowing down as it moves toward the right. Therefore, it has negative acceleration in our coordinate system, because its acceleration is toward the left. The car is also decelerating: the direction of its acceleration is opposite to its direction of motion. (c) This car is moving toward the left, but slowing down over time. Therefore, its acceleration is positive in our coordinate system because it is toward the right. However, the car is decelerating because its acceleration is opposite to its motion. (d) This car is speeding up as it moves toward the left. It has negative acceleration because it is accelerating toward the left. However, because its acceleration is in the same direction as its motion, it is speeding up (not decelerating).

Example 1.6.1: Calculating Acceleration: A Racehorse Leaves the Gate

A racehorse coming out of the gate accelerates from rest to a velocity of 15.0 m/s due west in 1.80 s. What is its average acceleration?


Figure 1.6.4: Two racehorses running toward the left. (credit: Jon Sullivan, PD Photo.org)

Strategy

First we draw a sketch and assign a coordinate system to the problem. This is a simple problem, but it always helps to visualize it. Notice that we assign east as positive and west as negative. Thus, in this case, we have negative velocity.

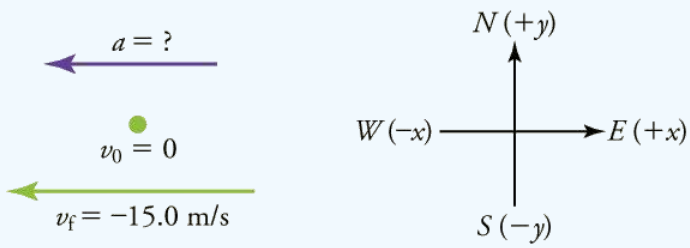


Figure 1.6.5.

We can solve this problem by identifying Δv and Δt from the given information and then calculating the average acceleration directly from the Equation ???:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}.$$

Solution

1. Identify the knowns. $v_0 = 0, v_f = -15.0 m/s$ (the negative sign indicates direction toward the west), $\Delta t = 1.80 s$.
2. Find the change in velocity. Since the horse is going from zero to $-15.0 m/s$, its change in velocity equals its final velocity:

$$\Delta v = v_f = -15.0\, m/s.$$

3. Plug in the known values (Δv and Δt) and solve for the unknown $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-15.0 m/s}{1.80 s} = -8.33 m/s^2 .$$

Discussion

The negative sign for acceleration indicates that acceleration is toward the west. An acceleration of $8.33\, m/s^2$ due west means that the horse increases its velocity by 8.33 m/s due west each second, that is, 8.33 meters per second per second, which we write as $8.33\, m/s^2$. This is truly an average acceleration, because the ride is not smooth. We shall see later that an acceleration of this magnitude would require the rider to hang on with a force nearly equal to his weight.

Instantaneous Acceleration

Instantaneous acceleration a, or the acceleration at a specific instant in time, is obtained by the same process as discussed for instantaneous velocity in Time, Velocity, and Speed—that is, by considering an infinitesimally small interval of time. How do we find instantaneous acceleration using only algebra? The answer is that we choose an average acceleration that is representative of the motion. Figure 1.6.6 shows graphs of instantaneous acceleration versus time for two very different motions. In Figure 1.6.6a, the acceleration varies slightly and the average over the entire interval is nearly the same as the instantaneous acceleration at any time. In this case, we should treat this motion as if it had a constant acceleration equal to the average (in this case about $1.8 m/s^2$). In Figure 1.6.6b, the acceleration varies drastically over time. In such situations it is best to consider smaller time intervals and choose an average acceleration for each. For example, we could consider motion over the time intervals from 0 to 1.0 s and from 1.0 to 3.0 s as separate motions with accelerations of $+3.0\, m/s^2$ and $-2.0\, m/s^2$, respectively.

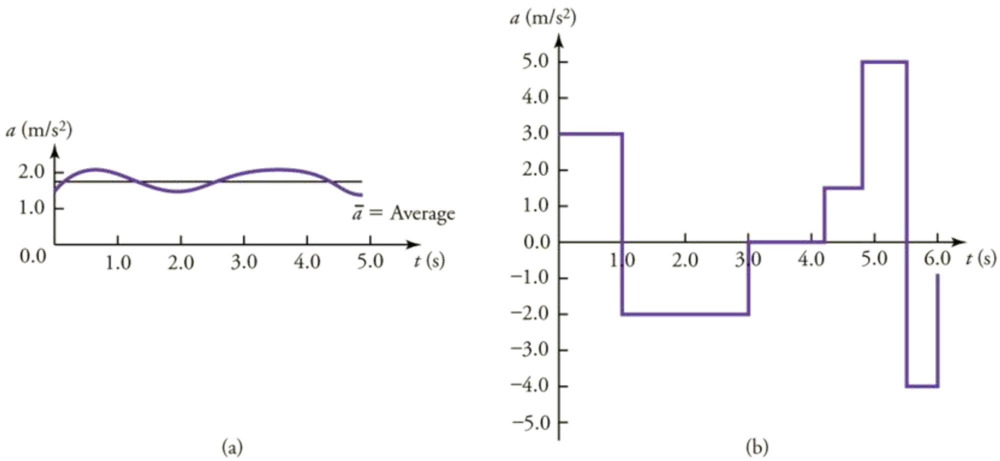


Figure 1.6.6: Graphs of instantaneous acceleration versus time for two different one-dimensional motions. (a) Here acceleration varies only slightly and is always in the same direction, since it is positive. The average over the interval is nearly the same as the acceleration at any given time. (b) Here the acceleration varies greatly, perhaps representing a package on a post office conveyor belt that is accelerated forward and backward as it bumps along. It is necessary to consider small time intervals (such as from 0 to 1.0 s) with constant or nearly constant acceleration in such a situation.

The next several examples consider the motion of the subway train shown in Figure 1.6.7. In Figure 1.6.7a the shuttle moves to the right, and in Figure 1.6.7b it moves to the left. The examples are designed to further illustrate aspects of motion and to illustrate some of the reasoning that goes into solving problems.

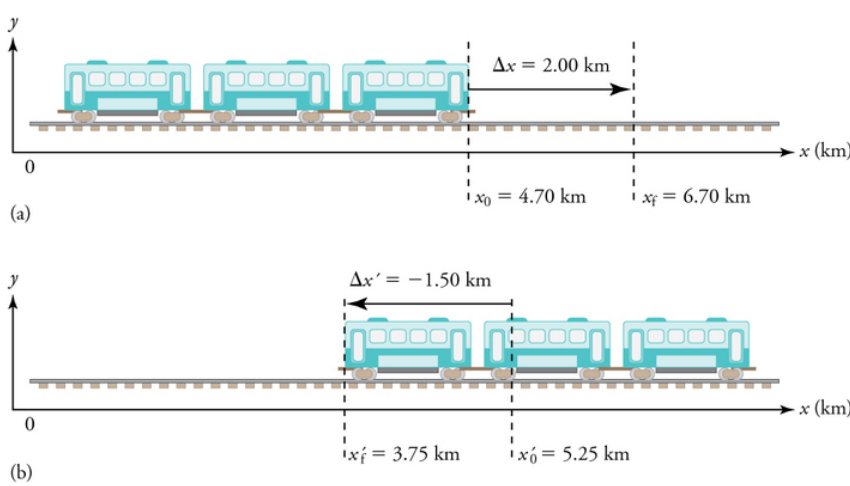


Figure 1.6.7: One-dimensional motion of a subway train considered in Examples 1.6.2 - 1.6.5. Here we have chosen the x -axis so that + means to the right and − means to the left for displacements, velocities, and accelerations. (a) The subway train moves to the right from x_0 to x_f. Its displacement Δx is +2.0 km. (b) The train moves to the left from x_0 to x_f. Its displacement Δx is $-1.5 km$. (Note that the prime symbol (′) is used simply to distinguish between displacement in the two different situations. The distances of travel and the size of the cars are on different scales to fit everything into the diagram.)

Example 1.6.2: Calculating Displacement - A Subway Train

What are the magnitude and sign of displacements for the motions of the subway train shown in parts (a) and (b) of Figure 1.6.7?

Strategy

A drawing with a coordinate system is already provided, so we don't need to make a sketch, but we should analyze it to make sure we understand what it is showing. Pay particular attention to the coordinate system. To find displacement, we use the equation $\Delta x = x_f - x_0$. This is straightforward since the initial and final positions are given.

Solution

1. Identify the knowns. In the figure we see that $x_f = 6.70\,km$ and $x_0 = 4.70\,km$ for part (a), and $x'_f = 3.75\,km$ and $x'_0 = 5.25\,km$ for part (b).
2. Solve for displacement in part (a).

$$\begin{aligned}\Delta x &= x_f - x_0 \\ &= 6.70\,km - 4.70\,km \\ &= +2.00\,km\end{aligned}$$

3. Solve for displacement in part (b).

$$\begin{aligned}\Delta x' &= x'_f - x'_0 \\ &= 3.75\,km - 5.25\,km \\ &= -1.50\,km\end{aligned}$$

Discussion

The direction of the motion in (a) is to the right and therefore its displacement has a positive sign, whereas motion in (b) is to the left and thus has a negative sign.

Example 1.6.3: Comparing Distance Traveled with Displacement - A Subway Train

What are the distances traveled for the motions shown in parts (a) and (b) of the subway train in Figure $1.6.7$?

Strategy

To answer this question, think about the definitions of distance and distance traveled, and how they are related to displacement. Distance between two positions is defined to be the magnitude of displacement, which was found in Example $1.6.2$. Distance traveled is the total length of the path traveled between the two positions (see Section on Displacement). In the case of the subway train shown in Figure $1.6.7$, the distance traveled is the same as the distance between the initial and final positions of the train.

Solution

1. The displacement for part (a) was +2.00 km. Therefore, the distance between the initial and final positions was 2.00 km, and the distance traveled was 2.00 km.

2. The displacement for part (b) was $-1.5 km$. Therefore, the distance between the initial and final positions was 1.50 km, and the distance traveled was 1.50 km.

Discussion

Distance is a scalar. It has magnitude but no sign to indicate direction.

Example 1.6.4: Calculating Acceleration: A Subway Train Speeding Up

Suppose the train in Figure $1.6.7a$ accelerates from rest to 30.0 km/h in the first 20.0 s of its motion. What is its average acceleration during that time interval?

Strategy

It is worth it at this point to make a simple sketch:

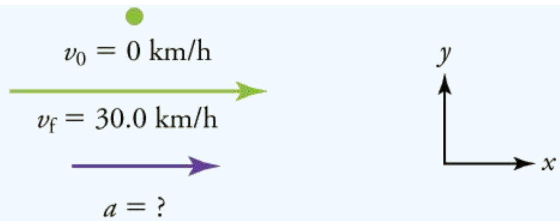


Figure 1.6.8: This problem involves three steps. First we must determine the change in velocity, then we must determine the change in time, and finally we use these values to calculate the acceleration.

Solution

1. Identify the knowns. $v_0 = 0$ (the trains starts at rest), $v_f = 30.0 km/h$, and $\Delta t = 20.0 s$.
2. Calculate Δv. Since the train starts from rest, its change in velocity is $\Delta v = +30.0 km/h$, where the plus sign means velocity to the right.
3. Plug in known values and solve for the unknown, $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+30.0 km/h}{20.0 s}$$

4. Since the units are mixed (we have both hours and seconds for time), we need to convert everything into SI units of meters and seconds. (See Physical Quantities and Units for more guidance.)

$$\bar{a} = \left(\frac{+30 km/h}{20.0 s}\right)\left(\frac{10^3 m}{1 km}\right)\left(\frac{1 h}{3600 s}\right) = 0.417 m/s^2$$

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train starts from rest and ends up with a velocity to the right (also positive). So acceleration is in the same direction as the change in velocity, as is always the case.

Example 1.6.5: Calculate Acceleration

A Subway Train Slowing Down: Now suppose that at the end of its trip, the train in Figure $1.6.7a$ slows to a stop from a speed of 30.0 km/h in 8.00 s. What is its average acceleration while stopping?

Strategy

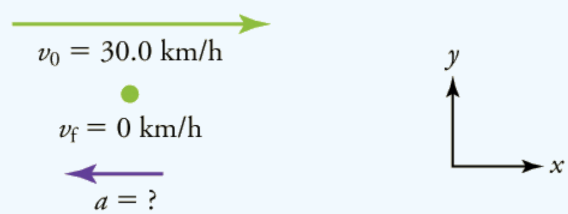


Figure 1.6.9:.

In this case, the train is decelerating and its acceleration is negative because it is toward the left. As in the previous example, we must find the change in velocity and the change in time and then solve for acceleration.

Solution

1. Identify the knowns. $v_0 = 30.0 km/h, v_f = 0 km/h$ (the train is stopped, so its velocity is 0), and $\Delta t = 8.00 s$.
2. Solve for the change in velocity, Δv.

$$\Delta v = v_f - v_0 = 0 - 30.0 km/h = -30.0 km/h$$

3. Plug in the knowns, Δv and Δt, and solve for $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-30.0 km/h}{8.00 s}$$

4. Convert the units to meters and seconds.

$$\bar{a}=\frac{\Delta v}{\Delta t}=\left(\frac{-30.0km/h}{8.00s}\right)\left(\frac{10^3m}{1km}\right)\left(\frac{1h}{3600s}\right)=-1.04m/s^2.$$

Discussion

The minus sign indicates that acceleration is to the left. This sign is reasonable because the train initially has a positive velocity in this problem, and a negative acceleration would oppose the motion. Again, acceleration is in the same direction as the change in velocity, which is negative here. This acceleration can be called a deceleration because it has a direction opposite to the velocity.

The graphs of position, velocity, and acceleration vs. time for the trains in Example 1.6.4 and 1.6.5 are displayed in Figure 1.6.10. (We have taken the velocity to remain constant from 20 to 40 s, after which the train decelerates.)

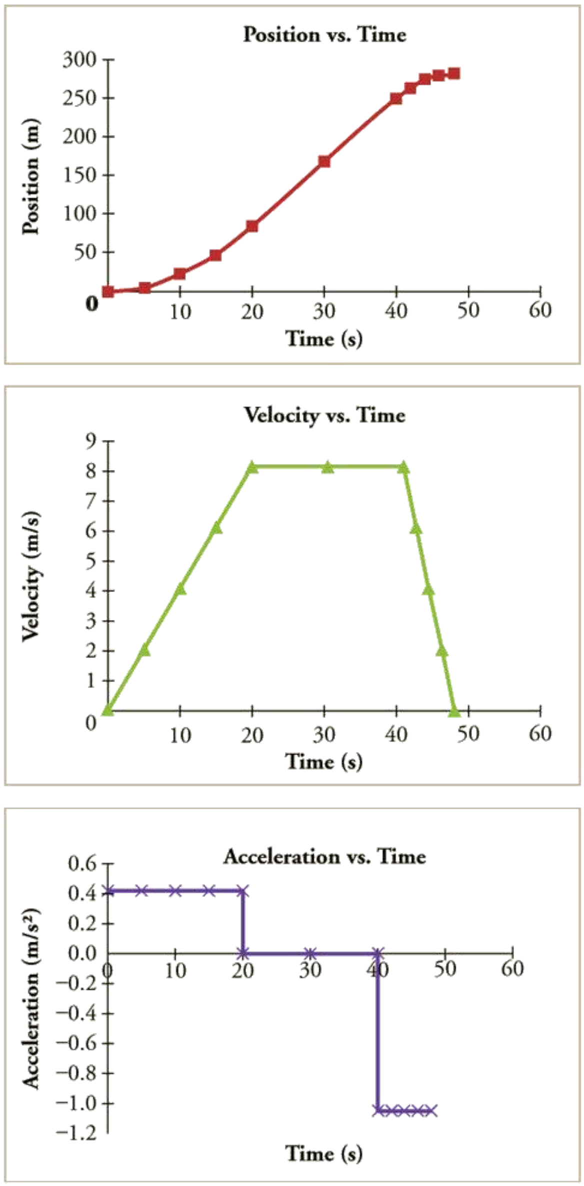


Figure 1.6.10: (a) Position of the train over time. Notice that the train’s position changes slowly at the beginning of the journey, then more and more quickly as it picks up speed. Its position then changes more slowly as it slows down at the end of the journey. In the middle of the journey, while the velocity remains constant, the position changes at a constant rate. (b) Velocity of the train over time. The train’s velocity increases as it accelerates at the beginning of the journey. It remains the same in the middle of the journey (where there is no acceleration). It decreases as the train decelerates at the end of the journey. (c) The acceleration of the train over time. The train has positive acceleration as it speeds up at the beginning of the journey. It has no acceleration as it travels at constant velocity in the middle of the journey. Its acceleration is negative as it slows down at the end of the journey.

Example 1.6.6: Calculating Average Velocity: The Subway Train

What is the average velocity of the train in part b of Example 1.6.2, and shown again below, if it takes 5.00 min to make its trip?

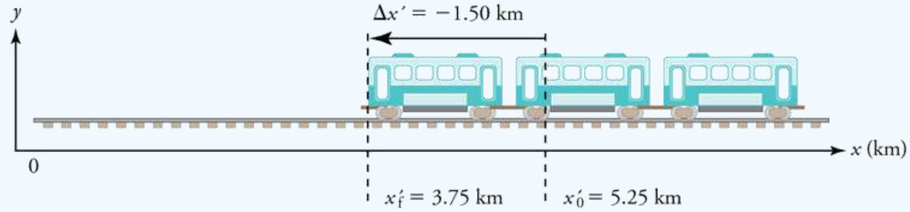


Figure 1.6.11

Strategy

Average velocity is displacement divided by time. It will be negative here, since the train moves to the left and has a negative displacement.

Solution

1. Identify the knowns.

$$x'_f = 3.75 km,\ x'_0 = 5.25 km,\ \Delta t = 5.00 min.$$

2. Determine displacement, $\Delta x'$. We found $\Delta x'$ to be $-1.5 km$ in Example 1.6.7.
3. Solve for average velocity.

$$\bar{v} = \frac{\Delta x'}{\Delta t} = \frac{-1.50 km}{5.00 min}$$

4. Convert units.

$$\bar{v} = \frac{\Delta x'}{\Delta t} = (\frac{-1.50 km}{5.00 min})(\frac{60 min}{1} h) = -18.0 km/h$$

Discussion

The negative velocity indicates motion to the left.

Example 1.6.7: Calculating Deceleration: The Subway Train

Finally, suppose the train in Figure 1.6.7 slows to a stop from a velocity of 20.0 km/h in 10.0 s. What is its average acceleration?

Strategy

Once again, let's draw a sketch:

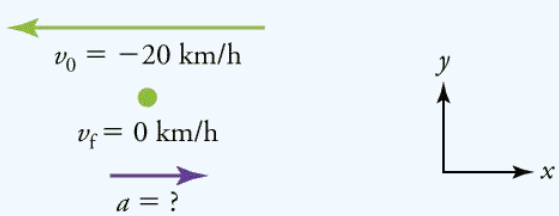


Figure 1.6.12: As before, we must find the change in velocity and the change in time to calculate average acceleration.

Solution

1. Identify the knowns. $v_0 = -20\ km/h$, $v_f = 0\ km/h$, $\Delta t = 10.0\ s$.
2. Calculate Δv. The change in velocity here is actually positive, since

$$\Delta v = v_f - v_0 = 0 - (-20\ km/h) = +20\ km/h.$$

3. Solve for $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+20.0\ km/h}{10.0\ s}$$

4. Convert units.

$$\bar{a} = \left(\frac{+20.0\ km/h}{10.0\ s}\right)\left(\frac{10^3\ m}{1\ km}\right)\left(\frac{1\ h}{3600\ s}\right) = +0.556\ m/^s2$$

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train initially has a negative velocity (to the left) in this problem and a positive acceleration opposes the motion (and so it is to the right). Again, acceleration is in the same direction as the change in velocity, which is positive here. As in Example $1.6.5$, this acceleration can be called a deceleration since it is in the direction opposite to the velocity.

Sign and Direction

Perhaps the most important thing to note about these examples is the signs of the answers. In our chosen coordinate system, plus means the quantity is to the right and minus means it is to the left. This is easy to imagine for displacement and velocity. But it is a little less obvious for acceleration. Most people interpret negative acceleration as the slowing of an object. This was not the case in Example $1.6.5$, where a positive acceleration slowed a negative velocity. The crucial distinction was that the acceleration was in the opposite direction from the velocity. In fact, a negative acceleration will increase a negative velocity. For example, the train moving to the left in Figure Figure $1.6.11$ is sped up by an acceleration to the left. In that case, both v and a are negative. The plus and minus signs give the directions of the accelerations. If acceleration has the same sign as the velocity, the object is speeding up. If acceleration has the opposite sign as the velocity, the object is slowing down.

Exercise $1.6.1$

An airplane lands on a runway traveling east. Describe its acceleration.

Answer

If we take east to be positive, then the airplane has negative acceleration, as it is accelerating toward the west. It is also decelerating: its acceleration is opposite in direction to its velocity.

PHET EXPLORATIONS: MOVING MAN SIMULATION

Learn about position, velocity, and acceleration graphs with the PhET Moving Man simulation. Move the little man back and forth with the mouse and plot his motion. Set the position, velocity, or acceleration and let the simulation move the man for you.

Summary

- Acceleration is the rate at which velocity changes. In symbols, **average acceleration** $\bar{a}$ is $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$.
- The SI unit for acceleration is 2 .
- Acceleration is a vector, and thus has a both a magnitude and direction.
- Acceleration can be caused by either a change in the magnitude or the direction of the velocity.
- Instantaneous acceleration is the acceleration at a specific instant in time.
- Deceleration is an acceleration with a direction opposite to that of the velocity.

Glossary

acceleration

the rate of change in velocity; the change in velocity over time

average acceleration

the change in velocity divided by the time over which it changes

instantaneous acceleration

acceleration at a specific point in time

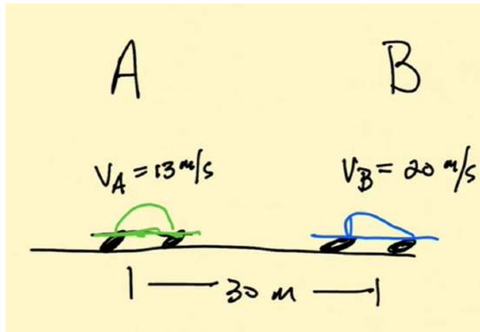
deceleration

acceleration in the direction opposite to velocity; acceleration that results in a decrease in velocity

---

1.7: Overtaking on the Highway (Video Solution)

Cars A and B are travelling in adjacent lanes along a straight road. At time, t = 0 their positions and speeds are as shown in the diagram below. If car A has a constant acceleration of 0.6 m/s^2 and car B has a constant deceleration of 0.46 m/s^2, determine when A will overtake B.



Summary

In this physics lecture, the instructor demonstrates how to solve a problem involving two cars traveling along a straight road with different velocities and accelerations. The problem asks when car A will overtake car B, given their initial positions, velocities, and acceleration/deceleration rates.

00:00:17 Introduction to the Physics Problem

00:01:43 Setting Up the Problem and Analyzing Car A

00:07:45 Analyzing Car B's Motion

00:12:00 Solving for the Overtaking Time

00:15:49 Conclusion and Educational Philosophy

 Transcript

1.8: Motion Equations for Constant Acceleration in One Dimension

Learning Objectives

By the end of this section, you will be able to:

- Calculate displacement of an object that is not accelerating, given initial position and velocity.
- Calculate final velocity of an accelerating object, given initial velocity, acceleration, and time.
- Calculate displacement and final position of an accelerating object, given initial position, initial velocity, time, and acceleration.

We might know that the greater the acceleration of, say, a car moving away from a stop sign, the greater the displacement in a given time. But we have not developed a specific equation that relates acceleration and displacement. In this section, we develop some convenient equations for kinematic relationships, starting from the definitions of displacement, velocity, and acceleration already covered.

Figure 1.8.1: Kinematic equations can help us describe and predict the motion of moving objects such as these kayaks racing in Newbury, England. (credit: Barry Skeates, Flickr)

Notation: *t*, *x*, *v*, *a*

First, let us make some simplifications in notation. Taking the initial time to be zero, as if time is measured with a stopwatch, is a great simplification. Since elapsed time is $\Delta t = t_f - t_0$, taking $t_0 = 0$ means that $\Delta t = t_f$, the final time on the stopwatch. When initial time is taken to be zero, we use the subscript 0 to denote initial values of position and velocity. That is, x_0 is the initial position and v_0 is *the initial velocity*. We put no subscripts on the final values. That is, t is *the final time*, x is *the final position*, and v is *the final velocity*. This gives a simpler expression for elapsed time—now, $\Delta t = t$. It also simplifies the expression for displacement, which is now $\Delta x = x - x_0$. Also, it simplifies the expression for change in velocity, which is now $\Delta v = v - v_0$. To summarize, using the simplified notation, with the initial time taken to be zero,

$$\Delta t = t$$
$$\Delta x = x - x_0$$
$$\Delta v = v - v_0$$

where *the subscript 0 denotes an initial value and the absence of a subscript denotes a final value* in whatever motion is under consideration.

We now make the important assumption that *acceleration is constant*. This assumption allows us to avoid using calculus to find instantaneous acceleration. Since acceleration is constant, the average and instantaneous accelerations are equal. That is,

$$\bar{a} = a = constant, \tag{1.8.1}$$

so we use the symbol a for acceleration at all times. Assuming acceleration to be constant does not seriously limit the situations we can study nor degrade the accuracy of our treatment. For one thing, acceleration is constant in a great number of situations. Furthermore, in many other situations we can accurately describe motion by assuming a constant acceleration equal to the average acceleration for that motion. Finally, in motions where acceleration changes drastically, such as a car accelerating to top speed and then braking to a stop, the motion can be considered in separate parts, each of which has its own constant acceleration.

SOLVING FOR DISPLACEMENT (Δx) AND FINAL POSITION (x) FROM AVERAGE VELOCITY WHEN ACCELERATION (a) IS CONSTANT

To get our first two new equations, we start with the definition of average velocity:

$$\bar{v} = \frac{\Delta x}{\Delta t} \tag{1.8.2}$$

Substituting the simplified notation for Δx and Δt yields

$$\bar{v} = \frac{x - x_0}{t}. \tag{1.8.3}$$

Solving for x yields

$$x = x_0 + \bar{v}t, \tag{1.8.4}$$

where the average velocity is

$$\bar{v} = \frac{v_0 + v}{2} \tag{1.8.5}$$

with constant a.

Equation 1.8.5 reflects the fact that, when acceleration is constant, v is just the simple average of the initial and final velocities. For example, if you steadily increase your velocity (that is, with constant acceleration) from 30 to 60 km/h, then your average velocity during this steady increase is 45 km/h. Using the equation $\bar{v} = \frac{v_0 + v}{2}$ to check this, we see that

$$\bar{v} = \frac{v_0 + v}{2} = \frac{30km/h + 60km/h}{2} = 45km/h,$$

which seems logical.

Example 1.8.1: Calculating Displacement - How Far does the Jogger Run?

A jogger runs down a straight stretch of road with an average velocity of 4.00 m/s for 2.00 min. What is his final position, taking his initial position to be zero?

Strategy

Draw a sketch.

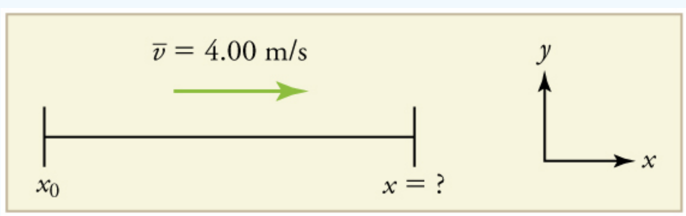


Figure 1.8.2

The final position is given by the equation

$x = x_0 + \bar{v}t$.

To find x, we identify the values of x_0, $\bar{v}$, and t from the statement of the problem and substitute them into the equation.

Solution

1. Identify the knowns. $\bar{v} = 4.00m/s$, $\Delta t = 2.00min$, and $x_0 = 0m$.
2. Enter the known values into the equation.

$$x = x_0 + \bar{v}t = 0 + (4.00m/s)(120s) = 480m$$

Discussion

Velocity and final displacement are both positive, which means they are in the same direction.

The equation $x = x_0 + \bar{v}t$ gives insight into the relationship between displacement, average velocity, and time. It shows, for example, that displacement is a linear function of average velocity. (By linear function, we mean that displacement depends on $\bar{v}$ rather than on $\bar{v}$ raised to some other power, such as $\bar{v}^2$. When graphed, linear functions look like straight lines with a constant slope.) On a car trip, for example, we will get twice as far in a given time if we average 90 km/h than if we average 45 km/h.

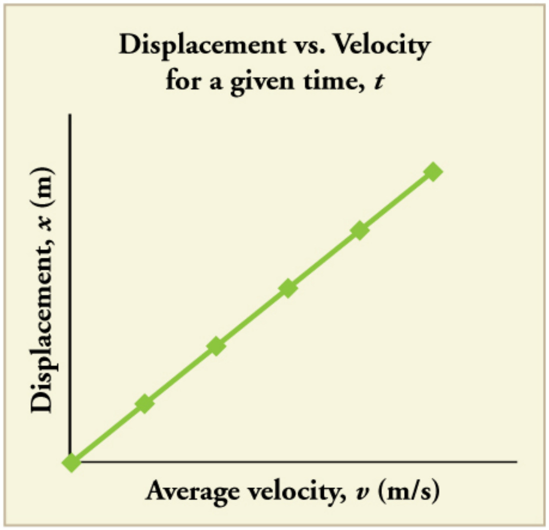


Figure 1.8.3: There is a linear relationship between displacement and average velocity. For a given time t, an object moving twice as fast as another object will move twice as far as the other object.

SOLVING FOR FINAL VELOCITY

We can derive another useful equation by manipulating the definition of acceleration.

$$a = \frac{\Delta v}{\Delta t}$$

Substituting the simplified notation for Δv and Δt gives us

$$a = \frac{v - v_0}{t} \tag{1.8.6}$$

(constant a).

Solving for v yields

$$v = v_0 + at \tag{1.8.7}$$

(constanta).

Example 1.8.2:Calculating Final Velocity: An Airplane Slowing Down after Landing

An airplane lands with an initial velocity of 70.0 m/s and then decelerates at $1.50 m/s^2$ for 40.0 s. What is its final velocity?

Strategy

Draw a sketch. We draw the acceleration vector in the direction opposite the velocity vector because the plane is decelerating.

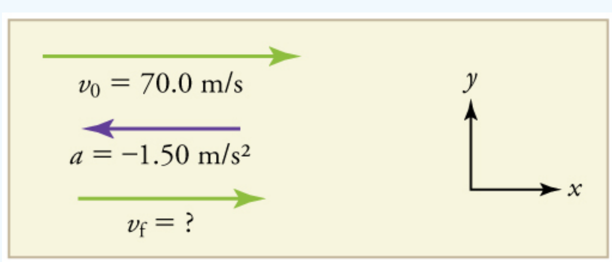


Figure 1.8.4

Solution

1. Identify the knowns. $v_0 = 70.0 m/s, a = -1.50 m/s^2, t = 40.0 s$.

2. Identify the unknown. In this case, it is final velocity, v_f.

3. Determine which equation to use. We can calculate the final velocity using the equation $v = v_0 + at$.

4. Plug in the known values and solve.

$$v = v_0 + at = 70.0 m/s + (-1.50 m/s^2)(40.0 s) = 10.0 m/s$$

Discussion

The final velocity is much less than the initial velocity, as desired when slowing down, but still positive. With jet engines, reverse thrust could be maintained long enough to stop the plane and start moving it backward. That would be indicated by a negative final velocity, which is not the case here.

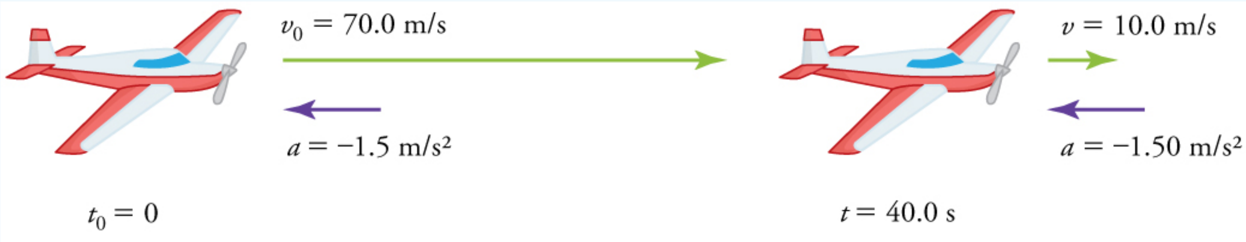


Figure 1.8.5: The airplane lands with an initial velocity of 70.0 m/s and slows to a final velocity of 10.0 m/s before heading for the terminal. Note that the acceleration is negative because its direction is opposite to its velocity, which is positive.

In addition to being useful in problem solving, the equation $v = v_0 + at$ gives us insight into the relationships among velocity, acceleration, and time. From it we can see, for example, that

- final velocity depends on how large the acceleration is and how long it lasts
- if the acceleration is zero, then the final velocity equals the initial velocity ($v = v_0$), as expected (i.e., velocity is constant)
- if a is negative, then the final velocity is less than the initial velocity

(All of these observations fit our intuition, and it is always useful to examine basic equations in light of our intuition and experiences to check that they do indeed describe nature accurately.)

MAKING CONNECTIONS: REAL-WORLD CONNECTION

Figure 1.8.6: The Space Shuttle Endeavor blasts off from the Kennedy Space Center in February 2010. (credit: Matthew Simantov, Flickr)

An intercontinental ballistic missile (ICBM) has a larger average acceleration than the Space Shuttle and achieves a greater velocity in the first minute or two of flight (actual ICBM burn times are classified—short-burn-time missiles are more difficult for an enemy to destroy). But the Space Shuttle obtains a greater final velocity, so that it can orbit the earth rather than come directly back down as an ICBM does. The Space Shuttle does this by accelerating for a longer time.

SOLVING FOR FINAL POSITION WHEN VELOCITY IS NOT CONSTANT (a≠0)

We can combine the equations above to find a third equation that allows us to calculate the final position of an object experiencing constant acceleration. We start with

$$v = v_0 + at.$$

Adding v_0 to each side of this equation and dividing by 2 gives

$$\frac{v_0 + v}{2} = v_0 + \frac{1}{2}at\ .$$

Since \\frac{(v_0+v}{2}=\bar{v}\) for constant acceleration, then

$$\bar{v} = v_0 + \frac{1}{2}at\ .$$

Now we substitute this expression for $\bar{v}$ into the equation for displacement, $x = x_0 + \bar{v}t$, yielding

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \ \text{(constant a)}.$$

Example 1.8.3: Calculating Displacement of an Accelerating Object - Dragsters

Dragsters can achieve average accelerations of $26.0 m/s^2$. Suppose such a dragster accelerates from rest at this rate for 5.56 s. How far does it travel in this time?

Figure 1.8.7: U.S. Army Top Fuel pilot Tony “The Sarge” Schumacher begins a race with a controlled burnout. (credit: Lt. Col. William Thurmond. Photo Courtesy of U.S. Army.)

Strategy

Draw a sketch.

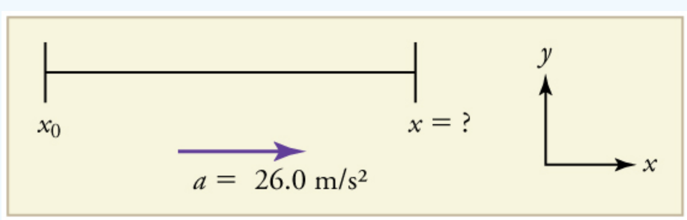


Figure 1.8.8

We are asked to find displacement, which is x if we take x_0 to be zero. (Think about it like the starting line of a race. It can be anywhere, but we call it 0 and measure all other positions relative to it.) We can use the equation $x = x_0 + v_0 t + \frac{1}{2}at^2$ once we identify v_0, a, and t from the statement of the problem.

Solution

1. Identify the knowns. Starting from rest means that $v_0 = 0$, a is given as $26.0 m/s^2$ and t is given as 5.56 s.

2. Plug the known values into the equation to solve for the unknown x:

$x=x_0+v_0t+\frac{1}{2}at^2$.

Since the initial position and velocity are both zero, this simplifies to

$x=\frac{1}{2}at^2$.

Substituting the identified values of a and t gives

$x=\frac{1}{2}(26.0m/s^2)(5.56s)^2,$

yielding

$x=402m.$

Discussion

If we convert 402 m to miles, we find that the distance covered is very close to one quarter of a mile, the standard distance for drag racing. So the answer is reasonable. This is an impressive displacement in only 5.56 s, but top-notch dragsters can do a quarter mile in even less time than this.

What else can we learn by examining the equation $x=x_0+v_0t+\frac{1}{2}at^2$? We see that:

- displacement depends on the square of the elapsed time when acceleration is not zero. In Example, the dragster covers only one fourth of the total distance in the first half of the elapsed time
- if acceleration is zero, then the initial velocity equals average velocity ($v_0=\bar{v}$) and $x=x_0+v_0t+\frac{1}{2}at^2$ becomes $x=x_0+v_0t$

SOLVING FOR FINAL VELOCITY WHEN VELOCITY IS NOT CONSTANT (a≠0)

A fourth useful equation can be obtained from another algebraic manipulation of previous equations.

If we solve $v=v_0+at$ for t, we get

$$t=\frac{v-v_0}{a}.$$

Substituting this and $\bar{v}=\frac{v_0+v}{2}$ into $x=x_0+\bar{v}t$, we get

$$v^2=v_0^2+2a(x-x_0) \text{ (constant a).}$$

Example 1.8.4: Calculating Final Velocity: Dragsters

Calculate the final velocity of the dragster in Example 1.8.3 without using information about time.

Strategy

Draw a sketch.

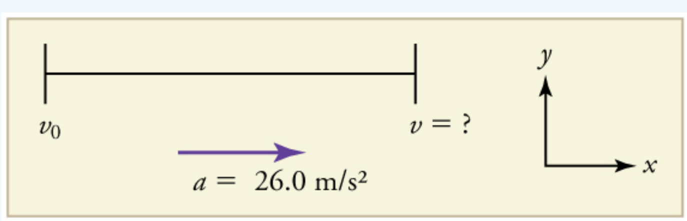


Figure 1.8.9

The equation $v^2=v_0^2+2a(x-x_0)$ is ideally suited to this task because it relates velocities, acceleration, and displacement, and no time information is required.

Solution

1. Identify the known values. We know that $v_0 = 0$, since the dragster starts from rest. Then we note that $x - x_0 = 402m$ (this was the answer in Example). Finally, the average acceleration was given to be $a = 26.0m/s^2$.

2. Plug the knowns into the equation $v^2 = v_0^2 + 2a(x - x_0)$ and solve for v.

$$v^2 = 0 + 2(26.0m/s^2)(402m).$$

Thus

$$v^2 = 2.09 \times 10^4 m^2/s^2.$$

To get v, we take the square root:

$$v = \sqrt{2.09 \times 10^4 m^2/s^2} = 145m/s.$$

Discussion

145 m/s is about 522 km/h or about 324 mi/h, but even this breakneck speed is short of the record for the quarter mile. Also, note that a square root has two values; we took the positive value to indicate a velocity in the same direction as the acceleration.

An examination of the equation $v^2 = v_0^2 + 2a(x - x_0)$ can produce further insights into the general relationships among physical quantities:

- The final velocity depends on how large the acceleration is and the distance over which it acts
- For a fixed deceleration, a car that is going twice as fast doesn't simply stop in twice the distance—it takes much further to stop. (This is why we have reduced speed zones near schools.)

Putting Equations Together

In the following examples, we further explore one-dimensional motion, but in situations requiring slightly more algebraic manipulation. The examples also give insight into problem-solving techniques. The box below provides easy reference to the equations needed.

SUMMARY OF KINEMATIC EQUATIONS (CONSTANT a)

$$x = x_0 + \bar{v}t$$

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Example 1.8.5: Calculating Displacement: How Far Does a Car Go When Coming to a Halt?

On dry concrete, a car can decelerate at a rate of $7.00m/s^2$, whereas on wet concrete it can decelerate at only $5.00m/s^2$. Find the distances necessary to stop a car moving at 30.0 m/s (about 110 km/h)

a. on dry concrete and
b. on wet concrete.
c. Repeat both calculations, finding the displacement from the point where the driver sees a traffic light turn red, taking into account his reaction time of 0.500 s to get his foot on the brake.

Strategy

Draw a sketch.

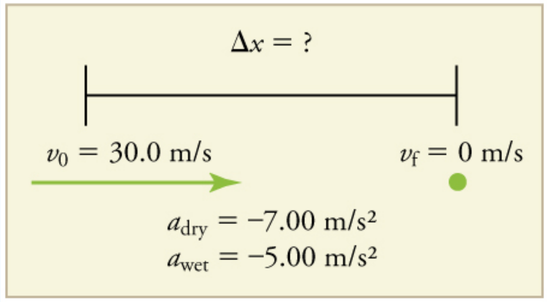


Figure 1.8.10

In order to determine which equations are best to use, we need to list all of the known values and identify exactly what we need to solve for. We shall do this explicitly in the next several examples, using tables to set them off.

Solution for (a)

1. Identify the knowns and what we want to solve for. We know that $v_0 = 30.0 m/s; v = 0; a = -7.00 m/s^2$ (a is negative because it is in a direction opposite to velocity). We take x_0 to be 0. We are looking for displacement Δx, or $x - x_0$.

2. Identify the equation that will help up solve the problem. The best equation to use is

$$v^2 = v_0^2 + 2a(x - x_0) \ .$$

This equation is best because it includes only one unknown, x. We know the values of all the other variables in this equation. (There are other equations that would allow us to solve for x, but they require us to know the stopping time, t, which we do not know. We could use them but it would entail additional calculations.)

3. Rearrange the equation to solve for x.

$$x - x_0 = \frac{v^2 - v_0^2}{2a}$$

4. Enter known values.

$$x - 0 = \frac{0^2 - (30.0 m/s)^2}{2(-7.00 m/s^2)}$$

Thus,

$$x = 64.3 m \text{ on dry concrete.}$$

Solution for (b)

This part can be solved in exactly the same manner as Part A. The only difference is that the deceleration is $-5.00 m/s^2$. The result is

$$x_{wet} = 90.0 m \text{ on wet concrete.}$$

Solution for (c)

Once the driver reacts, the stopping distance is the same as it is in Parts A and B for dry and wet concrete. So to answer this question, we need to calculate how far the car travels during the reaction time, and then add that to the stopping time. It is reasonable to assume that the velocity remains constant during the driver's reaction time.

1. Identify the knowns and what we want to solve for. We know that $\bar{v} = 30.0 m/s; t_{reaction} = 0.500 s; a_{reaction} = 0$. We take $x_{0-reaction}$ to be 0. We are looking for $x_{reaction}$.

2. Identify the best equation to use.

$x = x_0 + \bar{v}t$ works well because the only unknown value is x, which is what we want to solve for.

3. Plug in the knowns to solve the equation.

$$x = 0 + (30.0 m/s)(0.500 s) = 15.0 m.$$

This means the car travels 15.0 m while the driver reacts, making the total displacements in the two cases of dry and wet concrete 15.0 m greater than if he reacted instantly.

4. Add the displacement during the reaction time to the displacement when braking.

$$x_{braking} + x_{reaction} = x_{total}$$

a. 64.3 m + 15.0 m = 79.3 m when dry
b. 90.0 m + 15.0 m = 105 m when wet

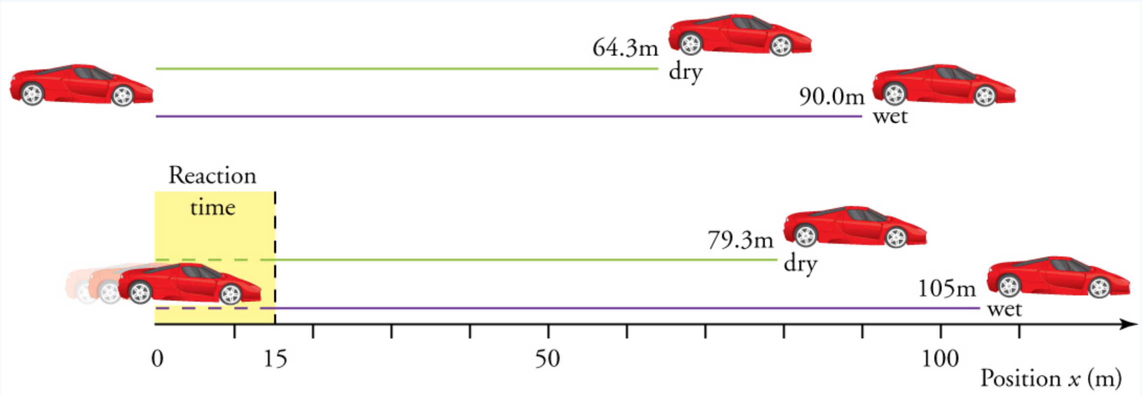


Figure 1.8.11: The distance necessary to stop a car varies greatly, depending on road conditions and driver reaction time. Shown here are the braking distances for dry and wet pavement, as calculated in this example, for a car initially traveling at 30.0 m/s. Also shown are the total distances traveled from the point where the driver first sees a light turn red, assuming a 0.500 s reaction time.

Discussion

The displacements found in this example seem reasonable for stopping a fast-moving car. It should take longer to stop a car on wet rather than dry pavement. It is interesting that reaction time adds significantly to the displacements. But more important is the general approach to solving problems. We identify the knowns and the quantities to be determined and then find an appropriate equation. There is often more than one way to solve a problem. The various parts of this example can in fact be solved by other methods, but the solutions presented above are the shortest.

Example 1.8.5: Calculating Time - A Car Merges into Traffic

Suppose a car merges into freeway traffic on a 200-m-long ramp. If its initial velocity is 10.0 m/s and it accelerates at \(\displaystyle .00 m/s^2\), how long does it take to travel the 200 m up the ramp? (Such information might be useful to a traffic engineer.)

Strategy

Draw a sketch.

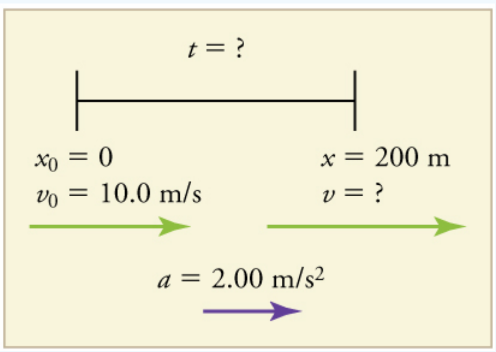


Figure 1.8.12: We are asked to solve for the time t. As before, we identify the known quantities in order to choose a convenient physical relationship (that is, an equation with one unknown, t).

Solution

1. Identify the knowns and what we want to solve for. We know that $v_0 = 10 m/s; a = 2.00 m/s^2$; and $x = 200 m$.

2. We need to solve for t. Choose the best equation. $x = x_0 + v_0 t + \frac{1}{2}at^2$ works best because the only unknown in the equation is the variable t for which we need to solve.

3. We will need to rearrange the equation to solve for t. In this case, it will be easier to plug in the knowns first.

$$200m = 0m + (10.0m/s)t + \frac{1}{2}(2.00m/s^2)t^2$$

4. Simplify the equation. The units of meters (m) cancel because they are in each term. We can get the units of seconds (s) to cancel by taking $t = ts$, where t is the magnitude of time and s is the unit. Doing so leaves

$$200 = 10t + t^2.$$

5. Use the quadratic formula to solve for t.

(a) Rearrange the equation to get 0 on one side of the equation.

$$t^2 + 10t - 200 = 0$$

This is a quadratic equation of the form

$$at^2 + bt + c = 0,$$

where the constants are $a = 1.00, b = 10.0,$and $c = -200$.

(b) Its solutions are given by the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

This yields two solutions for t, which are

$$t = 10.0 \text{ and } -20.0.$$

In this case, then, the time is $t = t$ in seconds, or

$$t = 10.0s \text{ and } -20.0s.$$

A negative value for time is unreasonable, since it would mean that the event happened 20 s before the motion began. We can discard that solution. Thus,

$$t = 10.0s.$$

Discussion

Whenever an equation contains an unknown squared, there will be two solutions. In some problems both solutions are meaningful, but in others, such as the above, only one solution is reasonable. The 10.0 s answer seems reasonable for a typical freeway on-ramp.

With the basics of kinematics established, we can go on to many other interesting examples and applications. In the process of developing kinematics, we have also glimpsed a general approach to problem solving that produces both correct answers and insights into physical relationships. Problem-Solving Basics discusses problem-solving basics and outlines an approach that will help you succeed in this invaluable task.

MAKING CONNECTIONS: TAKE-HOME EXPERIMENT—BREAKING NEWS

We have been using SI units of meters per second squared to describe some examples of acceleration or deceleration of cars, runners, and trains. To achieve a better feel for these numbers, one can measure the braking deceleration of a car doing a slow (and safe) stop. Recall that, for average acceleration, $\bar{a} = \Delta v/\Delta t$. While traveling in a car, slowly apply the brakes as you come up to a stop sign. Have a passenger note the initial speed in miles per hour and the time taken (in seconds) to stop. From this, calculate the deceleration in miles per hour per second. Convert this to meters per second squared and compare with other decelerations mentioned in this chapter. Calculate the distance traveled in braking.

Exercise 1.8.1

A manned rocket accelerates at a rate of $20 m/s^2$ during launch. How long does it take the rocket to reach a velocity of 400 m/s?

Answer

To answer this, choose an equation that allows you to solve for time t, given only a, v_0, and v.

$$v = v_0 + at$$

Rearrange to solve for t.

$$t = \frac{v - v_0}{a} = \frac{400 m/s - 0 m/s}{20 m/s^2} = 20 s$$

Summary

- To simplify calculations we take acceleration to be constant, so that $\bar{a} = a$ at all times.
- We also take initial time to be zero.
- Initial position and velocity are given a subscript 0; final values have no subscript. Thus,

$$\Delta t = t$$

$$\Delta x = x - x_0$$

$$\Delta v = v - v_0$$

- The following kinematic equations for motion with constant a are useful:

$$x = x_0 + \bar{v} t$$

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

- In vertical motion, y is substituted for x.

1.9: Romeo and Julliet Race To Each Other, When Shall They Meet (Video Solution))

Romeo is at x =0 at t=0 when he sees Juliet at x=6m. He begins to run towards her at v=5m/s. She in turn begins to accelerate towards him at a=-2.0m/s/s. When and where will they cross. Suppose she moves away from him with positive acceleration. Find a_{max} the maximum acceleration for which he will ever catch up with her. For this case find the time of their contact

Summary

This physics lecture focuses on solving a kinematics problem involving Romeo and Juliet when Romeo begins running at a constant velocity while Juliet accelerates toward him and later away from him. The lecture demonstrates how to determine when and where the two will meet, and calculates the maximum acceleration at which Juliet can move away while still being caught by Romeo.

00:00:15 Introduction to the Romeo and Juliet Physics Problem

00:01:23 Setting Up the Problem with Graphs

00:02:46 Solving for the Meeting Time and Position

00:07:25 Finding the Maximum Acceleration

00:11:36 Conclusion and Reflection

 Transcript

1.10: Problem-Solving Basics for One-Dimensional Kinematics

Learning Objectives

By the end of this section, you will be able to:

- Apply problem-solving steps and strategies to solve problems of one-dimensional kinematics.
- Apply strategies to determine whether or not the result of a problem is reasonable, and if not, determine the cause.

Problem-Solving Basics for One-Dimensional Kinematics

Problem-solving skills are obviously essential to success in a quantitative course in physics. More importantly, the ability to apply broad physical principles, usually represented by equations, to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and problem-solving abilities can be applied to new situations, whereas a list of facts cannot be made long enough to contain every possible circumstance. Such analytical skills are useful both for solving problems in this text and for applying physics in everyday and professional life.

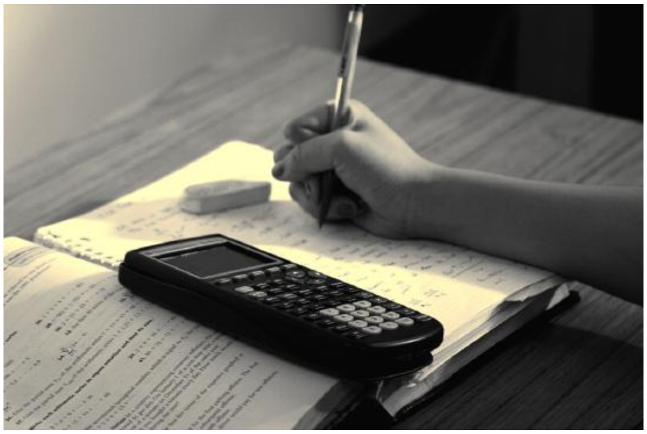


Figure 1.10.1: Problem-solving skills are essential to your success in Physics. (credit: scui3asteveo, Flickr)

Problem-Solving Steps

While there is no simple step-by-step method that works for every problem, the following general procedures facilitate problem solving and make it more meaningful. A certain amount of creativity and insight is required as well.

Step 1

Examine the situation to determine which physical principles are involved. It often helps to *draw a simple sketch* at the outset. You will also need to decide which direction is positive and note that on your sketch. Once you have identified the physical principles, it is much easier to find and apply the equations representing those principles. Although finding the correct equation is essential, keep in mind that equations represent physical principles, laws of nature, and relationships among physical quantities. Without a conceptual understanding of a problem, a numerical solution is meaningless.

Step 2

Make a list of what is given or can be inferred from the problem as stated (identify the knowns). Many problems are stated very succinctly and require some inspection to determine what is known. A sketch can also be very useful at this point. Formally identifying the knowns is of particular importance in applying physics to real-world situations. Remember, “stopped” means velocity is zero, and we often can take initial time and position as zero.

Step 3

Identify exactly what needs to be determined in the problem (identify the unknowns). In complex problems, especially, it is not always obvious what needs to be found or in what sequence. Making a list can help.

Step 4

Find an equation or set of equations that can help you solve the problem. Your list of knowns and unknowns can help here. It is easiest if you can find equations that contain only one unknown—that is, all of the other variables are known, so you can easily solve for the unknown. If the equation contains more than one unknown, then an additional equation is needed to solve the problem. In some problems, several unknowns must be determined to get at the one needed most. In such problems it is especially important to keep physical principles in mind to avoid going astray in a sea of equations. You may have to use two (or more) different equations to get the final answer.

Step 5

Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units. This step produces the numerical answer; it also provides a check on units that can help you find errors. If the units of the answer are incorrect, then an error has been made. However, be warned that correct units do not guarantee that the numerical part of the answer is also correct.

Step 6

Check the answer to see if it is reasonable: Does it make sense? This final step is extremely important—the goal of physics is to accurately describe nature. To see if the answer is reasonable, check both its magnitude and its sign, in addition to its units. Your judgment will improve as you solve more and more physics problems, and it will become possible for you to make finer and finer judgments regarding whether nature is adequately described by the answer to a problem. This step brings the problem back to its conceptual meaning. If you can judge whether the answer is reasonable, you have a deeper understanding of physics than just being able to mechanically solve a problem.

When solving problems, we often perform these steps in different order, and we also tend to do several steps simultaneously. There is no rigid procedure that will work every time. Creativity and insight grow with experience, and the basics of problem solving become almost automatic. One way to get practice is to work out the text's examples for yourself as you read. Another is to work as many end-of-section problems as possible, starting with the easiest to build confidence and progressing to the more difficult. Once you become involved in physics, you will see it all around you, and you can begin to apply it to situations you encounter outside the classroom, just as is done in many of the applications in this text.

Unreasonable Results

Physics must describe nature accurately. Some problems have results that are unreasonable because one premise is unreasonable or because certain premises are inconsistent with one another. The physical principle applied correctly then produces an unreasonable result. For example, if a person starting a foot race accelerates at $0.40 m/s^2$ for 100 s, his final speed will be $40 m/s$ (about 150 km/h)—clearly unreasonable because the time of 100 s is an unreasonable premise. The physics is correct in a sense, but there is more to describing nature than just manipulating equations correctly. Checking the result of a problem to see if it is reasonable does more than help uncover errors in problem solving—it also builds intuition in judging whether nature is being accurately described.

Use the following strategies to determine whether an answer is reasonable and, if it is not, to determine what is the cause.

Step 1

Solve the problem using strategies as outlined and in the format followed in the worked examples in the text. In the example given in the preceding paragraph, you would identify the givens as the acceleration and time and use the equation below to find the unknown final velocity. That is,

$$v = v0 + at = 0 + (0.40 m/s2)(100 s) = 40 m/s. \tag{1.10.1}$$

Step 2

Check to see if the answer is reasonable. Is it too large or too small, or does it have the wrong sign, improper units, …? In this case, you may need to convert meters per second into a more familiar unit, such as miles per hour.

$$(40 ms)(3.28 ftm)(1 mi5280 ft)(60 smin)(60 min1 h) = 89 mph \tag{1.10.2}$$

This velocity is about four times greater than a person can run—so it is too large.

Step 3

If the answer is unreasonable, look for what specifically could cause the identified difficulty. In the example of the runner, there are only two assumptions that are suspect. The acceleration could be too great or the time too long. First look at the acceleration and think about what the number means. If someone accelerates at 0.40 m/s2, their velocity is increasing by 0.4 m/s each second. Does this seem reasonable? If so, the time must be too long. It is not possible for someone to accelerate at a constant rate of 0.40 m/s2 for 100 s (almost two minutes).

Summary

- The six basic problem solving steps for physics are:

  Step 1. Examine the situation to determine which physical principles are involved.

  Step 2. Make a list of what is given or can be inferred from the problem as stated (identify the knowns).

  Step 3. Identify exactly what needs to be determined in the problem (identify the unknowns).

  Step 4. Find an equation or set of equations that can help you solve the problem.

  Step 5. Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units.

  Step 6. Check the answer to see if it is reasonable: Does it make sense?

1.11: Falling Objects

Learning Objectives

By the end of this section, you will be able to:

- Describe the effects of gravity on objects in motion.
- Describe the motion of objects that are in free fall.
- Calculate the position and velocity of objects in free fall.

Falling Objects

Falling objects form an interesting class of motion problems. For example, we can estimate the depth of a vertical mine shaft by dropping a rock into it and listening for the rock to hit the bottom. By applying the kinematics developed so far to falling objects, we can examine some interesting situations and learn much about gravity in the process.

Gravity

The most remarkable and unexpected fact about falling objects is that, if air resistance and friction are negligible, then in a given location all objects fall toward the center of Earth with the *same constant acceleration*, *independent of their mass*. This experimentally determined fact is unexpected, because we are so accustomed to the effects of air resistance and friction that we expect light objects to fall slower than heavy ones.

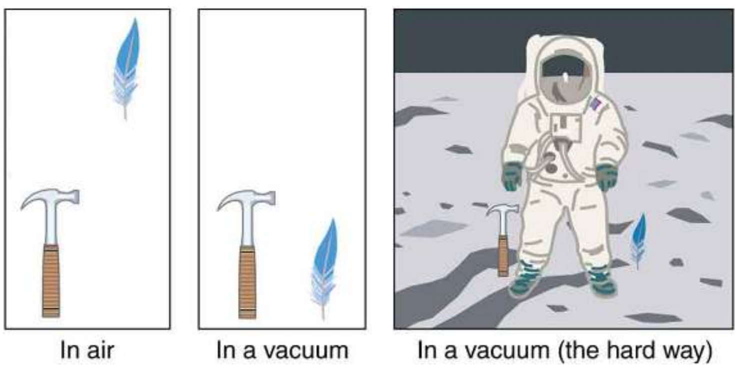


Figure 1.11.1: A hammer and a feather will fall with the same constant acceleration if air resistance is considered negligible. This is a general characteristic of gravity not unique to Earth, as astronaut David R. Scott demonstrated on the Moon in 1971, where the acceleration due to gravity is only .67 m/s2.

In the real world, air resistance can cause a lighter object to fall slower than a heavier object of the same size. A tennis ball will reach the ground after a hard baseball dropped at the same time. (It might be difficult to observe the difference if the height is not large.) Air resistance opposes the motion of an object through the air, while friction between objects—such as between clothes and a laundry chute or between a stone and a pool into which it is dropped—also opposes motion between them. For the ideal situations of these first few chapters, an object *falling without air resistance or friction* is defined to be in free-fall.

The force of gravity causes objects to fall toward the center of Earth. The acceleration of free-falling objects is therefore called the acceleration due to gravity. The acceleration due to gravity is *constant*, which means we can apply the kinematics equations to any falling object where air resistance and friction are negligible. This opens a broad class of interesting situations to us. The acceleration due to gravity is so important that its magnitude is given its own symbol, . It is constant at any given location on Earth and has the average value

$$g = 9.80 m/s^2. \tag{1.11.1}$$

Although varies from .78 m/s2 to 9.83 m/s2, depending on latitude, altitude, underlying geological formations, and local topography, the average value of 9.80 m/s2 will be used in this text unless otherwise specified. The direction of the acceleration due to gravity is downward (towards the center of Earth). In fact, its direction defines what we call vertical. Note that whether the acceleration a in the kinematic equations has the value +g or −g depends on how we define our coordinate system. If we define the upward direction as negative, then a=−g=−9.80 m/s2, and if we define the downward direction as positive, then =g=9.80 m/s2.

One-Dimensional Motion Involving Gravity

The best way to see the basic features of motion involving gravity is to start with the simplest situations and then progress toward more complex ones. So we start by considering straight up and down motion with no air resistance or friction. These assumptions mean that the velocity (if there is any) is vertical. If the object is dropped, we know the initial velocity is zero. Once the object has left contact with whatever held or threw it, the object is in free-fall. Under these circumstances, the motion is one-dimensional and has constant acceleration of magnitude g. We will also represent vertical displacement with the symbol y and use x for horizontal displacement.

KINEMATIC EQUATIONS FOR OBJECTS IN FREE-FALL WHERE ACCELERATION = -G

$$v = v_0 - gt$$

$$y = y_0 + v_0 t - \frac{1}{2} g t^2$$

$$v^2 = v_0^2 - 2g(y - y_0)$$

Example 1.11.1: Calculating Position and Velocity of a Falling Object: A Rock Thrown Upward

A person standing on the edge of a high cliff throws a rock straight up with an initial velocity of 13.0 m/s. The rock misses the edge of the cliff as it falls back to earth. Calculate the position and velocity of the rock 1.00 s, 2.00 s, and 3.00 s after it is thrown, neglecting the effects of air resistance.

Strategy

Draw a sketch.

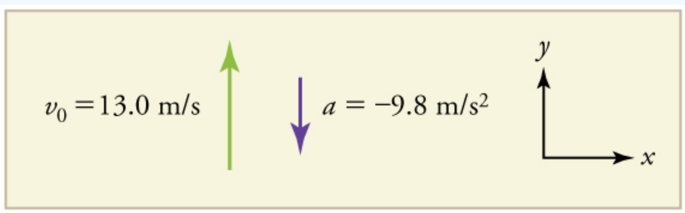


Figure 1.11.2

We are asked to determine the position y at various times. It is reasonable to take the initial position y_0 to be zero. This problem involves one-dimensional motion in the vertical direction. We use plus and minus signs to indicate direction, with up being positive and down negative. Since up is positive, and the rock is thrown upward, the initial velocity must be positive too. The acceleration due to gravity is downward, so a is negative. It is crucial that the initial velocity and the acceleration due to gravity have opposite signs. Opposite signs indicate that the acceleration due to gravity opposes the initial motion and will slow and eventually reverse it.

Since we are asked for values of position and velocity at three times, we will refer to these as y_1 and v_1; y_2 and v_2; and y_3 and v_3.

Solution for Position y_1

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0 m/s$; $a = -g = -9.80 m/s^2$; and $t = 1.00 s$.
2. Identify the best equation to use. We will use $y = y_0 + v_0 t + \frac{1}{2} a t^2$ because it includes only one unknown, y (or y_1, here), which is the value we want to find.
3. Plug in the known values and solve for y_1.

$$y_1 = 0 + (13.0 m/s)(1.00 s) + \frac{1}{2}(-9.80 m/s^2)(1.00 s)^2 = 8.10 m$$

Discussion

The rock is 8.10 m above its starting point at $t = 1.00 s$, since $y_1 > y_0$. It could be moving up or down; the only way to tell is to calculate v_1 and find out if it is positive or negative.

Solution for Velocity v_1

1. Identify the knowns. We know that $y_0 = 0; v_0 = 13.0 m/s; a = -g = -9.80 m/s^2$; and $t = 1.00 s$. We also know from the solution above that $y_1 = 8.10 m$.

2. Identify the best equation to use. The most straightforward is $v = v_0 - gt$ (from $v = v_0 + at$, where a=gravitational acceleration=−g).

3. Plug in the knowns and solve.

$$v_1 = v_0 - gt = 13.0 m/s - (9.80 m/s^2)(1.00 s) = 3.20 m/s$$

Discussion

The positive value for v_1 means that the rock is still heading upward at $t = 1.00 s$. However, it has slowed from its original 13.0 m/s, as expected.

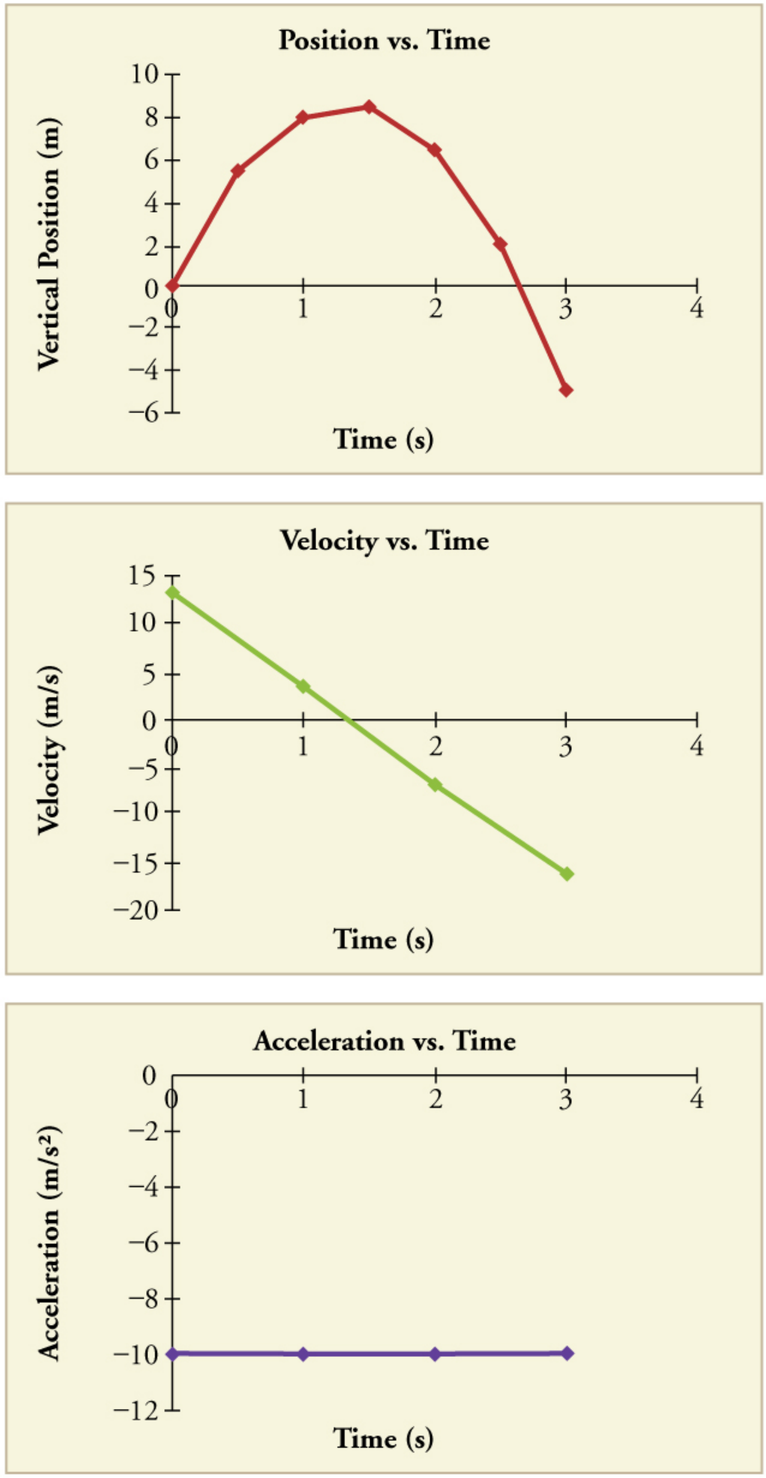
Solution for Remaining Times

The procedures for calculating the position and velocity at $t = 2.00 s$ and $3.00 s$ are the same as those above. The results are summarized in Table and illustrated in Figure.

Results

Time, *t*	Position, *y*	Velocity, *v*	Acceleration, *a*
.00 s	.10 m	.20 m/s\)	9.80 m/s^2\)
.00 s	.40 m	6.60 m/s\)	9.80 m/s^2\)
.00 s	5.10 m	16.4 m/s\)	9.80 m/s^2\)

Graphing the data helps us understand it more clearly.



PageIndex3: *Vertical position, vertical velocity, and vertical acceleration vs. time for a rock thrown vertically up at the edge of a cliff. Notice that velocity changes linearly with time and that acceleration is constant. Misconception Alert! Notice that the position vs. time graph shows vertical position only. It is easy to get the impression that the graph shows some horizontal motion—the shape of the graph looks like the path of a projectile. But this is not the case; the horizontal axis is time, not space. The actual path of the rock in space is straight up, and straight down.*

Discussion

The interpretation of these results is important. At 1.00 s the rock is above its starting point and heading upward, since y_1 and v_1 are both positive. At 2.00 s, the rock is still above its starting point, but the negative velocity means it is moving downward. At 3.00 s, both y_3 and v_3 are negative, meaning the rock is below its starting point and continuing to move downward. Notice that when the rock is at its highest point (at 1.5 s), its velocity is zero, but its acceleration is still $-9.80 m/s^2$. Its acceleration is $-9.80 m/s^2$ for the whole trip—while it is moving up and while it is moving down. Note that the values for y are the positions (or displacements) of the rock, not the total distances traveled. Finally, note that free-fall applies to upward motion as well as downward. Both have the same acceleration—the acceleration due to gravity, which remains constant the entire time. Astronauts training in the famous Vomit Comet, for example, experience free-fall while arcing up as well as down, as we will discuss in more detail later.The interpretation of these results is important. At 1.00 s the rock is above its starting point and heading upward, since y1 and v1 are both positive. At 2.00 s, the rock is still above its starting point, but the negative velocity means it is moving downward. At 3.00 s, both y3 and v3 are negative, meaning the rock is below its starting point and continuing to move downward. Notice that when the rock is at its highest point (at 1.5 s), its velocity is zero, but its acceleration is still −9.80 m/s2. Its acceleration is −9.80 m/s2 for the whole trip—while it is moving up and while it is moving down. Note that the values for y are the positions (or displacements) of the rock, not the total distances traveled. Finally, note that free-fall applies to upward motion as well as downward. Both have the same acceleration—the acceleration due to gravity, which remains constant the entire time. Astronauts training in the famous Vomit Comet, for example, experience free-fall while arcing up as well as down, as we will discuss in more detail later.

MAKING CONNECTIONS: TAKE-HOME EXPERIMENT—REACTION TIME

A simple experiment can be done to determine your reaction time. Have a friend hold a ruler between your thumb and index finger, separated by about 1 cm. Note the mark on the ruler that is right between your fingers. Have your friend drop the ruler unexpectedly, and try to catch it between your two fingers. Note the new reading on the ruler. Assuming acceleration is that due to gravity, calculate your reaction time. How far would you travel in a car (moving at 30 m/s) if the time it took your foot to go from the gas pedal to the brake was twice this reaction time?

Example 1.11.2:Calculating Velocity of a Falling Object: A Rock Thrown Down

What happens if the person on the cliff throws the rock straight down, instead of straight up? To explore this question, calculate the velocity of the rock when it is 5.10 m below the starting point, and has been thrown downward with an initial speed of 13.0 m/s.

Strategy

Draw a sketch.

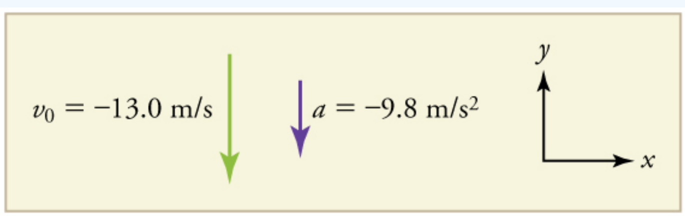


Figure 1.11.4

Since up is positive, the final position of the rock will be negative because it finishes below the starting point at $y_0 = 0$. Similarly, the initial velocity is downward and therefore negative, as is the acceleration due to gravity. We expect the final velocity to be negative since the rock will continue to move downward.

Solution

1. Identify the knowns. $y_0 = 0; y_1 = -5.10m; v_0 = -13.0m/s; a = -g = -9.80m/s^2$.

2. Choose the kinematic equation that makes it easiest to solve the problem. The equation $v^2 = v_0^2 + 2a(y - y_0)$ works well because the only unknown in it is v. (We will plug y_1 in for y.)

3. Enter the known values

$$v^2 = (-13.0 m/s)^2 + 2(-9.80 m/s^2)(-5.10 m - 0 m) = 268.96 m^2/s^2,$$

where we have retained extra significant figures because this is an intermediate result.

Taking the square root, and noting that a square root can be positive or negative, gives

$$v = \pm 16.4 m/s.$$

The negative root is chosen to indicate that the rock is still heading down. Thus,

$$v = -16.4 m/s.$$

Discussion

Note that *this is exactly the same velocity the rock had at this position when it was thrown straight upward with the same initial speed.* (See Example and Figure(a).) This is not a coincidental result. Because we only consider the acceleration due to gravity in this problem, the speed of a falling object depends only on its initial speed and its vertical position relative to the starting point. For example, if the velocity of the rock is calculated at a height of 8.10 m above the starting point (using the method from Example) when the initial velocity is 13.0 m/s straight up, a result of $\pm 3.20 m/s$ is obtained. Here both signs are meaningful; the positive value occurs when the rock is at 8.10 m and heading up, and the negative value occurs when the rock is at 8.10 m and heading back down. It has the same speed but the opposite direction.

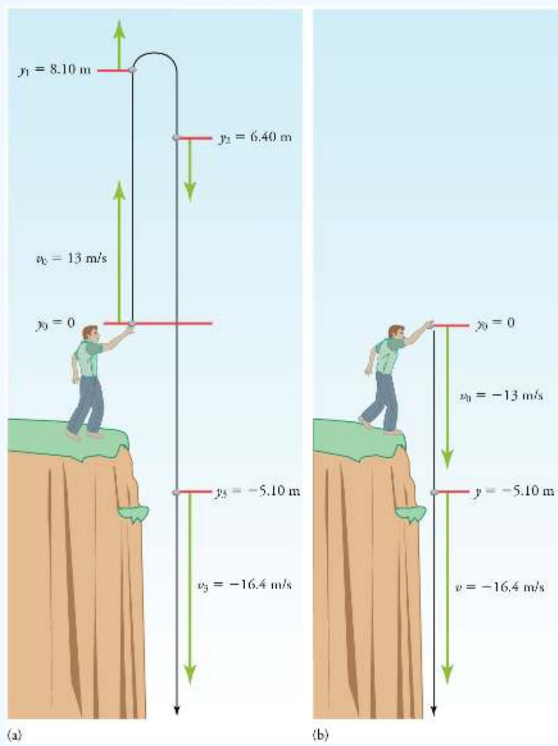


Figure $PageIndex5$: (a) A person throws a rock straight up, as explored in Example. The arrows are velocity vectors at 0, 1.00, 2.00, and 3.00 s. (b) A person throws a rock straight down from a cliff with the same initial speed as before, as in Example. Note that at the same distance below the point of release, the rock has the same velocity in both cases.

Another way to look at it is this: In Example, the rock is thrown up with an initial velocity of $13.0 m/s$. It rises and then falls back down. When its position is $y = 0$ on its way back down, its velocity is $-13.0 m/s$. That is, it has the same speed on its way down as on its way up. We would then expect its velocity at a position of $y = -5.10$ m to be the same whether we have thrown it upwards at $+13.0 m/s$ or thrown it downwards at $-13.0 m/s$. The velocity of the rock on its way down from $y = 0$ is the same whether we have thrown it up or down to start with, as long as the speed with which it was initially thrown is the same.

Example 1.11.3:Find g from Data on a Falling Object

The acceleration due to gravity on Earth differs slightly from place to place, depending on topography (e.g., whether you are on a hill or in a valley) and subsurface geology (whether there is dense rock like iron ore as opposed to light rock like salt beneath you.) The precise acceleration due to gravity can be calculated from data taken in an introductory physics laboratory course. An object, usually a metal ball for which air resistance is negligible, is dropped and the time it takes to fall a known distance is measured. See, for example, Figure. Very precise results can be produced with this method if sufficient care is taken in measuring the distance fallen and the elapsed time.

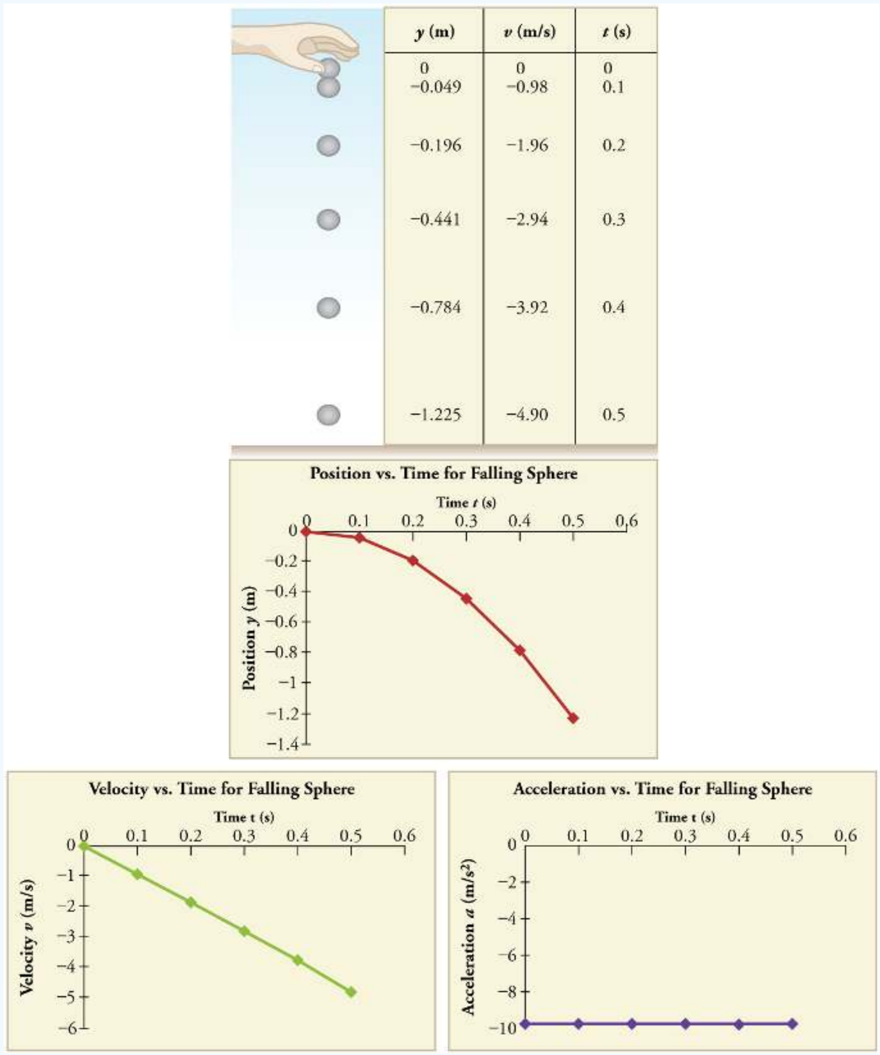


Figure 1.11.6: Positions and velocities of a metal ball released from rest when air resistance is negligible. Velocity is seen to increase linearly with time while displacement increases with time squared. Acceleration is a constant and is equal to gravitational acceleration.

Suppose the ball falls 1.0000 m in 0.45173 s. Assuming the ball is not affected by air resistance, what is the precise acceleration due to gravity at this location?

Strategy

Draw a sketch.

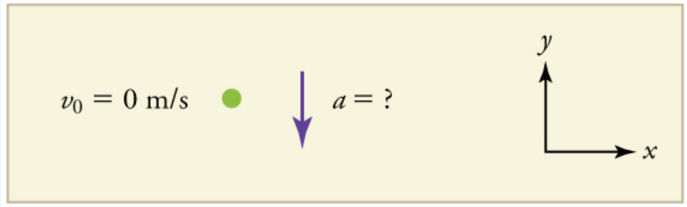


Figure 1.11.7:

We need to solve for acceleration a. Note that in this case, displacement is downward and therefore negative, as is acceleration.

Solution

1. Identify the knowns. $y_0 = 0; y = -1.0000m; t = 0.45173; v_0 = 0$.
2. Choose the equation that allows you to solve for a using the known values.

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

3. Substitute 0 for v_0 and rearrange the equation to solve for a. Substituting 0 for v_0 yields

$$y = y_0 + \frac{1}{2}at^2 \ .$$

Solving for a gives

$$a = \frac{2(y - y_0)}{t^2} \ .$$

4. Substitute known values yields

$$a = \frac{2(-1.0000m - 0)}{(0.45173s)^2} = -9.8010m/s^2,$$

so, because $a = -g$ with the directions we have chosen,

$$g = 9.8010m/s^2.$$

Discussion

The negative value for a indicates that the gravitational acceleration is downward, as expected. We expect the value to be somewhere around the average value of $9.80m/s^2$, so $9.8010m/s^2$ makes sense. Since the data going into the calculation are relatively precise, this value for g is more precise than the average value of $9.80m/s$; it represents the local value for the acceleration due to gravity.

Exercise 1.11.1

A chunk of ice breaks off a glacier and falls 30.0 meters before it hits the water. Assuming it falls freely (there is no air resistance), how long does it take to hit the water?

Answer

We know that initial position $y_0 = 0$, final position $y = -30.0m$, and $a = -g = -9.80m/s^2$. We can then use the equation $y = y_0 + v_0 t + \frac{1}{2}at^2$ to solve for t. Inserting $a = -g$, we obtain

$y = 0 + 0 - \frac{1}{2}gt^2$

$t^2 = \frac{2y}{-g}$

$t = \pm\sqrt{\frac{2y}{-g}} = \pm\sqrt{\frac{2(-30.0m)}{-9.80m/s^2}} = \pm\sqrt{6.12s^2} = 2.47s \approx 2.5s$

where we take the positive value as the physically relevant answer. Thus, it takes about 2.5 seconds for the piece of ice to hit the water.

PHET EXPLORATIONS: EQUATION GRAPHER

Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g. $y = bx$) to see how they add to generate the polynomial curve.

PhET Interactive Simulation

Figure 1.11.8: Equation Grapher

Summary

- An object in free-fall experiences constant acceleration if air resistance is negligible.
- On Earth, all free-falling objects have an acceleration due to gravity g, which averages

$$g = 9.80 m/s^2.$$

- Whether the acceleration a should be taken as $+g$ or $-g$ is determined by your choice of coordinate system. If you choose the upward direction as positive, $a = -g = -9.80 m/s^2$ is negative. In the opposite case, $a = +g = 9.80 m/s^2$ is positive. Since acceleration is constant, the kinematic equations above can be applied with the appropriate $+g$ or $-g$ substituted for a.
- For objects in free-fall, up is normally taken as positive for displacement, velocity, and acceleration.

Glossary

free-fall

the state of movement that results from gravitational force only

acceleration due to gravity

acceleration of an object as a result of gravity

1.12: Graphical Analysis of One-Dimensional Motion

Learning Objectives

By the end of this section, you will be able to:

- Describe a straight-line graph in terms of its slope and y-intercept.
- Determine average velocity or instantaneous velocity from a graph of position vs. time.
- Determine average or instantaneous acceleration from a graph of velocity vs. time.
- Derive a graph of velocity vs. time from a graph of position vs. time.
- Derive a graph of acceleration vs. time from a graph of velocity vs. time.

A graph, like a picture, is worth a thousand words. Graphs not only contain numerical information; they also reveal relationships between physical quantities. This section uses graphs of displacement, velocity, and acceleration versus time to illustrate one-dimensional kinematics.

Slopes and General Relationships

First note that graphs in this text have perpendicular axes, one horizontal and the other vertical. When two physical quantities are plotted against one another in such a graph, the horizontal axis is usually considered to be an independent variable and the vertical axis a dependent variable. If we call the horizontal axis the x-axis and the vertical axis the y-axis, as in Figure $1.12.1$, a straight-line graph has the general form

$$y = mx + b. \tag{1.12.1}$$

Here m is the slope, defined to be the rise divided by the run of the straight line (Figure $1.12.1$). The letter b is used for they-intercept, which is the point at which the line crosses the vertical axis.

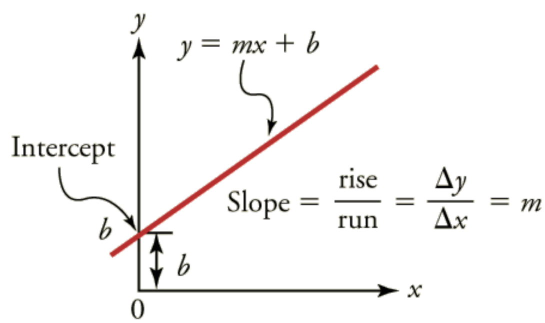


Figure 1.12.1: A straight-line graph. The equation for a straight line is $y = mx + b$.

Graph of Displacement vs. Time (*a* = 0, so *v* is constant)

Time is usually an independent variable that other quantities, such as displacement, depend upon. A graph of displacement versus time would, thus, have on the vertical axis and on the horizontal axis. Figure $1.12.2$ is just such a straight-line graph. It shows a graph of displacement versus time for a jet-powered car on a very flat dry lake bed in Nevada.

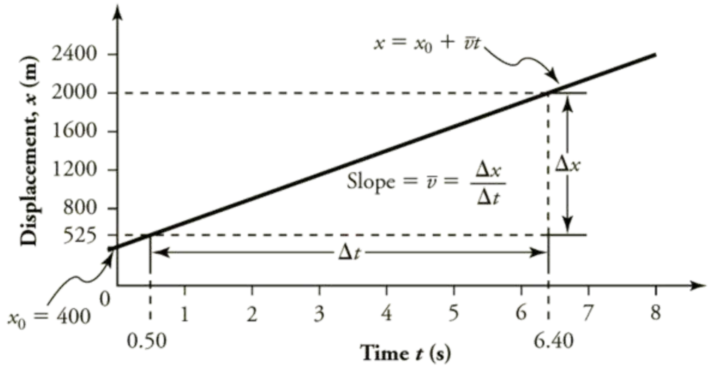


Figure 1.12.2: Graph of displacement versus time for a jet-powered car on the Bonneville Salt Flats.

Using the relationship between dependent and independent variables, we see that the slope in the graph above is average velocity $\bar{v}$ and the intercept is displacement at time zero—that is, x_0. Substituting these symbols into $y = mx + b$ gives

$$x = \bar{v}t + x_0 \tag{1.12.2}$$

or

$$x = x_0 + \bar{v}t. \tag{1.12.3}$$

Thus a graph of displacement versus time gives a general relationship among displacement, velocity, and time, as well as giving detailed numerical information about a specific situation.

THE SLOPE OF X VS. T

The slope of the graph of displacement x vs. time t is velocity v.

$$slope = \frac{\Delta x}{\Delta t} = v$$

Notice that this equation is the same as that derived algebraically from other motion equations in Motion Equations for Constant Acceleration in One Dimension.

From the figure we can see that the car has a displacement of 25 m at 0.50 s and 2000 m at 6.40 s. Its displacement at other times can be read from the graph; furthermore, information about its velocity and acceleration can also be obtained from the graph.

Example 1.12.1:Determining Average Velocity from a Graph of Displacement versus Time: Jet Car

Find the average velocity of the car whose position is graphed in Figure $1.12.2$

Strategy

The slope of a graph of x vs. t is average velocity, since slope equals rise over run. In this case, rise = change in position and run = change in time, so that

$$slope = \frac{\Delta x}{\Delta t} = \bar{v}.$$

Since the slope is constant here, any two points on the graph can be used to find the slope. (Generally speaking, it is most accurate to use two widely separated points on the straight line. This is because any error in reading data from the graph is proportionally smaller if the interval is larger.)

Solution

1. Choose two points on the line. In this case, we choose the points labeled on the graph: (6.4 s, 2000 m) and (0.50 s, 525 m). (Note, however, that you could choose any two points.)
2. Substitute the x and t values of the chosen points into the equation. Remember in calculating change (Δ) we always use final value minus initial value.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{2000m - 525m}{6.4s - 0.50s},$$

yielding

$$v- = 250m/s.$$

Discussion

This is an impressively large land speed (900 km/h, or about 560 mi/h): much greater than the typical highway speed limit of 60 mi/h (27 m/s or 96 km/h), but considerably shy of the record of 343 m/s (1234 km/h or 766 mi/h) set in 1997.

Graphs of Motion when is constant but ≠0

The graphs in Figure $1.12.3$ below represent the motion of the jet-powered car as it accelerates toward its top speed, but only during the time when its acceleration is constant. Time starts at zero for this motion (as if measured with a stopwatch), and the displacement and velocity are initially 200 m and 15 m/s, respectively.

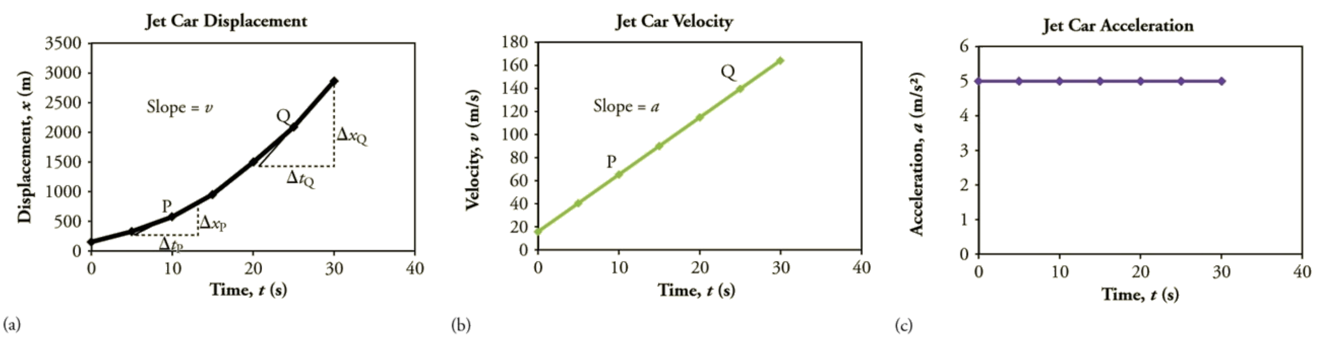


Figure $\mathbf{1.12.3}$***:*** *Graphs of motion of a jet-powered car during the time span when its acceleration is constant. (a) The slope of an* x *vs.* t *graph is velocity. This is shown at two points, and the instantaneous velocities obtained are plotted in the next graph. Instantaneous velocity at any point is the slope of the tangent at that point. (b) The slope of the* v *vs.* t *graph is constant for this part of the motion, indicating constant acceleration. (c) Acceleration has the constant value of* $5.0m/s^2$ *over the time interval plotted.*

Figure 1.12.4: A U.S. Air Force jet car speeds down a track. (credit: Matt Trostle, Flickr)

The graph of displacement versus time in Figure $1.12.3a$ is a curve rather than a straight line. The slope of the curve becomes steeper as time progresses, showing that the velocity is increasing over time. The slope at any point on a displacement-versus-time graph is the instantaneous velocity at that point. It is found by drawing a straight line tangent to the curve at the point of interest and taking the slope of this straight line. Tangent lines are shown for two points in Figure $1.12.3a$. If this is done at every point on the curve and the values are plotted against time, then the graph of velocity versus time shown in Figure $1.12.3b$ is obtained. Furthermore, the slope of the graph of velocity versus time is acceleration, which is shown in Figure $1.12.3c$

Example 1.12.2:

Calculate the velocity of the jet car at a time of 25 s by finding the slope of the \(\displaystyle vs. \(\displaystyle graph in the graph below

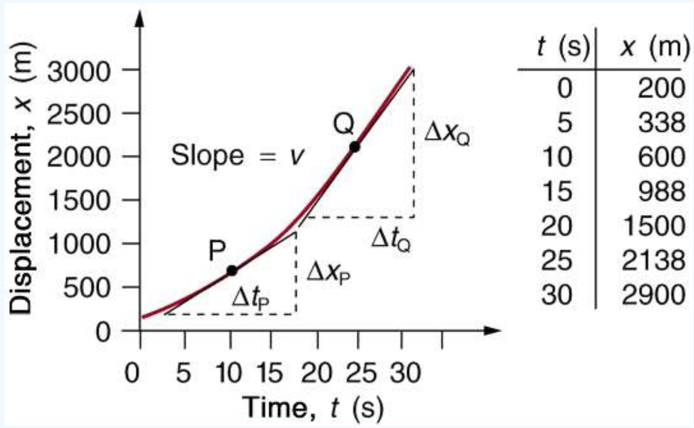


Figure 1.12.5: The slope of an \(vs. \(*graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point.*

Strategy

The slope of a curve at a point is equal to the slope of a straight line tangent to the curve at that point. This principle is illustrated in Figure, where Q is the point at $t = 25s$.

Solution

1. Find the tangent line to the curve at $t = 25s$.
2. Determine the endpoints of the tangent. These correspond to a position of 1300 m at time 19 s and a position of 3120 m at time 32 s.
3. Plug these endpoints into the equation to solve for the slope, .

$$slope = v_Q = \frac{\Delta x_Q}{\Delta t_Q} = \frac{(3120m - 1300m)}{(32s - 19s)}$$

Thus,

$$v_Q = \frac{1820m}{13s} = 140m/s.$$

Discussion

This is the value given in this figure's table for v at $t = 25s$. The value of 140 m/s for v_Q is plotted in Figure. The entire graph of v vs. t can be obtained in this fashion.

Carrying this one step further, we note that the slope of a velocity versus time graph is acceleration. Slope is rise divided by run; on a v vs. t graph, rise = change in velocity Δv and run = change in time Δt.

THE SLOPE OF V VS. T

The slope of a graph of velocity v vs. time t is acceleration a.

$$slope = \frac{\Delta v}{\Delta t} = a$$

Since the velocity versus time graph in Figure $1.12.3b$ is a straight line, its slope is the same everywhere, implying that acceleration is constant. Acceleration versus time is graphed in Figure(c).

Additional general information can be obtained from Figure and the expression for a straight line, $y = mx + b$.

In this case, the vertical axis y is V, the intercept b is v_0, the slope m is a, and the horizontal axis x is t. Substituting these symbols yields

$$v = v_0 + at.$$

A general relationship for velocity, acceleration, and time has again been obtained from a graph. Notice that this equation was also derived algebraically from other motion equations in Motion Equations for Constant Acceleration in One Dimension.

It is not accidental that the same equations are obtained by graphical analysis as by algebraic techniques. In fact, an important way to *discover* physical relationships is to measure various physical quantities and then make graphs of one quantity against another to see if they are correlated in any way. Correlations imply physical relationships and might be shown by smooth graphs such as those above. From such graphs, mathematical relationships can sometimes be postulated. Further experiments are then performed to determine the validity of the hypothesized relationships.

Graphs of Motion Where Acceleration is Not Constant

Now consider the motion of the jet car as it goes from 165 m/s to its top velocity of 250 m/s, graphed in Figure $1.12.6$. Time again starts at zero, and the initial position and velocity are 2900 m and 165 m/s, respectively. (These were the final position and velocity of the car in the motion graphed in Figure $1.12.4$) Acceleration gradually decreases from $5.0 m/s^2$ to zero when the car hits 250 m/s. The slope of the x vs. t graph increases until $t = 55 s$, after which time the slope is constant. Similarly, velocity increases until 55 s and then becomes constant, since acceleration decreases to zero at 55 s and remains zero afterward.

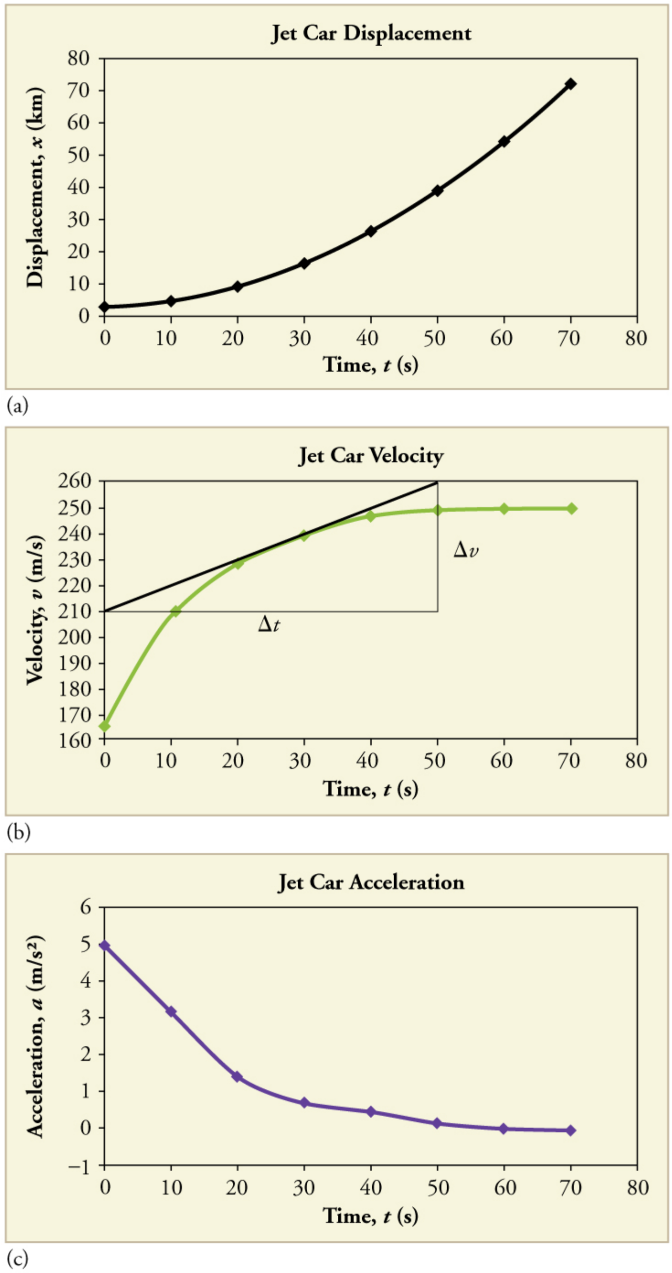


Figure 1.12.3 ends. (a) The slope of this graph is velocity; it is plotted in the next graph. (b) The velocity gradually approaches its top value. The slope of this graph is acceleration; it is plotted in the final graph. (c) Acceleration gradually declines to zero when velocity becomes constant.

Example 1.12.3:Calculating Acceleration from a Graph of Velocity versus Time

Calculate the acceleration of the jet car at a time of 25 s by finding the slope of the v vs. t graph in Figure $1.12.6b$

Strategy

The slope of the curve at $t = 25s$ is equal to the slope of the line tangent at that point, as illustrated in Figure $1.12.6b$

Solution

Determine endpoints of the tangent line from the figure, and then plug them into the equation to solve for slope, .

$$slope = \frac{\Delta v}{\Delta t} = \frac{(260m/s - 210m/s)}{(51s - 1.0s)}$$

$$a = \frac{50 m/s}{50 s} = 1.0 m/s^2.$$

Discussion

Note that this value for a is consistent with the value plotted in Figure(c) at $t = 25 s$.

A graph of displacement versus time can be used to generate a graph of velocity versus time, and a graph of velocity versus time can be used to generate a graph of acceleration versus time. We do this by finding the slope of the graphs at every point. If the graph is linear (i.e., a line with a constant slope), it is easy to find the slope at any point and you have the slope for every point. Graphical analysis of motion can be used to describe both specific and general characteristics of kinematics. Graphs can also be used for other topics in physics. An important aspect of exploring physical relationships is to graph them and look for underlying relationships.

Exercise 1.12.1:Check Your Understanding

A graph of velocity vs. time of a ship coming into a harbor is shown below.

a. Describe the motion of the ship based on the graph.
b. What would a graph of the ship's acceleration look like?

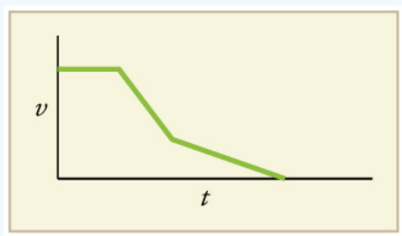


Figure 1.12.7

Answer a

(a) The ship moves at constant velocity and then begins to decelerate at a constant rate. At some point, its deceleration rate decreases. It maintains this lower deceleration rate until it stops moving.

Solution b

A graph of acceleration vs. time would show zero acceleration in the first leg, large and constant negative acceleration in the second leg, and constant negative acceleration.

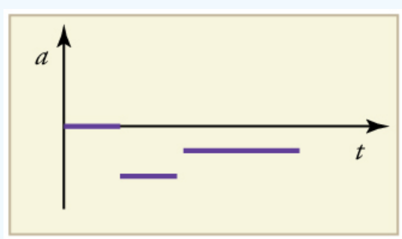


Figure 1.12.8

Summary

- Graphs of motion can be used to analyze motion.
- Graphical solutions yield identical solutions to mathematical methods for deriving motion equations.
- The slope of a graph of displacement x vs. time t is velocity v.
- The slope of a graph of velocity v vs. time t graph is acceleration a.
- Average velocity, instantaneous velocity, and acceleration can all be obtained by analyzing graphs.

Glossary

independent variable

the variable that the dependent variable is measured with respect to; usually plotted along the x-axis

dependent variable

the variable that is being measured; usually plotted along the y-axis

slope

the difference in y-value (the rise) divided by the difference in x-value (the run) of two points on a straight line

y-intercept

the y-value when $x=0$, or when the graph crosses the y-axis

1.E: Kinematics (Exercises)

Conceptual Questions

2.1: Displacement

1. Give an example in which there are clear distinctions among distance traveled, displacement, and magnitude of displacement. Specifically identify each quantity in your example.

2. Under what circumstances does distance traveled equal magnitude of displacement? What is the only case in which magnitude of displacement and displacement are exactly the same?

3. Bacteria move back and forth by using their flagella (structures that look like little tails). Speeds of up to $50\mu m/s (50 \times 10^{-6} m/s)$ have been observed. The total distance traveled by a bacterium is large for its size, while its displacement is small. Why is this?

2.2: Vectors, Scalars, and Coordinate Systems

4. A student writes, "*A bird that is diving for prey has a speed of* $-10 m/s$" What is wrong with the student's statement? What has the student actually described? Explain.

5. What is the speed of the bird in Exercise?

6. Acceleration is the change in velocity over time. Given this information, is acceleration a vector or a scalar quantity? Explain.

7. A weather forecast states that the temperature is predicted to be $-5^{\circ} C$ the following day. Is this temperature a vector or a scalar quantity? Explain.

2.3: Time, Velocity, and Speed

8. Give an example (but not one from the text) of a device used to measure time and identify what change in that device indicates a change in time.

9. There is a distinction between average speed and the magnitude of average velocity. Give an example that illustrates the difference between these two quantities.

10. Does a car's odometer measure position or displacement? Does its speedometer measure speed or velocity?

11. If you divide the total distance traveled on a car trip (as determined by the odometer) by the time for the trip, are you calculating the average speed or the magnitude of the average velocity? Under what circumstances are these two quantities the same?

12. How are instantaneous velocity and instantaneous speed related to one another? How do they differ?

2.4: Acceleration

13. Is it possible for speed to be constant while acceleration is not zero? Give an example of such a situation.

14. Is it possible for velocity to be constant while acceleration is not zero? Explain.

15. Give an example in which velocity is zero yet acceleration is not.

16. If a subway train is moving to the left (has a negative velocity) and then comes to a stop, what is the direction of its acceleration? Is the acceleration positive or negative?

17. Plus and minus signs are used in one-dimensional motion to indicate direction. What is the sign of an acceleration that reduces the magnitude of a negative velocity? Of a positive velocity?

2.6: Problem-Solving Basics for One-Dimensional Kinematics

18. What information do you need in order to choose which equation or equations to use to solve a problem? Explain.

19. What is the last thing you should do when solving a problem? Explain.

2.7: Falling Objects

20. What is the acceleration of a rock thrown straight upward on the way up? At the top of its flight? On the way down?

21. An object that is thrown straight up falls back to Earth. This is one-dimensional motion.

(a) When is its velocity zero?

(b) Does its velocity change direction?

(c) Does the acceleration due to gravity have the same sign on the way up as on the way down?

22. Suppose you throw a rock nearly straight up at a coconut in a palm tree, and the rock misses on the way up but hits the coconut on the way down. Neglecting air resistance, how does the speed of the rock when it hits the coconut on the way down compare with what it would have been if it had hit the coconut on the way up? Is it more likely to dislodge the coconut on the way up or down? Explain.

23. If an object is thrown straight up and air resistance is negligible, then its speed when it returns to the starting point is the same as when it was released. If air resistance were not negligible, how would its speed upon return compare with its initial speed? How would the maximum height to which it rises be affected?

24. The severity of a fall depends on your speed when you strike the ground. All factors but the acceleration due to gravity being the same, how many times higher could a safe fall on the Moon be than on Earth (gravitational acceleration on the Moon is about 1/6 that of the Earth)?

25. How many times higher could an astronaut jump on the Moon than on Earth if his takeoff speed is the same in both locations (gravitational acceleration on the Moon is about 1/6 of g on Earth)?

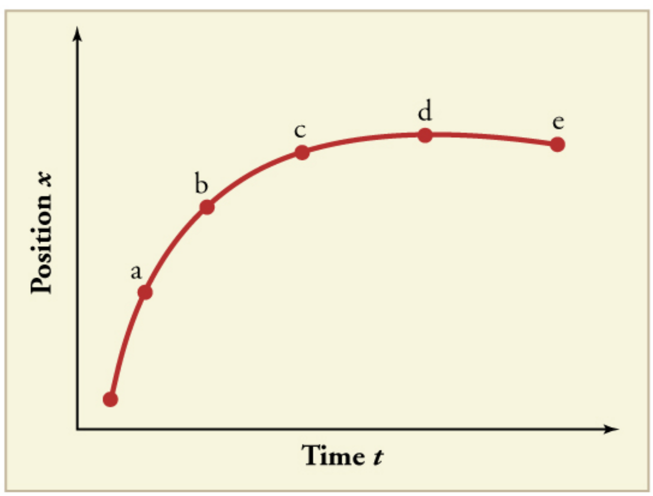
2.8: Graphical Analysis of One-Dimensional Motion

23. (a) Explain how you can use the graph of position versus time in Figure to describe the change in velocity over time. Identify

(b) the time (t_a, t_b, t_c, t_d, or t_e) at which the instantaneous velocity is greatest,

(c) the time at which it is zero, and

(d) the time at which it is negative.

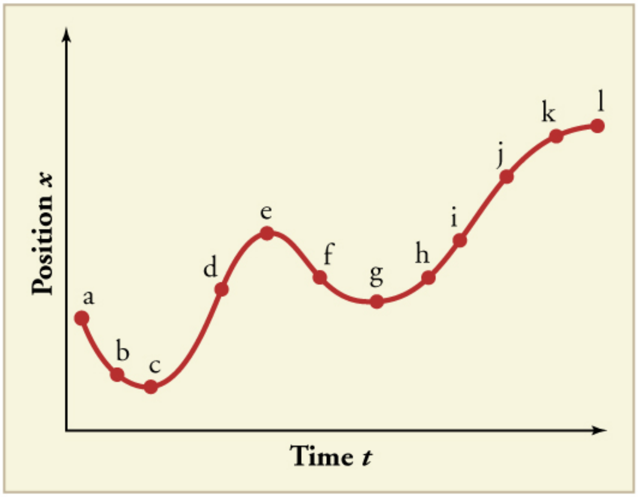


24. (a) Sketch a graph of velocity versus time corresponding to the graph of position versus time given in Figure.

(b) Identify the time or times (t_a, t_b, t_c, etc.) at which the instantaneous velocity is greatest.

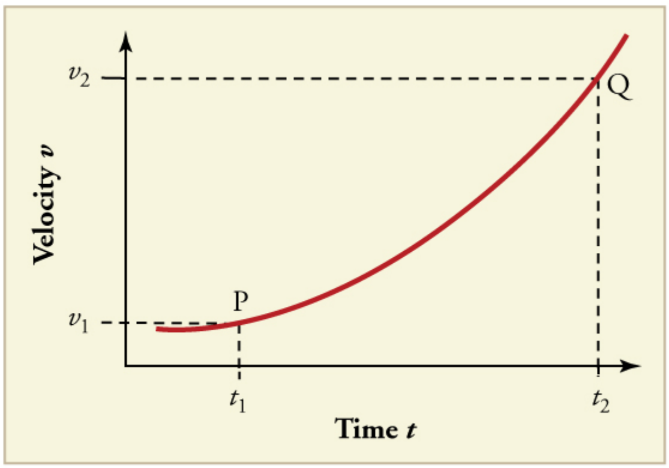
(c) At which times is it zero?

(d) At which times is it negative?



25. (a) Explain how you can determine the acceleration over time from a velocity versus time graph such as the one in Figure.

(b) Based on the graph, how does acceleration change over time?

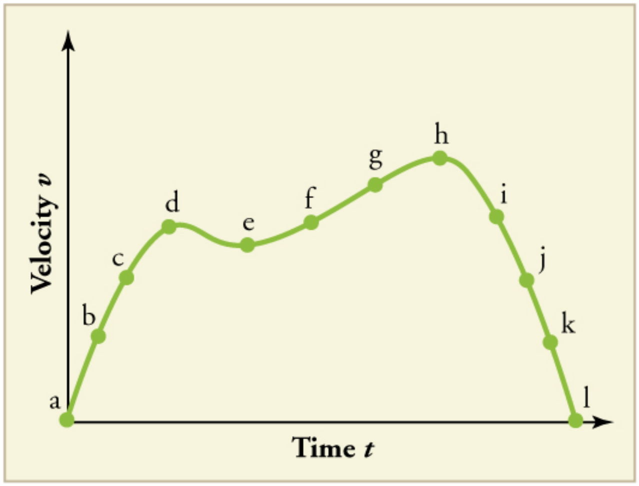


26. (a) Sketch a graph of acceleration versus time corresponding to the graph of velocity versus time given in Figure.

(b) Identify the time or times (t_a, t_b, t_c, etc.) at which the acceleration is greatest.

(c) At which times is it zero?

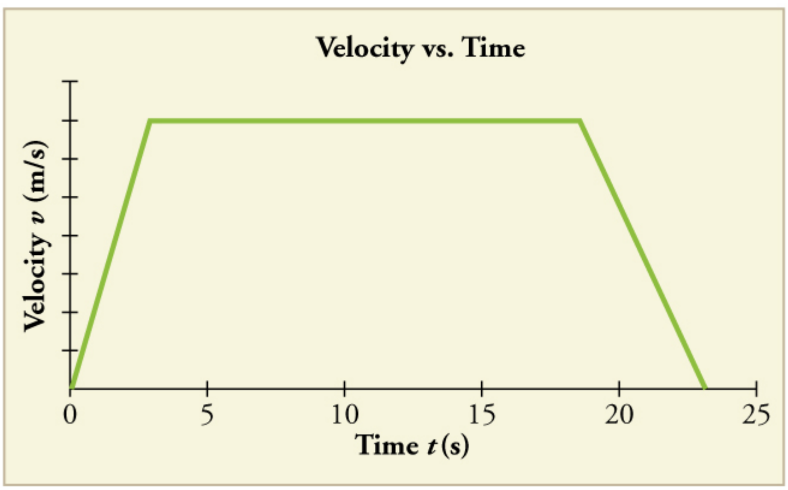
(d) At which times is it negative?



27. Consider the velocity vs. time graph of a person in an elevator shown in Figure. Suppose the elevator is initially at rest. It then accelerates for 3 seconds, maintains that velocity for 15 seconds, then decelerates for 5 seconds until it stops. The acceleration for the entire trip is not constant so we cannot use the equations of motion from Motion Equations for Constant Acceleration in One Dimension for the complete trip. (We could, however, use them in the three individual sections where acceleration is a constant.) Sketch graphs of

(a) position vs. time and

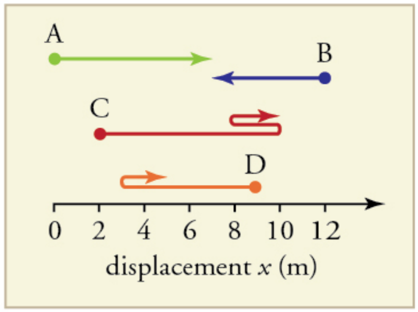
(b) acceleration vs. time for this trip.



28. A cylinder is given a push and then rolls up an inclined plane. If the origin is the starting point, sketch the position, velocity, and acceleration of the cylinder vs. time as it goes up and then down the plane.

Problems & Exercises

2.1: Displacement



29. Find the following for path A:

(a) The distance traveled.

(b) The magnitude of the displacement from start to finish.

(c) The displacement from start to finish.

Solution
(a) 7 m
(b) 7 m
(c) $+7m$

30. Find the following for path B:

(a) The distance traveled.

(b) The magnitude of the displacement from start to finish.

(c) The displacement from start to finish.

31. Find the following for path C:

(a) The distance traveled.

(b) The magnitude of the displacement from start to finish.

(c) The displacement from start to finish.

Solution
(a) 13 m
(b) 9 m
(c) $+9m$

32. Find the following for path D:

(a) The distance traveled.

(b) The magnitude of the displacement from start to finish

(c) The displacement from start to finish

2.3: Time, Velocity, and Speed

33. (a) Calculate Earth's average speed relative to the Sun.

(b) What is its average velocity over a period of one year?

Solution
(a) $3.0 \times 10^4 m/s$
(b) 0 m/s

34. A helicopter blade spins at exactly 100 revolutions per minute. Its tip is 5.00 m from the center of rotation.

(a) Calculate the average speed of the blade tip in the helicopter's frame of reference

(b) What is its average velocity over one revolution?

35. The North American and European continents are moving apart at a rate of about 3 cm/y. At this rate how long will it take them to drift 500 km farther apart than they are at present?

Solution
$2 \times 10^7 years$

36. Land west of the San Andreas fault in southern California is moving at an average velocity of about 6 cm/y northwest relative to land east of the fault. Los Angeles is west of the fault and may thus someday be at the same latitude as San Francisco, which is east of the fault. How far in the future will this occur if the displacement to be made is 590 km northwest, assuming the motion remains constant?

37. On May 26, 1934, a streamlined, stainless steel diesel train called the Zephyr set the world's nonstop long-distance speed record for trains. Its run from Denver to Chicago took 13 hours, 4 minutes, 58 seconds, and was witnessed by more than a million people along the route. The total distance traveled was 1633.8 km. What was its average speed in km/h and m/s?

Solution
$34.689 m/s = 124.88 km/h$

38. Tidal friction is slowing the rotation of the Earth. As a result, the orbit of the Moon is increasing in radius at a rate of approximately 4 cm/year. Assuming this to be a constant rate, how many years will pass before the radius of the Moon's orbit increases by $3.84 \times 10^6 m$(1%)?

39. A student drove to the university from her home and noted that the odometer reading of her car increased by 12.0 km. The trip took 18.0 min.

(a) What was her average speed?

(b) If the straight-line distance from her home to the university is 10.3 km in a direction 25.0^o south of east, what was her average velocity?

(c) If she returned home by the same path 7 h 30 min after she left, what were her average speed and velocity for the entire trip?

Solution
(a) $40.0 km/h$
(b) 34.3 km/h, 25^o S of E.
(c) average speed = $3.20 km/h, \bar{v} = 0$.

40. The speed of propagation of the action potential (an electrical signal) in a nerve cell depends (inversely) on the diameter of the axon (nerve fiber). If the nerve cell connecting the spinal cord to your feet is 1.1 m long, and the nerve impulse speed is 18 m/s, how long does it take for the nerve signal to travel this distance?

41. Conversations
ith astronauts on the lunar surface were characterized by a kind of echo in which the earthbound person's voice was so loud in the astronaut's space helmet that it was picked up by the astronaut's microphone and transmitted back to Earth. It is reasonable to assume that the echo time equals the time necessary for the radio wave to travel from the Earth to the Moon and back (that is, neglecting any time delays in the electronic equipment). Calculate the distance from Earth to the Moon given that the echo time was 2.56 s and that radio waves travel at the speed of light ($3.00 \times 10^8 m/s$)

Solution
384,000 km

42. A football quarterback runs 15.0 m straight down the playing field in 2.50 s. He is then hit and pushed 3.00 m straight backward in 1.75 s. He breaks the tackle and runs straight forward another 21.0 m in 5.20 s. Calculate his average velocity

(a) for each of the three intervals and

(b) for the entire motion.

43. The planetary model of the atom pictures electrons orbiting the atomic nucleus much as planets orbit the Sun. In this model you can view hydrogen, the simplest atom, as having a single electron in a circular orbit $1.06 \times 10^{-10} m$ in diameter.

(a) If the average speed of the electron in this orbit is known to be $2.20 \times 10^6 m/s$, calculate the number of revolutions per second it makes about the nucleus.

(b) What is the electron's average velocity?

Solution
(a) $6.61 \times 10^{15} rev/s$
(b) 0 m/s

2.4: Acceleration

44. A cheetah can accelerate from rest to a speed of 30.0 m/s in 7.00 s. What is its acceleration?

Solution
$4.29 m/s^2$

45. *Professional Application*

Dr. John Paul Stapp was U.S. Air Force officer who studied the effects of extreme deceleration on the human body. On December 10, 1954, Stapp rode a rocket sled, accelerating from rest to a top speed of 282 m/s (1015 km/h) in 5.00 s, and was brought jarringly back to rest in only 1.40 s! Calculate his (a) acceleration and (b) deceleration. Express each in multiples of $g(9.80 m/s^2)$ by taking its ratio to the acceleration of gravity.

46. A commuter backs her car out of her garage with an acceleration of $1.40 m/s^2$

(a) How long does it take her to reach a speed of 2.00 m/s?

(b) If she then brakes to a stop in 0.800 s, what is her deceleration?

Solution
(a) $1.43 s$
(b) $-2.50 m/s^2$

47. Assume that an intercontinental ballistic missile goes from rest to a suborbital speed of 6.50 km/s in 60.0 s (the actual speed and time are classified). What is its average acceleration in m/s^2 and in multiples of $g(9.80 m/s^2)$?

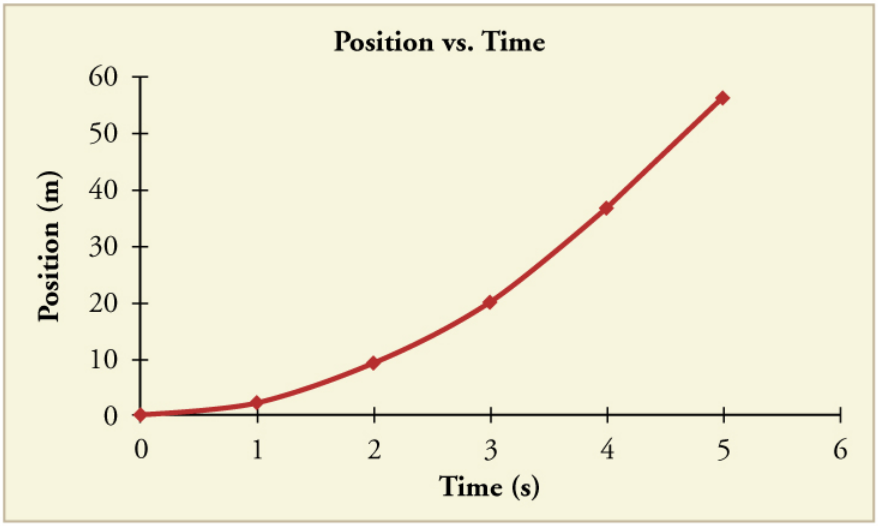
2.5: Motion Equations for Constant Acceleration in One Dimension

48. An Olympic-class sprinter starts a race with an acceleration of $4.50 m/s^2$

(a) What is her speed 2.40 s later?

(b) Sketch a graph of her position vs. time for this period.

Solution
(a) $10.8 m/s$
(b)



49. A well-thrown ball is caught in a well-padded mitt. If the deceleration of the ball is $2.10 \times 10^4 m/s^2$, and 1.85 ms $(1 ms = 10^{-3} s)$ elapses from the time the ball first touches the mitt until it stops, what was the initial velocity of the ball?

Solution
38.9 m/s (about 87 miles per hour)

50. A bullet in a gun is accelerated from the firing chamber to the end of the barrel at an average rate of $6.20 \times 10^5 m/s^2$ for $8.10 \times 10^{-4} s$. What is its muzzle velocity (that is, its final velocity)?

51. (a) A light-rail commuter train accelerates at a rate of $1.35 m/s^2$. How long does it take to reach its top speed of 80.0 km/h, starting from rest?

(b) The same train ordinarily decelerates at a rate of $1.65 m/s^2$. How long does it take to come to a stop from its top speed?

(c) In emergencies the train can decelerate more rapidly, coming to rest from 80.0 km/h in 8.30 s. What is its emergency deceleration in m/s^2?

Solution
(a) $16.5 s$
(b) $13.5 s$
(c) $-2.68 m/s^2$

52. While entering a freeway, a car accelerates from rest at a rate of $2.40 m/s^2$ for 12.0 s.

(a) Draw a sketch of the situation.

(b) List the knowns in this problem.

(c) How far does the car travel in those 12.0 s? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, check your units, and discuss whether the answer is reasonable.

(d) What is the car's final velocity? Solve for this unknown in the same manner as in part (c), showing all steps explicitly.

53. At the end of a race, a runner decelerates from a velocity of 9.00 m/s at a rate of $2.00 m/s^2$.

(a) How far does she travel in the next 5.00 s?

(b) What is her final velocity?

(c) Evaluate the result. Does it make sense?

Solution
(a) $20.0 m$

(b) $-1.00 m/s$
(c) This result does not really make sense. If the runner starts at 9.00 m/s and decelerates at $2.00 m/s^2$ then she will have stopped after 4.50 s. If she continues to decelerate, she will be running backwards.

54. *Professional Application:*

Blood is accelerated from rest to 30.0 cm/s in a distance of 1.80 cm by the left ventricle of the heart.

(a) Make a sketch of the situation.

(b) List the knowns in this problem.

(c) How long does the acceleration take? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking your units.

(d) Is the answer reasonable when compared with the time for a heartbeat?

55. In a slap shot, a hockey player accelerates the puck from a velocity of 8.00 m/s to 40.0 m/s in the same direction. If this shot takes $3.33 \times 10^{-2} s$, calculate the distance over which the puck accelerates.

Solution
0.799 m

56. A powerful motorcycle can accelerate from rest to 26.8 m/s (100 km/h) in only 3.90 s.

(a) What is its average acceleration?

(b) How far does it travel in that time?

57. Freight trains can produce only relatively small accelerations and decelerations.

(a) What is the final velocity of a freight train that accelerates at a rate of $0.0500 m/s^2$ for 8.00 min, starting with an initial velocity of 4.00 m/s?

(b) If the train can slow down at a rate of $0.550 m/s^2$, how long will it take to come to a stop from this velocity?

(c) How far will it travel in each case?

Solution
(a) $28.0 m/s$
(b) $50.9 s$
(c) 7.68 km to accelerate and 713 m to decelerate

58. A fireworks shell is accelerated from rest to a velocity of 65.0 m/s over a distance of 0.250 m.

(a) How long did the acceleration last?

(b) Calculate the acceleration.

59. A swan on a lake gets airborne by flapping its wings and running on top of the water.

(a) If the swan must reach a velocity of 6.00 m/s to take off and it accelerates from rest at an average rate of $0.350 m/s^2$, how far will it travel before becoming airborne?

(b) How long does this take?

Solution
(a) $51.4 m$
(b) $17.1 s$

60. *Professional Application:*

A woodpecker's brain is specially protected from large decelerations by tendon-like attachments inside the skull. While pecking on a tree, the woodpecker's head comes to a stop from an initial velocity of 0.600 m/s in a distance of only 2.00 mm.

(a) Find the acceleration in m/s^2 and in multiples of $g (g = 9.80 m/s^2)$.

(b) Calculate the stopping time.

(c) The tendons cradling the brain stretch, making its stopping distance 4.50 mm (greater than the head and, hence, less deceleration of the brain). What is the brain's deceleration, expressed in multiples of g?

61. An unwary football player collides with a padded goalpost while running at a velocity of 7.50 m/s and comes to a full stop after compressing the padding and his body 0.350 m.

(a) What is his deceleration?

(b) How long does the collision last?

Solution
(a) $-80.4 m/s^2$
(b) $9.33 \times 10^{-2} s$

62. In World War II, there were several reported cases of airmen who jumped from their flaming airplanes with no parachute to escape certain death. Some fell about 20,000 feet (6000 m), and some of them survived, with few life-threatening injuries. For these lucky pilots, the tree branches and snow drifts on the ground allowed their deceleration to be relatively small. If we assume that a pilot's speed upon impact was 123 mph (54 m/s), then what was his deceleration? Assume that the trees and snow stopped him over a distance of 3.0 m.

63. Consider a grey squirrel falling out of a tree to the ground.

(a) If we ignore air resistance in this case (only for the sake of this problem), determine a squirrel's velocity just before hitting the ground, assuming it fell from a height of 3.0 m.

(b) If the squirrel stops in a distance of 2.0 cm through bending its limbs, compare its deceleration with that of the airman in the previous problem.

Solution
(a) $7.7 m/s$
(b) $-15 \times 10^2 m/s^2$. This is about 3 times the deceleration of the pilots, who were falling from thousands of meters high!

64. An express train passes through a station. It enters with an initial velocity of 22.0 m/s and decelerates at a rate of $0.150 m/s^2$ as it goes through. The station is 210 m long.

(a) How long is the nose of the train in the station?

(b) How fast is it going when the nose leaves the station?

(c) If the train is 130 m long, when does the end of the train leave the station?

(d) What is the velocity of the end of the train as it leaves?

65. Dragsters can actually reach a top speed of 145 m/s in only 4.45 s—considerably less time than given in Example and Example.

(a) Calculate the average acceleration for such a dragster.

(b) Find the final velocity of this dragster starting from rest and accelerating at the rate found in (a) for 402 m (a quarter mile) without using any information on time.

(c) Why is the final velocity greater than that used to find the average acceleration?

Hint: Consider whether the assumption of constant acceleration is valid for a dragster. If not, discuss whether the acceleration would be greater at the beginning or end of the run and what effect that would have on the final velocity.

Solution
(a) $32.6 m/s^2$
(b) $162 m/s$
(c) $v > v_{max}$, because the assumption of constant acceleration is not valid for a dragster. A dragster changes gears, and would have a greater acceleration in first gear than second gear than third gear, etc. The acceleration would be greatest at the beginning, so it would not be accelerating at $32.6 m/s^2$ during the last few meters, but substantially less, and the final velocity would be less than 162 m/s.

66. A bicycle racer sprints at the end of a race to clinch a victory. The racer has an initial velocity of 11.5 m/s and accelerates at the rate of $0.500 m/s^2$ for 7.00 s.

(a) What is his final velocity?

(b) The racer continues at this velocity to the finish line. If he was 300 m from the finish line when he started to accelerate, how much time did he save?

(c) One other racer was 5.00 m ahead when the winner started to accelerate, but he was unable to accelerate, and traveled at 11.8 m/s until the finish line. How far ahead of him (in meters and in seconds) did the winner finish?

67. In 1967, New Zealander Burt Munro set the world record for an Indian motorcycle, on the Bonneville Salt Flats in Utah, with a maximum speed of 183.58 mi/h. The one-way course was 5.00 mi long. Acceleration rates are often described by the time it takes to reach 60.0 mi/h from rest. If this time was 4.00 s, and Burt accelerated at this rate until he reached his maximum speed, how long did it take Burt to complete the course?

Solution
104 s

68. (a) A world record was set for the men's 100-m dash in the 2008 Olympic Games in Beijing by Usain Bolt of Jamaica. Bolt "coasted" across the finish line with a time of 9.69 s. If we assume that Bolt accelerated for 3.00 s to reach his maximum speed, and maintained that speed for the rest of the race, calculate his maximum speed and his acceleration.

(b) During the same Olympics, Bolt also set the world record in the 200-m dash with a time of 19.30 s. Using the same assumptions as for the 100-m dash, what was his maximum speed for this race?

Solution
(a) $v = 12.2 m/s; a = 4.07 m/s^2$
(b) $v = 11.2 m/s$

2.7: Falling Objects

Assume air resistance is negligible unless otherwise stated.

69. Calculate the displacement and velocity at times of

(a) 0.500,

(b) 1.00,

(c) 1.50, and

(d) 2.00 s for a ball thrown straight up with an initial velocity of 15.0 m/s. Take the point of release to be $y_0 = 0$

Solution
(a) $y_1 = 6.28 m; v_1 = 10.1 m/s$
(b) $y_2 = 10.1 m; v_2 = 5.20 m/s$
(c) $y_3 = 11.5 m; v_3 = 0.300 m/s$
(d) $y_4 = 10.4 m; v_4 = -4.60 m/s$

70. Calculate the displacement and velocity at times of

(a) 0.500,

(b) 1.00,

(c) 1.50,

(d) 2.00, and

(e) 2.50 s for a rock thrown straight down with an initial velocity of 14.0 m/s from the Verrazano Narrows Bridge in New York City. The roadway of this bridge is 70.0 m above the water.

71. A basketball referee tosses the ball straight up for the starting tip-off. At what velocity must a basketball player leave the ground to rise 1.25 m above the floor in an attempt to get the ball?

Solution
$v_0 = 4.95 m/s$

72. A rescue helicopter is hovering over a person whose boat has sunk. One of the rescuers throws a life preserver straight down to the victim with an initial velocity of 1.40 m/s and observes that it takes 1.8 s to reach the water.

(a) List the knowns in this problem.

(b) How high above the water was the preserver released? Note that the downdraft of the helicopter reduces the effects of air resistance on the falling life preserver, so that an acceleration equal to that of gravity is reasonable.

73. A dolphin in an aquatic show jumps straight up out of the water at a velocity of 13.0 m/s.

(a) List the knowns in this problem.

(b) How high does his body rise above the water? To solve this part, first note that the final velocity is now a known and identify its value. Then identify the unknown, and discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking units, and discuss whether the answer is reasonable.

(c) How long is the dolphin in the air? Neglect any effects due to his size or orientation.

Solution
(a) $a = -9.80 m/s^2; v_0 = 13.0 m/s; y_0 = 0 m$
(b) $v = 0 m/s$. Unknown is distance y to top of trajectory, where velocity is zero. Use equation $v^2 = v_0^2 + 2a(y - y_0)$ because it contains all known values except for y, so we can solve for y. Solving for y gives

$$v^2 - v_0^2 = 2ay - y_0)$$

$$\frac{v^2 - v_0^2}{2a} = y - y_0$$

$$y = y0 + \frac{v^2 - v_0^2}{2a} = 0m + \frac{(0m/s)^2 - (13.0m/s)^2}{2(-9.80m/s^2)} = 8.62m$$

Dolphins measure about 2 meters long and can jump several times their length out of the water, so this is a reasonable result.
(c) $2.65 s$

74. A swimmer bounces straight up from a diving board and falls feet first into a pool. She starts with a velocity of 4.00 m/s, and her takeoff point is 1.80 m above the pool.

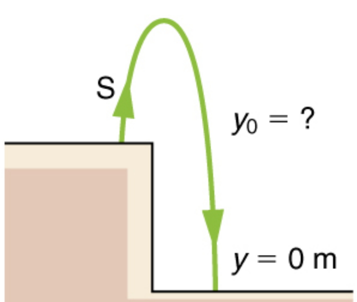
(a) How long are her feet in the air?

(b) What is her highest point above the board?

(c) What is her velocity when her feet hit the water?

75. (a) Calculate the height of a cliff if it takes 2.35 s for a rock to hit the ground when it is thrown straight up from the cliff with an initial velocity of 8.00 m/s.

(b) How long would it take to reach the ground if it is thrown straight down with the same speed?



Solution
(a) 8.26 m
(b) 0.717 s

76. A very strong, but inept, shot putter puts the shot straight up vertically with an initial velocity of 11.0 m/s. How long does he have to get out of the way if the shot was released at a height of 2.20 m, and he is 1.80 m tall?

77. You throw a ball straight up with an initial velocity of 15.0 m/s. It passes a tree branch on the way up at a height of 7.00 m. How much additional time will pass before the ball passes the tree branch on the way back down?

Solution
1.91 s

78. A kangaroo can jump over an object 2.50 m high.

(a) Calculate its vertical speed when it leaves the ground.

(b) How long is it in the air?

79. Standing at the base of one of the cliffs of Mt. Arapiles in Victoria, Australia, a hiker hears a rock break loose from a height of 105 m. He can't see the rock right away but then does, 1.50 s later.

(a) How far above the hiker is the rock when he can see it?

(b) How much time does he have to move before the rock hits his head?

Solution
(a) 94.0 m
(b) 3.13 s

80. An object is dropped from a height of 75.0 m above ground level.

(a) Determine the distance traveled during the first second.

(b) Determine the final velocity at which the object hits the ground. (c) Determine the distance traveled during the last second of motion before hitting the ground.

81. There is a 250-m-high cliff at Half Dome in Yosemite National Park in California. Suppose a boulder breaks loose from the top of this cliff.

(a) How fast will it be going when it strikes the ground?

(b) Assuming a reaction time of 0.300 s, how long will a tourist at the bottom have to get out of the way after hearing the sound of the rock breaking loose (neglecting the height of the tourist, which would become negligible anyway if hit)? The speed of sound is 335 m/s on this day.

Solution
(a) -70.0 m/s (downward)
(b) 6.10 s

82. A ball is thrown straight up. It passes a 2.00-m-high window 7.50 m off the ground on its path up and takes 0.312 s to go past the window. What was the ball's initial velocity? *Hint:* First consider only the distance along the window, and solve for the ball's velocity at the bottom of the window. Next, consider only the distance from the ground to the bottom of the window, and solve for the initial velocity using the velocity at the bottom of the window as the final velocity.

83. Suppose you drop a rock into a dark well and, using precision equipment, you measure the time for the sound of a splash to return.

(a) Neglecting the time required for sound to travel up the well, calculate the distance to the water if the sound returns in 2.0000 s.

(b) Now calculate the distance taking into account the time for sound to travel up the well. The speed of sound is 332.00 m/s in this well.

Solution
(a) $19.6m$
(b) $18.5m$

84. A steel ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.45 m.

(a) Calculate its velocity just before it strikes the floor.

(b) Calculate its velocity just after it leaves the floor on its way back up.

(c) Calculate its acceleration during contact with the floor if that contact lasts 0.0800 ms $(8.00 \times 10^{-5} s)$

(d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

85. A coin is dropped from a hot-air balloon that is 300 m above the ground and rising at 10.0 m/s upward. For the coin, find

(a) the maximum height reached,

(b) its position and velocity 4.00 s after being released, and

(c) the time before it hits the ground.

Solution
(a) 305 m
(b) 262 m, -29.2 m/s
(c) 8.91 s

86. A soft tennis ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.10 m.

(a) Calculate its velocity just before it strikes the floor.

(b) Calculate its velocity just after it leaves the floor on its way back up.

(c) Calculate its acceleration during contact with the floor if that contact lasts 3.50 ms $(3.50 \times 10^{-3} s)$

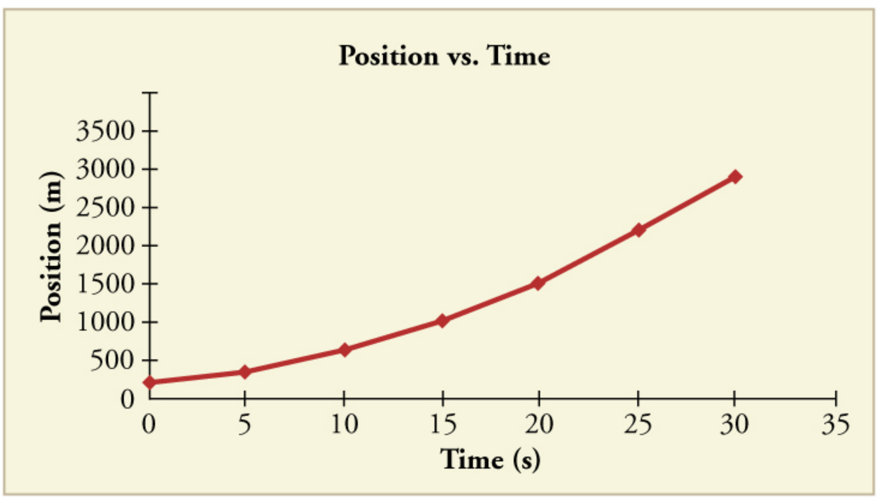
(d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

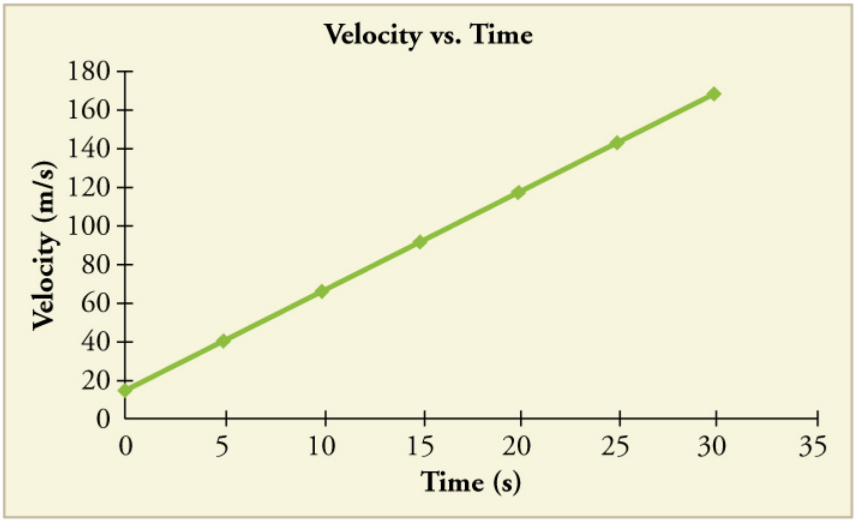
2.8: Graphical Analysis of One-Dimensional Motion

Note: There is always uncertainty in numbers taken from graphs. If your answers differ from expected values, examine them to see if they are within data extraction uncertainties estimated by you.

87. (a) By taking the slope of the curve in Figure, verify that the velocity of the jet car is 115 m/s at $t = 20$ s.

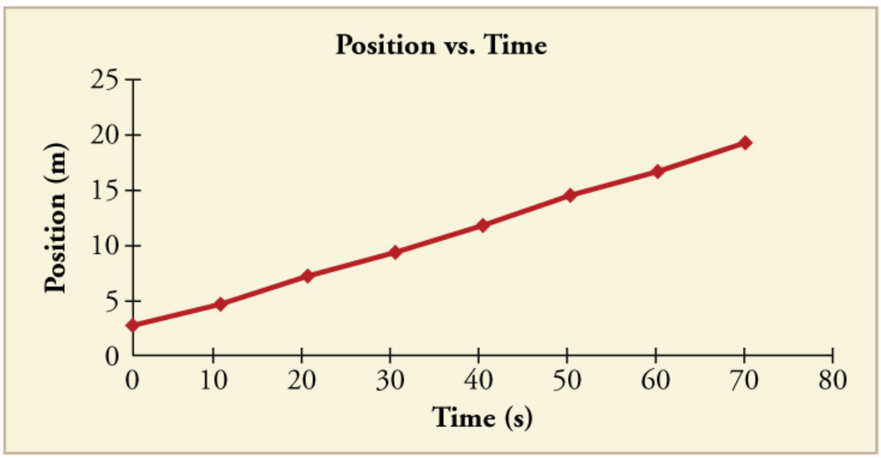
(b) By taking the slope of the curve at any point in Figure, verify that the jet car's acceleration is $5.0 m/s^2$





Solution
(a) $115 m/s$
(b) $5.0 m/s^2$

88. Using approximate values, calculate the slope of the curve in Figure to verify that the velocity at $t = 10.0s$ is 0.208 m/s. Assume all values are known to 3 significant figures.

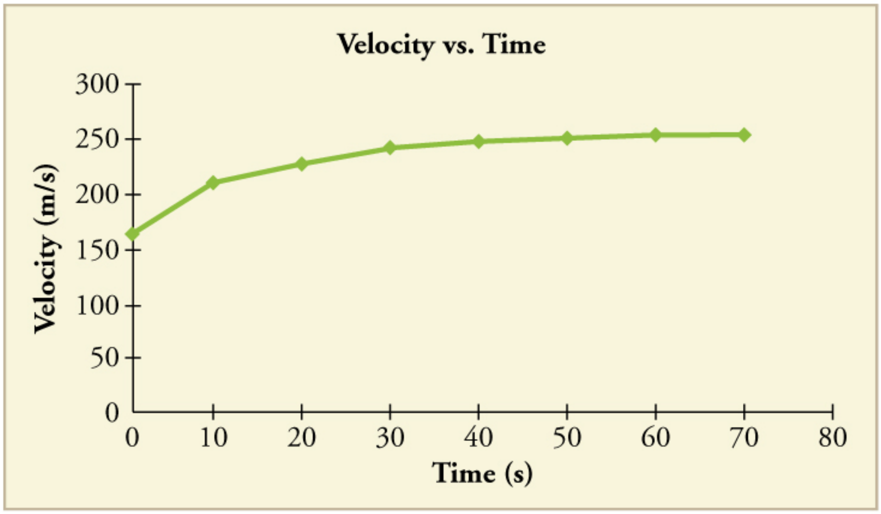


89. Using approximate values, calculate the slope of the curve in above Figure to verify that the velocity at $t = 30.0s$ is approximately 0.24 m/s.

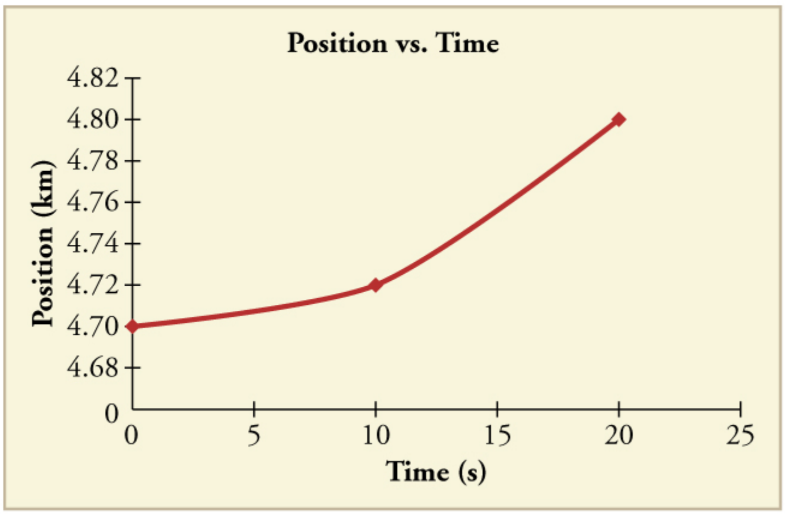
Solution

$$v = \frac{(11.7 - 6.95) \times 10^3 m}{(40.0 - 20.0)s} = 238 m/s$$

90. By taking the slope of the curve in Figure, verify that the acceleration is $3.2 m/s^2$ at $t = 10s$

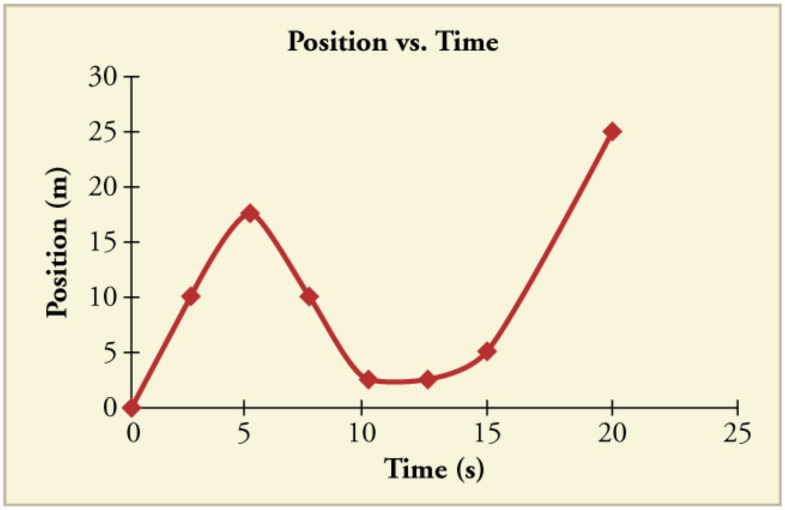


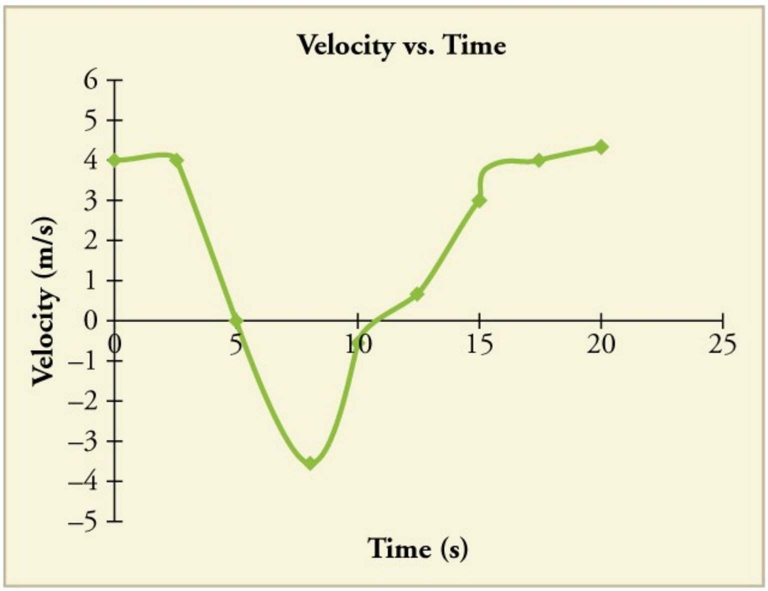
91. Construct the position graph for the subway shuttle train as shown in [link](a). Your graph should show the position of the train, in kilometers, from t = 0 to 20 s. You will need to use the information on acceleration and velocity given in the examples for this figure.

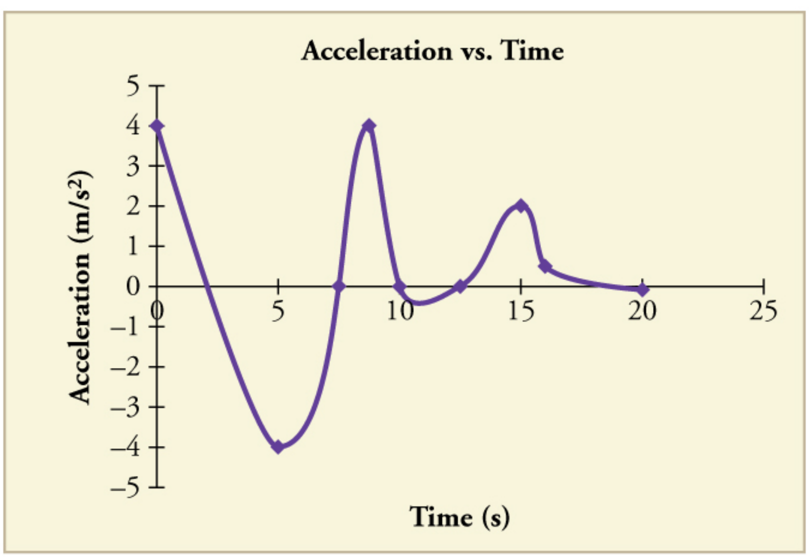


92. (a) Take the slope of the curve in Figure to find the jogger's velocity at $t = 2.5 s$.

(b) Repeat at 7.5 s. These values must be consistent with the graph in Figure.







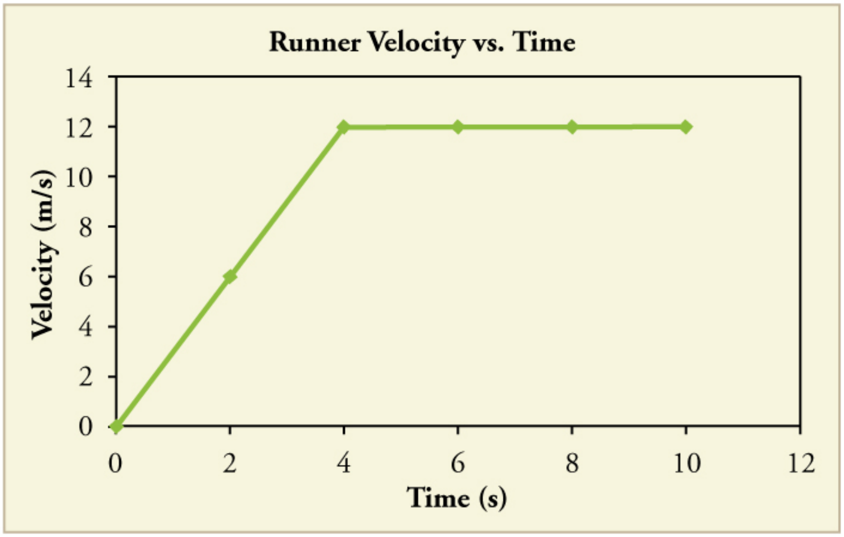
93. A graph of $v(t)$ is shown for a world-class track sprinter in a 100-m race. (See Figure).

(a) What is his average velocity for the first 4 s?

(b) What is his instantaneous velocity at $t = 5s$?

(c) What is his average acceleration between 0 and 4 s?

(d) What is his time for the race?



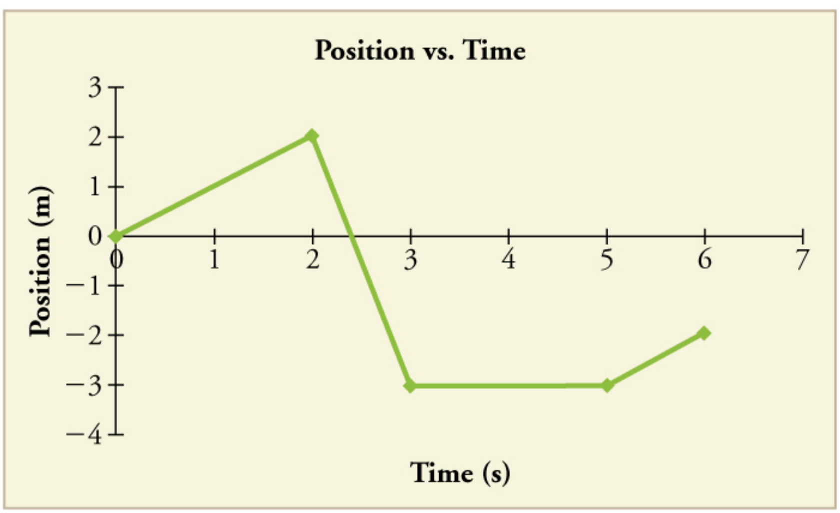
Solution
(a) $6m/s$
(b) $12m/s$
(c) $3m/s^2$
(d) $10s$

94. Figure shows the position graph for a particle for 6 s.

(a) Draw the corresponding Velocity vs. Time graph.

(b) What is the acceleration between 0 s and 2 s?

(c) What happens to the acceleration at exactly 2 s?



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CHAPTER OVERVIEW

2: Two-Dimensional Kinematics

The arc of a basketball, the orbit of a satellite, a bicycle rounding a curve, a swimmer diving into a pool, blood gushing out of a wound, and a puppy chasing its tail are but a few examples of motions along curved paths. In fact, most motions in nature follow curved paths rather than straight lines. Motion along a curved path on a flat surface or a plane (such as that of a ball on a pool table or a skater on an ice rink) is two-dimensional, and thus described by two-dimensional kinematics.

2.1: Prelude to Two-Dimensional Kinematics

The arc of a basketball, the orbit of a satellite, a bicycle rounding a curve, a swimmer diving into a pool, blood gushing out of a wound, and a puppy chasing its tail are but a few examples of motions along curved paths. In fact, most motions in nature follow curved paths rather than straight lines. Motion along a curved path on a flat surface or a plane (such as that of a ball on a pool table or a skater on an ice rink) is two-dimensional, and thus described by two-dimensional kinematics. Motion not confined to a plane, such as a car following a winding mountain road, is described by three-dimensional kinematics. Both two- and three-dimensional kinematics are simple extensions of the one-dimensional kinematics developed for straight-line motion in the previous chapter. This simple extension will allow us to apply physics to many more situations, and it will also yield unexpected insights about nature.

Figure 2.1.1: Everyday motion that we experience is, thankfully, rarely as tortuous as a rollercoaster ride like this—the Dragon Khan in Spain's Universal Port Aventura Amusement Park. However, most motion is in curved, rather than straight-line, paths. Motion along a curved path is two- or three-dimensional motion, and can be described in a similar fashion to one-dimensional motion. (credit: Boris23/Wikimedia Commons)

2.2: Kinematics in Two Dimensions - An Introduction

Learning Objectives

By the end of this section, you will be able to:

- Observe that motion in two dimensions consists of horizontal and vertical components.
- Understand the independence of horizontal and vertical vectors in two-dimensional motion.

Figure 2.2.1: Walkers and drivers in a city like New York are rarely able to travel in straight lines to reach their destinations. Instead, they must follow roads and sidewalks, making two-dimensional, zigzagged paths. (credit: Margaret W. Carruthers)

Two-Dimensional Motion: Walking in a City

Suppose you want to walk from one point to another in a city with uniform square blocks, as pictured in Figure 2.2.2.

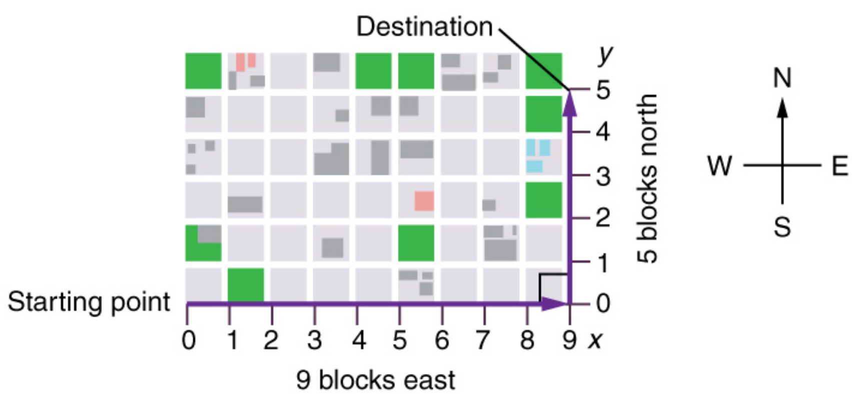


Figure 2.2.2: A pedestrian walks a two-dimensional path between two points in a city. In this scene, all blocks are square and are the same size.

The straight-line path that a helicopter might fly is blocked to you as a pedestrian, and so you are forced to take a two-dimensional path, such as the one shown. You walk 14 blocks in all, 9 east followed by 5 north. What is the straight-line distance?

An old adage states that the shortest distance between two points is a straight line. The two legs of the trip and the straight-line path form a right triangle, and so the Pythagorean theorem,

$$a^2 + b^2 = c^2 \tag{2.2.1}$$

can be used to find the straight-line distance.

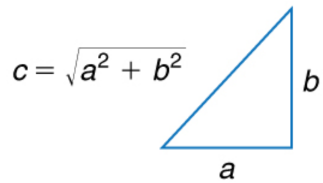


Figure 2.2.3: The Pythagorean theorem relates the length of the legs of a right triangle, labeled a and b, with the hypotenuse, labeled c. The relationship is given by: $a^2 + b^2 = c^2$. This can be rewritten, solving for $c : c = \sqrt{a^2 + b^2}$.

The hypotenuse of the triangle is the straight-line path, and so in this case its length in units of city blocks is $\sqrt{(9 blocks)^2 + (5 blocks)^2} = 10.3 blocks$, considerably shorter than the 14 blocks you walked. (Note that we are using three significant figures in the answer. Although it appears that "9" and "5" have only one significant digit, they are discrete numbers. In this case "9 blocks" is the same as "9.0 or 9.00 blocks." We have decided to use three significant figures in the answer in order to show the result more precisely.)

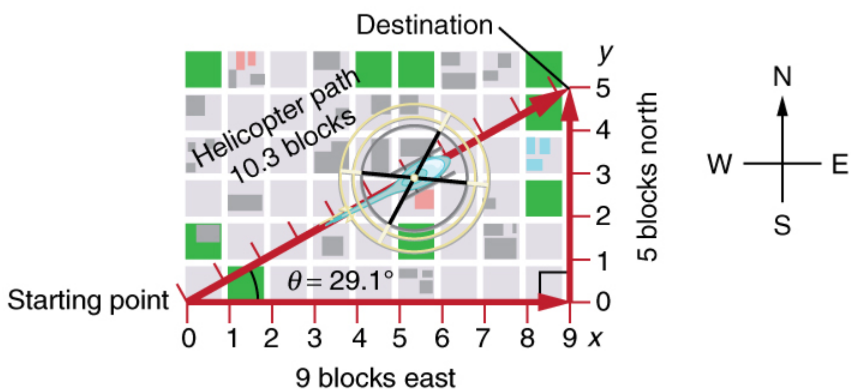


Figure 2.2.4: The straight-line path followed by a helicopter between the two points is shorter than the 14 blocks walked by the pedestrian. All blocks are square and the same size.

The fact that the straight-line distance (10.3 blocks) in Figure is less than the total distance walked (14 blocks) is one example of a general characteristic of vectors. (Recall that vectors are quantities that have both magnitude and direction.)

As for one-dimensional kinematics, we use arrows to represent vectors. The length of the arrow is proportional to the vector's magnitude. The arrow's length is indicated by hash marks in Figure and Figure. The arrow points in the same direction as the vector. For two-dimensional motion, the path of an object can be represented with three vectors: one vector shows the straight-line path between the initial and final points of the motion, one vector shows the horizontal component of the motion, and one vector shows the vertical component of the motion. The horizontal and vertical components of the motion add together to give the straight-line path. For example, observe the three vectors in Figure. The first represents a 9-block displacement east. The second represents a 5-block displacement north. These vectors are added to give the third vector, with a 10.3-block total displacement. The third vector is the straight-line path between the two points. Note that in this example, the vectors that we are adding are perpendicular to each other and thus form a right triangle. This means that we can use the Pythagorean theorem to calculate the magnitude of the total displacement. (Note that we cannot use the Pythagorean theorem to add vectors that are not perpendicular. We will develop techniques for adding vectors having any direction, not just those perpendicular to one another, in Vector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods.)

The Independence of Perpendicular Motions

The person taking the path shown in Figure walks east and then north (two perpendicular directions). How far he or she walks east is only affected by his or her motion eastward. Similarly, how far he or she walks north is only affected by his or her motion northward.

INDEPENDENCE OF MOTION

The horizontal and vertical components of two-dimensional motion are independent of each other. Any motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

This is true in a simple scenario like that of walking in one direction first, followed by another. It is also true of more complicated motion involving movement in two directions at once. For example, let's compare the motions of two baseballs. One baseball is dropped from rest. At the same instant, another is thrown horizontally from the same height and follows a curved path. A stroboscope has captured the positions of the balls at fixed time intervals as they fall.

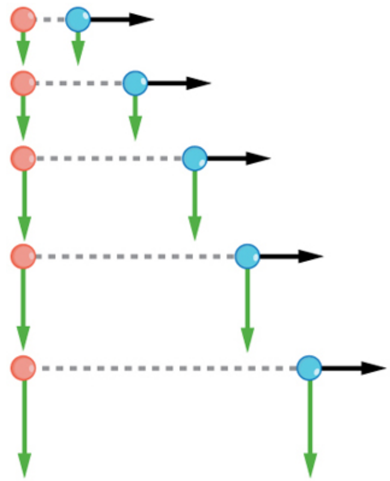


Figure 2.2.5: This shows the motions of two identical balls—one falls from rest, the other has an initial horizontal velocity. Each subsequent position is an equal time interval. Arrows represent horizontal and vertical velocities at each position. The ball on the right has an initial horizontal velocity, while the ball on the left has no horizontal velocity. Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls. This shows that the vertical and horizontal motions are independent.

It is remarkable that for each flash of the strobe, the vertical positions of the two balls are the same. This similarity implies that the vertical motion is independent of whether or not the ball is moving horizontally. (Assuming no air resistance, the vertical motion of a falling object is influenced by gravity only, and not by any horizontal forces.) Careful examination of the ball thrown horizontally shows that it travels the same horizontal distance between flashes. This is due to the fact that there are no additional forces on the ball in the horizontal direction after it is thrown. This result means that the horizontal velocity is constant, and affected neither by vertical motion nor by gravity (which is vertical). Note that this case is true only for ideal conditions. In the real world, air resistance will affect the speed of the balls in both directions.

The two-dimensional curved path of the horizontally thrown ball is composed of two independent one-dimensional motions (horizontal and vertical). The key to analyzing such motion, called *projectile motion*, is to *resolve* (break) it into motions along perpendicular directions. Resolving two-dimensional motion into perpendicular components is possible because the components are independent. We shall see how to resolve vectors in Vector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods. We will find such techniques to be useful in many areas of physics.

PHET EXPLORATIONS: LADYBUG MOTION 2D

Learn about position, velocity and acceleration vectors. Move the ladybug by setting the position, velocity or acceleration, and see how the vectors change. Choose linear, circular or elliptical motion, and record and playback the motion to analyze the behavior.

Figure 2.2.5: Ladybug Motion 2D

Summary

- The shortest path between any two points is a straight line. In two dimensions, this path can be represented by a vector with horizontal and vertical components.
- The horizontal and vertical components of a vector are independent of one another. Motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

Glossary

vector
: a quantity that has both magnitude and direction; an arrow used to represent quantities with both magnitude and direction

---

2.3: Vector Addition and Subtraction- Graphical Methods

Learning Objectives

By the end of this section, you will be able to:

- Understand the rules of vector addition, subtraction, and multiplication.
- Apply graphical methods of vector addition and subtraction to determine the displacement of moving objects.

Vector Addition and Subtraction: Graphical Methods

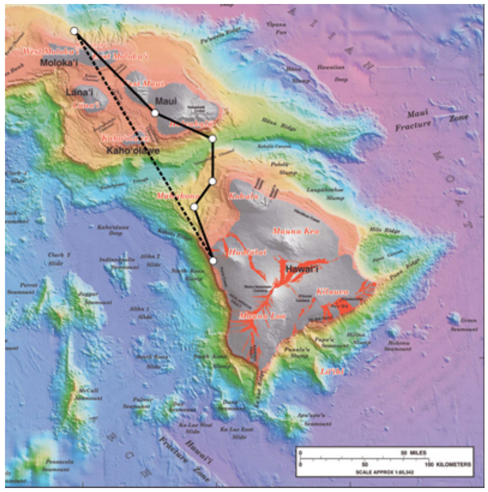


Figure 2.3.1: Displacement can be determined graphically using a scale map, such as this one of the Hawaiian Islands. A journey from Hawai'i to Moloka'i has a number of legs, or journey segments. These segments can be added graphically with a ruler to determine the total two-dimensional displacement of the journey. (credit: US Geological Survey)

Vectors in Two Dimensions

A vector is a quantity that has magnitude and direction. Displacement, velocity, acceleration, and force, for example, are all vectors. In one-dimensional, or straight-line, motion, the direction of a vector can be given simply by a plus or minus sign. In two dimensions (2-d), however, we specify the direction of a vector relative to some reference frame (i.e., coordinate system), using an arrow having length proportional to the vector's magnitude and pointing in the direction of the vector.

Figure shows such a *graphical representation of a vector*, using as an example the total displacement for the person walking in a city considered in Kinematics in Two Dimensions: An Introduction. We shall use the notation that a boldface symbol, such as D, stands for a vector. Its magnitude is represented by the symbol in italics, D, and its direction by θ.

VECTORS IN THIS TEXT

In this text, we will represent a vector with a boldface variable. For example, we will represent the quantity force with the vector F, which has both magnitude and direction. The magnitude of the vector will be represented by a variable in italics, such as F, and the direction of the variable will be given by an angle θ.

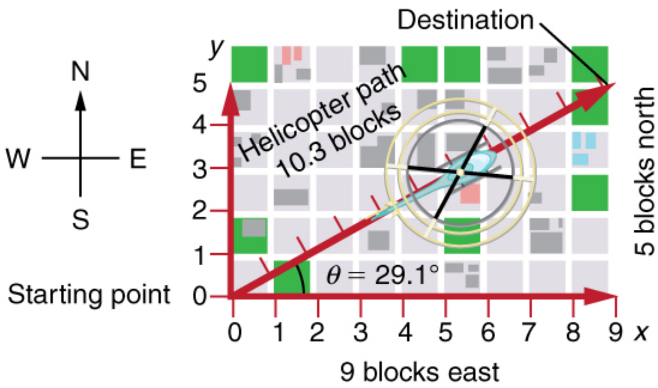


Figure $2.3.2$***:*** *A person walks 9 blocks east and 5 blocks north. The displacement is 10.3 blocks at an angle .1º north of east.*

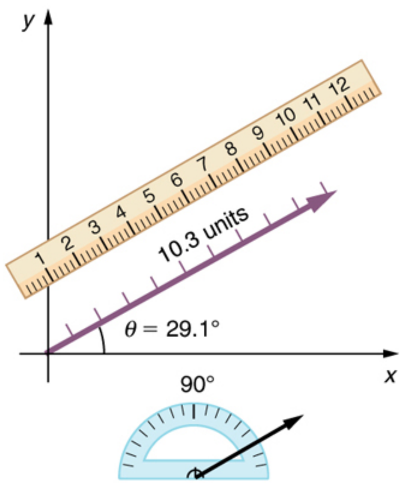


Figure graphically, draw an arrow to represent the total displacement vector D. Using a protractor, draw a line at an angle θ relative to the east-west axis. The length D of the arrow is proportional to the vector's magnitude and is measured along the line with a ruler. In this example, the magnitude D of the vector is 10.3 units, and the direction θ is 29.1º north of east.

Vector Addition: Head-to-Tail Method

The **head-to-tail method** is a graphical way to add vectors, described in Figure below and in the steps following. The **tail** of the vector is the starting point of the vector, and the **head** (or tip) of a vector is the final, pointed end of the arrow.

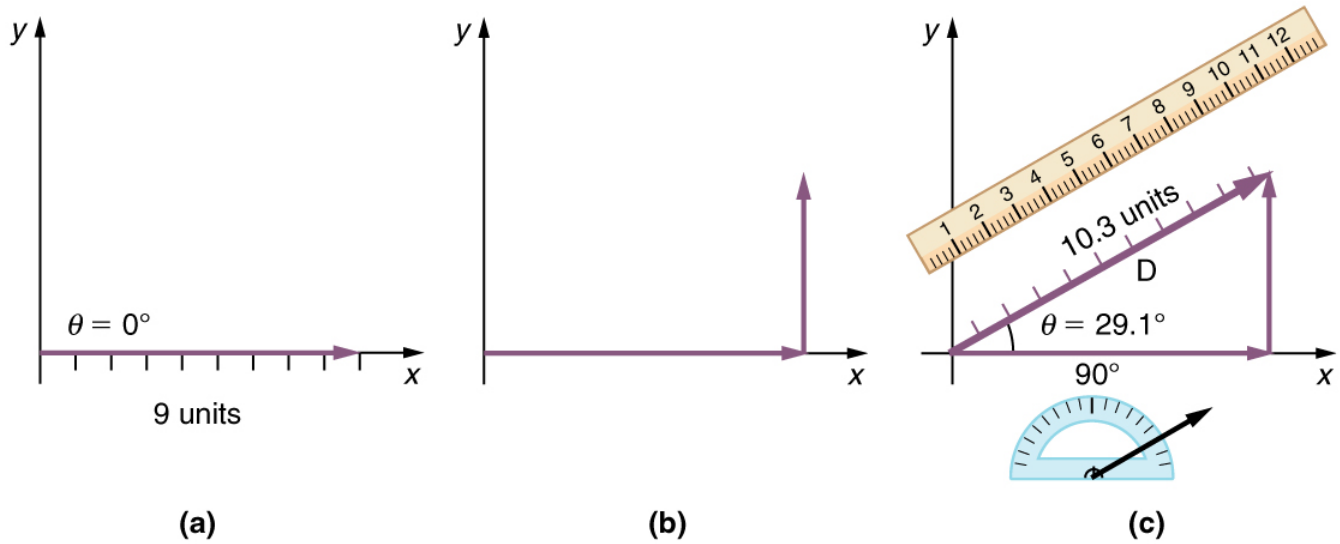


Figure. (a) Draw a vector representing the displacement to the east. (b) Draw a vector representing the displacement to the north. The tail of this vector should originate from the head of the first, east-pointing vector. (c) Draw a line from the tail of the east-pointing vector to the head of the north-pointing vector to form the sum or resultant vector D. The length of the arrow D is proportional to the vector's magnitude and is measured to be 10.3 units . Its direction, described as the angle with respect to the east (or horizontal axis) θ is measured with a protractor to be 29.1°.

Step 1. *Draw an arrow to represent the first vector (9 blocks to the east) using a ruler and protractor*.

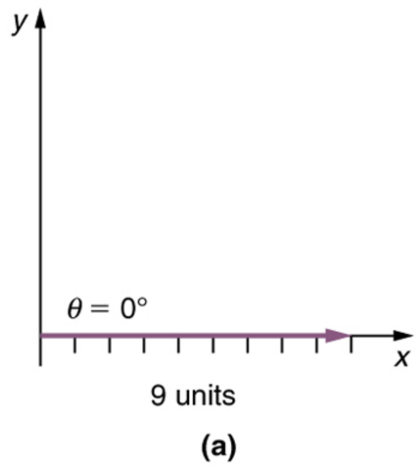
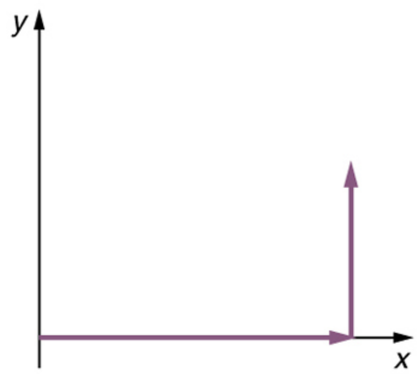


Figure 2.3.5

Step 2. *Now draw an arrow to represent the second vector (5 blocks to the north). Place the tail of the second vector at the head of the first vector.*

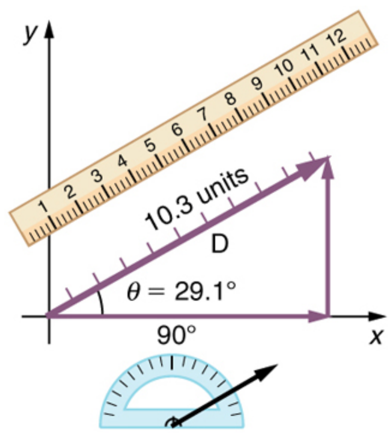


(b)

Figure 2.3.6

Step 3. *If there are more than two vectors, continue this process for each vector to be added. Note that in our example, we have only two vectors, so we have finished placing arrows tip to tail.*

Step 4. *Draw an arrow from the tail of the first vector to the head of the last vector*. This is the **resultant**, or the sum, of the other vectors.



(c)

Figure 2.3.7

Step 5. To get the **magnitude** of the resultant, *measure its length with a ruler. (Note that in most calculations, we will use the Pythagorean theorem to determine this length.)*

Step 6. To get the **direction** of the resultant, *measure the angle it makes with the reference frame using a protractor. (Note that in most calculations, we will use trigonometric relationships to determine this angle.)*

The graphical addition of vectors is limited in accuracy only by the precision with which the drawings can be made and the precision of the measuring tools. It is valid for any number of vectors.

Example 2.3.1:Adding Vectors Graphically Using the Head-to-Tail Method: A Woman Takes a Walk

Use the graphical technique for adding vectors to find the total displacement of a person who walks the following three paths (displacements) on a flat field. First, she walks 25.0 m in a direction north of east. Then, she walks 23.0 m heading north of east. Finally, she turns and walks 32.0 m in a direction **68.0°** south of east.

Strategy

Represent each displacement vector graphically with an arrow, labeling the first , the second , and the third , making the lengths proportional to the distance and the directions as specified relative to an east-west line. The head-to-tail method outlined above will give a way to determine the magnitude and direction of the resultant displacement, denoted .

Solution

(1) Draw the three displacement vectors.

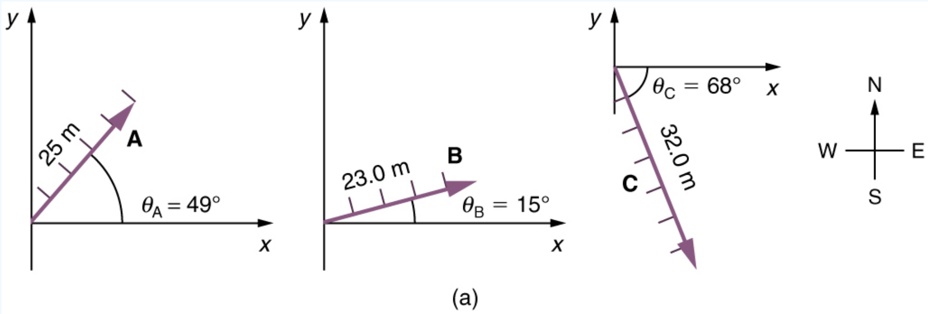


Figure 2.3.8

(2) Place the vectors head to tail retaining both their initial magnitude and direction.

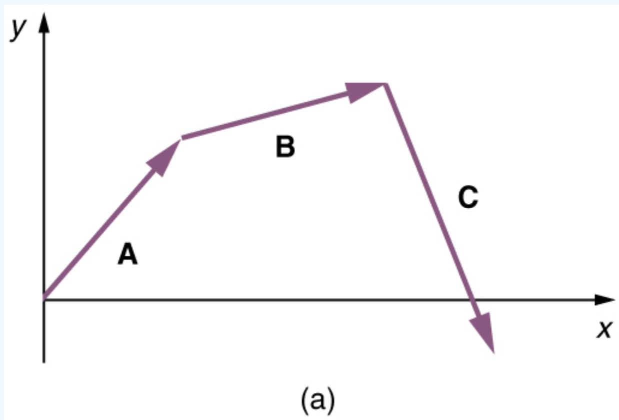


Figure 2.3.9

(3) Draw the resultant vector, .

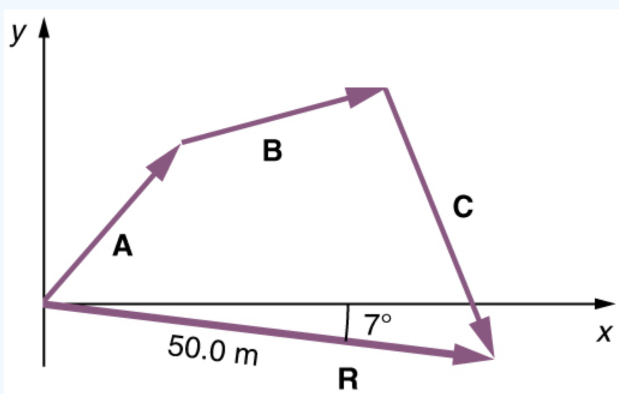


Figure 2.3.10

(4) Use a ruler to measure the magnitude of , and a protractor to measure the direction of . While the direction of the vector can be specified in many ways, the easiest way is to measure the angle between the vector and the nearest horizontal or vertical axis. Since the resultant vector is south of the eastward pointing axis, we flip the protractor upside down and measure the angle between the eastward axis and the vector.

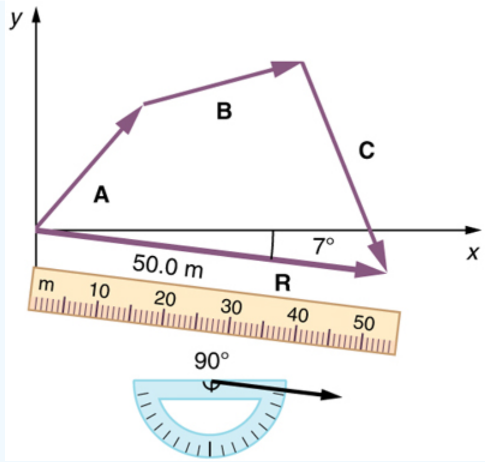


Figure 2.3.11

In this case, the total displacement is seen to have a magnitude of 50.0 m and to lie in a direction south of east. By using its magnitude and direction, this vector can be expressed as **=50.0 m** and =7.0º south of east.

Discussion

The head-to-tail graphical method of vector addition works for any number of vectors. It is also important to note that the resultant is independent of the order in which the vectors are added. Therefore, we could add the vectors in any order as illustrated in Figure and we will still get the same solution.

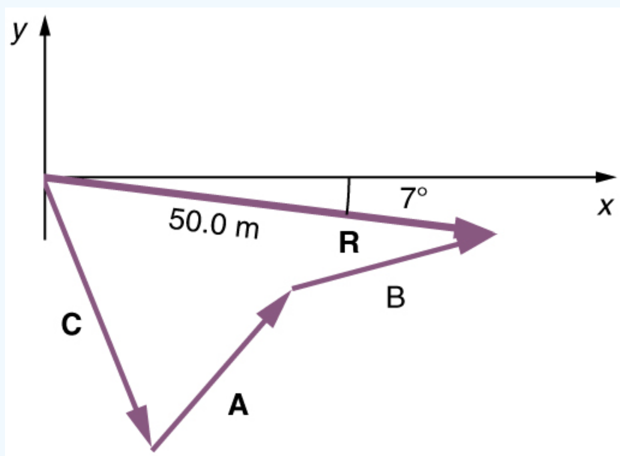


Figure 2.3.12

Here, we see that when the same vectors are added in a different order, the result is the same. This characteristic is true in every case and is an important characteristic of vectors. Vector addition is **commutative**. Vectors can be added in any order.

$$A+B=B+A.$$

(This is true for the addition of ordinary numbers as well—you get the same result whether you add **+3** or +2, for example).

Vector Subtraction

Vector subtraction is a straightforward extension of vector addition. To define subtraction (say we want to subtract from , written **– B** , we must first define what we mean by subtraction. The *negative* of a vector is defined to be ; that is, graphically *the negative of any vector has the same magnitude but the opposite direction*, as shown in Figure. In other words, has the same length as , but points in the opposite direction. Essentially, we just flip the vector so it points in the opposite direction.

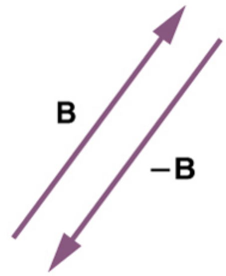


Figure 2.3.13: The negative of a vector is just another vector of the same magnitude but pointing in the opposite direction. So is the negative of ; it has the same length but opposite direction.

The *subtraction* of vector from vector is then simply defined to be the addition of to . Note that vector subtraction is the addition of a negative vector. The order of subtraction does not affect the results.

A – B = A + (–B).

This is analogous to the subtraction of scalars (where, for example, (–2)**)**. Again, the result is independent of the order in which the subtraction is made. When vectors are subtracted graphically, the techniques outlined above are used, as the following example illustrates.

Example 2.3.1:Subtracting Vectors Graphically: A Woman Sailing a Boat

A woman sailing a boat at night is following directions to a dock. The instructions read to first sail 27.5 m in a direction north of east from her current location, and then travel 30.0 m in a direction north of east (or west of north). If the woman makes a mistake and travels in the *opposite* direction for the second leg of the trip, where will she end up? Compare this location with the location of the dock.

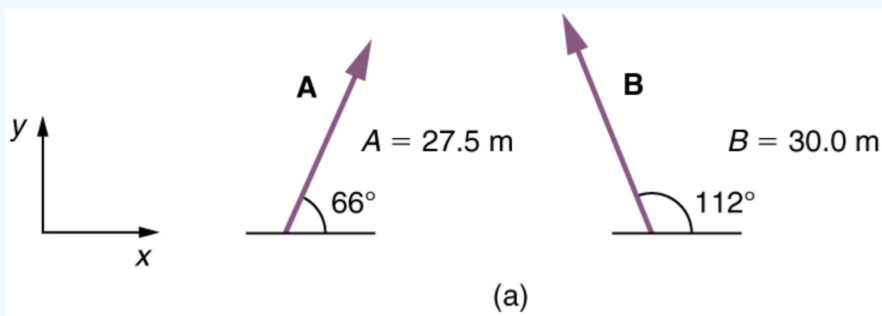


Figure 2.3.14

Strategy

We can represent the first leg of the trip with a vector , and the second leg of the trip with a vector . The dock is located at a location +B. If the woman mistakenly travels in the *opposite* direction for the second leg of the journey, she will travel a distance (30.0 m) in the direction **–112º=68º** south of east. We represent this as , as shown below. The vector has the same magnitude as but is in the opposite direction. Thus, she will end up at a location +(–B), or **–B**.

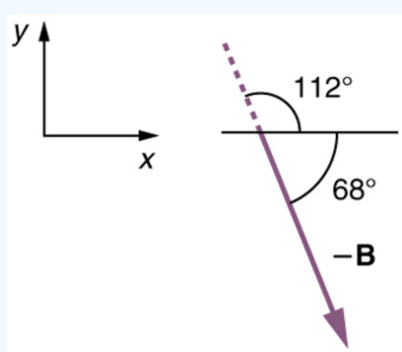


Figure 2.3.15

We will perform vector addition to compare the location of the dock, + B, with the location at which the woman mistakenly arrives, **+ (–B)**.

Solution

(1) To determine the location at which the woman arrives by accident, draw vectors and .

(2) Place the vectors head to tail.

(3) Draw the resultant vector .

(4) Use a ruler and protractor to measure the magnitude and direction of .

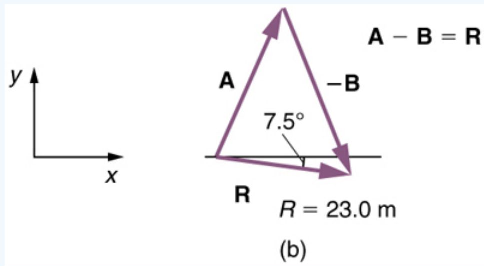


Figure 2.3.16

In this case, **=23.0 m** and **=7.5º** south of east.

(5) To determine the location of the dock, we repeat this method to add vectors and . We obtain the resultant vector **'**:

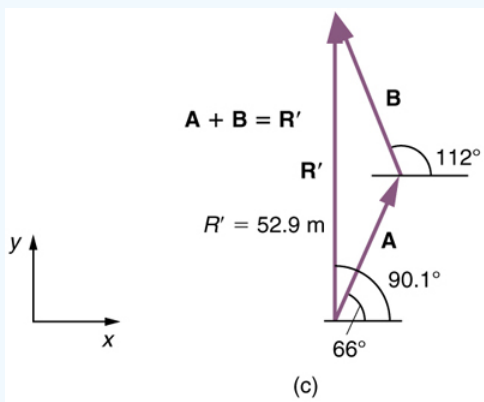


Figure 2.3.17

In this case **= 52.9 m** and **=90.1º** north of east.

We can see that the woman will end up a significant distance from the dock if she travels in the opposite direction for the second leg of the trip.

Discussion

Because subtraction of a vector is the same as addition of a vector with the opposite direction, the graphical method of subtracting vectors works the same as for addition.

Multiplication of Vectors and Scalars

If we decided to walk three times as far on the first leg of the trip considered in the preceding example, then we would walk **× 27.5 m**, or 82.5 m, in a direction **.0º** north of east. This is an example of multiplying a vector by a positive **scalar**. Notice that the magnitude changes, but the direction stays the same.

If the scalar is negative, then multiplying a vector by it changes the vector's magnitude and gives the new vector the *opposite*direction. For example, if you multiply by –2, the magnitude doubles but the direction changes. We can summarize these rules in the following way: When vector is multiplied by a scalar ,

- the magnitude of the vector becomes the absolute value of ,
- if is positive, the direction of the vector does not change,

- if is negative, the direction is reversed.

In our case, =3 and =27.5 m. Vectors are multiplied by scalars in many situations. Note that division is the inverse of multiplication. For example, dividing by 2 is the same as multiplying by the value (1/2). The rules for multiplication of vectors by scalars are the same for division; simply treat the divisor as a scalar between 0 and 1.

Resolving a Vector into Components

In the examples above, we have been adding vectors to determine the resultant vector. In many cases, however, we will need to do the opposite. We will need to take a single vector and find what other vectors added together produce it. In most cases, this involves determining the perpendicular **components** of a single vector, for example the *x- and y*-components, or the north-south and east-west components.

For example, we may know that the total displacement of a person walking in a city is 10.3 blocks in a direction **.0º** north of east and want to find out how many blocks east and north had to be walked. This method is called *finding the components (or parts)* of the displacement in the east and north directions, and it is the inverse of the process followed to find the total displacement. It is one example of finding the components of a vector. There are many applications in physics where this is a useful thing to do. We will see this soon in Projectile Motion, and much more when we cover **forces** in Dynamics: Newton's Laws of Motion. Most of these involve finding components along perpendicular axes (such as north and east), so that right triangles are involved. The analytical techniques presented in Vector Addition and Subtraction: Analytical Methods are ideal for finding vector components.

PHET EXPLORATIONS: MAZE GAME

Learn about position, velocity, and acceleration in the "Arena of Pain". Use the green arrow to move the ball. Add more walls to the arena to make the game more difficult. Try to make a goal as fast as you can.

Figure 2.3.18: Maze Game

Summary

- The **graphical method of adding vectors** and involves drawing vectors on a graph and adding them using the head-to-tail method. The resultant vector is defined such that +B=R**.** The magnitude and direction of are then determined with a ruler and protractor, respectively.
- The **graphical method of subtracting vector** from involves adding the opposite of vector , which is defined as **B**. In this case, **–B=A+(–B)=R**. Then, the head-to-tail method of addition is followed in the usual way to obtain the resultant vector .
- Addition of vectors is **commutative** such that **+B=B+A** .
- The **head-to-tail method** of adding vectors involves drawing the first vector on a graph and then placing the tail of each subsequent vector at the head of the previous vector. The resultant vector is then drawn from the tail of the first vector to the head of the final vector.
- If a vector is multiplied by a scalar quantity , the magnitude of the product is given by . If is positive, the direction of the product points in the same direction as ; if is negative, the direction of the product points in the opposite direction as .

Glossary

component (of a 2-d vector)

a piece of a vector that points in either the vertical or the horizontal direction; every 2-d vector can be expressed as a sum of two vertical and horizontal vector components

commutative

refers to the interchangeability of order in a function; vector addition is commutative because the order in which vectors are added together does not affect the final sum

direction (of a vector)
the orientation of a vector in space

head (of a vector)
the end point of a vector; the location of the tip of the vector’s arrowhead; also referred to as the “tip”

head-to-tail method
a method of adding vectors in which the tail of each vector is placed at the head of the previous vector

magnitude (of a vector)
the length or size of a vector; magnitude is a scalar quantity

resultant
the sum of two or more vectors

resultant vector
the vector sum of two or more vectors

scalar
a quantity with magnitude but no direction

tail
the start point of a vector; opposite to the head or tip of the arrow

---

2.4: Vector Addition and Subtraction- Analytical Methods

Learning Objectives

By the end of this section, you will be able to:

- Understand the rules of vector addition and subtraction using analytical methods.
- Apply analytical methods to determine vertical and horizontal component vectors.
- Apply analytical methods to determine the magnitude and direction of a resultant vector.

Analytical methods of vector addition and subtraction employ geometry and simple trigonometry rather than the ruler and protractor of graphical methods. Part of the graphical technique is retained, because vectors are still represented by arrows for easy visualization. However, analytical methods are more concise, accurate, and precise than graphical methods, which are limited by the accuracy with which a drawing can be made. Analytical methods are limited only by the accuracy and precision with which physical quantities are known.

Resolving a Vector into Perpendicular Components

Analytical techniques and right triangles go hand-in-hand in physics because (among other things) motions along perpendicular directions are independent. We very often need to separate a vector into perpendicular components. For example, given a vector like A in Figure $2.4.1$, we may wish to find which two perpendicular vectors, A_x and A_y, add to produce it.

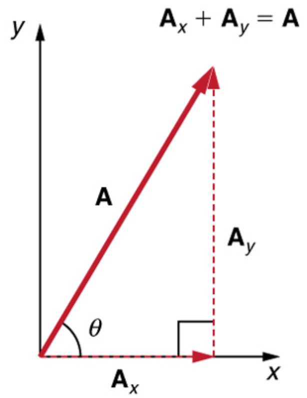


Figure 2.4.1:The vector A, with its tail at the origin of an x, y-coordinate system, is shown together with its x- and y-components, A_x and A_y. These vectors form a right triangle. The analytical relationships among these vectors are summarized below.

A_x and A_y are defined to be the components of A along the x- and y-axes. The three vectors A, A_x, and A_y form a right triangle:

$$A_x + A_y = A. \tag{2.4.1}$$

Note that this relationship between vector components and the resultant vector holds only for vector quantities (which include both magnitude and direction). The relationship does not apply for the magnitudes alone. For example, if $A_x = 3m$ east, $A_y = 4m$ north, and $A = 5m$north-east, then it is true that the vectors $A_x + A_y = A$. However, it is not true that the sum of the magnitudes of the vectors is also equal. That is,

$$3m + 4m \neq 5m \tag{2.4.2}$$

Thus,

$$A_x + A_y \neq A \tag{2.4.3}$$

If the vector A is known, then its magnitude A (its length) and its angle θ (its direction) are known. To find A_x and A_y, its x- and y-components, we use the following relationships for a right triangle.

$$A_x = Acos\theta \tag{2.4.4}$$

and

$$A_y = Asin\theta. \tag{2.4.5}$$

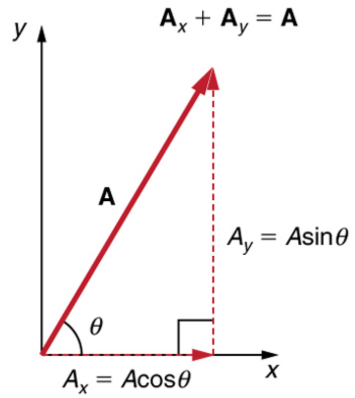


Figure 2.4.2: The magnitudes of the vector components A_x and A_y can be related to the resultant vector A and the angle θ with trigonometric identities. Here we see that $A_x = Acos\theta$ and $A_y = Asin\theta$.

Suppose, for example, that **A** is the vector representing the total displacement of the person walking in a city considered in Kinematics in Two Dimensions: An Introduction and Vector Addition and Subtraction: Graphical Methods.

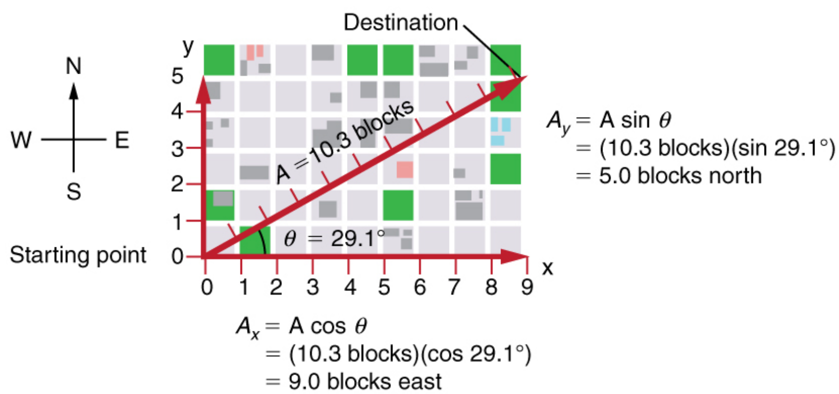


Figure 2.4.3: We can use the relationships $A_x = Acos\theta$ and $A_y = Asin\theta$ to determine the magnitude of the horizontal and vertical component vectors in this example.

Then $A = 10.3$ blocks and $\theta = 29.1^{\circ}$, so that

$$A_x = A\cos\theta = (10.3 blocks)(\cos 29.1^{\circ}) = 9.0 \quad blocks \tag{2.4.6}$$

$$A_y = A\sin\theta = (10.3 blocks)(\sin 29.1^{\circ}) = 5.0 \quad blocks \tag{2.4.7}$$

Calculating a Resultant Vector

If the perpendicular components A_x and A_y of a vector A are known, then A can also be found analytically. To find the magnitude A and direction θ of a vector from its perpendicular components A_x and A_y, we use the following relationships:

$$A = \sqrt{A_{x^2} + A_{y^2}} \tag{2.4.8}$$

$$\theta = tan^{-1}(A_y/A_x) \tag{2.4.9}$$

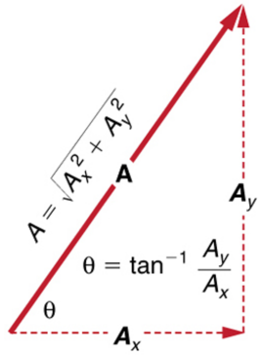


Figure 2.4.4: The magnitude and direction of the resultant vector can be determined once the horizontal and vertical components A_x and A_y have been determined.

Note that the equation $A=\sqrt{A_x^2+A_y^2}$ is just the Pythagorean theorem relating the legs of a right triangle to the length of the hypotenuse. For example, if A_x and A_y are 9 and 5 blocks, respectively, then $A=\sqrt{9^2+5^2}=10.3$ blocks, again consistent with the example of the person walking in a city. Finally, the direction is $\theta=tan^{-1}(5/9)=29.1^{\circ}$, as before.

DETERMINING VECTORS AND VECTOR COMPONENTS WITH ANALYTICAL METHODS

Equations $A_x=Acos\theta$ and $A_y=Asin\theta$ are used to find the perpendicular components of a vector—that is, to go from A and θ to A_x and A_y. Equations $A=\sqrt{A_x^2+A_y^2}$ and $\theta=tan^{-1}(A_y/A_x)$ are used to find a vector from its perpendicular components—that is, to go from A_x and A_y to A and θ. Both processes are crucial to analytical methods of vector addition and subtraction.

Adding Vectors Using Analytical Methods

To see how to add vectors using perpendicular components, consider Figure $2.4.5$, in which the vectors A and B are added to produce the resultant R.

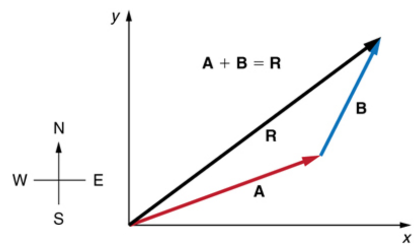


Figure 2.4.5: Vectors A and$B$ are two legs of a walk, and R is the resultant or total displacement. You can use analytical methods to determine the magnitude and direction of R.

If A and B represent two legs of a walk (two displacements), then R is the total displacement. The person taking the walk ends up at the tip of R. There are many ways to arrive at the same point. In particular, the person could have walked first in the x-direction and then in the y-direction. Those paths are the x- and y-components of the resultant, R_x and R_y. If we know R_x and R_y, we can find R and θ using the equations $A=\sqrt{A_x^2+A_y^2}$ and $\theta=tan^{-1}(A_y/A_x)$. When you use the analytical method of vector addition, you can determine the components or the magnitude and direction of a vector.

Step 1. *Identify the x- and y-axes that will be used in the problem. Then, find the components of each vector to be added along the chosen perpendicular axes.* Use the equations $A_x=Acos\theta$ and $A_y=Asin\theta$ to find the components. In Figure, these components are A_x, A_y, B_x, and B_y. The angles that vectors A and B make with the x-axis are θ_A and θ_B, respectively.

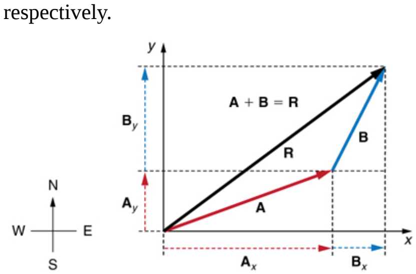


Figure 2.4.6: To add vectors A and B, first determine the horizontal and vertical components of each vector. These are the dotted vectors A_x, A_y, B_x and $B-y$ shown in the image.

Step 2. *Find the components of the resultant along each axis by adding the components of the individual vectors along that axis. That is, as shown in Figure* \(\PageIndex{7}\,

$$R_x = A_x + B_x \tag{2.4.10}$$

and

$$R_y = A_y + B_y. \tag{2.4.11}$$

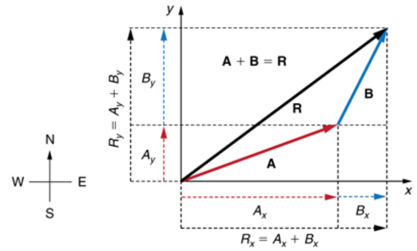


Figure 2.4.7: The magnitude of the vectors A_x and B_x add to give the magnitude R_x of the resultant vector in the horizontal direction. Similarly, the magnitudes of the vectors A_y and B_y add to give the magnitude R_y of the resultant vector in the vertical direction.

Components along the same axis, say the *x*-axis, are vectors along the same line and, thus, can be added to one another like ordinary numbers. The same is true for components along the *y*-axis. (For example, a 9-block eastward walk could be taken in two legs, the first 3 blocks east and the second 6 blocks east, for a total of 9, because they are along the same direction.) So resolving vectors into components along common axes makes it easier to add them. Now that the components of *R* are known, its magnitude and direction can be found.

Step 3. *To get the magnitude R of the resultant, use the Pythagorean theorem:*

$$R = \sqrt{R_x^2 + R_y^2} \tag{2.4.12}$$

Step 4. *To get the direction of the resultant:*

$$\theta = tan^{-1}(R_y/R_x) \tag{2.4.13}$$

The following example illustrates this technique for adding vectors using perpendicular components.

Example 2.4.1: Adding Vectors Using Analytical Methods

Add the vector A to the vector B shown in Figure, using perpendicular components along the *x*- and *y*-axes. The *x*- and *y*-axes are along the east–west and north–south directions, respectively. Vector A represents the first leg of a walk in which a person walks $53.0 m$ in a direction 20.0° north of east. Vector B represents the second leg, a displacement of $34.0 m$ in a direction 63.0° north of east.

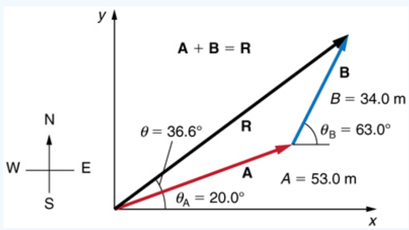


Figure 2.4.8: Vector A has magnitude $53.0 m$ and direction 20.0° north of the *x*-axis. Vector B has magnitude $34.0 m$ and direction 63.0° north of the x-axis. You can use analytical methods to determine the magnitude and direction of R.

Strategy

The components of A and B along the *x*- and *y*-axes represent walking due east and due north to get to the same ending point. Once found, they are combined to produce the resultant.

Solution

Following the method outlined above, we first find the components of A and B along the *x*- and *y*-axes. Note that $A = 53.0 m, \theta_A = 20.0^{\circ}, B = 34.0 m$, and $\theta_B = 63.0^{\circ}$. We find the *x*-components by using $A_x = A cos\theta$, which gives

$$A_x = A cos\theta_A = (53.0 m)(cos 20.0^{\circ})(53.0 m)(0.940) = 49.8 m \tag{2.4.14}$$

and

$$B_x = B cos\theta_B = (34.0 m)(cos 63.0^{\circ})(34.0 m)(0.454) = 15.4 m. \tag{2.4.15}$$

Similarly, the *y*-components are found using $A_y = A sin\theta_A$:

$$A_y = A sin\theta_A = (53.0 m)(sin 20.0^{\circ})(53.0 m)(0.342) = 18.1 m \tag{2.4.16}$$

and

$$B_y = B sin\theta_B = (34.0m)(sin 63.0^\circ)(34.0m)(0.891) = 30.3m. \tag{2.4.17}$$

The *x*- and *y*-components of the resultant are thus

$$R_x = A_x + B_x = 49.8m + 15.4m = 65.2m \tag{2.4.18}$$

and

$$R_y = A_y + B_y = 18.1m + 30.3m = 48.4m. \tag{2.4.19}$$

Now we can find the magnitude of the resultant by using the Pythagorean theorem:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(65.2)^2 + (48.4)^2 m} \tag{2.4.20}$$

so that

$$R = 81.2m. \tag{2.4.21}$$

Finally, we find the direction of the resultant:

$$\theta = tan^{-1}(R_y/R_x) = +tan^{-1}(48.4/65.2). \tag{2.4.22}$$

Thus,

$$\theta = tan^{-1}(0.742) = 36.6^\circ. \tag{2.4.23}$$

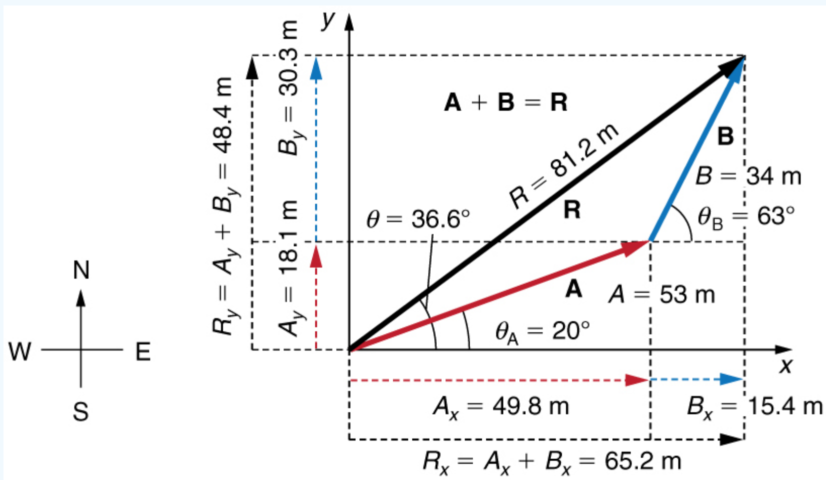


Figure 2.4.9: Using analytical methods, we see that the magnitude of R is $81.2m$ and its direction is 36.6° north of east.

Discussion

This example illustrates the addition of vectors using perpendicular components. Vector subtraction using perpendicular components is very similar—it is just the addition of a negative vector.

Subtraction of vectors is accomplished by the addition of a negative vector. That is, $A - B \equiv A + (-B)$. Thus, *the method for the subtraction of vectors using perpendicular components is identical to that for addition.* The components of $-B$ are the negatives of the components of B. The *x*- and *y*-components of the resultant $A - B = R$ are thus

$$R_x = A_x + (-B_x) \tag{2.4.24}$$

and

$$R_y = A_y + (-B_y) \tag{2.4.25}$$

and the rest of the method outlined above is identical to that for addition. (See Figure $2.4.10$)

Analyzing vectors using perpendicular components is very useful in many areas of physics, because perpendicular quantities are often independent of one another. The next module, Projectile Motion, is one of many in which using perpendicular components helps make the picture clear and simplifies the physics.

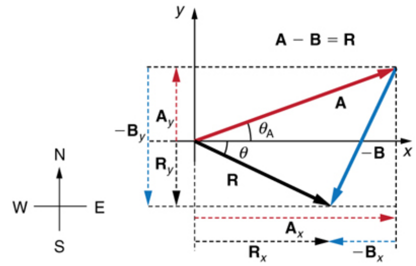
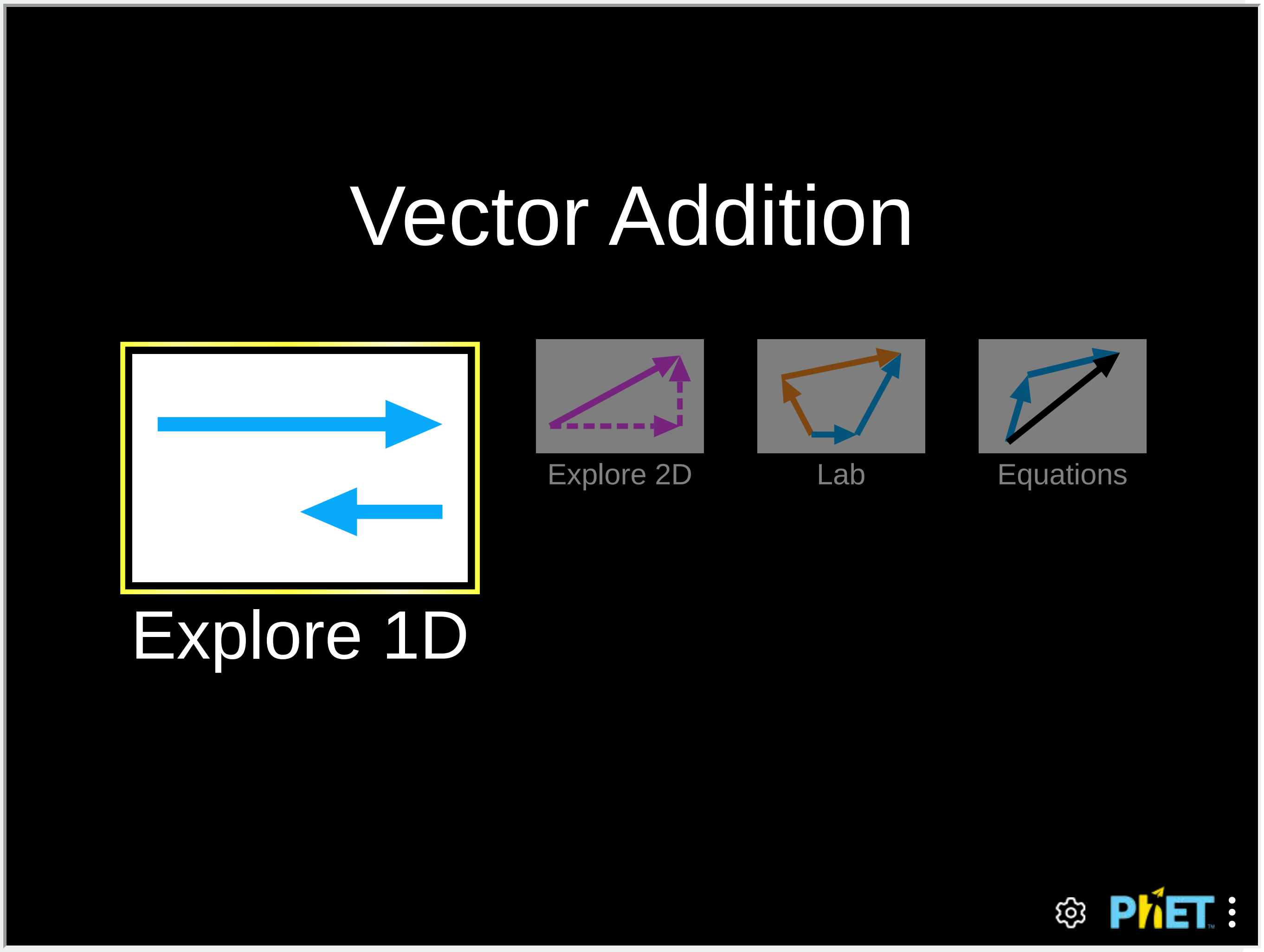


Figure 2.4.10. The components of $-B$ are the negatives of the components of B. The method of subtraction is the same as that for addition.

PHET EXPLORATIONS: VECTOR ADDITION

Learn how to add vectors. Drag vectors onto a graph, change their length and angle, and sum them together. The magnitude, angle, and components of each vector can be displayed in several formats.



Summary

- The analytical method of vector addition and subtraction involves using the Pythagorean theorem and trigonometric identities to determine the magnitude and direction of a resultant vector.
- The steps to add vectors A and B using the analytical method are as follows:

  Step 1: Determine the coordinate system for the vectors. Then, determine the horizontal and vertical components of each vector using the equations

$$A_x = Acos\theta$$
$$B_x = Bcos\theta$$

  and

$$A_y = Asin\theta$$
$$B_y = Bsin\theta.$$

Step 2: Add the horizontal and vertical components of each vector to determine the components Rx and Ry of the resultant vector, R:

$$R_x = A_x + B_x$$

and

$$R_y = A_y + B_y \ .$$

Step 3: Use the Pythagorean theorem to determine the magnitude, R, of the resultant vector R:

$$R = \sqrt{R_x^2 + R_y^2} \ .$$

Step 4: Use a trigonometric identity to determine the direction, θ, of **R**:

$$\theta = tan^{-1}(R_y/R_x) \, .$$

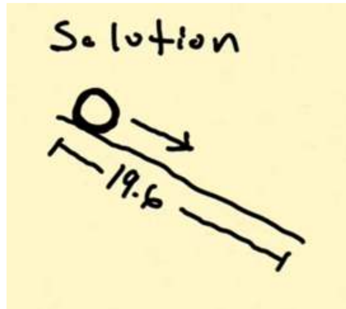
Glossary

analytical method

the method of determining the magnitude and direction of a resultant vector using the Pythagorean theorem and trigonometric identities

2.5: Your Generic Ball Rolling Down Hill Problem (Video Solution)

A ball rolling down a hill was displaced 19.6 m while uniformly accelerating from rest. If the final velocity was 5.00 m/s. what was the rate of acceleration?



Summary

The lecture presents a step-by-step solution to a kinematics problem involving uniform acceleration. It emphasizes key terms like "uniformly accelerating" and "rest" as critical to understanding the problem.

00:00:23 Introduction to the Kinematics Problem

00:01:31 Using a Velocity-Time Graph to Solve the Problem

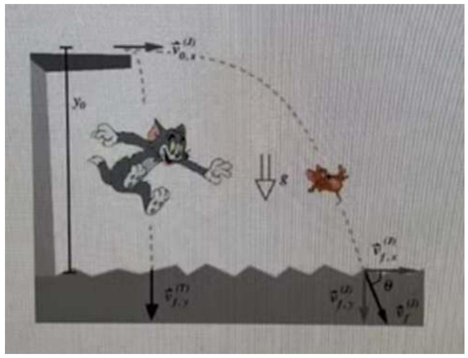
00:03:05 Calculating Time Using Distance Formula

00:04:33 Determining the Final Acceleration

Transcript

2.6: Tom and Jerry Dive Into a Pool, Who Splashes First (Video Solution))

Two students, Tom and Jerry, are jumping off a diving board into a swimming pool. Tom just drops straight downward off the board, and Jerry runs off the same board with some horizontal velocity v0.
a. Which diver, Tom or Jerry, has the greater splashdown speed? Or are they the same?
b. Calculate the splashdown speed for each diver, if the height of the board is 10 m, and Jerry's initial velocity is v0 = 4.5 m/s.



Summary

00:00:09 Introduction to a classical physics problem involving two divers, Tom and Jerry. Tom falls straight down from a diving board, while Jerry has an initial velocity in the x-direction. The question asks which diver has the greatest splashdown speed and requires calculating the splashdown speed for each diver given the height and Jerry's initial speed. The instructor, Prem-Raj Ruffin, begins by analyzing Tom's motion, noting that Tom starts with zero initial velocity in both x and y directions but speeds up as he falls.

00:01:30 Starting with the initial condition of Tom starting with zero initial velocity in both x and y directions but speeds up as he falls the instructor calculates Tom's final velocity, noting that acceleration due to gravity is 9.8 m/s².

00:08:32 The instructor moves on to analyze Jerry's motion.

00:13:22 The instructor talksabout a common mistake made in such problems, assuming that because Tom and Jerry have the same vertical velocity, they have the same impact speed.

00:18:06 The instructor summarizes the key insight: when comparing an object falling straight down to one with horizontal velocity, the object with horizontal velocity will always have a greater impact speed because it has velocity components in both directions. The instructor encourages students to practice this problem and similar ones repeatedly to reinforce physics concepts, comparing this practice to rehearsing a musical instrument. The instructor concludes by thanking the audience for submitting the problem and indicates that another physics problem will be discussed next time.

 Transcript

2.7: Projectile Motion

Learning Objectives

By the end of this section, you will be able to:

- Identify and explain the properties of a projectile, such as acceleration due to gravity, range, maximum height, and trajectory.
- Determine the location and velocity of a projectile at different points in its trajectory.
- Apply the principle of independence of motion to solve projectile motion problems.

Projectile motion is the **motion** of an object thrown or projected into the air, subject to only the acceleration of gravity. The object is called a **projectile**, and its path is called its **trajectory.** The motion of falling objects, as covered in Problem-Solving Basics for One-Dimensional Kinematics, is a simple one-dimensional type of projectile motion in which there is no horizontal movement. In this section, we consider two-dimensional projectile motion, such as that of a football or other object for which **air resistance** *is negligible*.

The most important fact to remember here is that motions along perpendicular axes are independent and thus can be analyzed separately. This fact was discussed in Kinematics in Two Dimensions: An Introduction, where vertical and horizontal motions were seen to be independent. The key to analyzing two-dimensional projectile motion is to break it into two motions, one along the horizontal axis and the other along the vertical. (This choice of axes is the most sensible, because acceleration due to gravity is vertical—thus, there will be no acceleration along the horizontal axis when air resistance is negligible.) As is customary, we call the horizontal axis the *x*-axis and the vertical axis the *y*-axis. Figure illustrates the notation for displacement, where s is defined to be the total displacement and x and y are its components along the horizontal and vertical axes, respectively. The magnitudes of these vectors are s, x, and y. (Note that in the last section we used the notation A to represent a vector with components A_x and A_y. If we continued this format, we would call displacement s with components s_x and s_y. However, to simplify the notation, we will simply represent the component vectors as x and y.)

Of course, to describe motion we must deal with velocity and acceleration, as well as with displacement. We must find their components along the x- and *y*-axes, too. We will assume all forces except gravity (such as air resistance and friction, for example) are negligible. The components of acceleration are then very simple: $a_y = -g = -9.80 m/s^2$(. (Note that this definition assumes that the upwards direction is defined as the positive direction. If you arrange the coordinate system instead such that the downwards direction is positive, then acceleration due to gravity takes a positive value.) Because gravity is vertical, $a_x = 0$. Both accelerations are constant, so the kinematic equations can be used.

REVIEW OF KINEMATIC EQUATIONS (CONSTANT)

$$x = x_0 + \bar{v}t$$
$$\bar{v} = \frac{v_0 + v}{2}$$
$$v = v_0 + at$$
$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a(x - x_0) \ .$$

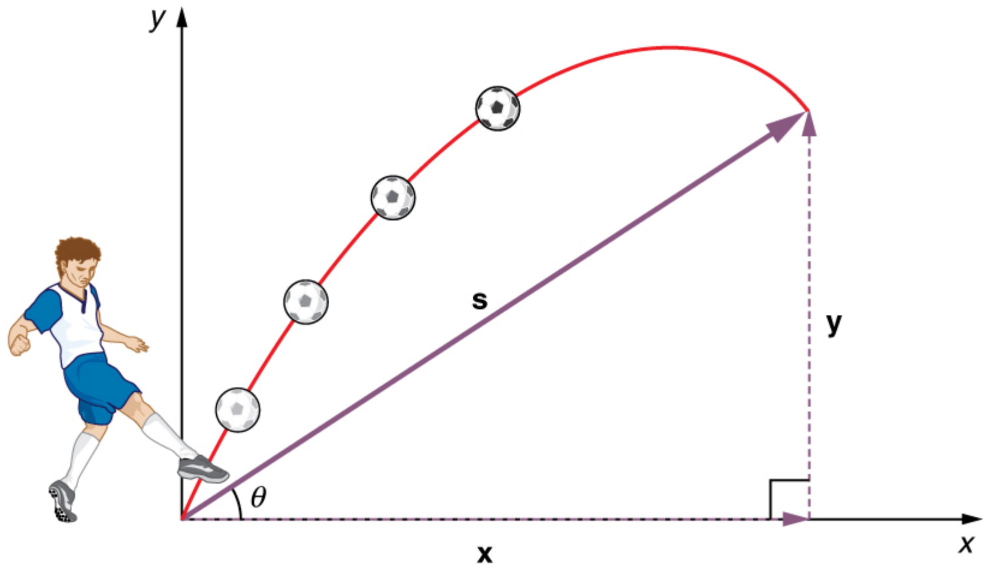


Figure 2.7.1: The total displacement s of a soccer ball at a point along its path. The vector s has components x and y along the horizontal and vertical axes. Its magnitude is s, and it makes an angle θ with the horizontal.

Given these assumptions, the following steps are then used to analyze projectile motion:

Step 1. *Resolve or break the motion into horizontal and vertical components along the x- and y-axes.* These axes are perpendicular, so $A_x = Acos\theta$ and $A_y = Asin\theta$ are used. The magnitude of the components of displacement s along these axes are x and y. The magnitudes of the components of the velocity v are $v_x = vcos\theta$ and $v_y = vsin\theta$, where v is the magnitude of the velocity and θ is its direction, as shown in Figure. Initial values are denoted with a subscript 0, as usual.

Step 2. *Treat the motion as two independent one-dimensional motions, one horizontal and the other vertical.* The kinematic equations for horizontal and vertical motion take the following forms:

$$\text{Horizontal Motion}(a_x = 0)$$
$$x = x_0 + v_x t$$
$$v_x = v_{0x} = v_x = \text{velocity is a constant} \ .$$
$$\text{Vertical Motion}(\text{assuming positive is up} a_y = -g = -9.80 m/s^2)$$
$$y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$$
$$v_y = v_{0y} - gt$$
$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$
$$v_y^2 = v_{0y}^2 - 2g(y - y_0) \ .$$

Step 3. *Solve for the unknowns in the two separate motions—one horizontal and one vertical.* Note that the only common variable between the motions is time *t*. The problem solving procedures here are the same as for one-dimensional **kinematics** and are illustrated in the solved examples below.

Step 4. *Recombine the two motions to find the total displacement s and velocity v.* Because the *x* - and *y* -motions are perpendicular, we determine these vectors by using the techniques outlined in the Vector Addition and Subtraction: Analytical Methods and employing $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = tan^{-1}(A_y/A_x)$ in the following form, where θ is the direction of the displacement s and θ_v is the direction of the velocity v:

Total displacement and velocity

$$s = \sqrt{x^2 + y^2}$$
$$\theta = tan^{-1}(y/x)$$
$$v = \sqrt{v_x^2 + v_y^2}$$
$$\theta_v = tan^{-1}(v_y/v_x) \, .$$

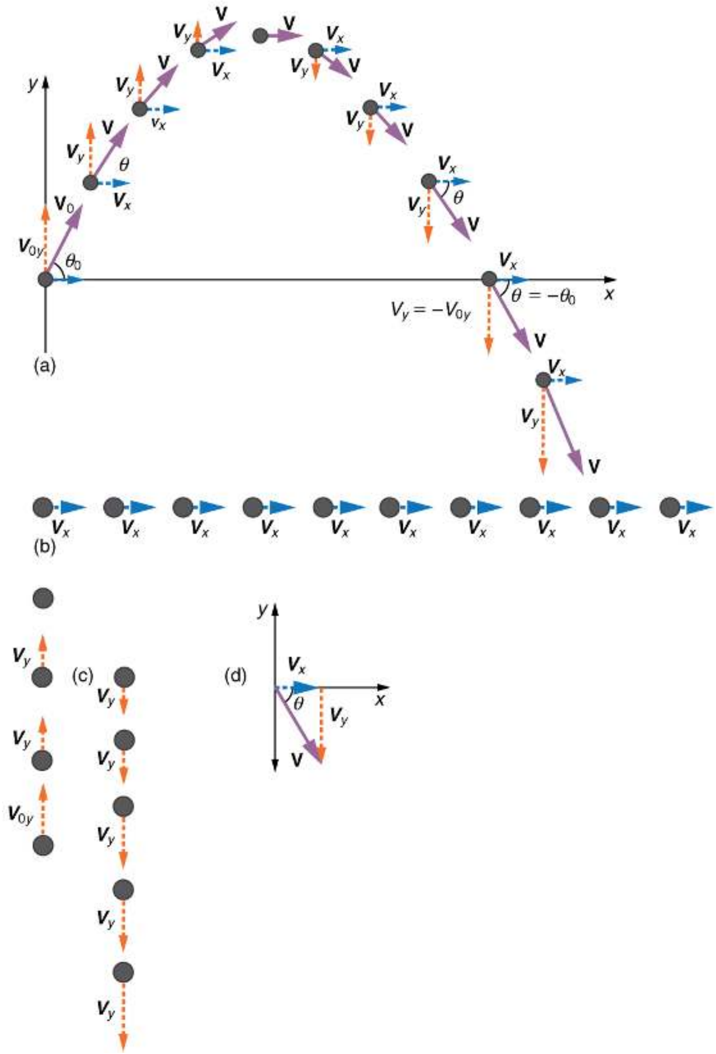


Figure 2.7.2: (a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes. (b) The horizontal motion is simple, because $a_x = 0$ and v_x is thus constant. (c) The velocity in the vertical direction begins to decrease as the object rises; at its highest point, the vertical velocity is zero. As the object falls towards the Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity. (d) The x - and y -motions are recombined to give the total velocity at any given point on the trajectory.

Example 2.7.1: A Fireworks Projectile Explodes High and Away

During a fireworks display, a shell is shot into the air with an initial speed of 70.0 m/s at an angle of **75.0º** above the horizontal, as illustrated in Figure. The fuse is timed to ignite the shell just as it reaches its highest point above the ground.

a. Calculate the height at which the shell explodes.
b. How much time passed between the launch of the shell and the explosion?
c. What is the horizontal displacement of the shell when it explodes?

Strategy

Because air resistance is negligible for the unexploded shell, the analysis method outlined above can be used. The motion can be broken into horizontal and vertical motions in which $a_x = 0$ and $a_y = -g$. We can then define x_0 and y_0 to be zero and solve for the desired quantities.

Solution for (a)

By "height" we mean the altitude or vertical position y above the starting point. The highest point in any trajectory, called the apex, is reached when $v_y = 0$. Since we know the initial and final velocities as well as the initial position, we use the following equation to find y:

$$v_y^2 = v_{0y}^2 - 2g(y - y_0)\ .$$

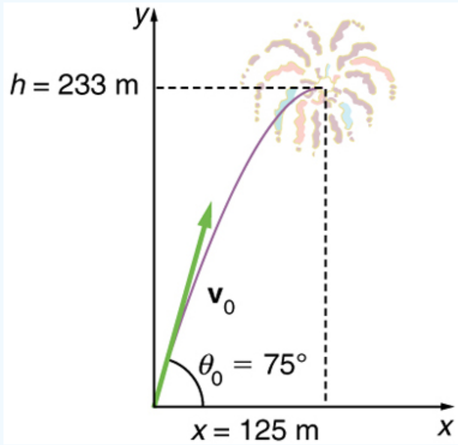


Figure 2.7.3: The trajectory of a fireworks shell. The fuse is set to explode the shell at the highest point in its trajectory, which is found to be at a height of 233 m and 125 m away horizontally.

Because y_0 and v_y are both zero, the equation simplifies to

$$0 = v_{0y}^2 - 2gy.$$

Solving for y gives

$$y = \frac{v_{0y}^2}{2g}.$$

Now we must find v_{0y}, the component of the initial velocity in the *y*-direction. It is given by $v_{0y} = v_0 sin\theta$, where v_{0y} is the initial velocity of 70.0 m/s, and $\theta_0 = 75.0^o$ is the initial angle. Thus,

$$v_{0y} = v_0 sin\theta_0 = (70.0m/s)(sin 75^o) = 67.6m/s.$$

and y is

$$y = \frac{(67.6m/s)^2}{2(9.80m/s^2)},$$

so that

$$y = 233m.$$

Discussion for (a)

Note that because up is positive, the initial velocity is positive, as is the maximum height, but the acceleration due to gravity is negative. Note also that the maximum height depends only on the vertical component of the initial velocity, so that any projectile with a 67.6 m/s initial vertical component of velocity will reach a maximum height of 233 m (neglecting air resistance). The numbers in this example are reasonable for large fireworks displays, the shells of which do reach such heights before exploding. In practice, air resistance is not completely negligible, and so the initial velocity would have to be somewhat larger than that given to reach the same height.

Solution for (b)

As in many physics problems, there is more than one way to solve for the time to the highest point. In this case, the easiest method is to use $y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$. Because y0 is zero, this equation reduces to simply

$$y = \frac{1}{2}(v_{0y} + v_y)t\ .$$

Note that the final vertical velocity, v_y, at the highest point is zero. Thus,

$$t = \frac{2y}{(v_{0y} + v_y)} = \frac{2(233m)}{(67.6m/s)} = 6.90s.$$

Discussion for (b)

This time is also reasonable for large fireworks. When you are able to see the launch of fireworks, you will notice several seconds pass before the shell explodes. (Another way of finding the time is by using $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$, and solving the quadratic equation for t.)

Solution for (c)

Because air resistance is negligible, $a_x = 0$ and the horizontal velocity is constant, as discussed above. The horizontal displacement is horizontal velocity multiplied by time as given by $x = x_0 + v_x t$, where x_0 is equal to zero:

$$x = v_x t,$$

where v_x is the *x*-component of the velocity, which is given by $v_x = v_0 cos\theta_0$. Now,

$$v_x = v_0 cos\theta_0 = (70.0 m/s)(cos 75.0^{\circ}) = 18.1 m/s.$$

The time t for both motions is the same, and so x is

$$x = (18.1 m/s)(6.90 s) = 125 m.$$

Discussion for (c)

The horizontal motion is a constant velocity in the absence of air resistance. The horizontal displacement found here could be useful in keeping the fireworks fragments from falling on spectators. Once the shell explodes, air resistance has a major effect, and many fragments will land directly below.

In solving part (a) of the preceding example, the expression we found for y is valid for any projectile motion where air resistance is negligible. Call the maximum height $y = h$; then,

$$h = \frac{v_{0y}^2}{2g}.$$

This equation defines the *maximum height* of a projectile and depends only on the vertical component of the initial velocity.

DEFINING A COORDINATE SYSTEM

It is important to set up a coordinate system when analyzing projectile motion. One part of defining the coordinate system is to define an origin for the x and y positions. Often, it is convenient to choose the initial position of the object as the origin such that $x_0 = 0$ and $y_0 = 0$. It is also important to define the positive and negative directions in the x and y directions. Typically, we define the positive vertical direction as upwards, and the positive horizontal direction is usually the direction of the object's motion. When this is the case, the vertical acceleration, g, takes a negative value (since it is directed downwards towards the Earth). However, it is occasionally useful to define the coordinates differently. For example, if you are analyzing the motion of a ball thrown downwards from the top of a cliff, it may make sense to define the positive direction downwards since the motion of the ball is solely in the downwards direction. If this is the case, g takes a positive value.

Example 2.7.2: Calculating Projectile Motion: Hot Rock Projectile

Kilauea in Hawaii is the world's most continuously active volcano. Very active volcanoes characteristically eject red-hot rocks and lava rather than smoke and ash. Suppose a large rock is ejected from the volcano with a speed of 25.0 m/s and at an angle above the horizontal, as shown in Figure. The rock strikes the side of the volcano at an altitude 20.0 m lower than its starting point. (a) Calculate the time it takes the rock to follow this path. (b) What are the magnitude and direction of the rock's velocity at impact?

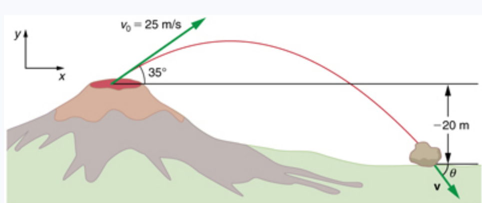


Figure 2.7.4: The trajectory of a rock ejected from the Kilauea volcano.

Strategy

Again, resolving this two-dimensional motion into two independent one-dimensional motions will allow us to solve for the desired quantities. The time a projectile is in the air is governed by its vertical motion alone. We will solve for t first. While the rock is rising and falling vertically, the horizontal motion continues at a constant velocity. This example asks for the final velocity. Thus, the vertical and horizontal results will be recombined to obtain v and θ_v at the final time t determined in the first part of the example.

Solution for (a)

While the rock is in the air, it rises and then falls to a final position 20.0 m lower than its starting altitude. We can find the time for this by using

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2 \ .$$

If we take the initial position y_0 to be zero, then the final position is $y = -20.0m$. Now the initial vertical velocity is the vertical component of the initial velocity, found from $v_{0y} = v_0 sin\theta_0 = (25.0m/s)(sin35.0^{\circ}) = 14.3m/s$. Substituting known values yields

$$-20.0m = (14.3m/s)t - (4.90m/s^2)t^2.$$

Rearranging terms gives a quadratic equation in t:

$$(4.90m/s^2)t^2 - (14.3m/s)t - (20.0m) = 0.$$

This expression is a quadratic equation of the form $at^2 + bt + c = 0$, where the constants are $a = 4.90$, $b =- 14.3$, and $c =- 20.0$. Its solutions are given by the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \ .$$

This equation yields two solutions: $t = 3.96$ and $t =- 1.03$. (It is left as an exercise for the reader to verify these solutions.) The time is $t = 3.96s$ or $-1.03s$. The negative value of time implies an event before the start of motion, and so we discard it. Thus,

$$t = 3.96s.$$

Discussion for (a)

The time for projectile motion is completely determined by the vertical motion. So any projectile that has an initial vertical velocity of 14.3 m/s and lands 20.0 m below its starting altitude will spend 3.96 s in the air.

Solution for (b)

From the information now in hand, we can find the final horizontal and vertical velocities v_x and v_y and combine them to find the total velocity v and the angle θ_0 it makes with the horizontal. Of course, v_x is constant so we can solve for it at any horizontal location. In this case, we chose the starting point since we know both the initial velocity and initial angle. Therefore:

$$v_x = v_0 cos\theta_0 = (25.0m/s)(cos35^{\circ}) = 20.5m/s.$$

The final vertical velocity is given by the following equation:

$$v_y = v_{0y} - gt \ ,$$

where v_{0y} was found in part (a) to be **14.3 m/s**. Thus,

$$v_y = 14.3m/s - (9.80m/s^2)(3.96s)$$

so that

$$v_y = -24.5m/s.$$

To find the magnitude of the final velocity v we combine its perpendicular components, using the following equation:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(20.5m/s)^2 + (-24.5m/s)^2} \ ,$$

which gives

$$v = 31.9 m/s.$$

The direction θv is found from the equation:

$$\theta_v = tan^{-1}(v_y/v_x)$$

so that

$$\theta_v = tan^{-1}(-24.5/20.5) = tan^{-1}(-1.19).$$

Thus,

$$\theta_v = -50.1^o.$$

Discussion for (b)

The negative angle means that the velocity is **50.1°** below the horizontal. This result is consistent with the fact that the final vertical velocity is negative and hence downward—as you would expect because the final altitude is 20.0 m lower than the initial altitude. (See Figure.)

One of the most important things illustrated by projectile motion is that vertical and horizontal motions are independent of each other. Galileo was the first person to fully comprehend this characteristic. He used it to predict the range of a projectile. On level ground, we define **range** to be the horizontal distance traveled by a projectile. Galileo and many others were interested in the range of projectiles primarily for military purposes—such as aiming cannons. However, investigating the range of projectiles can shed light on other interesting phenomena, such as the orbits of satellites around the Earth. Let us consider projectile range further.

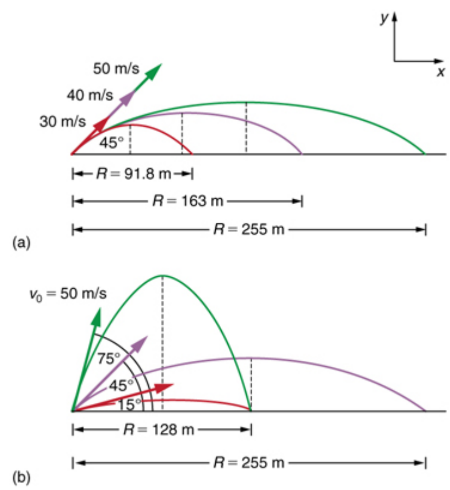


Figure 2.7.5: Trajectories of projectiles on level ground. (a) The greater the initial speed v_0, the greater the range for a given initial angle. (b) The effect of initial angle θ_0 on the range of a projectile with a given initial speed. Note that the range is the same for 15° and 75°, although the maximum heights of those paths are different.

How does the initial velocity of a projectile affect its range? Obviously, the greater the initial speed v_0, the greater the range, as shown in Figure(a). The initial angle θ_0 also has a dramatic effect on the range, as illustrated in Figure(b). For a fixed initial speed, such as might be produced by a cannon, the maximum range is obtained with $\theta_0 = 45^o$. This is true only for conditions neglecting air resistance. If air resistance is considered, the maximum angle is approximately 38^o. Interestingly, for every initial angle except 45^o, there are two angles that give the same range—the sum of those angles is 90^o. The range also depends on the value of the acceleration of gravity g. The lunar astronaut Alan Shepherd was able to drive a golf ball a great distance on the Moon because gravity is weaker there. The range R of a projectile on level ground for which air resistance is negligible is given by

$$R = \frac{v_0^2 sin 2\theta_0}{g},$$

where v_0 is the initial speed and θ_0 is the initial angle relative to the horizontal. The proof of this equation is left as an end-of-chapter problem (hints are given), but it does fit the major features of projectile range as described.

When we speak of the range of a projectile on level ground, we assume that R is very small compared with the circumference of the Earth. If, however, the range is large, the Earth curves away below the projectile and acceleration of gravity changes direction along the path. The range is larger than predicted by the range equation given above because the projectile has farther to fall than it would on level ground. (See Figure.) If the initial speed is great enough, the projectile goes into orbit. This is called exit velocity. This possibility was recognized centuries before it could be accomplished. When an object is in orbit, the Earth curves away from underneath the object at the same rate as it falls. The object thus falls continuously but never hits the surface. These and other aspects of orbital motion, such as the rotation of the Earth, will be covered analytically and in greater depth later in this text.

Once again we see that thinking about one topic, such as the range of a projectile, can lead us to others, such as the Earth orbits. In Addition of Velocities, we will examine the addition of velocities, which is another important aspect of two-dimensional kinematics and will also yield insights beyond the immediate topic.

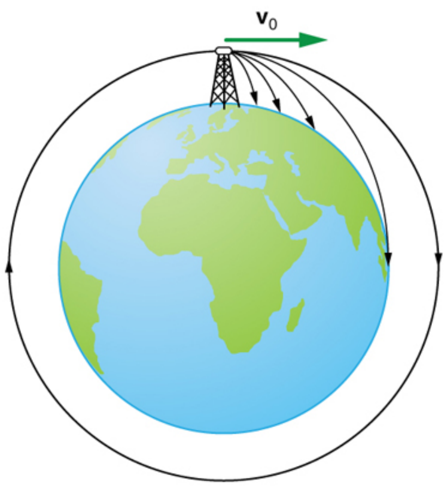


Figure 2.7.6: Projectile to satellite. In each case shown here, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the range increases and becomes longer than it would be on level ground because the Earth curves away underneath its path. With a large enough initial speed, orbit is achieved.

PHET EXPLORATIONS: PROJECTILE MOTION

Blast a Buick out of a cannon! Learn about projectile motion by firing various objects. Set the angle, initial speed, and mass. Add air resistance. Make a game out of this simulation by trying to hit a target.

Figure 2.7.7: Projectile Motion

Summary

- Projectile motion is the motion of an object through the air that is subject only to the acceleration of gravity.
- To solve projectile motion problems, perform the following steps:

1. Determine a coordinate system. Then, resolve the position and/or velocity of the object in the horizontal and vertical components. The components of position s are given by the quantities x and y, and the components of the velocity v are given by vx=vcosθ and vy=vsinθ, where v is the magnitude of the velocity and θ is its direction.

2. Analyze the motion of the projectile in the horizontal direction using the following equations:

$$\text{Horizontal motion}(a_x = 0)$$

$$x = x_0 + v_x t$$

$$v_x = v_{0x} = v_x = \text{velocity is a constant} \;.$$

3. Analyze the motion of the projectile in the vertical direction using the following equations:

$$\text{Vertical motion(Assuming positive direction is up;} a_y = -g = -9.80 m/s^2)$$

$$y=y_0+\frac{1}{2}(v_{0y}+v_y)t$$

$$v_y=v_{0y}-gt$$

$$y=y_0+v_{0y}t-\frac{1}{2}gt^2$$

$$v_y^2=v_{0y}^2-2g(y-y_0)\ .$$

4. Recombine the horizontal and vertical components of location and/or velocity using the following equations:

$$s=\sqrt{x^2+y^2}$$

$$\theta=tan^{-1}(y/x)$$

$$v=\sqrt{v_x^2+v_y^2}$$

$$\theta_v=tan^{-1}(v_y/v_x)\,.$$

- The maximum height h of a projectile launched with initial vertical velocity v_{0y} is given by

$$h=\frac{v_{0y}^2}{2g}\,.$$

- The maximum horizontal distance traveled by a projectile is called the **range**. The range R of a projectile on level ground launched at an angle θ_0 above the horizontal with initial speed v_0 is given by

$$R=\frac{v_0^2 sin 2\theta_0}{g}\,.$$

Glossary

air resistance

a frictional force that slows the motion of objects as they travel through the air; when solving basic physics problems, air resistance is assumed to be zero

kinematics

the study of motion without regard to mass or force

motion

displacement of an object as a function of time

projectile

an object that travels through the air and experiences only acceleration due to gravity

projectile motion

the motion of an object that is subject only to the acceleration of gravity

range

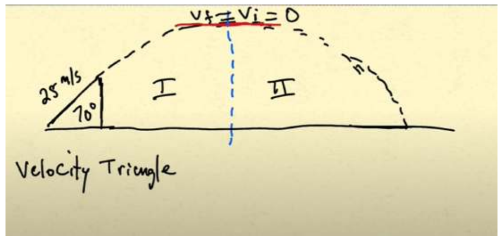
the maximum horizontal distance that a projectile travels

trajectory

the path of a projectile through the air

2.8: Snowball Fight Calculation (Video Solution)

One strategy in a snowball fight is to throw a first snowball at a high angle over level ground, while your opponent is watching the first one you throw a second one at a low angle and timed to arrive at your opponent before or at the same time as the first one. Assume both snowballs are thrown with a speed of 25.0 m/s. The first one is thrown at an angle of 70° with respect to the horizontal. At what angle should the second snowball be thrown if it is to land at the same point as the first? How many seconds later should the second snowball be thrown if it is to land at the same time as the first.



Summary

The video focuses on solving a physics problem involving projectile motion with two snowballs. The problem is methodically broke down the problem into two parts, analyzing the projectile motion using velocity triangles, kinematics equations, and the range equation. Students learn how to calculate the time to reach maximum height, total flight time, and horizontal distance traveled for both snowballs.

00:02:30 Students verify first snowball calculations using initial velocity of 25 m/s and angle of 70 degrees.

00:08:00 Students practice calculating the range equation to find the second snowball's angle.

00:14:19 Students independently solve the problem to verify the timing difference.

00:06:37 Students review and confirm all numerical calculations and units.

 Transcript

2.9: Addition of Velocities

Learning Objectives

By the end of this section, you will be able to:

- Apply principles of vector addition to determine relative velocity.
- Explain the significance of the observer in the measurement of velocity.

Relative Velocity

If a person rows a boat across a rapidly flowing river and tries to head directly for the other shore, the boat instead moves diagonallyrelative to the shore, as in Figure 2.9.1. The boat does not move in the direction in which it is pointed. The reason, of course, is that the river carries the boat downstream. Similarly, if a small airplane flies overhead in a strong crosswind, you can sometimes see that the plane is not moving in the direction in which it is pointed, as illustrated in Figure 2.9.2. The plane is moving straight ahead relative to the air, but the movement of the air mass relative to the ground carries it sideways.

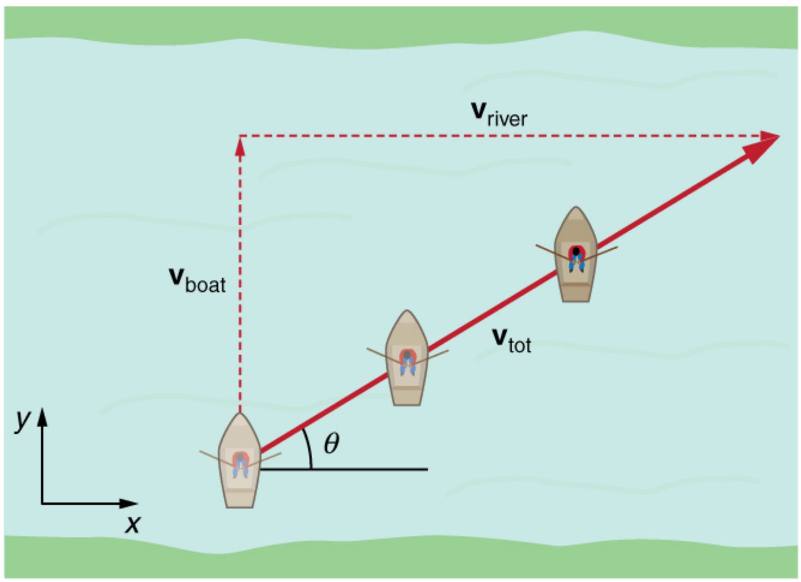


Figure 2.9.1: A boat trying to head straight across a river will actually move diagonally relative to the shore as shown. Its total velocity (solid arrow) relative to the shore is the sum of its velocity relative to the river plus the velocity of the river relative to the shore.

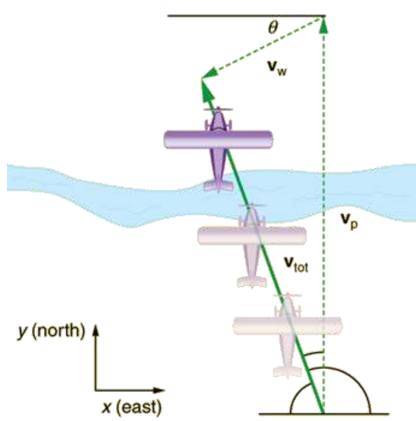


Figure 2.9.2: An airplane heading straight north is instead carried to the west and slowed down by wind. The plane does not move relative to the ground in the direction it points; rather, it moves in the direction of its total velocity (solid arrow).

In each of these situations, an object has a velocity relative to a medium (such as a river) and that medium has a velocity relative to an observer on solid ground. The velocity of the object relative to the observer is the sum of these velocity vectors, as indicated in

Figures $2.9.1$ and $2.9.2$. These situations are only two of many in which it is useful to add velocities. In this module, we first re-examine how to add velocities and then consider certain aspects of what relative velocity means.

How do we add velocities? Velocity is a vector (it has both magnitude and direction); the rules of vector addition discussed inVector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods apply to the addition of velocities, just as they do for any other vectors. In one-dimensional motion, the addition of velocities is simple—they add like ordinary numbers. For example, if a field hockey player is moving at 5 m/s straight toward the goal and drives the ball in the same direction with a velocity of 30 m/s relative to her body, then the velocity of the ball is 35 m/s relative to the stationary, profusely sweating goalkeeper standing in front of the goal.

In two-dimensional motion, either graphical or analytical techniques can be used to add velocities. We will concentrate on analytical techniques. The following equations give the relationships between the magnitude and direction of velocity (v and θ) and its components (v_x and v_y) along the x- and y-axes of an appropriately chosen coordinate system:

$$v_x = v\cos\theta \tag{2.9.1}$$

$$v_y = v\sin\theta \tag{2.9.2}$$

$$v = \sqrt{v^2x + v^2} \tag{2.9.3}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right). \tag{2.9.4}$$

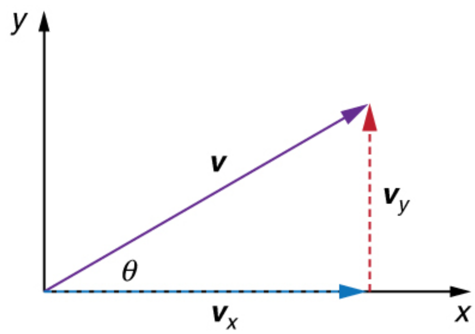


Figure $2.9.3$: The velocity, v, of an object traveling at an angle θ to the horizontal axis is the sum of component vectors v_x and v_y.

These equations are valid for any vectors and are adapted specifically for velocity. The first two equations are used to find the components of a velocity when its magnitude and direction are known. The last two are used to find the magnitude and direction of velocity when its components are known.

TAKE-HOME EXPERIMENT: RELATIVE VELOCITY OF A BOAT

Fill a bathtub half-full of water. Take a toy boat or some other object that floats in water. Unplug the drain so water starts to drain. Try pushing the boat from one side of the tub to the other and perpendicular to the flow of water. Which way do you need to push the boat so that it ends up immediately opposite? Compare the directions of the flow of water, heading of the boat, and actual velocity of the boat.

Example $2.9.1$: Adding Velocities - A Boat on a River

Refer to Figure $2.9.4$, which shows a boat trying to go straight across the river. Let us calculate the magnitude and direction of the boat's velocity relative to an observer on the shore, v_{tot}. The velocity of the boat, vboat , is 0.75 m/s in the y -direction relative to the river and the velocity of the river, v_{river} , is 1.20 m/s to the right.

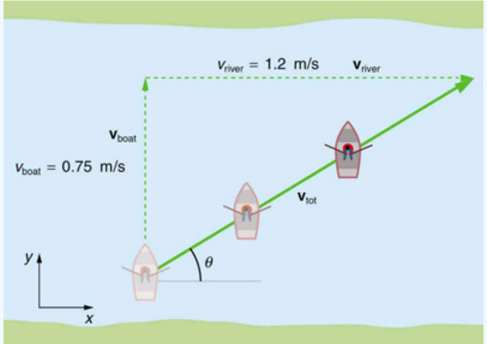


Figure 2.9.4: A boat attempts to travel straight across a river at a speed 0.75 m/s. The current in the river, however, flows at a speed of 1.20 m/s to the right. What is the total displacement of the boat relative to the shore?

Strategy

We start by choosing a coordinate system with its x-axis parallel to the velocity of the river, as shown in Figure. Because the boat is directed straight toward the other shore, its velocity relative to the water is parallel to the y-axis and perpendicular to the velocity of the river. Thus, we can add the two velocities by using the equations $v_{tot} = \sqrt{v_x^2 + v_y^2}$ and $\theta = tan^{-1}(v_y/v_x)$ directly.

Solution

The magnitude of the total velocity is

$$v_{tot} = \sqrt{v_x^2 + v_y^2},$$

where

$$v_x = v_{river} = 1.20 m/s$$

and

$$v_y = v_{boat} = 0.750 m/s.$$

Thus,

$$v_{tot} = \sqrt{(1.20 m/s)^2 + (0.750 m/s)^2}$$

yielding

$$v_{tot} = 1.42 m/s.$$

The direction of the total velocity θ is given by:

$$\theta = tan^{-1}(v_y/v_x) = tan^{-1}(0.750/1.20).$$

This equation gives

$$\theta = 32.0^{o}.$$

Discussion

Both the magnitude v and the direction θ of the total velocity are consistent with Figure. Note that because the velocity of the river is large compared with the velocity of the boat, it is swept rapidly downstream. This result is evidenced by the small angle (only **32.0º**) the total velocity has relative to the riverbank.

Example 2.9.2: Calculating Velocity - Wind Velocity Causes an Airplane to Drift

Calculate the wind velocity for the situation shown in Figure 2.9.5. The plane is known to be moving at 45.0 m/s due north relative to the air mass, while its velocity relative to the ground (its total velocity) is 38.0 m/s in a direction **20.0º** west of north.

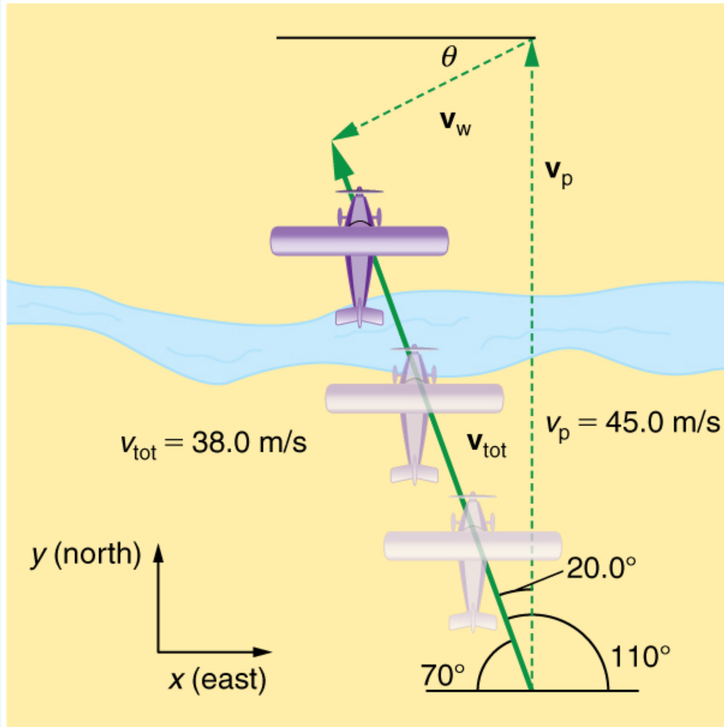


Figure 2.9.5: An airplane is known to be heading north at 45.0 m/s, though its velocity relative to the ground is 38.0 m/s at an angle west of north. What is the speed and direction of the wind?

Strategy

In this problem, somewhat different from the previous example, we know the total velocity v_{to}t and that it is the sum of two other velocities, v_w (the wind) and v_p (the plane relative to the air mass). The quantity v_p is known, and we are asked to find vw. None of the velocities are perpendicular, but it is possible to find their components along a common set of perpendicular axes. If we can find the components of v_w, then we can combine them to solve for its magnitude and direction. As shown in Figure, we choose a coordinate system with its *x*-axis due east and its *y*-axis due north (parallel to v_p). (You may wish to look back at the discussion of the addition of vectors using perpendicular components in Vector Addition and Subtraction: Analytical Methods.)

Solution

Because v_{tot} is the vector sum of the v_w and v_p, its *x*- and *y*-components are the sums of the *x*- and *y*-components of the wind and plane velocities. Note that the plane only has vertical component of velocity so $v_{px} = 0$ and $v_{py} = v_p$. That is,

$$v_{tot}x = v_{wx}$$

and

$$v_{toty} = v_{wy} + v_p \ .$$

We can use the first of these two equations to find v_{wx}:

$$v_{wx} = v_{totx} = v_{tot} cos 110^o.$$

Because $v_{tot} = 38.0 m/s$ and $cos 110^o = -0.342$ we have

$$v_{wx} = (38.0 m/s)(-0.342) = -13 m/s.$$

The minus sign indicates motion west which is consistent with the diagram.

Now, to find v_{wy} we note that

$$v_{toty} = v_{wy} + v_p$$

Here $v_{toty} = v_{tot} sin 110^o$; thus,

$$v_{wy} = (38.0 m/s)(0.940) - 45.0 m/s = -9.29 m/s.$$

This minus sign indicates motion south which is consistent with the diagram.

Now that the perpendicular components of the wind velocity v_{wx} and v_{wy} are known, we can find the magnitude and direction of v_w. First, the magnitude is

$$v_w = \sqrt{v_{wx}^2 + v_{wy}^2} = \sqrt{(-13.0m/s)^2 + (-9.29m/s)^2}$$

so that

$$v_w = 16.0m/s.$$

The direction is:

$$\theta = tan^{-1}(v_{wy}/v_{wx}) = tan^{-1}(-9.29/-13.0)$$

giving

$$\theta = 35.6^o.$$

Discussion

The wind's speed and direction are consistent with the significant effect the wind has on the total velocity of the plane, as seen in Figure. Because the plane is fighting a strong combination of crosswind and head-wind, it ends up with a total velocity significantly less than its velocity relative to the air mass as well as heading in a different direction.

Note that in both of the last two examples, we were able to make the mathematics easier by choosing a coordinate system with one axis parallel to one of the velocities. We will repeatedly find that choosing an appropriate coordinate system makes problem solving easier. For example, in projectile motion we always use a coordinate system with one axis parallel to gravity.

Relative Velocities and Classical Relativity

When adding velocities, we have been careful to specify that the *velocity is relative to some reference frame*. These velocities are called **relative velocities**. For example, the velocity of an airplane relative to an air mass is different from its velocity relative to the ground. Both are quite different from the velocity of an airplane relative to its passengers (which should be close to zero). Relative velocities are one aspect of **relativity**, which is defined to be the study of how different observers moving relative to each other measure the same phenomenon.

Nearly everyone has heard of relativity and immediately associates it with Albert Einstein (1879–1955), the greatest physicist of the 20th century. Einstein revolutionized our view of nature with his *modern* theory of relativity, which we shall study in later chapters. The relative velocities in this section are actually aspects of classical relativity, first discussed correctly by Galileo and Isaac Newton.**Classical relativity** is limited to situations where speeds are less than about 1% of the speed of light—that is, less than . Most things we encounter in daily life move slower than this speed.

Let us consider an example of what two different observers see in a situation analyzed long ago by Galileo. Suppose a sailor at the top of a mast on a moving ship drops his binoculars. Where will it hit the deck? Will it hit at the base of the mast, or will it hit behind the mast because the ship is moving forward? The answer is that if air resistance is negligible, the binoculars will hit at the base of the mast at a point directly below its point of release. Now let us consider what two different observers see when the binoculars drop. One observer is on the ship and the other on shore. The binoculars have no horizontal velocity relative to the observer on the ship, and so he sees them fall straight down the mast. (Figure $2.9.1$; blue curve) To the observer on shore, the binoculars and the ship have the *same* horizontal velocity, so both move the same distance forward while the binoculars are falling. This observer sees the red curved path shown in Figure $2.9.6$. Although the paths look different to the different observers, each sees the same result—the binoculars hit at the base of the mast and not behind it. To get the correct description, it is crucial to correctly specify the velocities relative to the observer.

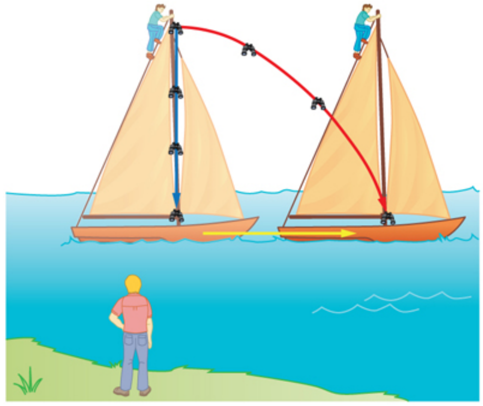


Figure 2.9.6: Classical relativity. The same motion as viewed by two different observers. An observer on the moving ship sees the binoculars dropped from the top of its mast fall straight down. An observer on shore sees the binoculars take the curved path, moving forward with the ship. Both observers see the binoculars strike the deck at the base of the mast. The initial horizontal velocity is different relative to the two observers. (The ship is shown moving rather fast to emphasize the effect.)

Example 2.9.3: Calculating Relative Velocity: An Airline Passenger Drops a Coin

An airline passenger drops a coin while the plane is moving at 260 m/s. What is the velocity of the coin when it strikes the floor 1.50 m below its point of release: (a) Measured relative to the plane? (b) Measured relative to the Earth?

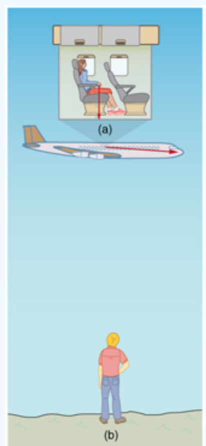


Figure 2.9.7: The motion of a coin dropped inside an airplane as viewed by two different observers. (a) An observer in the plane sees the coin fall straight down. (b) An observer on the ground sees the coin move almost horizontally.

Strategy

Both problems can be solved with the techniques for falling objects and projectiles. In part (a), the initial velocity of the coin is zero relative to the plane, so the motion is that of a falling object (one-dimensional). In part (b), the initial velocity is 260 m/s horizontal relative to the Earth and gravity is vertical, so this motion is a projectile motion. In both parts, it is best to use a coordinate system with vertical and horizontal axes.

Solution for (a)

Using the given information, we note that the initial velocity and position are zero, and the final position is 1.50 m. The final velocity can be found using the equation:

$$v_y^2 = v_{0y}^2 - 2g(y - y_0).$$

Substituting known values into the equation, we get

$$v_y^2 = 0^2 - 2(9.80 m/s^2)(-1.50 m - 0 m) = 29.4 m^2/s^2$$

yielding

$$v_y = -5.42 m/s.$$

We know that the square root of 29.4 has two roots: 5.42 and -5.42. We choose the negative root because we know that the velocity is directed downwards, and we have defined the positive direction to be upwards. There is no initial horizontal velocity relative to the plane and no horizontal acceleration, and so the motion is straight down relative to the plane.

Solution for (b)

Because the initial vertical velocity is zero relative to the ground and vertical motion is independent of horizontal motion, the final vertical velocity for the coin relative to the ground is $v_y = -5.42 m/s$, the same as found in part (a). In contrast to part (a), there now is a horizontal component of the velocity. However, since there is no horizontal acceleration, the initial and final horizontal velocities are the same and $v_x = 260 m/s$. The *x*- and *y*-components of velocity can be combined to find the magnitude of the final velocity:

$$v = \sqrt{v_x^2 + v_y^2}\ .$$

Thus,

$$v = \sqrt{(260 m/s)^2 + (-5.42 m/s)^2}$$

yielding

$$v = 260.06 m/s.$$

The direction is given by:

$$\theta = tan^{-1}(v_y/v_x) = tan^{-1}(-5.42/260)$$

so that

$$\theta = tan^{-1}(-0.0208) = -1.19^o.$$

Discussion

In part (a), the final velocity relative to the plane is the same as it would be if the coin were dropped from rest on the Earth and fell 1.50 m. This result fits our experience; objects in a plane fall the same way when the plane is flying horizontally as when it is at rest on the ground. This result is also true in moving cars. In part (b), an observer on the ground sees a much different motion for the coin. The plane is moving so fast horizontally to begin with that its final velocity is barely greater than the initial velocity. Once again, we see that in two dimensions, vectors do not add like ordinary numbers—the final velocity v in part (b) is not $(260 - 5.42) m/s$; rather, it is $260.06 m/s$. The velocity's magnitude had to be calculated to five digits to see any difference from that of the airplane. The motions as seen by different observers (one in the plane and one on the ground) in this example are analogous to those discussed for the binoculars dropped from the mast of a moving ship, except that the velocity of the plane is much larger, so that the two observers see very different paths. (See Figure.) In addition, both observers see the coin fall 1.50 m vertically, but the one on the ground also sees it move forward 144 m (this calculation is left for the reader). Thus, one observer sees a vertical path, the other a nearly horizontal path.

MAKING CONNECTIONS: RELATIVITY AND EINSTEIN

Because Einstein was able to clearly define how measurements are made (some involve light) and because the speed of light is the same for all observers, the outcomes are spectacularly unexpected. Time varies with observer, energy is stored as increased mass, and more surprises await.

PHET EXPLORATIONS: MOTION IN 2D

Try the new "Ladybug Motion 2D" simulation for the latest updated version. Learn about position, velocity, and acceleration vectors. Move the ball with the mouse or let the simulation move the ball in four types of motion (2 types of linear, simple harmonic, circle).

Figure 2.9.1: Motion in 2D

Summary

- Velocities in two dimensions are added using the same analytical vector techniques, which are rewritten as

$$v_x = v cos\theta$$
$$v_y = v sin\theta$$
$$v = \sqrt{v_x^2 + v^2 + y}$$
$$\theta = tan^{-1}(v_y/v_x).$$

- Relative velocity is the velocity of an object as observed from a particular reference frame, and it varies dramatically with reference frame.
- **Relativity** is the study of how different observers measure the same phenomenon, particularly when the observers move relative to one another. **Classical relativity** is limited to situations where speed is less than about 1% of the speed of light (3000 km/s).

Glossary

classical relativity
: the study of relative velocities in situations where speeds are less than about 1% of the speed of light—that is, less than 3000 km/s

relative velocity
: the velocity of an object as observed from a particular reference frame

relativity
: the study of how different observers moving relative to each other measure the same phenomenon

velocity
: speed in a given direction

vector addition
: the rules that apply to adding vectors together

2.E: Two-Dimensional Kinematics (Exercises)

Conceptual Questions

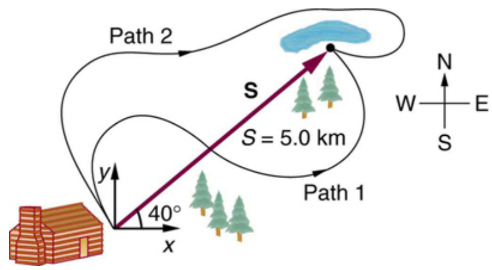
3.2: Vector Addition and Subtraction: Graphical Methods

1. Which of the following is a vector: a person's height, the altitude on Mt. Everest, the age of the Earth, the boiling point of water, the cost of this book, the Earth's population, the acceleration of gravity?

2. Give a specific example of a vector, stating its magnitude, units, and direction.

3. What do vectors and scalars have in common? How do they differ?

4. Two campers in a national park hike from their cabin to the same spot on a lake, each taking a different path, as illustrated below. The total distance traveled along Path 1 is 7.5 km, and that along Path 2 is 8.2 km. What is the final displacement of each camper?



5. If an airplane pilot is told to fly 123 km in a straight line to get from San Francisco to Sacramento, explain why he could end up anywhere on the circle shown in Figure. What other information would he need to get to Sacramento?

6. Suppose you take two steps A and **B** (that is, two nonzero displacements). Under what circumstances can you end up at your starting point? More generally, under what circumstances can two nonzero vectors add to give zero? Is the maximum distance you can end up from the starting point **A+B** the sum of the lengths of the two steps?

7. Explain why it is not possible to add a scalar to a vector

8. If you take two steps of different sizes, can you end up at your starting point? More generally, can two vectors with different magnitudes ever add to zero? Can three or more?

3.3: Vector Addition and Subtraction: Analytical Methods

9. Suppose you add two vectors **A** and **B**. What relative direction between them produces the resultant with the greatest magnitude? What is the maximum magnitude? What relative direction between them produces the resultant with the smallest magnitude? What is the minimum magnitude?

10. Give an example of a nonzero vector that has a component of zero.

11. Explain why a vector cannot have a component greater than its own magnitude.

12. If the vectors **A** and **B** are perpendicular, what is the component of **A** along the direction of **B**? What is the component of **B** along the direction of **A**?

3.4: Projectile Motion

13. Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither **0º** nor **90º**):

(a) Is the velocity ever zero?

(b) When is the velocity a minimum? A maximum?

(c) Can the velocity ever be the same as the initial velocity at a time other than at $t = 0$?

(d) Can the speed ever be the same as the initial speed at a time other than at $t = 0$?

14. Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither 0º nor 90º):

(a) Is the acceleration ever zero?

(b) Is the acceleration ever in the same direction as a component of velocity?

(c) Is the acceleration ever opposite in direction to a component of velocity?

15. For a fixed initial speed, the range of a projectile is determined by the angle at which it is fired. For all but the maximum, there are two angles that give the same range. Considering factors that might affect the ability of an archer to hit a target, such as wind, explain why the smaller angle (closer to the horizontal) is preferable. When would it be necessary for the archer to use the larger angle? Why does the punter in a football game use the higher trajectory?

16. During a lecture demonstration, a professor places two coins on the edge of a table. She then flicks one of the coins horizontally off the table, simultaneously nudging the other over the edge. Describe the subsequent motion of the two coins, in particular discussing whether they hit the floor at the same time.

3.5: Addition of Velocities

17. What frame or frames of reference do you instinctively use when driving a car? When flying in a commercial jet airplane?

18. A basketball player dribbling down the court usually keeps his eyes fixed on the players around him. He is moving fast. Why doesn't he need to keep his eyes on the ball?

19. If someone is riding in the back of a pickup truck and throws a softball straight backward, is it possible for the ball to fall straight down as viewed by a person standing at the side of the road? Under what condition would this occur? How would the motion of the ball appear to the person who threw it?

20. The hat of a jogger running at constant velocity falls off the back of his head. Draw a sketch showing the path of the hat in the jogger's frame of reference. Draw its path as viewed by a stationary observer.

21. A clod of dirt falls from the bed of a moving truck. It strikes the ground directly below the end of the truck. What is the direction of its velocity relative to the truck just before it hits? Is this the same as the direction of its velocity relative to ground just before it hits? Explain your answers.

Problems & Exercises

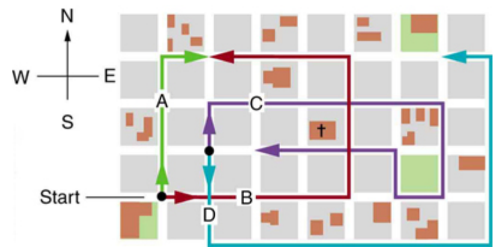
3.2: Vector Addition and Subtraction: Graphical Methods

Use graphical methods to solve these problems. You may assume data taken from graphs is accurate to three digits.

22. Find the following for path A in Figure:

(a) the total distance traveled, and

(b) the magnitude and direction of the displacement from start to finish.



The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.

solution:

(a) **480 m**

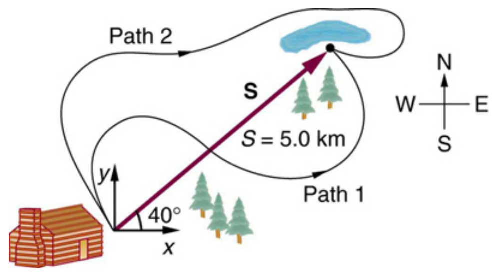
(b) **379 m**, **18.4º** east of north

23. Find the following for path B in Figure:

(a) the total distance traveled, and

(b) the magnitude and direction of the displacement from start to finish.

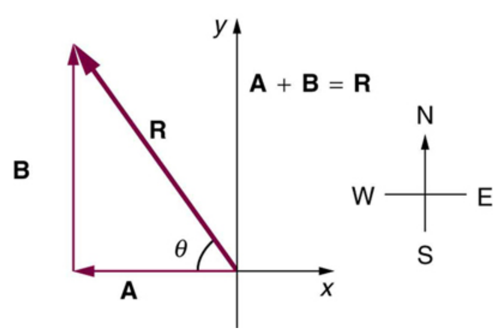
24. Find the north and east components of the displacement for the hikers shown in Figure.



Solution

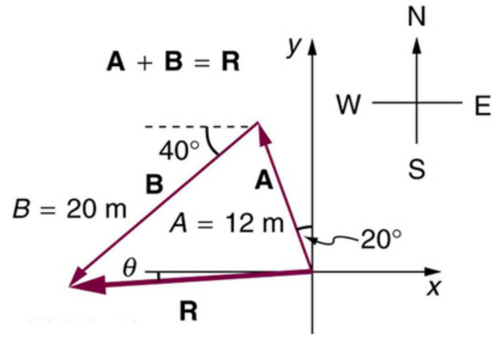
north component 3.21 km, east component 3.83 km

25. Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements **A** and **B** , as in Figure, then this problem asks you to find their sum **R=A+B**.)



*The two displacements **A** and **B** add to give a total displacement **R** having magnitude R and direction θ.*

26. Suppose you first walk 12.0 m in a direction **20º** west of north and then 20.0 m in a direction **40.0º** south of west. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements **A** and **B**, as in Figure, then this problem finds their sum **R** = **A** + **B**.)



Solution
$19.5 m, 4.65°$ south of west

27. Repeat the problem above, but reverse the order of the two legs of the walk; show that you get the same final result. That is, you first walk leg **B**, which is 20.0 m in a direction exactly **40º** south of west, and then leg **A** size 12{A} {}, which is 12.0 m in a direction exactly **20º** west of north. (This problem shows that **A+B=B+A**.)

28. (a) Repeat the problem two problems prior, but for the second leg you walk 20.0 m in a direction **40.0º** north of east (which is equivalent to subtracting **B** from **A** —that is, to finding **R'=A−B**).

(b) Repeat the problem two problems prior, but now you first walk 20.0 m in a direction **40.0º** south of west and then 12.0 m in a direction **20.0º** east of south (which is equivalent to subtracting **A** from **B** —that is, to finding **R''=B−A=−R'**). Show that this is the case.

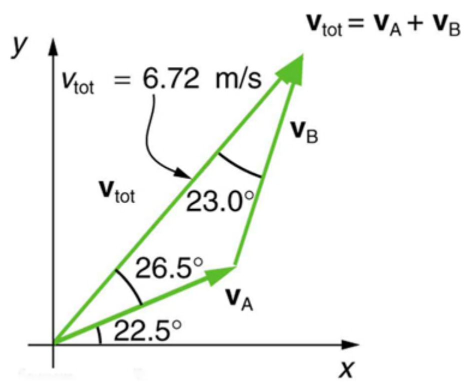
Solution
(a) $26.6 m, 65.1°$ north of east
(b) $26.6 m, 65.1°$ south of west

29. Show that the order of addition of three vectors does not affect their sum. Show this property by choosing any three vectors **A**, **B** and **C**, all having different lengths and directions. Find the sum **A + B + C** then find their sum when added in a different order and show the result is the same. (There are five other orders in which **A**, **B**, and **C** can be added; choose only one.)

30. Show that the sum of the vectors discussed in Example gives the result shown in Figure.

Solution
$52.9 m, 90.1°$ with respect to the x-axis.

31. Find the magnitudes of velocities v_A and v_B in Figure



The two velocities v_A and v_B add to give a total v_{tot}.

32. Find the components of v_{tot} along the x- and y-axes in Figure.

Solution
x-component 4.41 m/s
y-component 5.07 m/s

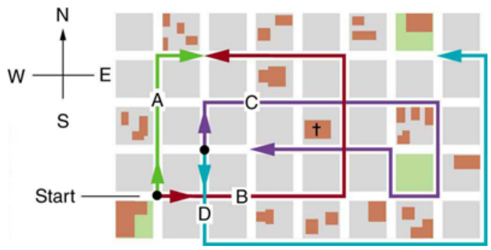
33. Find the components of v_{tot} along a set of perpendicular axes rotated 30° counterclockwise relative to those in Figure.

3.3: Vector Addition and Subtraction: Analytical Methods

34. Find the following for path C in Figure:

(a) the total distance traveled and

(b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.



The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.

Solution

(a) 1.56 km

(b) 120 m east

35. Find the following for path D in Figure:

(a) the total distance traveled and

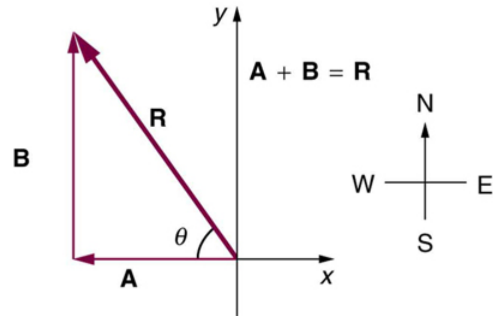
(b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.

36. Find the north and east components of the displacement from San Francisco to Sacramento shown in Figure.

Solution

North-component 87.0 km, east-component 87.0 km

37. Solve the following problem using analytical techniques: Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements **A** and **B**, as in Figure, then this problem asks you to find their sum **R=A+B**.)



*The two displacements **A** and **B** add to give a total displacement **R** having magnitude R and direction θ.*

Note that you can also solve this graphically. Discuss why the analytical technique for solving this problem is potentially more accurate than the graphical technique.

38. Repeat Exercise using analytical techniques, but reverse the order of the two legs of the walk and show that you get the same final result. (This problem shows that adding them in reverse order gives the same result—that is, **B + A = A + B**.) Discuss how taking another path to reach the same point might help to overcome an obstacle blocking you other path.

Solution
30.8 m, 35.8 west of north

You drive **7.50 km** in a straight line in a direction **15º**.

(a) Find the distances you would have to drive straight east and then straight north to arrive at the same point. (This determination is equivalent to find the components of the displacement along the east and north directions.)

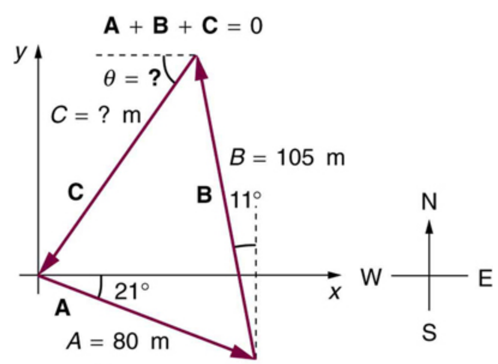
(b) Show that you still arrive at the same point if the east and north legs are reversed in order.

39. a) Do Exercise again using analytical techniques and change the second leg of the walk to **25.0 m** straight south. (This is equivalent to subtracting **B** from **A** —that is, finding **R'=A – B**)

(b) Repeat again, but now you first walk 25.0 m size 12{"25" "." "0 m"} {} north and then **18.0 m** size 12{"18" "." "0 m"} {} east. (This is equivalent to subtract **A** size 12{A} {} from **B** size 12{B} {} —that is, to find **A=B+C** size 12{A=B+C} {}. Is that consistent with your result?)

Solution
(a) $30.8 m, 54.2^{\circ}$south of west
(b) $30.8 m, 54.2^{\circ}$north of east

40. A new landowner has a triangular piece of flat land she wishes to fence. Starting at the west corner, she measures the first side to be 80.0 m long and the next to be 105 m. These sides are represented as displacement vectors **A** from **B** in Figure. She then correctly calculates the length and orientation of the third side C. What is her result?



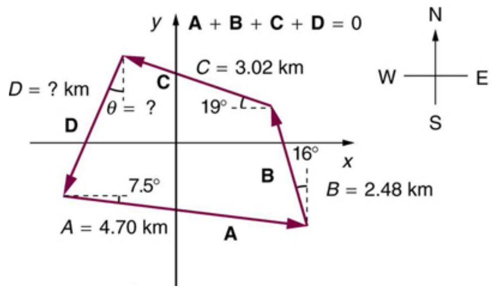
41. You fly **32.0 km** size 12{"32" "." "0 km"} {} in a straight line in still air in the direction **35.0º** size 12{"35"°} {} south of west.

(a) Find the distances you would have to fly straight south and then straight west to arrive at the same point. (This determination is equivalent to finding the components of the displacement along the south and west directions.)

(b) Find the distances you would have to fly first in a direction **45.0º** south of west and then in a direction **45.0º** west of north. These are the components of the displacement along a different set of axes—one rotated **45º**.

Solution
18.4 km south, then 26.2 km west(b) 31.5 km at **45.0º** south of west, then 5.56 km at **45.0º** west of north

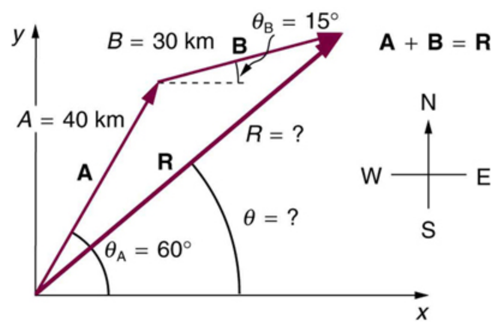
42. A farmer wants to fence off his four-sided plot of flat land. He measures the first three sides, shown as **A**, **B**, and **C** in Figure, and then correctly calculates the length and orientation of the fourth side D. What is his result?



43. In an attempt to escape his island, Gilligan builds a raft and sets to sea. The wind shifts a great deal during the day, and he is blown along the following straight lines: **2.50 km 45.0º** north of west; then **4.70 km 60.0º** south of east; then **1.30km 25.0º** south of west; then **5.10 km** straight east; then **1.70km 5.00º** east of north; then **7.20 km 55.0º** south of west; and finally **2.80 km 10.0º** north of east. What is his final position relative to the island?

Solution
$7.34 km, 63.5^o$south of east

44. Suppose a pilot flies **40.0 km** in a direction **60º** north of east and then flies **30.0 km** in a direction **15º** north of east as shown in Figure. Find her total distance R from the starting point and the direction θ of the straight-line path to the final position. Discuss qualitatively how this flight would be altered by a wind from the north and how the effect of the wind would depend on both wind speed and the speed of the plane relative to the air mass.



3.4: Projectile Motion

45. A projectile is launched at ground level with an initial speed of 50.0 m/s at an angle of **30.0º** above the horizontal. It strikes a target above the ground 3.00 seconds later. What are the x and y distances from where the projectile was launched to where it lands?

Solution
$x = 1.30m \times 10$
$y = 30.9m.$

46. A ball is kicked with an initial velocity of 16 m/s in the horizontal direction and 12 m/s in the vertical direction.

(a) At what speed does the ball hit the ground?

(b) For how long does the ball remain in the air?

(c)What maximum height is attained by the ball?

47. A ball is thrown horizontally from the top of a 60.0-m building and lands 100.0 m from the base of the building. Ignore air resistance.

(a) How long is the ball in the air?

(b) What must have been the initial horizontal component of the velocity?

(c) What is the vertical component of the velocity just before the ball hits the ground? (d) What is the velocity (including both the horizontal and vertical components) of the ball just before it hits the ground?

Solution
(a) 3.50 s
(b) 28.6 m/s
(c) 34.3 m/s
(d) 44.7 m/s, **50.2°** below horizontal

48. (a) A daredevil is attempting to jump his motorcycle over a line of buses parked end to end by driving up a **32°** ramp at a speed of **40.0 m/s** (**144 km/h**). How many buses can he clear if the top of the takeoff ramp is at the same height as the bus tops and the buses are 20.0 m long?

(b) Discuss what your answer implies about the margin of error in this act—that is, consider how much greater the range is than the horizontal distance he must travel to miss the end of the last bus. (Neglect air resistance.)

49. An archer shoots an arrow at a 75.0 m distant target; the bull's-eye of the target is at same height as the release height of the arrow.

(a) At what angle must the arrow be released to hit the bull's-eye if its initial speed is 35.0 m/s? In this part of the problem, explicitly show how you follow the steps involved in solving projectile motion problems.

(b) There is a large tree halfway between the archer and the target with an overhanging horizontal branch 3.50 m above the release height of the arrow. Will the arrow go over or under the branch?

Solution
(a) **18.4°**
(b) The arrow will go over the branch.

50. A rugby player passes the ball 7.00 m across the field, where it is caught at the same height as it left his hand.

(a) At what angle was the ball thrown if its initial speed was 12.0 m/s, assuming that the smaller of the two possible angles was used?

(b) What other angle gives the same range, and why would it not be used?

(c) How long did this pass take?

51. Verify the ranges for the projectiles in Figure(a) for $\theta = 45°$ and the given initial velocities.

Solution
$R = \frac{v_0^2}{sin2\theta_0 g}$
For $\theta = 45°$, $R = \frac{v_0^2}{g}$
$R = 91.8m$ for $v_0 = 30m/s$; $R = 163m$ for $v_0 = 40m/s$; $R = 255m$ for $v_= 50m/s$.

52. Verify the ranges shown for the projectiles in Figure(b) for an initial velocity of 50 m/s at the given initial angles.

53. The cannon on a battleship can fire a shell a maximum distance of 32.0 km.

(a) Calculate the initial velocity of the shell.

(b) What maximum height does it reach? (At its highest, the shell is above 60% of the atmosphere—but air resistance is not really negligible as assumed to make this problem easier.)

(c) The ocean is not flat, because the Earth is curved. Assume that the radius of the Earth is $6.37 \times 10^3 km$. How many meters lower will its surface be 32.0 km from the ship along a horizontal line parallel to the surface at the ship? Does your answer imply that error introduced by the assumption of a flat Earth in projectile motion is significant here?

Solution
(a) 560 m/s
(b) $8.00 \times 10^3 m$
(c) 80.0 m. This error is not significant because it is only 1% of the answer in part (b).

54. An arrow is shot from a height of 1.5 m toward a cliff of height H size 12{H} {}. It is shot with a velocity of 30 m/s at an angle of **60º** above the horizontal. It lands on the top edge of the cliff 4.0 s later.

(a) What is the height of the cliff?

(b) What is the maximum height reached by the arrow along its trajectory?

(c) What is the arrow's impact speed just before hitting the cliff?

55. In the standing broad jump, one squats and then pushes off with the legs to see how far one can jump. Suppose the extension of the legs from the crouch position is 0.600 m and the acceleration achieved from this position is 1.25 times the acceleration due to gravity, g size 12{g} {}. How far can they jump? State your assumptions. (Increased range can be achieved by swinging the arms in the direction of the jump.)

Solution
1.50 m, assuming launch angle of **45º**

56. The world long jump record is 8.95 m (Mike Powell, USA, 1991). Treated as a projectile, what is the maximum range obtainable by a person if he has a take-off speed of 9.5 m/s? State your assumptions.

57. Serving at a speed of 170 km/h, a tennis player hits the ball at a height of 2.5 m and an angle θ below the horizontal. The base line is 11.9 m from the net, which is 0.91 m high. What is the angle θ such that the ball just crosses the net? Will the ball land in the service box, whose service line is 6.40 m from the net?

Solution
$\theta = 6.1^{\circ}$
yes, the ball lands at 5.3 m from the net

58. A football quarterback is moving straight backward at a speed of 2.00 m/s when he throws a pass to a player 18.0 m straight downfield.

(a) If the ball is thrown at an angle of **25º** relative to the ground and is caught at the same height as it is released, what is its initial speed relative to the ground?

(b) How long does it take to get to the receiver?

(c) What is its maximum height above its point of release?

59. Gun sights are adjusted to aim high to compensate for the effect of gravity, effectively making the gun accurate only for a specific range.

(a) If a gun is sighted to hit targets that are at the same height as the gun and 100.0 m away, how low will the bullet hit if aimed directly at a target 150.0 m away? The muzzle velocity of the bullet is 275 m/s.

(b) Discuss qualitatively how a larger muzzle velocity would affect this problem and what would be the effect of air resistance.

Solution
(a) −0.486 m
(b) The larger the muzzle velocity, the smaller the deviation in the vertical direction, because the time of flight would be smaller. Air resistance would have the effect of decreasing the time of flight, therefore increasing the vertical deviation.

60. An eagle is flying horizontally at a speed of 3.00 m/s when the fish in her talons wiggles loose and falls into the lake 5.00 m below. Calculate the velocity of the fish relative to the water when it hits the water.

61. An owl is carrying a mouse to the chicks in its nest. Its position at that time is 4.00 m west and 12.0 m above the center of the 30.0 cm diameter nest. The owl is flying east at 3.50 m/s at an angle 30.0º size 12{"30º"} below the horizontal when it accidentally drops the mouse. Is the owl lucky enough to have the mouse hit the nest? To answer this question, calculate the horizontal position of the mouse when it has fallen 12.0 m.

Solution
4.23 m. No, the owl is not lucky; he misses the nest.

62. Suppose a soccer player kicks the ball from a distance 30 m toward the goal. Find the initial speed of the ball if it just passes over the goal, 2.4 m above the ground, given the initial direction to be **40º** above the horizontal.

63. Can a goalkeeper at her/ his goal kick a soccer ball into the opponent's goal without the ball touching the ground? The distance will be about 95 m. A goalkeeper can give the ball a speed of 30 m/s.

Solution
No, the maximum range (neglecting air resistance) is about 92 m.

64. The free throw line in basketball is 4.57 m (15 ft) from the basket, which is 3.05 m (10 ft) above the floor. A player standing on the free throw line throws the ball with an initial speed of 8.15 m/s, releasing it at a height of 2.44 m (8 ft) above the floor. At what angle above the horizontal must the ball be thrown to exactly hit the basket? Note that most players will use a large initial angle rather than a flat shot because it allows for a larger margin of error. Explicitly show how you follow the steps involved in solving projectile motion problems.

65. In 2007, Michael Carter (U.S.) set a world record in the shot put with a throw of 24.77 m. What was the initial speed of the shot if he released it at a height of 2.10 m and threw it at an angle of **38.0º** above the horizontal? (Although the maximum distance for a projectile on level ground is achieved at **45º** when air resistance is neglected, the actual angle to achieve maximum range is smaller; thus, **38º** will give a longer range than **45º** in the shot put.)

Solution
15.0 m/s

66. A basketball player is running at **5.00 m/s** directly toward the basket when he jumps into the air to dunk the ball. He maintains his horizontal velocity.

(a) What vertical velocity does he need to rise 0.750 m above the floor?

(b) How far from the basket (measured in the horizontal direction) must he start his jump to reach his maximum height at the same time as he reaches the basket?

67. A football player punts the ball at a **45.0º** angle. Without an effect from the wind, the ball would travel 60.0 m horizontally.

(a) What is the initial speed of the ball?

(b) When the ball is near its maximum height it experiences a brief gust of wind that reduces its horizontal velocity by 1.50 m/s. What distance does the ball travel horizontally?

Solution
(a) 24.2 m/s
(b) The ball travels a total of 57.4 m with the brief gust of wind.

68. Prove that the trajectory of a projectile is parabolic, having the form $y = ax + bx^2$. To obtain this expression, solve the equation $x = v_{0x}t$ for t and substitute it into the expression for $y = v_{0y}t – (1/2)gt^2$ (These equations describe the x and y positions of a projectile that starts at the origin.) You should obtain an equation of the form $y = ax + bx^2$ where a and b are constants.

69. Derive $R = \frac{v_0^2 sin 2\theta_0}{g}$ for the range of a projectile on level ground by finding the time t at which y becomes zero and substituting this value of t into the expression for $x - x_0$, noting that $R = x - x_0$

Solution
$y - y_0 = 0 = v_{0y}t - \frac{1}{2}gt^2 = (v_0 sin\theta)t - \frac{1}{2}gt^2$
so that $t = \frac{2(v0 sin\theta)}{g}$
$x - x_0 = v_0xt = (v_0 cos\theta)t = R,$ and substituting for t gives:
$R = v_0 cos\theta(\frac{2v_0 sin\theta}{g}) = \frac{2v_0^2 sin\theta cos\theta}{g}$

since $2sin\theta cos\theta = sin2\theta$, the range is:

$R = \frac{v_0^2 sin2\theta}{g}$

70. *Unreasonable Results*

(a) Find the maximum range of a super cannon that has a muzzle velocity of 4.0 km/s.

(b) What is unreasonable about the range you found?

(c) Is the premise unreasonable or is the available equation inapplicable? Explain your answer.

(d) If such a muzzle velocity could be obtained, discuss the effects of air resistance, thinning air with altitude, and the curvature of the Earth on the range of the super cannon.

71. *Construct Your Own Problem*

Consider a ball tossed over a fence. Construct a problem in which you calculate the ball's needed initial velocity to just clear the fence. Among the things to determine are; the height of the fence, the distance to the fence from the point of release of the ball, and the height at which the ball is released. You should also consider whether it is possible to choose the initial speed for the ball and just calculate the angle at which it is thrown. Also examine the possibility of multiple solutions given the distances and heights you have chosen.

3.5: Addition of Velocities

72. Bryan Allen pedaled a human-powered aircraft across the English Channel from the cliffs of Dover to Cap Gris-Nez on June 12, 1979.

(a) He flew for 169 min at an average velocity of 3.53 m/s in a direction 45^o south of east. What was his total displacement?

(b) Allen encountered a headwind averaging 2.00 m/s almost precisely in the opposite direction of his motion relative to the Earth. What was his average velocity relative to the air?

(c) What was his total displacement relative to the air mass?

Solution

(a) $35.8 km, 45^o$south of east

(b) $5.53 m/s, 45^o$south of east

(c) $56.1 km, 45^o$south of east

73. A seagull flies at a velocity of 9.00 m/s straight into the wind.

(a) If it takes the bird 20.0 min to travel 6.00 km relative to the Earth, what is the velocity of the wind?

(b) If the bird turns around and flies with the wind, how long will he take to return 6.00 km?

(c) Discuss how the wind affects the total round-trip time compared to what it would be with no wind.

74. Near the end of a marathon race, the first two runners are separated by a distance of 45.0 m. The front runner has a velocity of 3.50 m/s, and the second a velocity of 4.20 m/s.

(a) What is the velocity of the second runner relative to the first?

(b) If the front runner is 250 m from the finish line, who will win the race, assuming they run at constant velocity?

(c) What distance ahead will the winner be when she crosses the finish line?

Solution

(a) 0.70 m/s faster

(b) Second runner wins

(c) 4.17 m

75. Verify that the coin dropped by the airline passenger in the Example travels 144 m horizontally while falling 1.50 m in the frame of reference of the Earth.

76. A football quarterback is moving straight backward at a speed of 2.00 m/s when he throws a pass to a player 18.0 m straight downfield. The ball is thrown at an angle of 25.0^o relative to the ground and is caught at the same height as it is released. What is the initial velocity of the ball *relative to the quarterback* ?

Solution
$17.0 m/s, 22.1^{\circ}$

77. A ship sets sail from Rotterdam, The Netherlands, heading due north at 7.00 m/s relative to the water. The local ocean current is 1.50 m/s in a direction 40.0° north of east. What is the velocity of the ship relative to the Earth?

78. (a) A jet airplane flying from Darwin, Australia, has an air speed of 260 m/s in a direction 5.0° south of west. It is in the jet stream, which is blowing at 35.0 m/s in a direction 15° south of east. What is the velocity of the airplane relative to the Earth?

(b) Discuss whether your answers are consistent with your expectations for the effect of the wind on the plane's path.

Solution
(a) $230 m/s, 8.0^{\circ}$south of west
(b) The wind should make the plane travel slower and more to the south, which is what was calculated.

79. (a) In what direction would the ship in Exercise have to travel in order to have a velocity straight north relative to the Earth, assuming its speed relative to the water remains $7.00 m/s$?

(b) What would its speed be relative to the Earth?

80. (a) Another airplane is flying in a jet stream that is blowing at 45.0 m/s in a direction 20º size 12{"20º"} {} south of east (as in Exercise). Its direction of motion relative to the Earth is 45.0º size 12{"45.0º"} {} south of west, while its direction of travel relative to the air is 5.00º size 12{5.00º} {} south of west. What is the airplane's speed relative to the air mass? (b) What is the airplane's speed relative to the Earth?

Solution
(a) 63.5 m/s
(b) 29.6 m/s

81. A sandal is dropped from the top of a 15.0-m-high mast on a ship moving at 1.75 m/s due south. Calculate the velocity of the sandal when it hits the deck of the ship:

(a) relative to the ship and

(b) relative to a stationary observer on shore.

(c) Discuss how the answers give a consistent result for the position at which the sandal hits the deck.

82. The velocity of the wind relative to the water is crucial to sailboats. Suppose a sailboat is in an ocean current that has a velocity of 2.20 m/s in a direction 30.0° east of north relative to the Earth. It encounters a wind that has a velocity of 4.50 m/s in a direction of 50.0° south of west relative to the Earth. What is the velocity of the wind relative to the water?

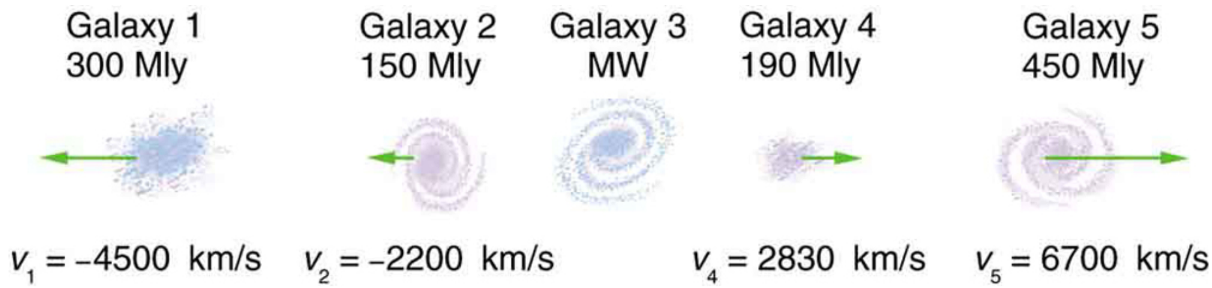
Solution
$6.68 m/s, 53.3^{\circ}$south of west

83. The great astronomer Edwin Hubble discovered that all distant galaxies are receding from our Milky Way Galaxy with velocities proportional to their distances. It appears to an observer on the Earth that we are at the center of an expanding universe. Figure illustrates this for five galaxies lying along a straight line, with the Milky Way Galaxy at the center. Using the data from the figure, calculate the velocities:

(a) relative to galaxy 2 and

(b) relative to galaxy 5.

The results mean that observers on all galaxies will see themselves at the center of the expanding universe, and they would likely be aware of relative velocities, concluding that it is not possible to locate the center of expansion with the given information.



Five galaxies on a straight line, showing their distances and velocities relative to the Milky Way (MW) Galaxy. The distances are in millions of light years (Mly), where a light year is the distance light travels in one year. The velocities are nearly proportional to the distances. The sizes of the galaxies are greatly exaggerated; an average galaxy is about 0.1 Mly across.

84. (a) Use the distance and velocity data in Figure to find the rate of expansion as a function of distance.

(b) If you extrapolate back in time, how long ago would all of the galaxies have been at approximately the same position? The two parts of this problem give you some idea of how the Hubble constant for universal expansion and the time back to the Big Bang are determined, respectively.

Solution

(a) $H_{average} = 14.9\frac{km/s}{Mly}$

(b) 20.2 billion years

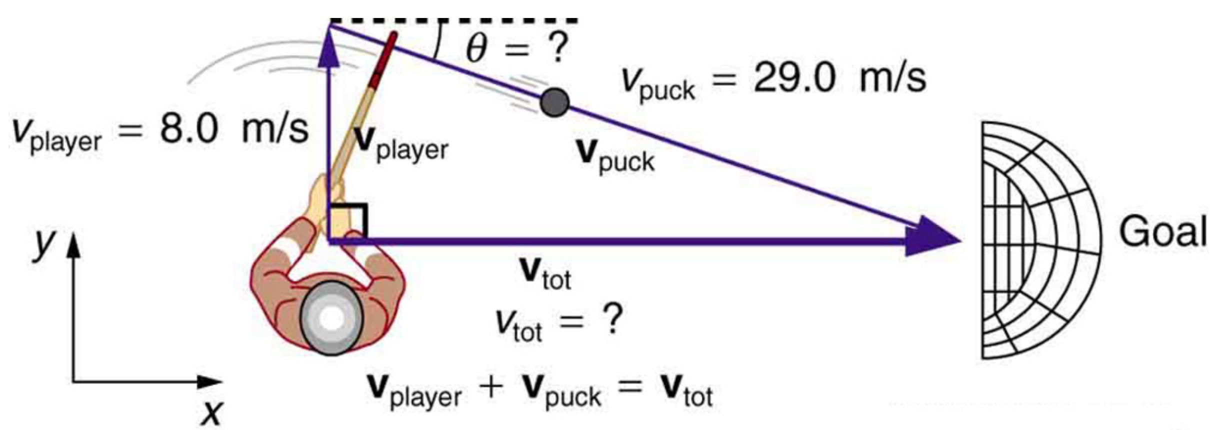
85. An athlete crosses a 25-m-wide river by swimming perpendicular to the water current at a speed of 0.5 m/s relative to the water. He reaches the opposite side at a distance 40 m downstream from his starting point. How fast is the water in the river flowing with respect to the ground? What is the speed of the swimmer with respect to a friend at rest on the ground?

86. A ship sailing in the Gulf Stream is heading 25.0° west of north at a speed of 4.00 m/s relative to the water. Its velocity relative to the Earth is $4.80 m/s 5.00^{\circ}$ west of north. What is the velocity of the Gulf Stream? (The velocity obtained is typical for the Gulf Stream a few hundred kilometers off the east coast of the United States.)

Solution

$1.72 m/s, 42.3^{\circ}$ north of east

87. An ice hockey player is moving at 8.00 m/s when he hits the puck toward the goal. The speed of the puck relative to the player is 29.0 m/s. The line between the center of the goal and the player makes a 90.0° angle relative to his path as shown in Figure. What angle must the puck's velocity make relative to the player (in his frame of reference) to hit the center of the goal?



An ice hockey player moving across the rink must shoot backward to give the puck a velocity toward the goal.

88. *Unreasonable Results*

Suppose you wish to shoot supplies straight up to astronauts in an orbit 36,000 km above the surface of the Earth.

(a) At what velocity must the supplies be launched?

(b) What is unreasonable about this velocity?

(c) Is there a problem with the relative velocity between the supplies and the astronauts when the supplies reach their maximum height?

(d) Is the premise unreasonable or is the available equation inapplicable? Explain your answer.

89. *Unreasonable Results*

A commercial airplane has an air speed of $280 m/s$ due east and flies with a strong tailwind. It travels 3000 km in a direction 5° south of east in 1.50 h.

(a) What was the velocity of the plane relative to the ground?

(b) Calculate the magnitude and direction of the tailwind's velocity.

(c) What is unreasonable about both of these velocities?

(d) Which premise is unreasonable?

90. *Construct Your Own Problem*

Consider an airplane headed for a runway in a cross wind. Construct a problem in which you calculate the angle the airplane must fly relative to the air mass in order to have a velocity parallel to the runway. Among the things to consider are the direction of the runway, the wind speed and direction (its velocity) and the speed of the plane relative to the air mass. Also calculate the speed of the airplane relative to the ground. Discuss any last minute maneuvers the pilot might have to perform in order for the plane to land with its wheels pointing straight down the runway.

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